PRIORITY PRICING OF INTERRUPTIBLE ELECTRIC SERVICE WITH AN EARLY WARNING OPTION

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Memorandum No. UCB/ERL/IGCT M91/94

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Abstract

Priority pricing of interruptible electric service induces each customer to self-select a rationing priority that matches the rank order of its interruption loss. This paper extends the theory by considering the possibility of early warning. We consider a two period model in which customers may either be irreversibly warned in the first period and interrupted in the second period, or may be interrupted in the second period without warning when the shortfall exceeds the warned load. Customers are characterized by their privately known cost of interruption with and without warning. The aggregate distribution of these costs is known to the utility. The proposed tariff structure is in the form of a menu that allows a customer to choose either the early warning option and pay a fixed fee, or select not to be warned along with a level of compensation when interrupted. The chosen compensation determines its service priority and corresponding price. The tariff induces self-selection that is consistent with the socially optimal warning and rationing plan.

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1. Background

Over 70 percent of investor-owned electric utilities in the United States offer some form of voluntary interruptible or curtailable electric service (Ebasco, 1987). Interruptible electric service refers to any customer load that is subject to partial or complete elimination for a period of time upon adequate notice from the electric utility. Typically, "adequate notice" ranges from 10 minutes to several hours or even one full day. This is the *warning time* a customer receives prior to actual interruption. The conditions of warning time are included in interruptible service tariffs, which may also specify the maximum number of interruptions allowed per day, per month, or per year; the maximum duration of any particular interruption; and the maximum number of interrupted hours per year. Christensen Associates (1988) describes interruptible service in more detail.

Analysis of interruptible service tariffs has focused on varying the demand charge for customers with different interruption costs. Marchand (1974), Tschirhart and Jen (1979), Woo and Toyama (1986), and Viswanathan and Tse (1989) consider one-dimensional models that differentiate on reliability, that is, probability or frequency of interruption. The models of Panzar and Sibley (1978), Hamlen and Jen (1983), and Woo (1990) include the amount of interruption as well as the frequency. Using a load duration curve model, Chao, Oren, Smith, and Wilson (1986) consider both frequency and duration, but not amount. Smith (1989) and Oren (1990) consider two-dimensional models that incorporate both frequency and duration of individual interruptions.

None of these models include warning time, an important element of actual interruptible service programs and tariffs such as Niagara Mohawk's Voluntary Interruptible Pricing Program, New England Electric Service's Cooperative Interruptible Service Program, and Southern California Edison's I-3 tariff schedule. In the analysis presented here, warning time is included through a simplified two-period model. Customers may be warned in the first period or interrupted without warning in the second period. We first analyze the socially optimal warning and rationing plan under perfect information about customer interruption losses. Then we consider the implementation of that allocation through customer self-selection, and derive a simple tariff schedule that will achieve this goal.

2. Description and Notation

Different customers and end uses incur different losses when an interruption of electric power occurs. However, no customer prefers a longer interruption to a shorter one, and no customer prefers a sudden, unexpected interruption to an interruption with some advance warning. Customers ordinarily assume that they will receive electric power, so a customer's interruption loss or *outage cost* is an "additive adjustment to the surplus...derived from its normal electric power consumption" (Smith, 1989). Similarly, the prices discussed in this paper are additive adjustments to customer bills "for avoided [or contracted] interruptions, as opposed to consumption."

With a control and metering technology that is able to separate end uses, each kilowatt (kW) of demand may be addressed separately. Consequently, each customer is considered to have one kW of demand; alternatively, each kW of demand is regarded as an independent decision-making unit. Furthermore, demand is non-stochastic.[1] Supply is stochastic, so the system *shortfall* is the difference between *N*, the number of customers, and the realized value of supply.

Each customer may then be characterized by i) its loss if interrupted suddenly and unexpectedly, and ii) its benefit from advance warning of an impending interruption. As modeled here, any and all warnings to customers may be issued only once, at a fixed length of time before the impending shortfall, say one period. We will assume here that warning is irreversible, so that a warned customer incurs its interruption loss less its early warning benefit. This net loss is herein referred to as a customer's *warning cost* (c_S).

[1]Both unit demand and its non-stochastic nature are common assumptions of priority pricing research. For example, see Wilson (1989).

At the time warnings are issued (*S*), the magnitude of an impending shortfall is uncertain, but it is fully revealed when the actual interruptions commence (*T*). If the magnitude of the actual shortfall exceeds the number of customers that were warned, this difference between supply and demand is satisfied by interrupting additional customers, without warning (or on very short notice), at time *T*. The interruption loss suffered by a customer interrupted without warning is referred to as its *base cost* (c_T). Any customer not warned at time *S* is on *standby*, and may be interrupted without warning at time *T*.

The customer population is heterogeneous with respect to c_S and c_T and characterized in terms of a distribution over c_S and c_T in the domain $0 \leq c_S \leq c_T$. Base costs are bounded above by *C*. For notational convenience, customer costs are scaled such that *C* equals one.

The electric utility must decide which customers should be warned and which should be on standby, and which standby customers should be interrupted, if necessary. This is the central planning problem faced by the utility. However, interruption losses are private information. Each customer knows its own particular outage costs, but the utility does not know the outage costs of any particular customer, only the aggregate distribution of outage costs in the customer population. Thus, a socially efficient tariff structure must induce customers to reveal their true outage costs through their selections from a menu of service options. The electric utility then uses the revealed preferences to allocate warnings and interruptions. Because this scheme calls for customer choice from a menu of service options, the tariff designer must recognize that each customer chooses from such a menu so as to maximize its own consumer surplus. This is the self-selection criterion (Mussa and Rosen, 1978).

The problem at hand is to design an interruptible service tariff. There are two aspects to this problem: allocating warnings and interruptions among customers (the central planning problem), and implementing this allocation through pricing, to account for customer self-selection. The goal is to minimize expected total customer interruption cost.

Shortfall duration is suppressed; this may be interpreted either as assuming that all shortfalls have a fixed duration or as taking expectations over duration. For notational convenience, both the distribution of interruption costs in the customer population and the distribution of shortfall magnitude are assumed to be continuous. If these distributions were discrete or mixed, clumping of customers might occur, but the sense of the analysis would remain unchanged.

We use the following notation:

N Number of customers

$D(x,y)$ Population distribution
Percentage of customers with *base cost*—cost of being interrupted without warning—less than or equal to x, and with *warning cost*—cost of being interrupted with one period advance warning—less than or equal to y.

$d(x,y)$ Population density

S Time when warnings are issued

T Time when interruptions commence

$G(z)$ Distribution of short-term uncertainty in shortfall magnitude at time S
Probability that magnitude of shortfall will be no greater than z.

$g(z)$ Density of short-term uncertainty in shortfall magnitude

3. Central Planning Problem

Existence of Utility's Decision Curve

The utility's objective is to minimize the total expected social cost of an impending shortfall. We first consider the case where the utility has complete information about each customer's costs. The utility must decide which customers are warned and which are standby, and among standby customers, which to interrupt, if any, in order to balance supply and demand at time T. We first show that the optimal set of customers to be warned is connected, and that the boundary between the set of warned customers and the set of standby customers may be expressed as a (smooth) curve u. Then, at time T, standby customers may be interrupted as needed, in increasing order of base costs.

Consider customer A, with base cost a_T and warning cost a_S. Suppose $\mathcal{W}$, the set of warned customers under the optimal warning policy, contains A. Then any customer with costs $c_S \leq a_S$ and $c_T \geq a_T$ (region I of Figure 3.1) should also be warned; otherwise, the total expected interruption cost can be reduced by warning such a customer instead of A. In this sense, region I *dominates* A.

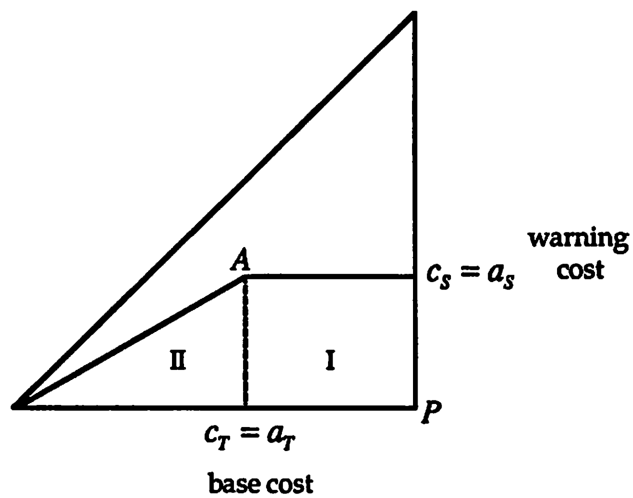

Figure 3.1: Regions I and II Dominate *A*

The fact that region I dominates A yields three properties of $\mathcal{W}$ and its boundary:

- $\mathcal{W}$ is connected

  The line segment joining A and P—the point (1,0)—is a continuous path lying in $\mathcal{W}$. Hence any two points A and B in $\mathcal{W}$ may be connected by the path along the line segments $\overline{AP}$ and $\overline{PB}$. Therefore, $\mathcal{W}$ is connected.

- the boundary of $\mathcal{W}$ can be described by a function $u(\cdot)$ with c_T as its domain and c_S as its range

  Suppose $u(a_T) = a_S$. All customers (a_T, c_S), $c_S < a_S$, lie in region I and should be warned, while any customer (a_T, c_S), $c_S > a_S$, cannot be warned, lest all customers between (a_T, a_S) and (a_T, c_S) also be warned, in which case $u(a_T) = c_S$ and not a_S.

- u is nondecreasing

  Suppose $A \in \mathcal{W}$. For $c_T > a_T$, (c_T, a_S) is in region I, hence, in $\mathcal{W}$. Therefore, $u(c_T) \geq a_S$.

Region II also dominates A. If $A \in \mathcal{W}$ but some customer Q with base cost $c_T < a_T$ and warning benefit factor $c_S/c_T \leq a_S/a_T$ (region II of Figure 3.1) is not warned, then the total expected interruption cost can be reduced either by warning Q instead of A or by warning Q in addition to A.

The fact that region II dominates A yields continuity of u:

• u is continuous

Suppose $0 < \delta < \varepsilon$. From Region II, $u(a_T+\delta) \leq (a_T+\delta)\, u(a_T)/a_T = u(a_T) + u(a_T)\, \delta/a_T$.
Hence, $0 \leq u(a_T+\delta) - u(a_T) \leq \delta\, u(a_T)/a_T \leq \delta < \varepsilon$.

This completes the development of the utility's planning curve. Assuming u to be twice differentiable facilitates using the calculus of variations to derive the optimal decision curve. The derivation is described next.

The Utility's Optimal Decision Curve

Figure 3.2 qualitatively illustrates the partitioning of a customer population in meeting the shortfall. As discussed above, all customers under the decision curve u are warned at time S, while the balance of the shortfall is met at time T by interrupting standby customers in order of their base costs, c_T.

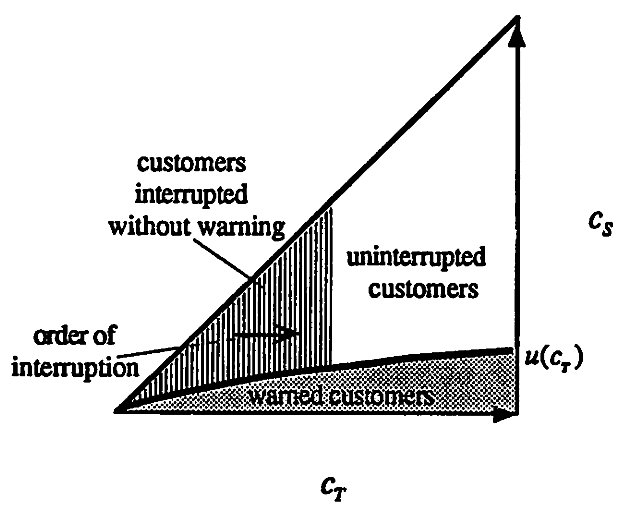

Figure 3.2: Optimal Warning and Interruption Policy

Now we compute the expected total interruption cost, at the decision time *S*, in terms of the decision curve to be optimized, *u*. The total cost of interrupting all customers that are warned is (3.1).

$$N\int_0^1\int_0^{u(x)} y\,d(x,y)\,dy\,dx \tag{3.1}$$

The number of standby customers interrupted is a random variable that depends on the short-term uncertainty in the magnitude of the shortfall. The expected total cost of interrupting standby customers is (3.2).

$$\int_0^1\left(N\int_0^v x\int_{u(x)}^x d(x,y)\,dy\,dx\right)g\left(N\,D(v,v)+N\int_v^1\int_0^{u(x)} d(x,y)\,dy\,dx\right)\left(N\int_{u(v)}^v d(v,y)\,dy\right)dv \tag{3.2}$$

Adding these two quantities, and using the substitution (3.3),

$$w(v)=N\int_0^v\int_0^s\int_0^{u(x)} d(x,y)\,dy\,dx\,ds \tag{3.3}$$

yields expected total interruption cost (3.4).

$$\int_0^1\left\{\begin{array}{c} uw''-N\int_0^{u(v)}\int_0^x d(v,y)\,dy\,dx \\ + \\ \left(NvD(v,v)-vw'-Nv\int_0^v D(s,s)\,ds+w\right)g\left(N\,D(v,v)+w'(1)-w'\right)\left(N\int_0^v d(v,y)\,dy-w''\right)\end{array}\right\}dv \tag{3.4}$$

Finding the curves *u* (and *w*) is a problem in the calculus of variations. With the integrand in (3.4) is represented as *I* and the Lagrange multiplier $\lambda(v)$ is attached to constraint (3.3), the Euler equations are (3.5a-b), where ξ denotes the difference between the left and right sides of (3.3).

$$0=I_u+\lambda\,\xi_u=w''-N\int_0^{u(v)} d(v,y)\,dy+\lambda(v)\left(-N\int_0^v\int_0^s d(x,u(x))\,dx\,ds\right) \tag{3.5a}$$

$$0=I_w+\frac{d}{dv}\left(\frac{d\,I_{w''}}{dv}-I_{w'}\right)+\lambda\,\xi_w \tag{3.5b}$$

Differentiating (3.3) twice with respect to *v* yields w'' equal to the second term on the right side of (3.5a), with the result that the Lagrange multiplier $\lambda(v)$ is zero for all values of *v*. Algebraic manipulation of the partial derivatives of *I* yields (3.6).

$$u'' = \frac{d}{dv}\left(\frac{dI_{w''}}{dv} - I_{w'}\right) \tag{3.6}$$

The second term of (3.5b) is replaced with (3.6), while I_w is written explicitly, transforming the Euler equation (3.5b) into (3.7), an expression in terms of the original function u.

$$0 = g\left(N D(v,v) + N\int_v^1\int_0^{u(x)} d(x,y)\,dy\,dx\right)\left(N\int_{u(v)}^{v} d(v,y)\,dy\right) + u'' \tag{3.7}$$

(3.7) indicates that u'' is not positive, hence u is concave.

Integrating (3.7) yields (3.8), an implicit expression for the desired curve u in terms of the given functions D and G. The constant of integration is zero, as determined by the transversality condition, $u'(1) = 0$.

$$u'(v) = \overline{G}\left(N D(v,v) + N\int_v^1\int_0^{u(x)} d(x,y)\,dy\,dx\right) \tag{3.8}$$

The expression on the right side of (3.8) is the probability that a standby customer with base cost v will be interrupted. The function u is obtained by solving the differential equation (3.8) with boundary condition $u(0) = 0$. After some mathematical manipulation, one obtains (3.9a).

$$u(v) = v\overline{G}(h(v;u)) + \int_0^v z\,dG(h(z;u)) \tag{3.9a}$$

where

$$h(v;u) = N D(v,v) + N\int_v^1\int_0^{u(x)} d(x,y)\,dy\,dx \tag{3.9b}$$

(3.9a) has an intuitive interpretation. The first term on the right side of (3.9a) is the expected interruption cost of a standby customer with base cost v. The second term is an externality cost imputed on standby customers with base costs less than v by standby customers with base cost v. In terms of social cost, the electric utility is indifferent between warning and not warning customers on the boundary, $(v,u(v))$, $0 \le v \le 1$. This result is discussed further in section 4.

An Illustrative Example: Uniform Distribution of Costs and Shortfall Magnitude

To illustrate the above result, we consider the case where customer outage costs are uniformly distributed. This is represented by $d(x,y) = 2, 0 \le y \le x \le 1$. We also assume the shortfall magnitude to be uniformly distributed, which may be interpreted as a non-informative forecast. This is represented by $g(z)=1/N$ for $0 \le z \le N$.

With these uniform distributions in effect, the expected total interruption cost may be represented as (3.10).

$$N\int_0^1 \left\{ (u(v))^2 + 4\left(\int_0^v x(x-u(x))\,dx\right)(v-u(v)) \right\} dv \qquad (3.10)$$

The Euler equation for the calculus of variations problem reduces to (3.11a), with initial condition (3.11b) and transversality condition (3.11c).

$$u'' - 2u = -2v \qquad (3.11a)$$
$$u(0) = 0 \qquad (3.11b)$$
$$u'(1) = 0 \qquad (3.11c)$$

The solution to (3.11a-c) is (3.12). Figure 3.3 displays the solution graphically.

$$u(v) = v - \frac{\sinh\sqrt{2}v}{\sqrt{2}\cosh\sqrt{2}} \qquad (3.12)$$

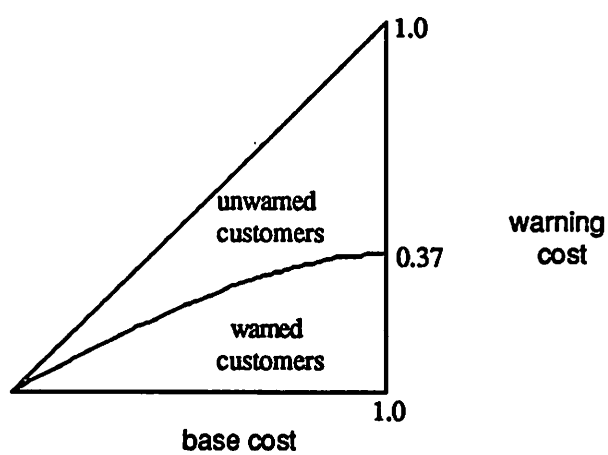

Figure 3.3: Optimal Decision Curve for Uniform Assumptions

In this example, 46 percent of all customers are warned. Interrupted standby customers are an additional 15 percent of all customers. The total expected interruption cost is $0.15N$, and 45 percent of this cost is borne by warned customers. Table 3.1 compares the results for this scenario under three models: priority rationing with early warning, as presented here; priority rationing without early warning, the standard priority pricing model; and random rationing. Priority rationing with early warning results in the smallest social cost, although more customers are interrupted. As indicated in the column labeled *relative cost*, priority rationing without early warning has total expected interruption cost 81 percent greater than the model presented here, while random rationing costs more than twice as much.

Table 3.1: Interruption Amounts and Costs for Uniform Scenario

Model	Amount Interrupted	Total Expected Interruption Cost	Relative Cost
priority rationing with early warning	$0.61N$	$0.15N$	100%
priority rationing without early warning	$0.50N$	$0.27N$	181
random rationing	$0.50N$	$0.33N$	226

4. Optimal Pricing Function

If each customer's outage costs were known to the utility, warnings would be allocated in accord with the optimal decision curve, u, derived above. Any additional demand that needed to be interrupted without warning would come from the remaining customers, interrupted in order of their base costs. However, outage costs are private information; while the utility knows the distribution of outage costs in the population, and hence can determine the function u, it cannot identify the outage costs of particular customers, which are needed to administer centrally the selective warning and interruptions called for by the optimum rationing plan.[2]

[2] Utilities employ customer surveys to ascertain the aggregate distribution of outage costs.

The objective of the tariff designer is to develop a price schedule that will induce customers to self-select a service option that will result in the optimal allocation prescribed by the solution to the central planning problem. To achieve this objective, we will consider a tariff schedule in which a customer can choose whether to be warned or standby in case of an impending shortfall, and also chooses a priority level if opting for standby status. If choosing to be warned, the customer pays nothing. If choosing standby, the customer pays $p(x)$ for priority level x, and receives compensation x if subsequently interrupted without warning. $p(x)$ may be interpreted as a demand charge or an insurance premium.

Each customer chooses the service option that minimizes its expected net interruption cost. A customer with base cost a_T and warning cost a_S will sustain a cost a_S if choosing to be warned, while if choosing standby at priority level x, the customer's expected net cost depends on its probability of being interrupted. To allocate interruptions, the utility will use the revealed priority levels as chosen by the customers. It is assumed that customers know this when they choose service options. In other words, for each priority level in the standby price menu, either the utility provides the probability of interruption along with the price, or customers correctly forecast that probability. In addition, each customer behaves as though all other customers were induced to reveal their true priority levels, and its own decision will not affect the utility's allocation scheme. This is an assumption of rational expectations.

With this scheme, the customer's problem is represented as (4.1).

$$\min_x \left\{ a_S, \overline{G}\left(N D(x,x) + N \int_x^1 \int_0^{u(c_T)} d(c_T, c_S)\, dc_S\, dc_T \right)(a_T - x) + p(x) \right\} \tag{4.1}$$

The curve u in (4.1) refers to the optimal curve found in section 3. The first-order necessary condition for the optimization problem (4.1) is (4.2a); the second-order necessary condition is (4.2b). Substitutions have been made using (3.9b).

$$0 = p'(x) - \overline{G}(h(x)) - g(h(x))h'(x)(a_T - x) \tag{4.2a}$$

$$0 \le p''(x) + 2g(h(x))h'(x) + g'(h(x))(h'(x))^2(a_T - x) + g(h(x))h''(x)(a_T - x) \tag{4.2b}$$

The tariff schedule

$$p(x) = u(x) + \text{constant} \tag{4.3}$$

satisfies (4.2a-b) at the value x = a_T. Thus, (4.3) supports the existence of a rational expectations equilibrium. However, this equilibrium may not be the unique one, and the tariff schedule (4.3) may yield other equilibria that are not socially optimal. By announcing the corresponding probability of interruption (3.8) along with the tariff at each priority level, the electric utility may facilitate customer formulation of (4.1), resulting in truthful revelation and the rational expectations equilibrium.

This scheme compensates customers for their true interruption losses. Therefore, customers on the utility's decision curve—those with base cost v and warning cost $u(v)$, $0 \leq v \leq 1$—will be indifferent in net expected interruption cost between selecting warning and sustaining interruption loss $u(v)$, and selecting standby at compensation level v and paying $u(v)$. Any customer with base cost v choosing standby pays $u(v)$, which according to (3.9a) is the sum of its expected interruption loss and an externality cost. The compensation scheme reimburses the customer for its own actual interruption losses, resulting in a net payment that is the externality cost.

Of course, if a constant is added to the price for customers who choose standby, it is also added to the price for customers who choose to be warned. This keeps the customers on the utility's decision curve indifferent in net expected interruption cost between choosing warning and standby. The constant may be positive or negative. Its value may be set to achieve one or more goals, including i) *revenue neutrality*, to ensure that the utility does not profit more from the priority pricing tariff than from random rationing, or a previously existing tariff; and ii) *Pareto superiority*, to provide all customers at least as better off under the priority pricing tariff as under random rationing, or a previously existing tariff.

From the representation of u in (3.9a) and the ensuing interpretation, we see that the result (4.3) is a manifestation of marginal cost pricing under an externality. By choosing standby, a customer with base cost v increases the probability that standby customers with smaller base costs will be interrupted. A customer choosing standby at compensation level v is therefore charged both for its own expected interruption cost and for the extra interruptions it imposes on other customers. The monopoly power of the regulated electric utility enables collection of the priority charge for the externality, so customers self-select the social optimum.

5. Conclusion

The priority pricing menu developed here extends the models in the priority service literature by allowing an early warning option. If no warning is allowed, the decision curve u disappears, and the standard one-dimensional priority service model applies. Compared with the standard model, our model results in smaller expected social cost due to capacity shortfalls.

In our analysis, we utilized a continuum of priority levels. As shown by Wilson (1989), several discrete priority levels are often enough to capture most of the efficiency gains of priority pricing. Real priority menus thus offer a handful of choices, yet retain most of the social welfare benefits indicated by our analysis. Similarly, the continuous decision curve u may be approximated by a piecewise linear function. An alternative practical implementation is to stipulate a parametric functional form and use the values of u at chosen priority levels to fit the parameter values.

Our analysis is for a single shortfall. An electric utility may experience a number of shortfalls during an operating year. The exact number of shortfalls, and exactly when each shortfall will occur, is not known in advance. Furthermore, shortfalls may be of different types: a number of short-term uncertainty distributions, G, may exist; shortfall k may have distribution G_k. This may complicate solving for the optimal allocation and implementing this allocation through priority pricing.

One of our assumptions is that warned customers incur their full interruption loss minus the warning benefit; in other words, once issued, warnings cannot be voided. Allowing the electric utility to void warnings reduces the impact of short-term uncertainty in shortfall magnitude, resulting in greater social welfare. Allowing warnings to be voided may change the optimal allocation and the resulting priority pricing menu.

Finally, our analysis of interruptible electric service with an early warning option may apply to other services with reservation options. Travel services such as airlines, car rentals, and hotels all offer differential prices that depend on advance reservation by the customer. Profit maximization distinguishes those situations from the analysis presented here.

References

Chao, Hung-po, Shmuel S. Oren, Stephen A. Smith, and Robert B. Wilson. (1986) "Multilevel demand subscription pricing for electric power." *Energy Economics* 8: 199-217.

Christensen Associates, Laurits R., Inc. (1988) *Customer Response to Interruptible and Curtailable Rates. Volume 1: Methodology and Results*. Electric Power Research Institute Report EM-5630.

Ebasco Business Consulting Company. (1987) *Innovative Rate Design Survey*. Electric Power Research Institute Report RP2381-5.

Hamlen, William A., Jr., and Frank Jen. (1983) "An Alternative Model of Interruptible Service Pricing and Rationing." *Southern Economic Journal* 49: 1108-1121.

Marchand, M. G. (1974) "Pricing Power Supplied on an Interruptible Basis." *European Economic Review* 5: 263-274.

Mussa, M., and S. Rosen. (1978) "Monopoly and Product Quality." *Journal of Economic Theory* 18: 310-317.

Oren, Shmuel S. (1990) "Pricing of Interruption and Resumption Priorities for Triangular Shortfalls." Chapter 10 of *Service Design in the Electric Power Industry*. Electric Power Research Institute Report P-6543.

Panzar, J., and D. Sibley. (1978) "Public Utility Pricing Under Risk: The Case of Self-Rationing." *American Economic Review* 68: 888-895.

Smith, Stephen A. (1989) "Efficient Menu Structures for Pricing Interruptible Electric Power Service." *Journal of Regulatory Economics* 1: 203-223.

Tschirhart, J., and Frank Jen. (1979) "Behavior of a Monopoly Offering Interruptible Service." *Bell Journal of Economics* 10: 244-258.

Viswanathan, N., and Edison T.S. Tse. (1989) "Monopolistic Provision of Congested Service with Incentive Based Allocation of Priorities." *International Economic Review* 30: 153-174.

Wilson, Robert. (1989) "Efficient and Competitive Rationing." *Econometrica* 57: 1-40.

Woo, Chi-Keung. (1990) "Efficient Electricity Pricing with Self-Rationing." *Journal of Regulatory Economics* 2: 69-81.

Woo, Chi-Keung, and Nate Toyama. (1986) "Service Reliability and the Optimal Interruptible Rate Option in Residential Electricity Pricing." *Energy Journal* 7(3): 123-136.