INDIRECT ADAPTIVE NONLINEAR CONTROL OF INDUCTION MOTORS

by

Raja R. Kadiyala

Memorandum No. UCB/ERL M92/23

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Indirect Adaptive Nonlinear Control of Induction Motors *

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March 2, 1992

Abstract

An indirect adaptive control law based on certainty equivalence is designed for a model of the induction motor with the assumption that the magnetic subsystem is linear. This nonlinear control law asymptotically renders the induction motor system input-output linear and also achieves input-output decoupling. In addition, we find that for the specific case of the induction motor we are able to prove parameter convergence and asymptotic tracking of a an open set of reference trajectories using the indirect adaptive controller. This differs from the generic case where we cannot guarantee parameter convergence. The indirect adaptive control methodology also does not suffer from the drawback of overparameterization, as in the direct adaptive control technique. In addition simulations are also given comparing nonadaptive and indirect adaptive nonlinear controllers.

*Research supported in part by the Army under grants DAAL-88-K-0106 and DAAL-91-G-0191, and NASA under grant NAG 2-243

1 Introduction

There has been much recent research in the use of adaptive control techniques for improving the input output linearization by state feedback of nonlinear systems with parametric uncertainty. Techniques of direct adaptive control (with no explicit identification) were proposed and developed in [Kanellakopoulos *et al.*, 1989, Sastry and Isidori, 1987, Taylor *et al.*, 1989] (see also [Sastry and Bodson, 1989]). Nonlinear indirect adaptive control was initiated in [Bastin and Campion, 1989, Campion and Bastin, 1989, Pomet and Praly, 1988]. It is motivated by the fact that, with exact knowledge of the plant parameters, a nonlinear state feedback law and a suitable set of coordinates can be chosen to produce linear input-output behavior. In the case of parameter uncertainty, intuition suggests that parameter estimates which are converging to their true values can be used to asymptotically linearize the system. This heuristic is known as the *certainty equivalence principle.* Indirect adaptive control differs from direct adaptive control in that it relies on an observation error to update the plant parameters rather than relying on an output error.

Indirect adaptive control can be broken down into two parts. First, a parameter identifier is attached to the plant and adjusts the parameter estimates on line. These estimated parameters are then used in the linearizing control law (see figure 1).

The use of nonadaptive feedback linearization for the control of an induction motor has been quite popular and has been used successfully in [Krzeminski, 1987, Luca and Ulivi, 1987] and others. Furthermore, direct adaptive nonlinear controllers have been developed in both [Georgiou and Normand-Cyrot, 1989] and [Marino *et al.*, 1990]. However, in general, the direct adaptive control scheme requires overparametrization, ie. extra parameters must be added in the controller (see [Sastry and Isidori, 1987]). The indirect adaptive control scheme does not suffer from this drawback.

In this paper we use the techniques constructed in [Teel *et al.*, 1991] as applied to a fifth order symmetric induction motor model with linear magnetic circuits. This model was used in [Marino *et al.*, 1990]. We start by presenting a review of nonadaptive feedback linearization in section 2 and continue with a description of a nonlinear identifier and the indirect adaptive nonlinear control scheme in sections 3, 4. With this background theory set, we then proceed with a derivation of the fifth order induction

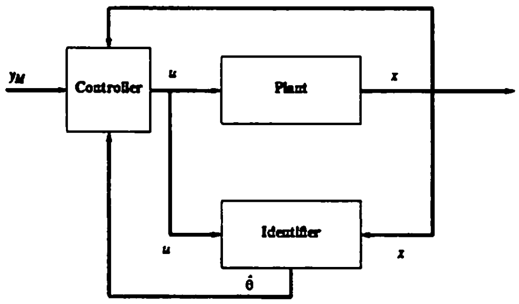

Figure 1: Block Diagram of an Indirect Adaptive Controller

motor model in section 5 and carry through the calculations necessary for the identification scheme and the indirect adaptive controller. We end with simulations comparing a nonadaptive feedback linearization and an indirect adaptive controller.

2 Nonadaptive Feedback Linearization

To fix notation, we review following [Isidori, 1989], the basic linearizing theory. Consider the two-input two-output system

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)u_1 + g_2(x)u_2 \\ y_1 &= h_1(x) \\ y_2 &= h_2(x) \end{aligned} \tag{1}$$

with $x \in \mathbf{R}^n$,$u \in \mathbf{R}$ and f, g_i, h_i smooth. Differentiating y_1 and y_2 with respect to time, one obtains

$$\begin{aligned} \dot{y_1} &= L_f h_1 + L_{g_1} h_1 u_1 + L_{g_2} h_1 u_2 \\ \dot{y_2} &= L_f h_2 + L_{g_1} h_2 u_1 + L_{g_2} h_2 u_2. \end{aligned} \tag{2}$$

Here $L_f h_j$, $L_{g_i} h_j$ stand for the Lie derivatives of h_j with respect to f, g_i respectively ($L_f h(x) = \frac{\partial h}{\partial x} f(x)$). Continue differentiating each y_i until some u_j appear (ie. $L_{g_j} h_i \neq 0$ for $j = 1$ or $j = 2$). We then have

$$\begin{aligned} {y_1}^{(\gamma_1)} &= L_f^{\gamma_1} h_1 + L_{g_1} L_f^{\gamma_1 - 1} h_1 u_1 + L_{g_2} L_f^{\gamma_1 - 1} h_1 u_2 \\ {y_2}^{(\gamma_2)} &= L_f^{\gamma_2} h_2 + L_{g_1} L_f^{\gamma_2 - 1} h_2 u_1 + L_{g_2} L_f^{\gamma_2 - 1} h_2 u_2. \end{aligned} \tag{3}$$

Define the $2x2$ decoupling matrix

$$A(x) = \begin{bmatrix} L_{g_1}L_f^{\gamma_1-1}h_1u_1 & L_{g_2}L_f^{\gamma_1-1}h_1u_2 \\ L_{g_1}L_f^{\gamma_2-1}h_2u_1 & L_{g_2}L_f^{\gamma_2-1}h_2u_2 \end{bmatrix} \tag{4}$$

and the $2x1$ vector

$$b(x) = \begin{bmatrix} L_f^{\gamma_1}h_1 \\ L_f^{\gamma_2}h_2 \end{bmatrix}. \tag{5}$$

The system (1) is said to have well defined vector relative degree $[\gamma_1\ \gamma_2]$ if $L_{g_i}L_f^j h_k u_i \equiv 0$ for ($i,k = 1,2$ and $0 \leq j \leq \gamma_k - 2$) and the matrix $A(x)$ is uniformly (in x) nonsingular.

If a system has well defined vector relative degree than the control law

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -A^{-1}(x)b(x) + A^{-1}(x)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{6}$$

yields

$$\begin{bmatrix} {y_1}^{(\gamma_1)} \\ {y_2}^{(\gamma_2)} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \tag{7}$$

We will not consider the case where the vector relative degree is not defined (ie. $A(x)$ is singular for some x).

For a system with a vector relative degree $[\gamma_1\ \gamma_2]$, it is easy to verify that at each $x^0 \in \mathbf{R}^n$ there exists a neighborhood U^0 of x^0 such that the mapping

$$\Phi : U^0 \longrightarrow \mathbf{R}^n$$

defined as

$$\begin{array}{rcccl} \Phi_1(x) & = & \xi_1 & = & h_1(x) \\ \Phi_2(x) & = & \xi_2 & = & L_fh_1(x) \\ & & \vdots & & \\ \Phi_{\gamma_1}(x) & = & \xi_{\gamma_1} & = & L_f^{\gamma_1-1}h_1(x) \\ \Phi_{\gamma_1+1}(x) & = & \xi_{\gamma_1+1} & = & h_2(x) \\ \Phi_{\gamma_1+2}(x) & = & \xi_{\gamma_1+2} & = & L_fh_2(x) \\ & & \vdots & & \\ \Phi_{\gamma_1+\gamma_2}(x) & = & \xi_{\gamma_1+\gamma_2} & = & L_f^{\gamma_2-1}h_2(x) \end{array} \tag{8}$$

with

$$\eta = [\Phi_{\gamma_1+\gamma_2+1}, \ldots, \Phi_n]^T$$

is a diffeomorphism onto its image. The system may be written in *normal form* ([Isidori, 1989]) as

$$\begin{array}{rcl}
\dot{\xi}_1 & = & \xi_2 \\
 & \vdots & \\
\dot{\xi}_{\gamma_1-1} & = & \xi_{\gamma_1} \\
\dot{\xi}_{\gamma_1} & = & b_1(\xi,\eta)+a_{(1,1)}(\xi,\eta)u_1+a_{(1,2)}(\xi,\eta)u_2 \\
\dot{\xi}_{\gamma_1+1} & = & \xi_{\gamma_1+2} \\
 & \vdots & \\
\dot{\xi}_{\gamma_1+\gamma_2-1} & = & \xi_{\gamma_1+\gamma_2} \\
\dot{\xi}_{\gamma_1+\gamma_2} & = & b_2(\xi,\eta)+a_{(2,1)}(\xi,\eta)u_1+a_{(2,2)}(\xi,\eta)u_2 \\
\dot{\eta} & = & q(\xi,\eta)+P(\xi,\eta)u \\
y_1 & = & \xi_1 \\
y_2 & = & \xi_{\gamma_1+1}
\end{array} \tag{9}$$

where $b_i(\xi,\eta)$ are elements of the vector in (5) and $a_{(i,j)}(\xi,\eta)$ are the entries of $A(x)$ given in (4). We assume that η remains bounded (this assumption is meet for the induction motor).

2.1 Non-Adaptive Tracking

We now apply the normal form to the tracking problem. We desire to have each $y_i(t)$ track a given $y_{i_M}(t)$. We start by choosing v_i in (6) as

$$\begin{array}{rcl}
v_i & = & y_{i_M}^{(\gamma_i)}+\alpha_{(i,1)}(y_{i_M}^{(\gamma_i-1)}-y_i{}^{(\gamma_i-1)}) \\
 & + & \ldots+\alpha_{(i,\gamma_i)}(y_{i_M}-y_i)
\end{array} \tag{10}$$

with $\alpha_{(i,1)},\ldots,\alpha_{(i,\gamma_i)}$ chosen so that

$$s^{\gamma_i}+\alpha_{(i,1)}s^{\gamma_i-1}+\ldots+\alpha_{(i,\gamma_i)} \tag{11}$$

is a Hurwitz polynomial.

It can be shown that this control results in asymptotic tracking and bounded states ξ provided $y_{i_M},\dot{y}_{i_M},\ldots,y_{i_M}^{(\gamma_i-1)}$ are bounded.

3 Identifier Structure

Consider the two-input system

$$\dot{x}=f(x,\theta^*)+f_0(x)+g_1(x)u_1+g_2(x)u_2 \tag{12}$$

with $x \in \mathbf{R}^n$,$\theta^* \in \mathbf{R}^p$ and f, f_0, g_i are assumed to be smooth vector fields on $\mathbf{R}^n$. Furthermore, let $f(x, \theta^*)$ have the form

$$f(x,\theta^*) \quad = \quad \textstyle\sum_{i=1}^{p} \theta_i^* f_i(x) \tag{13}$$

Here θ_i^*, $i = 1, \ldots, p$, are unknown parameters, which appear linearly, and the smooth vector fields $f_i(x)$ are known. If we formulate the regressor

$$w^T(x,u) = [f_1(x), \ldots, f_p(x)] \tag{14}$$

then (12) can be written as

$$\dot{x} = w^T(x,u)\theta^* + f_0(x) + g_1(x)u_1 + g_2(x)u_2. \tag{15}$$

3.1 Observer-based Identifier

To estimate the unknown parameters, we will use the identifier system

$$\begin{aligned} \dot{\hat{x}} &= A(\hat{x} - x) + w^T(x,u)\hat{\theta} \\ &+ f_0(x) + g_1(x)u_1 + g_2(x)u_2 \\ \dot{\hat{\theta}} &= -w(x,u)P(\hat{x} - x). \end{aligned} \tag{16}$$

Here $A \in \mathbf{R}^{n \times n}$ is a Hurwitz matrix and $P \in \mathbf{R}^{n \times n} > 0$ is a solution to the Lyapunov equation

$$A^T P + PA = -Q, \quad Q > 0. \tag{17}$$

This identifier is reminiscent of one proposed in [Kudva and Narendra, 1973], [Kreisselmeier, 1977]. Note that $A = -\sigma I$ is a special case of the identifier. If we define $e_1 = \hat{x} - x$, the observer state error, and $\phi = \hat{\theta} - \theta^*$, the parameter error, and assume θ^* to be constant, but unknown, then we have the error system

$$\begin{aligned} \dot{e}_1 &= Ae_1 + w^T(x,u)\phi \\ \dot{\phi} &= -w(x,u)Pe_1. \end{aligned} \tag{18}$$

One should note the similarity of the error equation above with that of the error equation of a full order observer, although all the states are available by assumption.

Theorem 3.1 Stability of Observer-based Identifier
Consider the observer-based identifier of equation (18),

then *1.* $\phi \in L_\infty$,

2. $e_1 \in L_\infty \cap L_2$,

3. If $w(x, u)$ *is bounded,*
then $\dot{e}_1 \in L_\infty$ *and* $\lim_{t\to\infty} e_1(t) = 0$.

Remarks:

1. The proof is a standard Lyapunov argument on the function

$$V(e_1, \phi) = e_1^T P e_1 + \phi^T \phi. \tag{19}$$

2. The condition on the boundedness of w is a stability condition. In particular, if the system is bounded-input bounded-state (bibs) stable with bounded input, then w is bounded. (see [Sastry and Bodson, 1989]).

3. Theorem 3.1 makes no statement about parameter convergence. As is standard in the literature, one can conclude from (18) that e_1 and ϕ both converge exponentially to zero if w is sufficiently rich, ie., $\exists \alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_1 I \geq \int_s^{s+\delta} w w^T dt \geq \alpha_2 I. \tag{20}$$

This condition is usually impossible to verify explicitly ahead of time since w is a function of x. If we assume that the regressor is bounded it is clearly not necessary to have the upper bound in (20). Henceforth, when we use this result we will assume that the regressor is bounded.

4 Indirect Adaptive Tracking

Let us consider our choice for the control law. The certainty equivalence principle suggests that we pick the appropriate linearizing control law but with the unknown parameters replaced by their estimates. We choose

$$\begin{aligned}
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} &= -A^{-1}{}_{\hat\theta}(x) b_{\hat\theta}(x) + A^{-1}{}_{\hat\theta}(x) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\
A_{\hat\theta}(x) &= \begin{bmatrix} L_{g_{1\hat\theta}} L^{\gamma_1 - 1}{}_{f_{\hat\theta}} h_1 u_1 & L_{g_{2\hat\theta}} L_{f_{\hat\theta}}{}^{\gamma_1 - 1} h_1 u_2 \\ L_{g_{1\hat\theta}} L^{\gamma_2 - 1}{}_{f_{\hat\theta}} h_2 u_1 & L_{g_{2\hat\theta}} L_{f_{\hat\theta}}{}^{\gamma_2 - 1} h_2 u_2 \end{bmatrix} \\
b_{\hat\theta}(x) &= \begin{bmatrix} L_{f_{\hat\theta}}{}^{\gamma_1} h_1 \\ L_{f_{\hat\theta}}{}^{\gamma_2} h_2 \end{bmatrix}.
\end{aligned} \tag{21}$$

To achieve tracking we pick $\hat v_i$ in the form of (10). However, we do not have exact expressions for the derivatives of y_i which involve unknown parameters. Instead we will use estimates of the derivatives of y_i obtained from the parameter estimates:

$$\begin{aligned}
\hat v_i &= y_{i_M}^{(\gamma_i)} + \alpha_{(i,1)}(y_{i_M}^{(\gamma_i - 1)} - \hat y_i{}^{(\gamma_i - 1)}) \\
&+ \ldots + \alpha_{(i,\gamma_i)}(y_{i_M} - \hat y_i) \\
\hat y_i{}^{(j)} &= L^j{}_{f_{\hat\theta}} h_i(x)
\end{aligned} \tag{22}$$

5 Induction Machine Model

The model for a 2-phase symmetric induction machine with linear magnetics is derived in [Marino *et al.*, 1990]. To set notation we have $R_{\{r,s\}}, i_{\{sa,sb,ra,rb\}}, \psi_{\{sa,sb,ra,rb\}}$ representing the resistance, current and flux linkage with respect to the stator (s), rotor (r) and the stationary stator reference frame (a,b). The inputs are taken as the voltages to the stator, namely v_{sa}, v_{sb} and $x = [\omega, \psi_{ra}, \psi_{rb}, i_{sa}, i_{sb}]^T$.

We have, in the familiar $\dot x = f(x) + g_1(x) v_{sa} + g_2(x) v_{sb}$ form

$$f(x) =$$

$$\begin{bmatrix}
\frac{n_p M}{J L_r}(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) - \frac{T_L}{J} \\
\frac{R_r}{L_r}(M i_{sa} - \psi_{ra}) - n_p \omega \psi_{rb} \\
n_p \omega \psi_{ra} + \frac{R_r}{L_r}(M i_{sb} - \psi_{rb}) \\
\frac{M}{\sigma L_r}\left(\frac{R_r}{L_r}\psi_{ra} + n_p \omega \psi_{rb}\right) - \left(\frac{M^2 R_r + R_s L_r{}^2}{\sigma L_r{}^2}\right) i_{sa} \\
\frac{M}{\sigma L_r}\left(\frac{R_r}{L_r}\psi_{rb} - n_p \omega \psi_{ra}\right) - \left(\frac{M^2 R_r + R_s L_r{}^2}{\sigma L_r{}^2}\right) i_{sb}
\end{bmatrix} \tag{23}$$

and

$$\begin{aligned} g_1 &= \begin{bmatrix} 0 \; 0 \; 0 \; \frac{1}{\sigma} \; 0 \end{bmatrix}^T \\ g_2 &= \begin{bmatrix} 0 \; 0 \; 0 \; 0 \; \frac{1}{\sigma} \end{bmatrix}^T \end{aligned} \tag{24}$$

where $\sigma = L_s - \frac{M^2}{L_r}$ and n_p is the number of pole pairs. Now choose the outputs for tracking as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \omega \\ \psi_{ra}^2 + \psi_{rb}^2 \end{bmatrix}. \tag{25}$$

5.1 Input-Output Linearization of the Induction Motor

With the dynamics and outputs defined as above, we find that the system has vector relative degree of [2 2], hence the η dynamics are one dimensional. Thus the change of coordinates may be defined as

$$\Phi(x) = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \eta \end{pmatrix} = \begin{pmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \\ \arctan\left(\frac{\psi_{ra}}{\psi_{rb}}\right) \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{n_p M}{J L_r}\left(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}\right) - \frac{T_L}{J} \\ \psi_{ra}^2 + \psi_{rb}^2 \\ \frac{2R_r}{L_r}\left[M\left(\psi_{ra} i_{sa} + \psi_{rb} i_{sb}\right) - \psi_{ra}^2 - \psi_{rb}^2\right] \\ \arctan\left(\frac{\psi_{ra}}{\psi_{rb}}\right) \end{pmatrix} \tag{26}$$

where η was picked to complete the change of coordinates. Since $det\left(\frac{\partial \Phi(x)}{\partial x}\right) = \frac{4MR_r}{L_r}\left(\psi_{ra}^2 + \psi_{rb}^2\right)$ we must have the quantity $\left(\psi_{ra}^2 + \psi_{rb}^2\right)$ nonzero, which is true provided the motor is rotating. For $\Phi(x)$ to be a diffeomorphism we must also have the angle $\arctan\left(\frac{\psi_{ra}}{\psi_{rb}}\right) \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

With this change of coordinates defined, we now proceed with the calculations to render the induction motor input-output linear and decoupled. If one differentiates $\dot{y}_1$ and $\dot{y}_2$ once each we obtain

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} L^2{}_f h_1(x) \\ L^2{}_f h_2(x) \end{bmatrix} + A(x) \begin{bmatrix} v_{sa} \\ v_{sb} \end{bmatrix} \tag{27}$$

where

$$
\begin{aligned}
A(x) &= \begin{bmatrix} L_{g_1}L_f h_1 & L_{g_2}L_f h_1 \\ L_{g_1}L_f h_2 & L_{g_2}L_f h_2 \end{bmatrix} \\
&= \frac{1}{\sigma L_r}\begin{bmatrix} -\frac{n_p M \psi_{rb}}{J} & \frac{n_p M \psi_{ra}}{J} \\ 2MR_r\psi_{ra} & 2MR_r\psi_{rb} \end{bmatrix}
\end{aligned} \tag{28}
$$

which is nonsingular provided $(\psi_{ra}^2 + \psi_{rb}^2)$ is nonzero.

Choosing the state feedback of

$$
\begin{bmatrix} v_{sa} \\ v_{sb} \end{bmatrix} = A^{-1}(x)\left\{\begin{bmatrix} -L^2{}_f h_1(x) \\ -L^2{}_f h_2(x) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right\} \tag{29}
$$

we get

$$
\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{30}
$$

which is input-output linear and decoupled. We may then apply a linear feedback of the form of (10, 22) to asymptotically track desired trajectories.

At this stage we point out a few important properties that hold for the induction motor model

- The parameters T_L and R_r enter linearly.
- The decoupling matrix $A(x)$ only depends on R_r and not T_L. The only way $A(x)$ can become singular through variations in R_r is if $R_r = 0$, hence the vector relative degree is well defined with respect to parameter variations.
- η remains bounded since it is a bounded function.
- The functions $f_i(x)$ multiplying the parameters are linear and hence Lipschitz.

5.2 Partitioning the Model for Adaptation

We wish to adapt to the load torque and the rotor resistance, T_L and R_r. As can be seen from (23) (24), both of these parameters enter linearly in $f(x)$

only, hence we need only partition f(x) as:

$$
\begin{aligned}
f_0(x) &= \begin{bmatrix} \frac{n_p M}{J L_r}\left(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}\right) \\ -n_p \omega \psi_{rb} \\ n_p \omega \psi_{ra} \\ \frac{M}{\sigma L_r} n_p \omega \psi_{rb} - \frac{R_s}{\sigma} i_{sa} \\ -\frac{M}{\sigma L_r} n_p \omega \psi_{ra} - \frac{R_s}{\sigma} i_{sb} \end{bmatrix} \\
f_1(x) &= \left[-\tfrac{1}{J}\ 0\ 0\ 0\ 0\right]^T \qquad (31) \\
f_2(x) &= \begin{bmatrix} 0 \\ -\frac{1}{L_r}\psi_{ra} + \frac{M}{L_r} i_{sa} \\ -\frac{1}{L_r}\psi_{rb} + \frac{M}{L_r} i_{sb} \\ \frac{M}{\sigma {L_r}^2}\left(\psi_{ra} - M i_{sa}\right) \\ \frac{M}{\sigma {L_r}^2}\left(\psi_{rb} - M i_{sb}\right) \end{bmatrix}
\end{aligned}
$$

with $[{\theta_1}^* \ {\theta_2}^*]^T = [T_L \ R_r]^T$ we have

$$
\begin{aligned}
\dot{x} &= f_0(x) + f_1(x){\theta_1}^* + f_2(x){\theta_2}^* \\
&+ \ g_1(x) v_{sa} + g_2(x) v_{sb}
\end{aligned} \qquad (32)
$$

and the regressor for the system is simply

$$
w^T(x) = [f_1(x) \ \ f_2(x)]\,. \qquad (33)
$$

5.3 Conditioning of the Regressor

Since we need persistency of excitation to guarantee that the parameter error is driven to zero, let us examine the regressor more closely. By straight forward calculation we have for $ww^T(x)$

$$
\begin{bmatrix} \frac{1}{J^2} & 0 \\ 0 & \left(\begin{array}{c} \frac{M^2 + \sigma^2 {L_r}^2}{\sigma^2 {L_r}^4} \times \\ \left((\psi_{ra} - M i_{sa})^2 + (\psi_{rb} - M i_{sb})^2 \right) \end{array} \right) \end{bmatrix}. \qquad (34)
$$

Clearly this matrix is positive semi-definite and has rank of at least one (in fact with the parameters and trajectories used in the simulations this matrix was full rank throughout). For the case of persistency of excitation we need only concern ourselves with the conditioning of $\int ww^T(x)d\tau$ over some window of time.

We will assume that this integral is a matrix of full rank. This assumption is not constraining in any sense since $ww^T(x)$ is singular only if $\psi_{rb} = Mi_{sb}$ *and* $\psi_{ra} = Mi_{sa}$, which will not happen over an extended period of time provided the motor is rotating. Hence requiring these two equations to not hold identically over some period of time is not in the least bit restrictive and is met in all but a singular case.

5.4 Adaptive Input-Output Linearization of the Induction Motor

With the nonadaptive linearization framework set in section 5.1, we now proceed with the adaptive law. We first must identify the unknown parameters T_L and R_r. Using the equations from section 3 we have

$$\begin{aligned} \dot{\hat{x}} &= A(\hat{x} - x) + w^T(x,u)\hat{\theta} \\ &+ f_0(x) + g_1(x)v_{sa} + g_2(x)v_{sb} \\ \dot{\hat{\theta}} &= -w(x,u)P(\hat{x} - x) \end{aligned} \tag{35}$$

where $\hat{\theta} = [\hat{T}_L \ \hat{R}_r]^T$.

Note that since we know $f_0(x), g_1(x), g_2(x)$ exactly, there is no need to include it in the regressor. As can be seen in (21), the same control law as in section 5.1 is used except $\hat{T}_L$ and $\hat{R}_r$ are used in place of the true values. Thus the control law for the indirect method is very easy to implement since we use the same law as the nonadaptive case except parameter estimates are used for the unknowns. The identifier is the only additional piece that need be added.

Furthermore, since we have persistency of excitation and since the properties listed in section 5.1 hold for the induction motor we can guarantee asymptotic tracking of desired trajectories and the parameter error being driven to zero. The proof is based on a MIMO version of the one in [Teel *et al.*, 1991].

Parameter	Value	Units
ω_{nom}	180	rad/sec
T_{max}	12	$N \cdot m$
J	0.089	$kg \cdot m^2$
n_p	2	
R_s	0.435	Ω
R_r	0.816	Ω
L_s	0.002	H
L_r	0.002	H
M	0.069	H

Table 1: Parameters for a 3-Hp Induction Motor

6 Simulation Results

A 3-Hp induction motor was simulated using the indirect adaptive scheme with the motor parameters given in table 1. These values may be found in [Krause, 1986, p. 190].

In the following we are allowing the parameters to be time varying. While it is assumed that the true parameters are constant but unknown for the proof to carry through, we simulate the more realistic scenario of the parameters varying with time.

The rotor resistance, R_r, can vary $\pm 50\%$ from its nominal value. In light of this we allowed the actual resistance to ramp from 50% of its nominal value to 150%. This simulates the rotor coils heating up, causing the resistance to increase. The initial estimate for R_r was set to the nominal value.

The load torque will be a function of the rotor speed. We model this in a similar fashion to [Nath and Berg, 1981] where T_L is related to ω quadratically. More specifically we have $T_L = 0.0012(0.05 + 0.3\omega^2)$. The initial estimate of T_L was 8 $N \cdot m$.

All of the simulations were carried out using $MATRIX_X$ version 2.4 with the variable step Kutta-Merson integration algorithm.

As can be seen in figures 2 and 3, the indirect adaptive control scheme worked extremely well. The trajectories for the indirect controller were virtually indistinguishable from the desired trajectories of ω and $\psi_{ra}^2 + \psi_{rb}^2$.

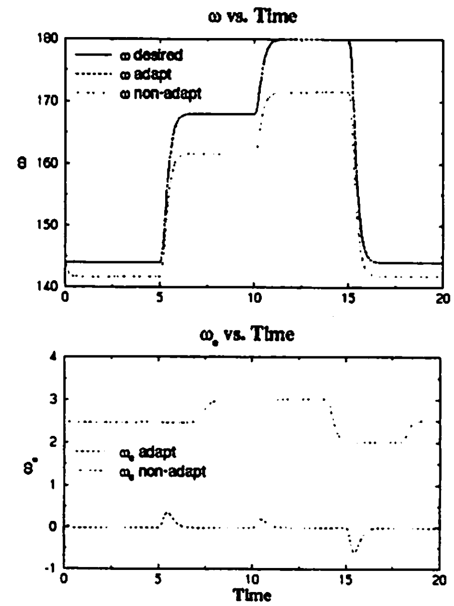

Figure 2: Tracking Results and Error for ω

Furthermore, from figure 4 one sees that the estimates for T_L and R_r reach their true values from initial offsets of 4 and 0.3, respectively, and then track the true parameters throughout.

Observing the error for the nonadaptive controller in the bottom half of figure 2, one sees that the shape of this waveform is similar to the desired trajectory for $\psi_{ra}^2 + \psi_{rb}^2$. This is due to the fact that for the nonadaptive controller we do not achieve output decoupling since there is parameter mismatch. As a result, the outputs interact. This is the main advantage of using the adaptive scheme since one may always argue that a simple PID loop will regulate the offset in the tracking error. This would result in the steady state error being driven to zero (for constant trajectories), but the outputs will never be decoupled if there is any parameter mismatch.

7 Conclusion

A nonlinear indirect adaptive controller was designed for a fifth order induction motor model. The simulation results were quite good and achieved both asymptotic tracking and output decoupling and the parameter estimates for

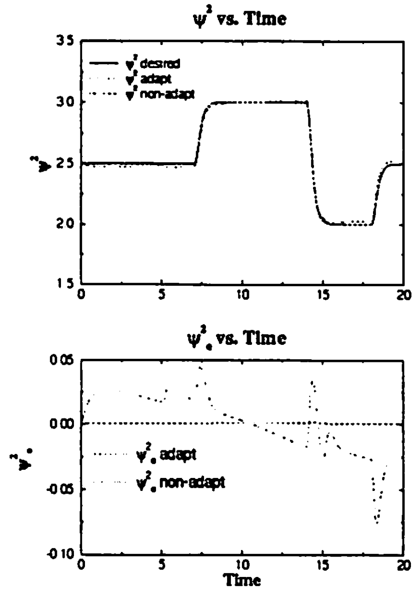

Figure 3: Tracking Results and Error for $\psi_{ra}^2 + \psi_{rb}^2$

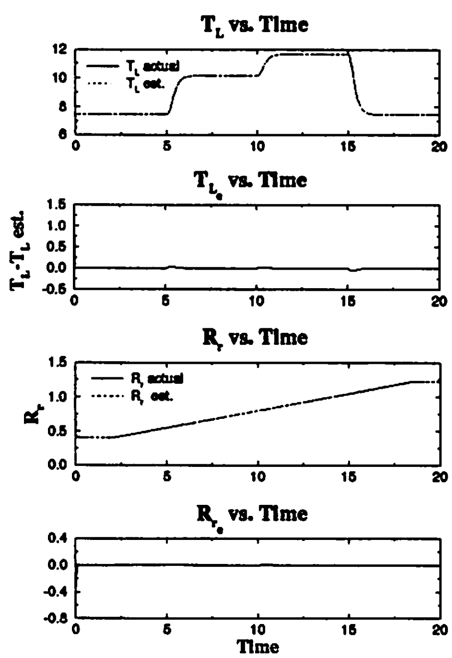

Figure 4: Parameter Estimates and Errors

T_L and R_r converged to their true values. It should be noted that we assumed the rotor flux linkages, ψ_{ra} and ψ_{rb}, were measured. This typically would involve flux coils to be installed in the rotor. One may also use the observers developed in [Verghese and Sanders, 1988] to estimate these quantities.

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