SOUND AND MUSIC FROM CHUA'S CIRCUIT

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Abstract

Nonlinear Dynamics have been very inspiring for musicians, but have rarely been considered specifically for sound synthesis. We discuss here the signals produced by Chua's circuit from an acoustical and musical point of view. We have designed a real-time simulation of Chua's circuit on a digital workstation allowing for easy experimentation with the properties and behaviors of the circuit and of the sounds. A surprisingly rich and novel family of musical sounds has been obtained. The audification of the local properties of the parameter space allows for easy determination of very complex structures which could not be computed analytically and would not be simple to determine by other methods. Finally we have found that the time-delayed Chua's circuit can model the basic behavior of an interesting class of musical instruments.

1. Introduction

Nonlinear Dynamics have been very inspiring for musicians[1] and have found interesting applications, for instance in the works of Pressing[2] and Wessel[3]. J. Pressing notes that several features make nonlinear maps potentially interesting as generators of musical design: fixed points, limit cycles, bifurcations, chaos and strange attractors. He has worked with the logistic map, a modified Metz map, the predator prey map and quadratic maps. The map output is used to control pitch selection, "interonset time", envelope attack time, dynamics, tempo, textural density and section length.

Wessel is using the output of Chua's circuit[4] in the same way to set pitch, duration and dynamics. The remarkable trajectories from this circuit shown in [5] and [6] are very interesting candidates for melodies and rhythms. Some transformations are needed between the musician's intention and the parameters that operate on the nonlinear system. Wessel propose using neural networks to learn and implement such transformations.

It seems that nonlinear dynamics have been less often considered specifically for sound synthesis[7]. Here we examine sounds obtained by use of Chua's circuit[8,9]. The basic oscillator circuit (Fig. 1) contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor, and a single nonlinear resistor N_r, called Chua's diode[10]. The v-i characteristic f of N_r is shown in Figure 2. The slopes on the different regions are designated by m_0, m_1 and m_2 respectively. Simple as it is, this circuit exhibits a surprisingly large variety of bifurcations and chaos[11].

2. Sound from Chua's circuit

The state equations for Chua's circuit can be rewritten in a dimensionless form as:

$$\dot{x} = \alpha[y - x - f(x)]$$

$$\dot{y} = x - y + z$$

$$\dot{z} = -\beta y$$

For many values of the parameters α and β, the circuit exhibits stable orbits[12] and the corresponding state variable values are periodic functions of time. As shown in [5] the circuit has interesting trajectories and periodic orbits. Similarly, the signals of sustained sounds from music instruments are generally periodic to a very good approximation. Therefore it is tempting to consider a state variable value as an acoustic signal to be amplified and sent to loudspeakers[13]. As explained below, for more flexibility we have done a real-time simulation of the circuit on a digital computer with audio capabilities and we have designed a graphical-user interface to control it interactively.

According to the parameter values in [11], signals with period-1, period-2, etc.... can be obtained and lead to *harmonic* sounds. In the case of chaotic signals that can be obtained with some other values, the corresponding sound can be qualified as *noisy*. A convenient way to characterize a sound is by the short-time spectrum of the signal, which, at least for sustained sounds, gives a rather good image of the way the sound is perceived. As an example, Figure 3 shows the short-time spectrum of some signals from Chua's circuit for $m_o = -1/7$, $m_1 = 2/7$, $\beta = 14.2857$ and for different values of α. These cases can be compared with the different regions displayed and commented on in [12], in particular in the frontispiece where the same parameter values are discussed. The results of the simple Euler simulation used here differ a little from the theoretical results, but very similar behaviors are found.

For $\alpha = 8.0$, 8.2 and 8.24 harmonic signals, hence harmonic sounds, are obtained with fundamental frequencies approximately 100Hz (this unit is arbitrary as explained below), 50Hz and 25Hz respectively, corresponding to period-1, period-2 and period-4 in the dynamical system. For $\alpha = 8.33$, 8.36 and 8.5 *noisy* signals are obtained together with some sinusoidal components which are harmonics of the same fundamental frequencies as above. This simultaneous presence of sinusoidal components and noise in the signal is very interesting since this occurs for the majority of classical instruments and since this is relatively difficult to model in a way which is useful for musical purposes[14].

However, for musically interesting use, a synthesis algorithm has to provide control parameters allowing for expressive timbre modifications, i.e., essentially spectrum content modifications, as required by the performer. In order to get such flexibility, we now consider a slightly modified circuit, known as the time-delayed Chua's circuit[15] (in this issue).

3. Sound from the time-delayed Chua's circuit

Let us look at Chua's circuit displayed in Figure 1. Sharkovsky et al. add a DC bias voltage source in series with the Chua's diode and replace the capacitor C2 and the inductance L by a lossless transmission line. The resulting time-delayed Chua's circuit is shown in Figure 4. In a first simplification, the slopes m_0 and m_2 of the characteristic of N_r are set equal. The study of this dynamical system is difficult, but with $C_1=0$, it reduces to a nonlinear difference equation. The solution consists of the sum of an incident wave a(t-x/v) and a reflected wave b(t+x/v) such that:

$$a(t-x/v) = -b(t-x/v) = \Phi(t-x/v), \text{ and}$$
$$\Phi(t) = \gamma(\Phi(t-2T)),$$

and Magenza[22] note that if h(t) is simplified into a dirac impulse generalised function δ_t (the sign inversion is included in γ) then:

$$q_o(t)=\gamma(q_o(t-T)),$$

and similarly for q_i. The signal value $q_o(t)$ depends only on the value at t-T. If $q_o(t)=Q_o$ is constant on [-T, 0] then it is constant on any interval [(n-1)T, nT] with a value:

$$Q_n=\gamma(Q_{n-1}).$$

We now examine this iterated map as the basic model of a clarinet-like instrument. If p=0, then q_o should stay zero. Thus the origin O is a fixed point of the map. In order for the system to oscillate around O, as we expect a musical instrument to do, the slope s_1 of a *smooth* map about O has to be less than -1 (Fig. 8). In order for the signal not to grow to infinity, the slope of γ has to become greater than -1 at some distance from O. Let us choose γ as two segments with slopes $s_1<-1$ and $s_2>-1$ for Q>0 and γ symmetric around O (Fig. 8). Remarkably, this is the same map as in the time-delayed Chua's circuit. The s_1-s_2 map is also justified by control considerations of the basic clarinet-like instrument as explained below. We thus have shown that the time-delayed Chua's circuit is a model of an interesting class of musical instruments, namely those, like the clarinet, consisting of a massless reed coupled to a linear system.

6. Roles of the slopes s1 and s2

It can easily be seen that $|s_1|$ controls the *transient onset velocity*, the greater $|s_1|$, the faster the onset. We have here a clear control parameter for the onset behavior of our instrument. If h(t)=δ, the signal is a square wave. If h(t) is a low pass kernel, then the signal is *rounded.* This rounding can be controlled by $|s_2|$: the closer $|s_2|$ is to unity, the less high frequencies are in q_o. This can also be viewed as follows: in the square wave case the system uses only two points of the map and in the rounded case it uses more points spread more regularly on the map.

As a first result, we have found that two important characteristics of the sound, transient onset velocity and richness, are controlled by the slopes s_1 and s_2. If one wants a smooth map, one may choose a simple cubic $\gamma(x)= ax^3+s_1x$ (Fig. 9), where a is determined again according to the slopes s_1 in O and s_2 at the point of abscissa x_0 such that: $-x_0 = ax_0^3+s_1x_0$.

7. Periodicity and harmonic content

For musical purposes we expect to have control of the period of the waveform since its inverse is the *pitch* of the sound. In the continuous case, Chow et al.[23] have studied similar equations of the form:

$$x(t) = f(h(t) \otimes x(t-T)).$$

It is shown that under some fairly general conditions on the map f and the kernel h, the period 2 (corresponding to the first *mode* in a clarinet, i.e. to a period of 2T) is asymptotically stable. This means that we can expect to play and keep some steady tone from the instrument. It is also shown that, if f is symmetrical around O the signal x(t) has the symmetry $x(t+T) = - x(t)$. Then the signal is composed of odd harmonics only. This is an essential characteristic of the clarinet sound. Under some conditions, periods having durations which are integer fractions of 2T are also possible.

As explained above, if the slope s_2 is greater than 1, the fixed point of the map $f^2 = f \circ f$ becomes unstable. We observe period doubling and, for greater values, chaotic behavior. From the sound synthesis point of view, this is very interesting. Period doubling corresponds to subharmonics. In the case of chaos, the signal sounds like noise added to the periodic tone of the instrument but with some relationship between partials and noise.

8. Digital simulation and user interface

For more flexibility, we have used a simulation of the time-delayed Chua's circuit on a digital computer. We have implemented our simulation on a Silicon Graphics Indigo workstation which is very well adapted for that purpose. It has good quality 16 bit audio ports and good graphic capabilities for user interface (Xwindow and Motif). Furthermore, it is fast enough for real-time simulation of the various circuits that we have studied. In order to keep up with real-time at a reasonable sampling frequency (such as 22050 Hz) integration is done by means of the simple forward Euler method. For given parameter values, the circuit has a precise period duration. We may allow arbitrary time stretching or compression in order to obtain any period duration, i.e. any pitch, wanted for the acoustic signal.

Real-time simulations were implemented using HTM, a tool for rapid prototyping of musical sound synthesis algorithms and control strategies[24]. We have written a Motif-C++ graphical-user interface allowing for easy experimentation with the parameter values. An example of such an interface is shown on Figure 10. The faders control the two slopes of the piecewise linear map γ, the output amplitude and the fundamental frequency. The extreme values of the faders can be adjusted by editing the corresponding character strings. Buttons allow one to start the dynamical system by opening the data channel to the synthesis program, to display a graph of the output signal or of the map and to monitor the signal amplitude (meter).

9. Conclusion

We have studied the signals produced by Chua's circuit from an acoustical and musical point of view. We have designed a real time simulation of Chua's circuit on an affordable digital workstation allowing for interactive changes of the parameters, for simultaneous listening of the corresponding sounds and for easy experimentation with the properties and behaviors of the circuit and of the sounds. Very interesting results have been found. First, in the different regions of the parameter space, periodic and chaotic signals provide a rich and novel family of musical sounds.

It is easy to trace and characterize these different regions by listening to the sound while changing parameters. This audification of the local properties of the parameter space allows an easy determination of very complex structures which could not be computed analytically and would not be simple to determine by other ways

Finally we have found that the time-delayed Chua's circuit is a model of the basic behavior of an interesting class of musical instruments, namely those, like the clarinet, consisting of a massless reed coupled to a linear system. Such models are essential for the development and musical use of physical models of classical or new instruments. We expect to extend Chua's circuit to other instruments such as brass, voice, flute, and strings.

Acknowledgments

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References

1. "Special session on Chaos in Music", *124th ASA meeting*, New Orleans, Oct. 1992.
2. Jeff Pressing, "Nonlinear Maps as Generators of Musical Design", *Computer Music Journal*, Vol. 12, No. 2, Summer 1988.
3. D. Wessel, "Connectionist models for musical control of nonlinear dynamical systems", presented at the *124th ASA meeting*, New Orleans, Oct. 1992.
4. L.O. Chua, "The genesis of Chua's circuit", *Archiv fur Elektronik und Ubertragungstechnik*, vol. 46, no 4, pp. 250-257, 1992.
5. Shuxian Wu, "Chua's Circuit Family", *Proc. of the IEEE*, Vol. 75, No.8, August 1987, pp. 1022-1032.
6. L.O. Chua, "A zoo of strange attractors from the Chua's circuit", *Proc. 35th Midwest Symposium on Circuits and Systems*, Whasington, D.C., August 9-12, 1992.
7. R. Schumacher and C. Chafe, "Detection of Aperiodicity in Nearly Periodic Signals", *Proc. of IEEE Int. Conf. on Acoust., Speech and Sig. Proc.*, Albuquerque, NM, April 1990.
8. L.O. Chua and L. Huynh, "Bifurcation analysis of Chua's circuit", *Proc. 35th Midwest Symposium on Circuits and Systems*, Whasington, D.C., August 9-12, 1992.
9. L. O. Chua and G.-N. Lin, "Canonical Realization of Chua's Circuit Family", *IEEE trans. Circuits & Syst.*, Vol. CAS-37 (July. 1990) No. 7, pp 885-902.
10. M.P. Kennedy, "Robust op amp implementation of Chua's circuit", *Frequenz*, 46, no 3-4, 1992, pp.66-80.
11. R.N. Madan, "Learning chaotic phenomena from Chua's circuit", *Proc. 35th Midwest Symposium on Circuits and Systems*, Whasington, D.C., August 9-12, 1992.
12. L. O. Chua, M. Komuro and T. Matsumoto, "The Double Scroll Family", *IEEE trans. Circuits & Syst.*, Vol. CAS-33 (Nov. 1986) No. 11, pp 1073-1118.
13. X. Rodet, "Time delayed nonlinear dynamical systems as musical instruments", presented at the *124th ASA meeting*, New Orleans, Oct. 1992.

14. X. Rodet, P. Depalle & G. Poirot, "Speech Analysis and Synthesis Methods Based on Spectral Envelopes and Voiced/Unvoiced Functions", *Proc. European Conference on Speech Tech.*, Edinburgh, U.K., Sept. 87.
15. A.N. Sharkovsky, Yu. Mastrenko, Ph. Deregel, and L.O.Chua, "Dry Turbulence from a time-delayed Chua's Circuit",to appear in *Journal of Circuits, Systems and Computers*, Special Issue on Chua's Circuit: a Paradigm for Chaos, March 1993.
16. X. Rodet, "Nonlinear Oscillator Models of Musical Instrument Excitation", *Proc. International Computer Music Conference*, San Jose, Oct. 1992.
17. *Proceedings of the Colloquium on Physical Modeling*, Grenoble, France, Oct. 1990.
18. H.K.Khalil, *Nonlinear Systems*, Macmillan Publishing Company, New York, 1992.
19. X. Rodet & P. Depalle, "A physical model of lips and trumpet", in *Proc. International Computer Music Conference*, San Jose, Oct. 1992.
20. P. Depalle, X. Rodet & D. Matignon, "State-Space Models for Sound Synthesis", *IEEE ASSP Workshop on Appl. of Digital Signal Processing to Audio and Acoustics*, Mohonk, New Paltz, New York, Nov. 1991
21. M.E. McIntyre et al., "On the Oscillations of Musical Instruments", *JASA* 74 (5), Nov. 83.
22. C. Magenza, "Excitation non linéaire d'un conduit acoustique cylindrique", *Thèse*, Université du Maine, France, 85.
23. S. N. Chow & D. Green Jr., "Some results on Singular Delay-Differential Equations", in *Chaos, Fractals and Dynamics*, edited by P. Fischer & W. R. Smith, Marcel Dekker, 1983.
24. A. Freed, "Tools for rapid prototyping of Music Sound Synthesis Algorithms", *Proc. International Computer Music Conference*, San Jose, 92.

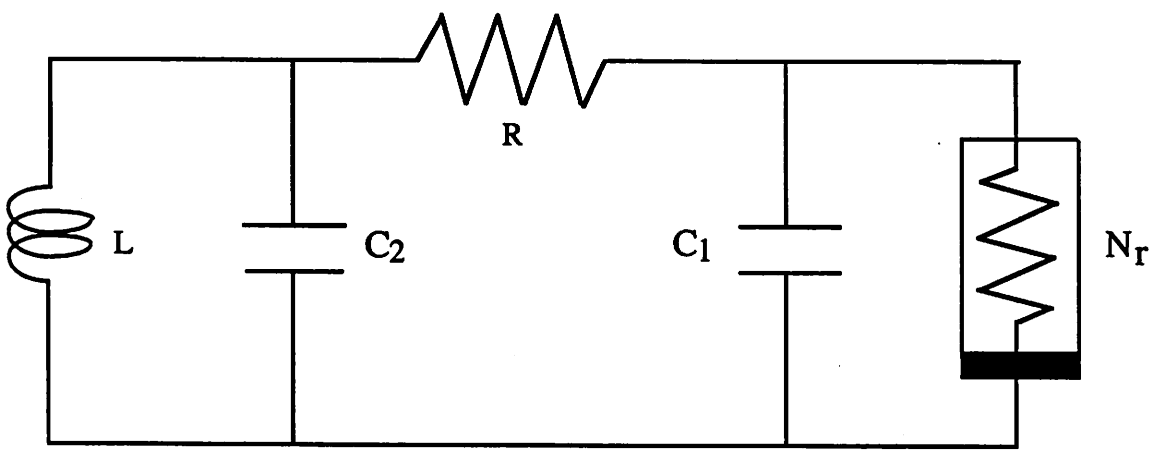


Figure 1: Chua's circuit.

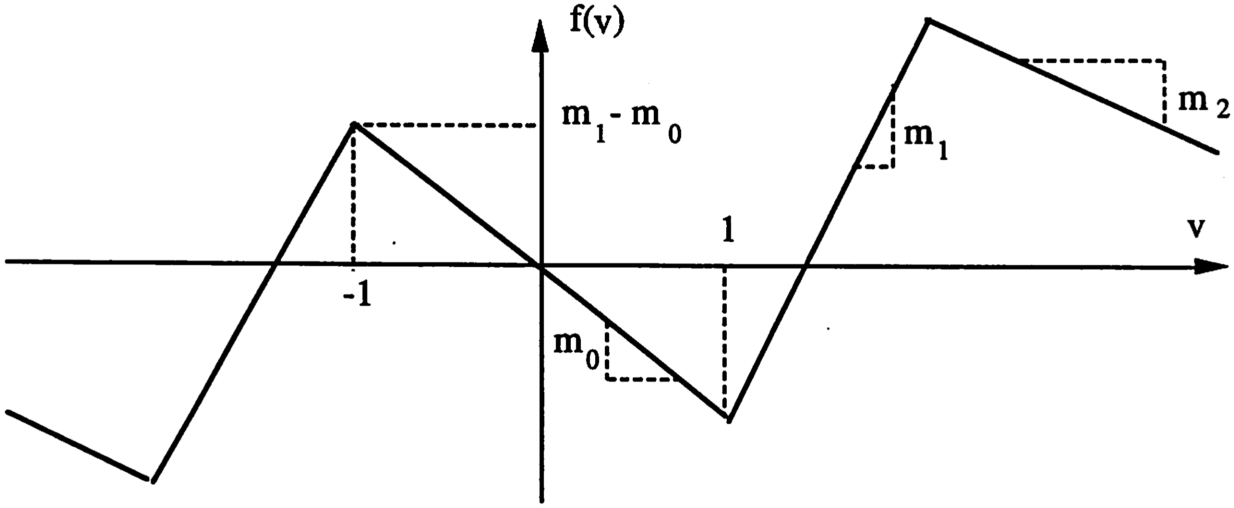


Figure 2: Chua's diode characteristic.

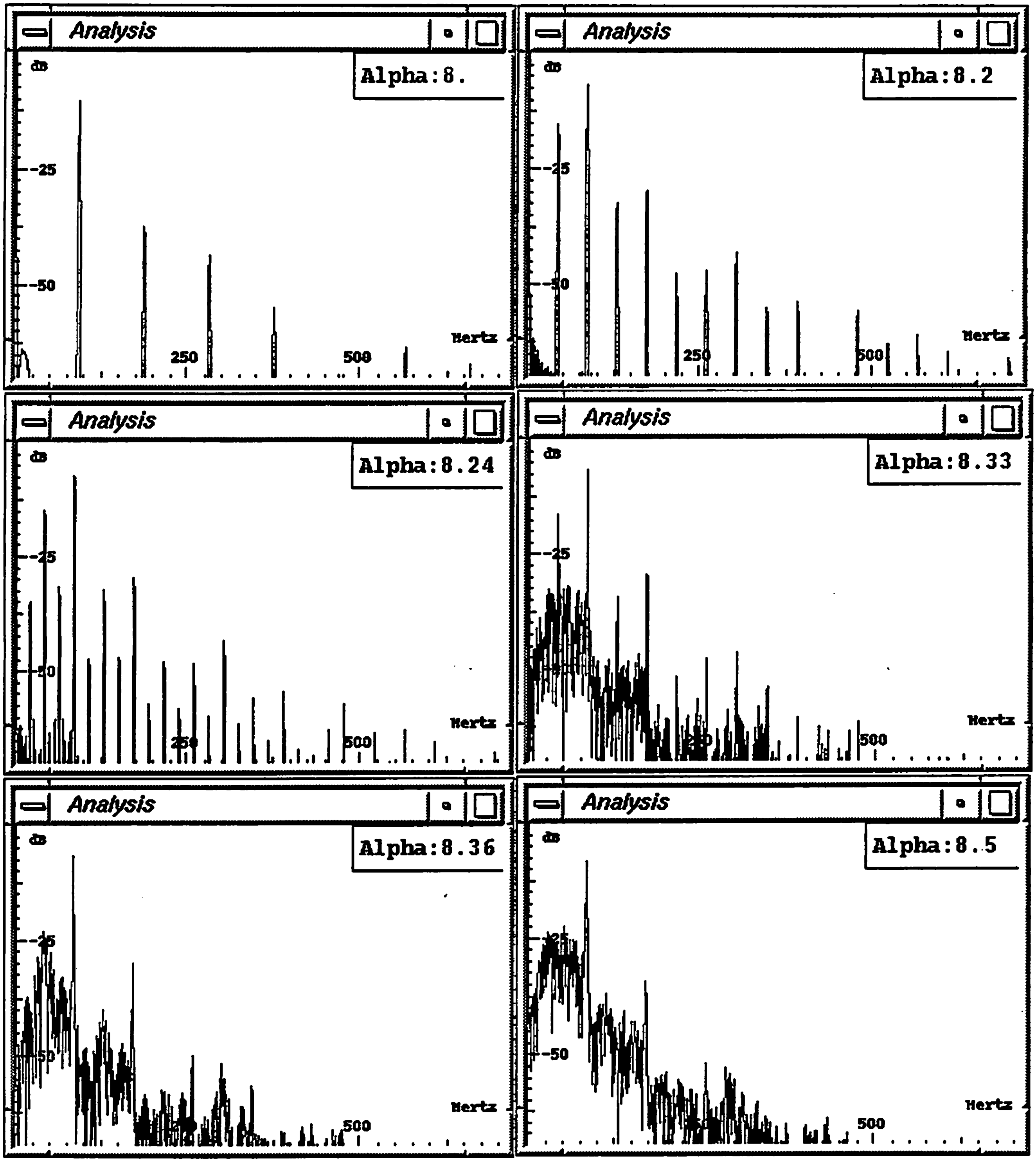


Figure 3: Short-time spectrum of signals from the digital simulation of Chua's circuit for $m_0=-1/7$, $m1=2/7$, $\beta=14.2857$ and for different values of alpha.

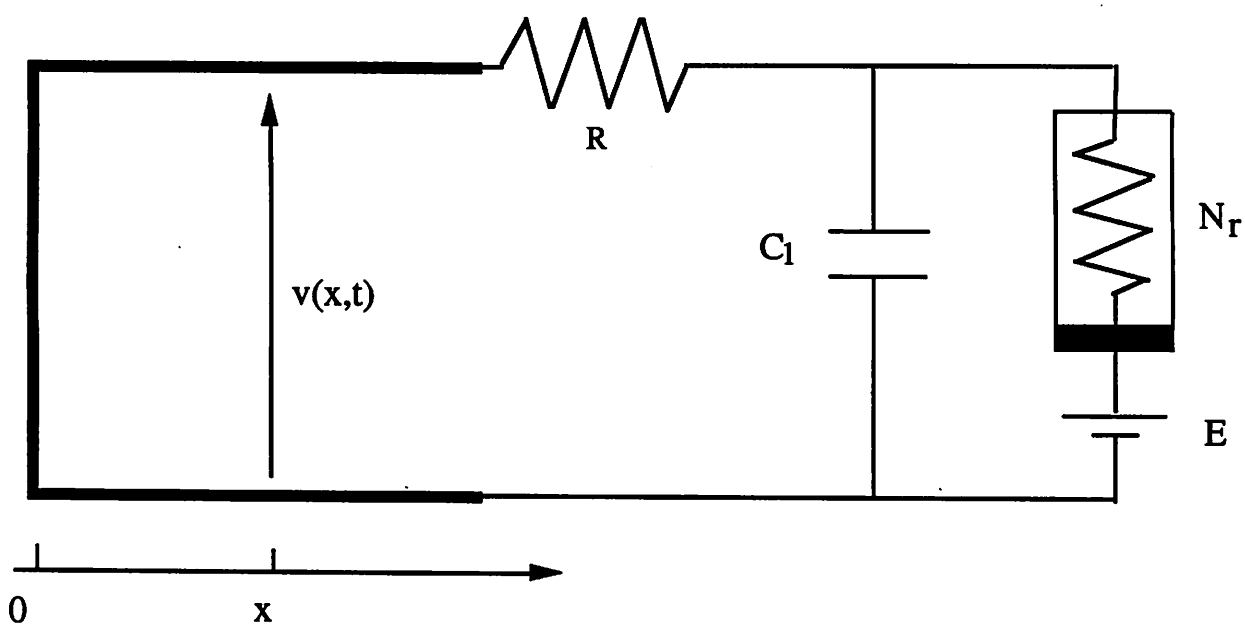


Figure 4: Time-delayed Chua's circuit.

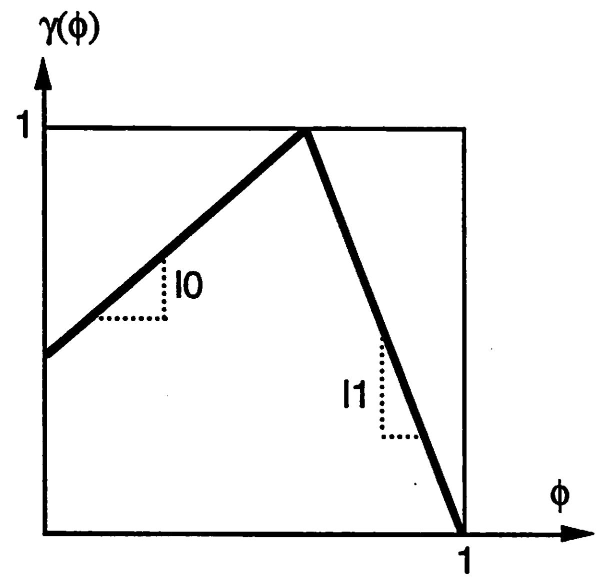


Figure 5: The two segments map γ.

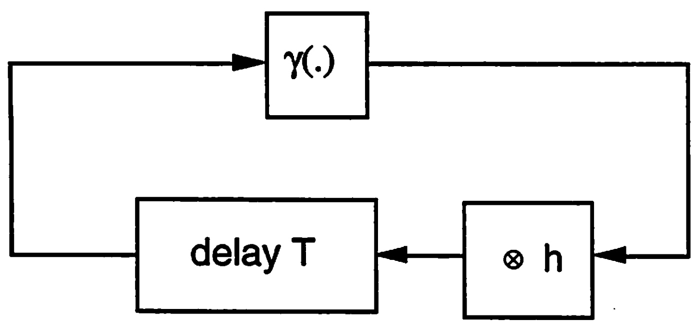


Figure 6: A simple time-delayed nonlinear system, also a basic clarinet model.

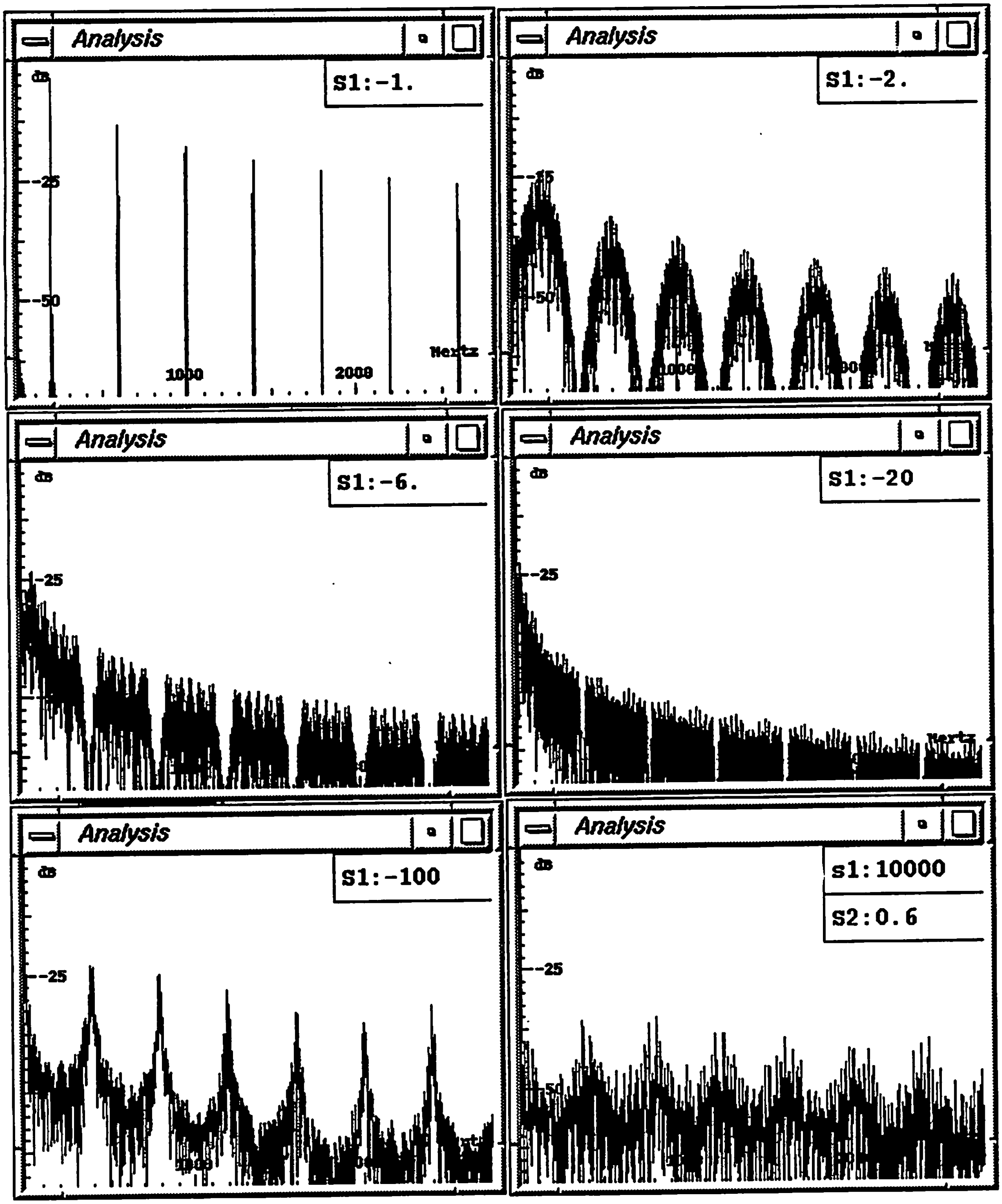


Figure 7: Short-time spectrum of signals from the digital simulation of the time-delayed Chua's circuit for l_0=0.99 and l_1 between -1 and -10000.

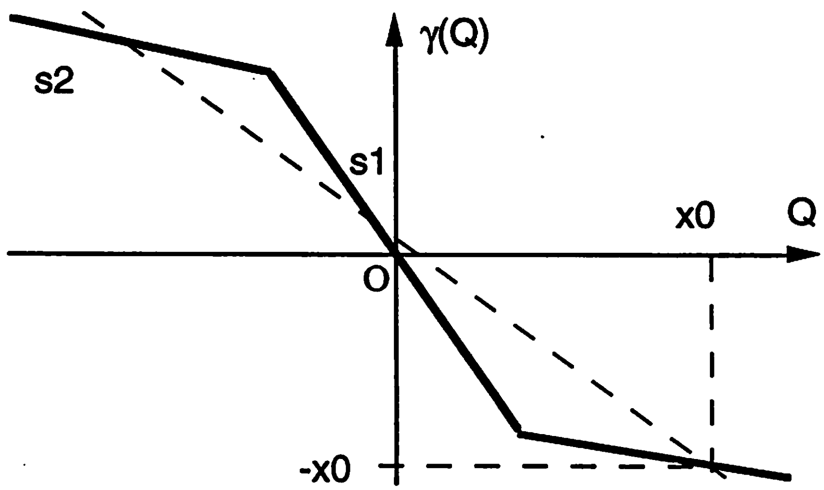


Figure 8: A piecewise linear map with slopes s1 and s2.

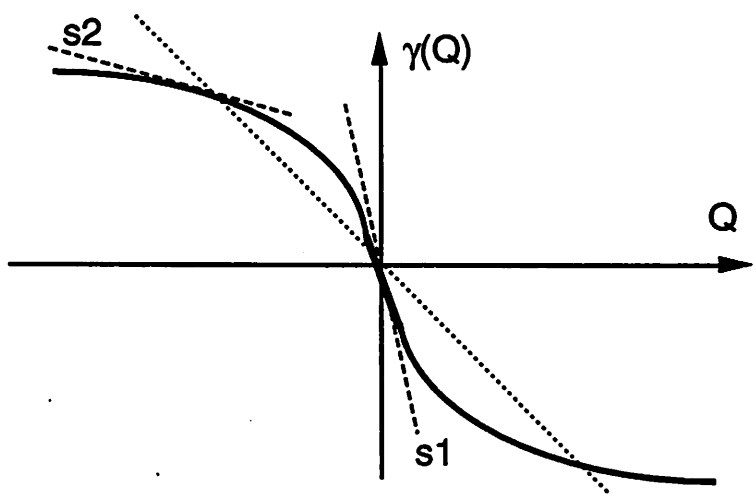
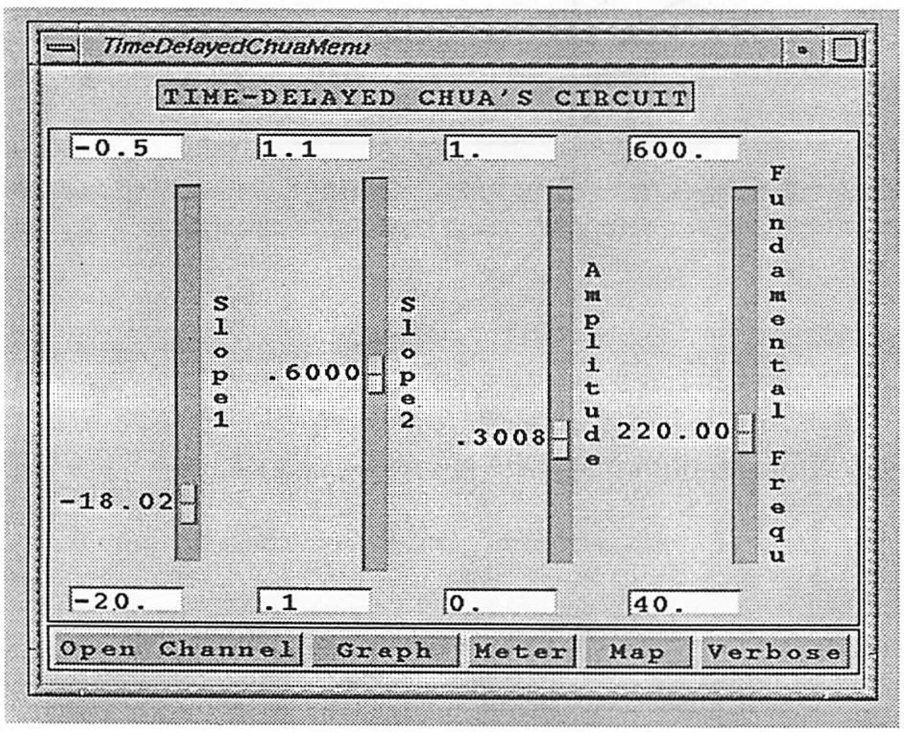


Figure 9: A cubic smooth map with slopes s1 and s2.



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