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Abstract

The interaction of magnetic field line stochasticity with drift islands can lead to Arnold diffusion of the drift island centers in a toroidal magnetic field. The basic formulas for this interaction have been derived and the diffusion rate determined. The formalism has been applied to calculating the parallel diffusion of an electron beam, which is subject both to magnetic stochasticity and a time varying electric field. It is shown that the resulting Arnold diffusion can be readily measurable.

I. Introduction

It is well known that a toroidal magnetic field in which the axisymmetry is broken exhibits magnetic islands with stochastic field lines near the island separatrices. This axisymmetric field-line structure has been analyzed for the cases helical applied currents,[1,2] external perturbations of symmetric applied currents[3] and internal helical plasma currents arising from instabilities.[4,5] It has also been shown that non-axisymmetric electric fields generate drift islands in the motion of particles that are traveling along magnetic lines of force.[6,7,8] Such electric fields may be either static or time varying, and may arise either from imposed potentials or from self-consistent potentials, for example, from drift waves. These drift islands scale with the Larmor radius and therefore, for electrons, are generally small compared to the magnetic islands. However, they also exhibit stochasticity in the neighborhood of the separatrices.

If the fields are time-varying, then it has been shown that forces which change the parallel velocity of the charged particles changes the resonance condition, which is also a function of the rotational transform ι.[7,8] Since ι is a function of radius, the effect is to shift the center of a drift island with respect to the magnetic surfaces. Collisional processes play such a role, and therefore diffuse the resonance centers. This effect strongly enhances the radial diffusion which then may account for anomalous heat loss observed to occur in the electron channel. Detailed calculations have been made to describe this anomalous diffusion.[8]

In this paper we consider a more subtle process in which the intrinsic stochasticity near the magnetic island separatrices interacts with the drift resonance to change the parallel velocity of the charged particles. This, in turn, leads to the radial shift of the drift resonance, as described above. The process, including the time dependence, takes place in three degrees of freedom, and the diffusion in parallel velocity is along a resonance. This process, known as Arnold diffusion, is exponentially slow in a ratio of characteristic frequencies.[9,10] However, since the frequencies may have similar magnitudes, the diffusion rate may also be significant.

In our analysis of the diffusion of the resonance center we shall use a formalism developed for mappings with a diffusing parameter.[11] This formalism has also been applied to the diffusion of drift island resonances resulting from collisional processes that change the parallel velocity.[12] Here the parallel velocity is only indirectly affected by its coupling to the radial motion.

In Section II we present the mechanism in a general form that is applicable to various types of applied fields. For example, a local time-varying electric potential can be applied to the torus, such that the resonances are associated with the Fourier decomposition of the fields. Alternatively, the electric fields may arise from natural drift wave activity in a toroidally confined plasma. Similarly, the magnetic islands can arise either from the natural structure of the confining magnetic field or from resistive instabilities. In Section III we consider the specific problem of measuring the Arnold diffusion of an electron beam probing the structure of a magnetic island when it is perturbed by an externally excited time-varying electric field. The vacuum magnetic field is represented by a local shear and perturbation strength, resulting in a magnetic island. The perturbing electric field is represented by its appropriate Fourier component. The resulting diffusion of the parallel beam velocity is a novel form of Arnold diffusion that can be measured in a straightforward manner and can be controlled by the external electric field.

II. The General Formalism

The magnetic islands are taken to arise from a field line Hamiltonian of the form[5,12]

$$H = H_0(J) + H_1(J, \theta, \phi) \tag{1}$$

where H_0 is the Hamiltonian in the absence of toroidal variation

$$H_0(J) = \int_0^J \iota(J) dJ \tag{2}$$

with ι the rotational transform,

$$\iota = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{d\phi}, \tag{3}$$

which gives the number of θ revolutions for each ϕ revolution, and J the action or flux variable

$$J = \int_0^{2\pi} \frac{r^2(\theta)}{2}\, d\theta. \tag{4}$$

The angles θ and ϕ are the poloidal and toroidal angles, respectively, and r is the radial distance from the axis to the field line. The perturbation H_1 can most generally be expressed in the form

$$H_1 = \sum_k \epsilon^k \frac{J^{k/2}}{r_0^k} e^{ik\theta} \sum_{m,n \neq 0} A_{mn} e^{i(m\theta - n\phi)} \tag{5}$$

where $\epsilon = r_0/R$ is the inverse aspect ratio. The first summation is the expansion of the toroidicity, and the second double summation is the expansion of the ϕ-varying perturbation with harmonic amplitudes A_{mn} (with $A_{-m,-n} = A_{mn}$ as a reality condition on the fields).

To be explicit, we assume that the main island amplitude is determined from a specific m, n term on the $\iota = n/m$ surface, which is resonant at zero order in ϵ. Transforming to new variables

$$\hat{\theta} = m\theta - n\phi$$

$$\hat{J} = J/m$$

and expanding $\hat{J} = \hat{J}_0 + \Delta\hat{J}$ where $\iota(J_0) = n/m$, the perturbed Hamiltonian is then

$$\Delta\hat{H} = m^2 \left.\frac{d\iota}{dJ}\right|_{J+J_0} \frac{(\Delta\hat{J})^2}{2} + A_{mn}\cos\hat{\theta} \quad + \quad \text{H.O.T.} \tag{6}$$

where the A_{mn} and $A_{-m,-n}$ terms from (5) have been summed to give the $\cos\hat{\theta}$ term which is in the pendulum form. The central frequency and maximum amplitude of the island are obtained from (6) as

$$\hat{\Omega}_M = \left(m^2 \frac{d\iota}{dJ}\bar{A}_{mn}\right)^{1/2} \tag{7a}$$

$$\Delta\hat{J}_M = 2\left(\bar{A}_{mn}/\,m^2\frac{d\iota}{dJ}\right)^{1/2} \tag{7b}$$

The higher order terms (H.O.T.) generate the stochastic layer around the island separatrix and also lead to any distortion of the island shape from that of a pendulum. We will not calculate either of these effects explicitly.

In this work we ignore the deviation of the drift surfaces from the magnetic surfaces due to magnetic forces. These deviations are of the order of Rr_L/r_0^2,[6] with the Larmor radius r_L assumed small compared to other dimensions. The resonant part of the particle drift due to electric fields is then governed by the equations[9,12]

$$\frac{d\Theta}{dt} = k_{\|}(r)v_{\|} - \omega \tag{8}$$

$$\frac{dv_{\|}}{dt} = \frac{e}{m} k_{\|}(r) \Phi_0 \sin\Theta \tag{9}$$

$$\frac{dr}{dt} = \frac{k_\perp}{B} \Phi_0 \sin\Theta + \left.\frac{dr}{dt}\right|_{\text{Mag}} \tag{10}$$

We have kept only a single Fourier component of the electrical potential, $\Phi = \Phi_0 \cos\Theta$. The equations are coupled to the magnetic field line variation through $k_{\|}(r)$ and through $\left.\frac{dr}{dt}\right|_{\text{Mag}}$. The former gives the zero and first order Θ-variation, and the latter gives the first order variation in r due to the magnetic island. We then expand about the resonance

$$v_{\|} k_{\|}(r_0) - \omega = 0. \tag{11}$$

Using the notation

$$x \equiv r - r_0 \tag{12}$$

$$\Delta v_{\|} \equiv v_{\|} - v_{\|0} \tag{13}$$

and expanding the magnetic island flux to first order, from (6),

$$\Delta J = r_0 x_{\text{Mag}}. \tag{14}$$

Equations (8)-(10) become, to first order,

$$\frac{d\Theta}{dt} = v_{\|0} \frac{dk_{\|}}{dr} x + k_{\|0} \Delta v_{\|} \tag{15}$$

$$\frac{d\Delta v_{\|}}{dt} = \frac{e}{m} k_{\|0} \Phi_0 \sin\Theta \tag{16}$$

$$\frac{dx}{dt} = \frac{k_{\perp 0}}{B} \Phi_0 \sin\Theta + \dot{x}_{\text{Mag}}. \tag{17}$$

Even in the absence of magnetic field-line stochasticity $\dot{x}_{\text{Mag}} \equiv \left.\frac{dx}{dt}\right|_{\text{Mag}}$ is an explicit function of time, through (6). We can then construct a time dependent Hamiltonian for the drift motion, from (15)-(17), which is a two degree of freedom (2D) dynamical system, having stochastic layers around the separatrices. However, this stochasticity tends to be bounded by KAM surfaces and therefore is not of great physical interest. Alternatively, if we take the initial conditions such that the particle lies in the stochastic separatrix layer of the magnetic island, the $\dot{x}_{\text{Mag}}$ term then has a stochastic component which drives the Arnold diffusion. It is this situation that we consider here.

In order to analyze the coupled equations, it is convenient to separate the drift variables from the magnetic island variables. To do this, we define a new variable

$$y = x - \frac{M}{e}\frac{k_{\perp 0}}{k_{\parallel 0}}\frac{1}{B}\Delta v_{\parallel}. \tag{18}$$

Then, multiplying (16) by $\frac{M}{eB}\frac{k_{\perp}}{k_{\parallel}}$ and subtracting (16) from (17), we obtain

$$\dot{y} = \dot{x}_{\text{Mag}} \tag{19}$$

which gives the uncoupled magnetic variation. Substituting the new variable in (15)

$$\dot{\Theta} = \left(v_{\parallel 0}\frac{dk_{\parallel}}{dr}\frac{M}{eB}\frac{k_{\perp 0}}{k_{\parallel 0}}\frac{1}{B} + k_{\parallel 0}\right)\Delta v_{\parallel} + v_{\parallel 0}\frac{dk_{\parallel}}{dr}y \tag{20}$$

and (19) remains as before. We now put the equations into the standard form[12] by making the simple changes in variables

$$I = \left(v_{\parallel 0}\frac{dk_{\parallel}}{dr}\frac{M}{eB}\frac{k_{\perp 0}}{k_{\parallel 0}} + k_{\parallel 0}\right)\Delta v_{\parallel} \tag{21}$$

$$P = v_{\parallel 0}\frac{dk_{\parallel}}{dr}y \tag{22}$$

such that the equations (16), (20), and (19) become, respectively,

$$\dot{I} = K\sin\Theta \tag{23}$$

$$\dot{\Theta} = I + P(t) \tag{24}$$

$$\dot{P} = v_{\parallel 0}0\frac{dk_{\parallel}}{dr}\dot{x}_{\text{Mag}} \tag{25}$$

where

$$K = \frac{e}{M} k_{\parallel 0} \Phi_0 \left(v_{\parallel 0} \frac{dk_\parallel}{dr} \frac{M}{eB} \frac{k_{\perp 0}}{k_{\parallel 0}} + k_{\parallel 0} \right). \tag{26}$$

Equations (23) and (24) are the pendulum equations with an explicit time dependent drive. They have been analyzed previously with $\dot{P}$ a random variable.[12] We now generalize for $\dot{P}$ the motion in a separatrix layer.

From (23) and (24) we construct the time-dependent Hamiltonian

$$H = \frac{I^2}{2} + P(t)I + K \cos \Theta \tag{27}$$

The Arnold diffusion is obtained from the explicit time dependence

$$\frac{\partial H}{\partial t} = I\dot{P} = Iv_{\parallel 0} \frac{dk_\parallel}{dr} \dot{x}_{\text{Mag}} \tag{28}$$

From (6) we obtain the explicit time dependence

$$r_0 \dot{x}_{\text{Mag}} = A_{mn} \sin \hat{\theta}(t) \tag{29}$$

where $\hat{\theta}(t)$ is approximated by the separatrix motion of (6)[10]

$$\hat{\theta}(t) = \hat{\theta}_{sx}(t) = 4 \tan^{-1} [\exp(\hat{\Omega}_M t) - \pi] \tag{30}$$

where, from (7a),

$$\hat{\Omega}_M = \left(m^2 \frac{dt}{dJ} A_{mn} \right)^{1/2}. \tag{31}$$

Substituting (29) in (28), and writing out I as an explicit function of time in the small oscillation portion of the drift island space, we have

$$\frac{\partial H}{\partial t} \simeq I_{\max} \sin(\Omega_E t + \chi) v_{\parallel 0} \frac{dk_\parallel}{dr} \frac{A_{mn}}{r_0} \sin \hat{\theta}(t) \tag{32}$$

To put (32) in the usual form for calculating the Melnikov-Arnold (MA) integral we expand in sum and difference variables, and keeping the term which contributes to the integral we obtain

$$\frac{\partial H}{\partial t} = I_{\max} v_{\parallel 0} \frac{\partial k_{\parallel}}{\partial r} \frac{A_{mn}}{r_0} \frac{1}{2} \cos(Q_0 s + \chi + \hat{\theta}(s)) \tag{33}$$

where we have changed the independent variable to $s = \Omega_M t$ and $Q_0 = \Omega_E/\Omega_M$ the ratio of the linearized drift to magnetic island frequencies. Explicit integration with respect to s gives the MA integral[9,10]

$$\begin{aligned} \Delta H &= I_{\max} \frac{v_{\parallel 0}}{\Omega_M} \frac{\partial k_{\parallel}}{\partial r} \frac{A_{mn}}{r_0} \sin\chi \int_{-\infty}^{\infty} \cos(Q_0 s + \hat{\theta}(s)) ds \\ &= I_{\max} \frac{v_{\parallel 0}}{\Omega_B} \frac{\partial k_{\parallel}}{\partial r} \frac{A_{mn}}{r_0} \sin\chi \mathcal{A}(Q_0). \end{aligned} \tag{34}$$

Here $\mathcal{A}(Q_0)$ is the MA integral

$$\mathcal{A}(Q_0) = 4\pi Q_0 \frac{\sinh(\pi Q_0/2)}{\sinh(\pi Q_0)} \tag{35}$$

and χ is a phase that is randomized on each half-period of the separatrix motion

$$T_{sx} = \frac{1}{\Omega_M} \ln \frac{32}{w_1}, \tag{36}$$

where w_1 is the thickness of the separatrix layer, normalized to the separatrix Hamiltonian,

$$w_1 = \frac{\Delta \hat{H}_{sx} - \Delta \hat{H}_1}{\Delta \hat{H}_{sx}}. \tag{37}$$

To calculate ΔH, the higher order terms in the magnetic Hamiltonian need to be used, but they only appear logarithmically in (36). We shall make explicit calculations in the next section. The above treatment, of the calculation of ΔH is described in more detail in Refs. 9 and 10. The final step in calculating the diffusion coefficient is to square ΔH and average over the random phase χ.

$$D_H = \frac{\langle(\Delta H)^2\rangle_\chi}{T_{sx}} \tag{38}$$

Depending on the problem, a final transformation must be made to the variable of interest. In the next section, we apply the formalism to a practical problem.

III. Arnold Diffusion of an Electron Beam

We consider the problem of measuring the Arnold diffusion of an electron beam, used to measure magnetic surfaces, when the beam is perturbed by a time-varying electric field. In the absence of the electric field the beam explores a magnetic structure caused by external currents. For example, the helical-toroidal currents of a stellarator or torsatron.[1,13] Because of the lack of axisymmetry, magnetic islands are generated which degrade the edge confinements. The external currents may be trimmed to reduce the size of the edge islands. The variation of islands with external currents are studied numerically.[13] The actual experiment can differ significantly from the numerical codes, necessitating the electron beam studies of the magnetic islands.

The study of Arnold diffusion is considered to be an ancillary experiment to the electron beam exploration of magnetic islands. The time-varying electric field is externally applied in order to produce the Arnold diffusion, which can then be measured with the electron beam source and detector. It is assumed that the variation of external currents that are used to minimize islands can also be used to obtain islands of the size which maximizes the diffusion rate.

From (7), the amplitude and frequency of the magnetic island are related by

$$\Delta \hat{J}_M = \frac{2\hat{\Omega}_M}{m^2 \frac{d\iota}{dJ}} \tag{39}$$

and using the defining relations $\hat{J} = J/m$ we have,

$$\Delta J_M = \frac{2\hat{\Omega}_M}{m \frac{d\iota}{dJ}} \tag{40}$$

where $d\iota/dJ$ is evaluated at the resonance. To compare this to the distance between island chains, we consider the case of $n = 1$. The distance between island chains, in action δJ, is then given by

$$\frac{1}{m+1} - \frac{1}{m} = -\frac{d\iota}{dJ}\delta J$$

such that

$$\delta J = -\frac{1}{m^2 \frac{d\iota}{dJ}}. \tag{41}$$

Taking the ratio of (40) to (41) the local gradient of rotational transform cancels and we obtain

$$\frac{\Delta J}{\delta J} = 2m\hat{\Omega}_M. \tag{42}$$

Assuming neighboring island chains of approximately equal size, for no overlap of the stochastic layers that would join the chains, we use the standard rule[15]

$$\frac{2\Delta J}{\delta J} \leq \frac{2}{3}$$

to obtain from (42),

$$m\hat{\Omega}_M \leq 1/6. \tag{43}$$

We use $m\hat{\Omega} = 1/6$ as the magnetic island frequency for our Arnold diffusion calculation.

The frequency of the electric oscillation, ω_E is given

$$\omega_E = \left[\frac{e}{M} k_{\|0}\Phi_0 \left(v_{\|0}\frac{dk_\|}{dr}\frac{M}{eB}\frac{k_{\perp 0}}{k_{\|0}} + k_{\|0}\right)\right]_{1/2} \simeq k_{\|0}\left(\frac{e}{M}\Phi_0\right)^{1/2} \tag{44}$$

Scaling this to the rotation frequency $v_{\|0}/2\pi R$, we have

$$\Omega_E \simeq \frac{2\pi R}{v_\|} k_{\|0}\left(\frac{e}{M}\Phi_0\right)^{1/2} \simeq \left(\frac{\Phi_0}{\Phi_b}\right)^{1/2} \tag{45}$$

where we have used the approximation $v_{\|0} \approx \left(\frac{e}{M}\Phi_b\right)^{1/2}$. An approximate maximization of the rate of Arnold diffusion is to maximize the MA integral (35). Within a small factor this is done by setting $Q_0 \equiv \Omega_E/\Omega_M = 1$. Using (43), with $\hat{\Omega}_M = m\Omega_M$, we find

$$\Omega_E = \frac{1}{6m^2} \tag{46}$$

For example, if we look for an interaction between the $m = 2$ and $m + 1 = 3$ island chains, $\Omega_E = 1/24$ and with $\Phi_b = 1$ kV, from (45) we obtain $\Phi_0 \simeq 2$ volts. We estimate the drift island width using the first terms on the right in (15) and (17) to construct a Hamiltonian

$$H = \frac{x^2}{2} + \frac{k_\perp\Phi_0}{v_{\|0}\frac{dk_\|}{dr}B}\cos\Theta = \text{Const} \tag{47}$$

such that, the maximum drift island excursion is

$$x_E = 2\left(\frac{k_\perp\Phi_0}{v_{\|0}\frac{dk_\|}{dr}B}\right)^{1/2} \tag{48}$$

Scaling $k_\perp \sim 1/r_0$ and $dk_\|/dr \sim 1/Rr_0$, (48) becomes

$$x_E \simeq 2\left(Rr_L^*\frac{\Phi_0}{\Phi_b}\right)^{1/2} \tag{49}$$

where $r_L^* = v_{\|0}/\omega_L$. For drift excursions to overlap neighboring island chains we set

$$x_E = \delta x_M \tag{50}$$

where δx_M is estimated from (41) to be

$$\delta x_m \simeq \frac{r_0}{m^2} \tag{51}$$

Continuing the previous example, with $R = 1$m and $r_0 = 10$ cm, we substitute (49) and (51) in (50), and using (45) and (46) we find

$$r_L^* \simeq \frac{r_0^2}{144R} = .69\text{cm}$$

The corresponding $B = 110$ Gauss.

A final calculation is to determine the rate of diffusion of $v_\parallel$. From (38), returning to physical variables, and scaling as previously, we find

$$D_{v_\parallel} \approx \frac{1}{2}\frac{(\delta x)^2}{r_0^2}\frac{1}{\Omega_M^2}\frac{(\Delta v_{\parallel M})^2}{T_{sx}}\mathcal{A}(Q_0) \tag{52}$$

We relate δx, the change in field line position, to Δx_M, the magnetic island size, through the Hamiltonian, to obtain

$$\delta x = \frac{\pi}{2}\frac{(\Delta x_m)^2}{r_0}$$

with

$$\Delta x = \frac{2}{3}\frac{r_0}{m^2}$$

for our characteristic island size, as previously. We rewrite T_{sx} as

$$T_{sx} \equiv \frac{T_{sx}}{T_M}\frac{1}{\Omega_M}$$

where using (43) $\Omega_M = 1/6m$. We evaluate $\mathcal{A}_{MA}(1) = 2.6$. Substituting these quantities into (52), we find to order unity

$$D_{v_\parallel} = \frac{(\Delta v_{\parallel M})^2}{\tau}; \qquad \tau \simeq \frac{m^6}{4}\frac{T_{sx}}{T_M} \tag{53}$$

To find the diffusion time τ_D required to change $v_{\parallel 0}$ by its characteristic value

$$\tau_D = \frac{v_{\parallel 0}^2}{(\Delta v_{\parallel M})^2}\tau \simeq \frac{\Phi_b}{\Phi_0}\tau \tag{54}$$

with $m = 2$ and $T_{sx}/T_M \simeq 4$, we find $\tau = 64$ in units of electron traversal of the torus. With $\Phi_b/\Phi_0 = 500$ we find $\tau_D = 32,000\tau_{tr.}$, where $\tau_{tr.} = 2\pi R/v_{\parallel 0} \simeq 0.5\mu$sec. This time is still very short compared to any collisional spreading of the beam on background gas.

IV. Conclusion and Discussion

We have identified a new mechanism for diffusion in a toroidal magnetic field. Stochasticity around magnetic islands can interact with the particle motion on E×B drift islands to diffuse the parallel velocity. This in turn moves the resonant center of the drift island. The process is a form of Arnold diffusion, in which the stochasticity of one degree of freedom drives another degree of freedom through a coupling term. If parameters exist or are chosen such that the oscillation frequencies of the two degrees of freedom are comparable, then the resulting Melnikov-Arnold integral, which governs the diffusion rate, is of order unity, and the rate of diffusion can be large. Radial transport, however, is more difficult to obtain, depending on the existence of chains of magnetic islands which can be successively joined by the drift island oscillations, together with the diffusion of the drift island center resulting from the diffusion of the parallel velocity.

We found that the diffusion could be measured with an electron beam of the type that might be used to explore magnetic surfaces. If the magnetic field can be adjusted such that some region of the flux surface has sets of magnetic islands that are close together, but not overlapping, then the introduction of a time-varying electric field perturbation of reasonable magnitude produces diffusion of the parallel beam velocity. Furthermore, the parameters could be chosen such that drift islands are comparable to the magnetic islands. This allows the diffused portion of the beam to be detected in the neighborhood of a magnetic island different from the one on which the beam is injected. A diffusion time is found of the order of milliseconds which, as calculated for a specific set of parameters, appears to be quite reasonable for doing a time resolved experiment.

Although a number of approximations were made in estimating the specific amplitudes and time constants, there is sufficient flexibility that changes of a few factors of two would not change the overall conclusions. The matching of the magnetic and drift frequencies can be obtained by varying the strength of the perturbing electric field. The matching of the magnetic and drift island amplitudes is adjustable with the beam velocity and the toroidal magnetic field strength. The adjustment of the magnetic island strength depends on the particular magnetic field configuration under investigation, but usually some relatively low m-number island chains can be found that satisfy the neighboring island criterion.

The accuracy of the calculations can be improved, but, as they are machine dependent, they should be performed on a specific device on which the experiment is to be performed. Because of the long-lived character of a beam probe, the value of the Melnikov-Arnold integral can be considerably smaller (at least a factor of 10) and still have reasonable diffusion times. This allows for some flexibility in choice of parameters and some leeway in the accuracy of the calculated diffusion time.

Although we have concentrated our attention on the ability to measure the Arnold diffusion that is externally induced in a torus, it is also possible for the effect to naturally occur, which may lead to unwanted diffusion. For example, a combination of magnetic perturbations and drift waves can lead to diffusion of the parallel electron velocity in a plasma. Such diffusion can be more rapid than collisional processes, causing electrons to drift in and out of banana orbits, or, more dangerously, superbananas. Such a process might be another source of anomalous electron diffusion. As discussed in the introduction, the collisional diffusion of drift island centers has already been studied.[7,8,12] The *diffusion* of drift island centers due to magnetic stochasticity, which appears on a time scale faster than collisions, may, in some configurations, be a larger effect.

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