PRIORITY ORDERING IN INTERRUPTIBLE ELECTRIC POWER SERVICE WITH IRREVERSIBLE EARLY NOTIFICATION

by

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Priority Ordering in Interruptible Electric Power Service with Irreversible Early Notification

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Abstract

We develop a model of interruptible electric power service that includes early notification. Notification time is included through a multi-period model. The allocation of notifications is described as a stochastic control problem: *whom to notify when.*

The general problem has great information requirements and an optimal allocation may not yield an incentive compatible price menu. Additional structure is imposed on customer outage costs and the evolution of uncertainty in the shortfall magnitude. Optimal priority orderings are sought. Optimality conditions and qualitative properties are derived. A numerical example illustrates the results.

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1. Background

Over 70 percent of investor-owned electric utilities in the United States offer some form of voluntary interruptible or curtailable electric power service (Ebasco, 1987). Interruptible electric service refers to any customer load that is subject to partial or complete elimination for a period of time upon adequate notice from the electric utility. Typically, "adequate notice" ranges from 10 minutes to one full day. This is the *notification time* a customer receives prior to actual interruption. The conditions of notification time are included in interruptible service tariffs, which may also specify the maximum number of interruptions allowed per day, per month, or per year; the maximum duration of any particular interruption; and the maximum number of interrupted hours per year. Christensen Associates (1988) describes interruptible service in more detail.

Analysis of interruptible service tariffs has focused on varying the demand charge for customers with different interruption costs. Marchand (1974), Tschirhart and Jen (1979), Woo and Toyama (1986), Chao and Wilson (1987), Viswanathan and Tse (1989), and Oren and Smith (1992) consider one-dimensional models that differentiate on reliability, that is, probability or frequency of interruption. The models of Panzar and Sibley (1978), Hamlen and Jen (1983), and Woo (1990) include the amount of interruption as well as the frequency. Oren and Doucet (1990) model customers differing with both interruption cost and location on the distribution network. Using a load duration curve model, Chao, Oren, Smith, and Wilson (1986) consider both frequency and duration, but not amount. Smith (1989) and Oren (1990) consider two-dimensional models that incorporate both frequency and duration of individual interruptions.

None of these models includes notification time, an important element of actual interruptible service programs and tariffs such as Niagara Mohawk's Voluntary Interruptible Pricing Program, New England Electric Service's Cooperative Interruptible Service Program, and Southern California Edison's I-3 tariff schedule. Notification time is included in the analysis presented here.

Optimal allocation of notifications is discussed, and a priority index rule is presented for several special cases. A numerical example illustrates the result.

2. Description and Notation

Different customers and end uses incur different losses when an interruption of electric power occurs. However, no customer prefers a longer interruption to a shorter one, and no customer prefers a sudden, unexpected interruption to an interruption with some advance notification. Customers ordinarily assume that they will receive electric power, so a customer's interruption loss is an "additive adjustment to the surplus...derived from its normal electric power consumption" (Smith, 1989). Similarly, the prices discussed in this paper are additive adjustments to customer bills "for avoided [or contracted] interruptions, as opposed to consumption."

With a control and metering technology that is able to separate end uses, each kilowatt (kW) of demand may be addressed separately. Consequently, each customer is considered to have one kW of demand; alternatively, each kW of demand is regarded as an independent decision-making unit. Furthermore, demand is non-stochastic.[1] Supply is stochastic, so the system *shortfall* is the difference between N, the number of customers, and the realized value of supply.

Shortfall duration is suppressed; this may be interpreted as taking expectations over duration. Each customer may then be characterized by i) its loss if interrupted suddenly and unexpectedly, and ii) its benefit from advance notification of an impending interruption.

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[1]Both unit demand and its non-stochastic nature are common assumptions of priority pricing research. For example, see Wilson (1989). Non-stochastic demand may be a reasonable assumption for large industrial and commercial customers with high load factors (average load divided by peak load). Residential customers tend to have low load factors and residential loads tend to be much more sensitive to ambient weather conditions. If only industrial and commercial customers are considered interruptible, the stochastic demands of residential customers may be lumped with stochastic supply into the random variable that is shortfall magnitude.

As modeled here, a customer may be notified at any time t, up to and including T, the instant the shortfall commences. A customer "notified" at time T is interrupted with no advance notification.

A customer's outage cost depends upon the time at which the customer makes an irrevocable decision not to use electric power at time T. Customers utilize notification of impending interruptions to take advance actions in order to mitigate the effect and cost of an interruption. For example, customers may cancel shifts, reschedule production processes, or fire up backup generators. These irrevocable actions result in customer costs that are effectively sunk and irremediable, akin to unit commitment costs in traditional supply-side electric operations. Once a customer takes such an irrevocable action and incurs the commitment cost, the customer's marginal value of receiving power (at the posted energy cost) is assumed to be zero.

It is assumed that customers make immediate use of early notification. Once notified at time t of an impending interruption at time T, a customer takes some irrevocable action at time t to mitigate the interruption loss. Thus, a customer receiving early notification at time t incurs its interruption loss less an early notification benefit resulting from its irremedial actions at time t. This net loss is herein referred to as a customer's *notification cost* at time t. Because a notified customer has no marginal benefit of receiving power, it is socially efficient to interrupt such a customer. In other words, notifications lead irreversibly to interruptions.

The interruption loss suffered by a customer interrupted without advance notification is referred to as that customer's *base cost*. Customers are indexed by their base cost: with N customers, customer 1 has the smallest base cost, c_1, while customer N has the largest base cost, c_N. The fraction of customers with base cost less than or equal to c is denoted as $D(c)$. The interruption loss suffered by customer i when notified at time t—customer i's notification cost at time t—is denoted as $w_i(t)$. Hence, $w_i(T)$ equals c_i. Since advance notification of an impending interruption reduces customer losses, $w_i(t)$ is nondecreasing in t.

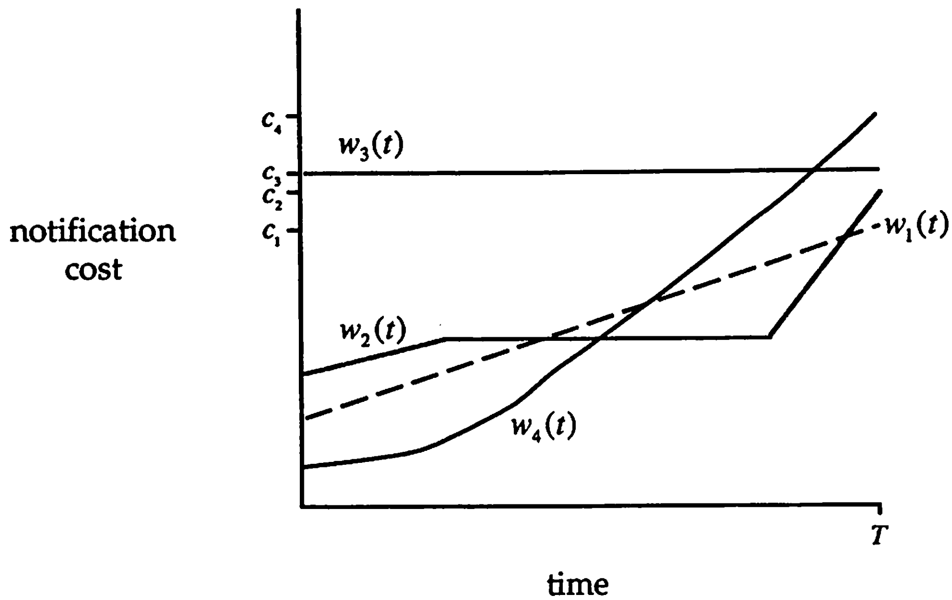

Figure 2.1: Notification Cost vs. Time

At each moment, for each unnotified customer, the electric utility must choose between issuing a notification and simply waiting. From the systems perspective, the state indicates *which customers were notified when*. The system state at time t includes the N-vector $\mathbf{x}(t)$, one element for each customer, each element with the value infinity (∞) or a value from $[0,t]$. Infinity indicates that the customer has not been notified by time t. For example, suppose customer 3 was notified at time 5; then $x_3(t) = \infty$ for $t < 5$ and $x_3(t) = 5$ for $t \geq 5$.

Since customers are indexed by their base costs, element $x_1(t)$ indicates whether the customer with the smallest base cost has been notified by time t, and if so, when; $x_N(t)$ indicates the same for the customer with the largest base cost. $x_i(T)$ indicates whether customer i receives power during the shortfall. The interruption loss suffered by customer i is zero if $x_i(T) = \infty$ and $w_i(x_i(T))$ if $x_i(T) < \infty$.

When notifications are issued, the magnitude of an impending shortfall is uncertain. The actual magnitude of the shortfall is revealed at time T. Supply must be equal to or greater than demand: if the number of customers notified previous to time T is less than the actual magnitude of the shortfall, the number of customers notified at time T equals the difference.

As time increases toward T, information about the magnitude of the shortfall is revealed. The probability distribution F_t represents the information known at time t about the magnitude of the shortfall. F_t and $\mathbf{x}(t)$ together comprise the system state. More detailed description of F_t is needed to determine a notification policy; this is considered in section 4.

Because the number of customers who must be notified is better known when the shortfall is imminent, delaying notifications leads to better allocation of interruptions. However, delaying notifications reduces the notification time of those customers that will be interrupted, thereby increasing their realized interruption costs. A fundamental tradeoff exists between knowledgeable allocation and interruption cost.

An additional obstacle to knowledgeable allocation exists. The notification costs w_i are private information. Each customer knows its own particular notification costs, but the electric utility does not know any particular customer's notification costs, only an aggregate distribution of notification costs in the customer population. Thus, a socially efficient tariff structure must induce customers to reveal their true notification costs through their selections from a menu of service options. The electric utility then uses the revealed preferences to allocate interruptions, with the goal of implementing an optimal notification policy. In this paper, we consider the allocation of notifications for multiple notification periods.[2] The goal is to minimize expected total customer interruption cost.

3. Whom to Notify When

Deciding *whom to notify when* is the allocation problem faced by the electric utility. In its most general form, this is a cumbersome stochastic dynamic programming problem. Nevertheless, with some additional structure, simple decision policies emerge.

The difficulties of the general problem are hinted at by the following stylized example, with two customers and two time periods. Notification costs at times S and T for customers Able and Baker are described in Table 3.1.

Table 3.1: Notification Costs for Two Customers

Customer	Notification Cost at time S	Notification Cost at time T
Able	1	2
Baker	10	100

Notifying Able is always less expensive than notifying Baker. However, each of the four possible actions at time S may be optimal, depending on the probability distribution for the magnitude of the shortfall. Specifically, let p_n be the probability, at time S, that the shortfall magnitude will be n. There are four possible actions at time S: notify neither Able nor Baker, notify Able but not Baker, notify Baker but not Able, and notify both. The conditions for when each action is optimal are presented in Table 3.2 and displayed in Figure 3.1.

[2] In Strauss and Oren (forthcoming), we develop a model in which all notified customers are given early notification simultaneously, at one and only one possible time. Tariff structures are discussed there, as well as the allocation of early notifications.

Table 3.2: Optimality Conditions for Possible Actions

Action	Expected Cost	Optimal When		
notify neither	$2p_1 + (2+100)p_2$	$p_0 \geq 1/2$,	$p_1 \leq 5(1-10p_2)$	
notify Able, not Baker	$1 + 100p_2$	$p_0 \leq 1/2$,	$p_1 \geq 40/98$,	$p_2 \leq 9/98$
notify Baker, not Able	$10 + 2p_2$		$p_1 \geq 5(1-10p_2)$,	$9/98 \leq p_2 \leq 1/2$
notify both	11			$p_2 \geq 1/2$

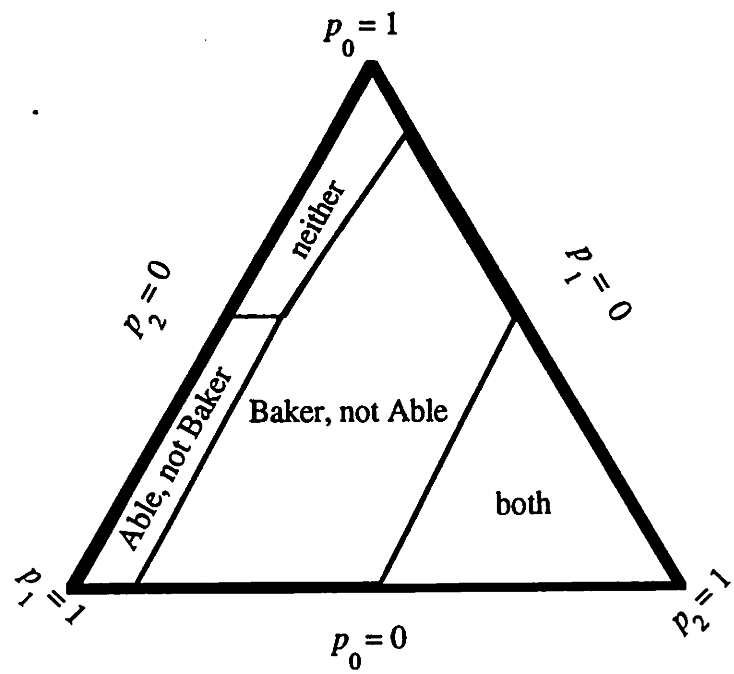

Figure 3.1: Optimality Regions

In this example, no optimal priority ordering exists. A *priority ordering* is a ranked list of customers, wherein customer i+1 would never be notified before customer i. An optimal priority ordering is a priority ordering that may be used by a utility planner to minimize expected total interruption cost. Even though Able's largest interruption cost is much less than Baker's smallest interruption cost, it is sometimes optimal to notify Baker and not Able, depending on the probability distribution for the magnitude of the shortfall.

The lack of an optimal priority ordering complicates characterization of the optimal notification policy. If an optimal priority ordering existed, the allocation problem at time t would be simply to choose a depth $M(t)$, and all customers with rank $i \leq M(t)$ would be notified at time t. Of course, $M(t) \leq M(\tau)$ for $t < \tau$, since notifications are irreversible. If an optimal priority ordering existed, *whom to notify when* would be reduced to *how much to notify when.*

The lack of an optimal priority ordering may also prevent implementation of the optimal policy through a self-selection pricing mechanism, as discussed by Oren (1990) and Smith (1989). In addition, regulatory agencies and electric customers may believe utility operation to be inefficient or to unfairly favor some customers over others; without an optimal priority ordering, it may be more difficult to convince such observers that the system is being operated efficiently and without favor.

While an optimal priority ordering does not exist in general, as demonstrated by the example above, there are situations in which such an ordering exists. Two situations when an optimal priority ordering exists are described below. One situation is when notification costs are multiplicative; the other is when notification costs are submodular. In each case, it is optimal to order customers by base cost such that $c_i \leq c_{i+1}$. Using the notation developed in section 2: if (3.1) holds for a priority ordering, then that priority ordering is an optimal one when notification costs are either multiplicative or submodular.

$$c_i < c_j \;\Rightarrow\; x_i(T) \leq x_j(T) \tag{3.1}$$

Multiplicative Notification Costs

Let notification costs have a multiplicative structure, that is, $w(c,t) = h(t)c$. The function h is the early notification benefit function. For example, when $h(t)$ is $\gamma^{(T-t)}$, $0 < \gamma < 1$, notification costs increase geometrically as notification time decreases. γ, the ratio of notification costs at times t and $t+1$, may be interpreted as the *early notification benefit factor*, akin to an interest rate.

Suppose (3.1) were not true, that is, there is some i and j such that $c_i < c_j$ but $t_i > t_j$, where $x_i(T) = t_i$ and $x_j(T) = t_j$. The expected cost of the decision to notify j but not i at time t_j is compared with the expected costs of two alternatives, as in the Able-Baker example: *a*) switching the times when customers i and j are notified, that is, $x_i(T) = t_j$ and $x_j(T) = t_i$; and *b*) notifying both customers i and j at the earlier time, that is, $x_i(T) = t_j$ and $x_j(T) = t_j$.

If the original decision has smaller expected cost, at time t_j, than alternative a, then (3.2a) is true; if smaller than alternative b, (3.2b) is true.

$$h(t_j)c_j + \pi h(t_i)c_i < h(t_j)c_i + \pi h(t_i)c_j \tag{3.2a}$$
$$h(t_j)c_j + \pi h(t_i)c_i < h(t_j)c_i + h(t_i)c_j \tag{3.2b}$$

π is the probability, at time t_j, that $x_i(T) = t_i$, given the original state $(\mathbf{x}(t_j), F_{t_j})$ at time t_j, where $x_i(t_j) = \infty$ and $x_j(t_j) = t_j$. In other words, π is the probability that, of customers i and j, if only one is notified at time t_j, the other will be notified at time t_i.

However, (3.2a-b) cannot both be true simultaneously. Hence, either the original decision has expected cost at least as much as alternative a, or at least as much as alternative b. Furthermore, both alternatives a and b are consistent with (3.1).

Submodular Notification Costs

Suppose customer notification costs are submodular, that is,

$$w(c_i,t) + w(c_j,\tau) \le w(c_i,\tau) + w(c_j,t), \quad i<j, \; t<\tau \tag{3.3}$$

For example, customer notification costs of the form $\log(c+t)$ are submodular. Submodularity means that customers with lower base costs are more sensitive to notification time: the percentage savings in notification costs realized by notifying these customers is greater than the percentage savings realized by notifying customers with higher base costs. Submodularity in this context of minimizing customer interruption loss serves as a structural assumption analogous to supermodularity in the standard pricing context.[3]

Since w is nondecreasing in t, submodularity implies that a customer with higher base cost always has higher notification cost (3.4). Figure 3.2 illustrates submodular notification costs.

$$c_i < c_j \;\Rightarrow\; w(c_i,t) \le w(c_j,t),\;\; 0 \le t \le T \tag{3.4}$$

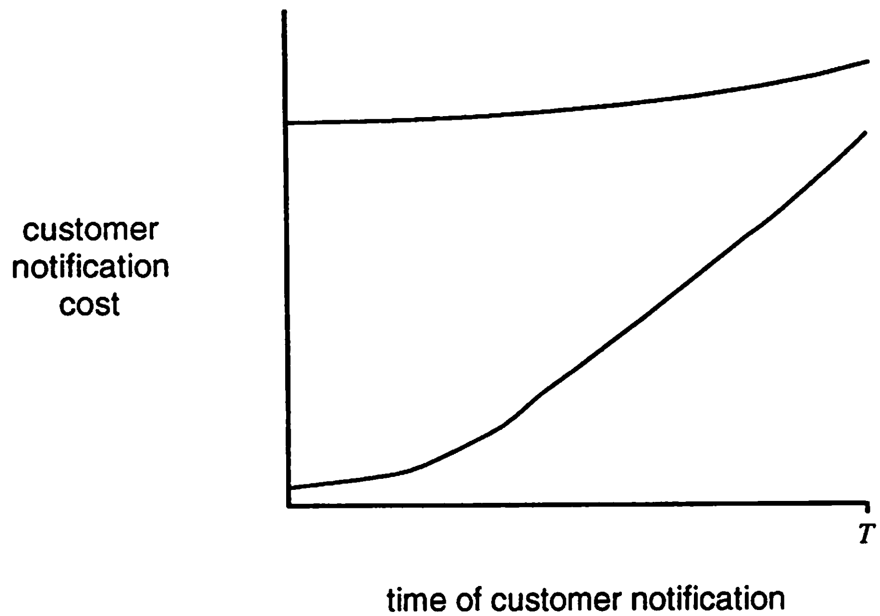

Figure 3.2: Submodular Notification Costs

As in the argument for multiplicative notification costs, suppose there is some i and j such that $c_i < c_j$ but $t_i > t_j$, where $x_i(T) = t_i$ and $x_j(T) = t_j$. If the expected cost of this decision at time t_j is smaller than the expected cost of alternative a, (3.5) is true. As before, π is the probability that, of customers i and j, if only one is notified at time t_j, the other will be notified at time t_i.

[3] Compare with Wilson (1989).

$$w(c_j,t_j)+\pi w(c_i,t_i) < w(c_i,t_j)+\pi w(c_j,t_i) \tag{3.5}$$

Rearranging terms and bounding π by one yields (3.6).

$$w(c_j,t_j)-w(c_i,t_j) < w(c_j,t_i)-w(c_i,t_i) \tag{3.6}$$

(3.6) contradicts (3.3),the submodular property of w. Hence notifiying j and not i never has smaller expected cost than notifying i and not j. Thus, notifying customers in order of increasing base costs is an optimal priority ordering.

4. How Much to Notify When

When an optimal priority ordering exists, the allocation problem is reduced to determining how much to notify at each decision time. Suppose notifying customers in order of increasing base costs is an optimal priority ordering, as above. Then at each decision time t, the allocation problem is to determine some value c such that all customers with base cost no greater than c have been notified on or before time t.

Ordering customers by base cost simplifies the system state. Let $x(t)$ indicate the largest base cost among customers notified prior to time t. The trajectory $x(t)$, $0 \le t \le T$, fully describes which customers are notified when. Since notifications are irreversible, $x(t)$ must be non-decreasing.

In addition to tracking which customers are notified when, the system state must reflect the uncertainty in the shortfall magnitude at time t. This uncertainty is modeled as follows. The shortfall magnitude, L, is a random variable with distribution G. However, there are K candidate distributions—$G_1,...,G_K$—and exactly which distribution L is drawn from is not revealed until time T. $Z(t)$, a random variable taking on values in $\{1,...,K\}$, represents a forecast, at time t, for the distribution of L. $Z(T) = k$ indicates that L is drawn from distribution k. The forecasts $\{Z(t), 0 \le t \le T\}$ form a Markov process with transition matrix $P(t)$.

Specifying the uncertainty in this way permits a broad range of reasonable models. For example, the candidate distributions for L may have the same functional form but differ in the mean.

The complete state at time t is $(x(t),Z(t))$. The allocation problem may now be represented as a dynamic programming problem. The optimal value function $V(t,x,i)$ indicates the expected interruption cost incurred by customers notified at or after time t, given $x(t) = x$ and $Z(t) = i$. The discrete time formulation has optimality equation (4.1) and terminal condition (4.2). Base costs are scaled so that the largest base cost is one.

$$V_t(x,i) = \min_{y \geq x}\left\{N\int_x^y w(c,t)\, d(c)\, dc + \sum_j p_{ij}(t)\, V_{t+1}(y,j)\right\} \tag{4.1}$$

$$V_T(x,i) = \int_x^1 \left(N\int_x^y w(c,t)\, d(c)\, dc\right) dG_i(N\,D(y)) \tag{4.2}$$

To simplify further explication, in the remainder of this paper interruption cost function w is assumed to be multiplicative (as opposed to submodular). When notification costs are multiplicative, the least-cost policy is as follows (the derivation appears in the appendix). For each possible forecast $Z_t = i$ at time t, there is some target value $a_t(i)$ such that (4.3) is satisfied.

$$h(t) = \sum_{j_{t+1}} \cdots \sum_{j_T} p_{i,j_{t+1}}(t) \cdots p_{j_{T-1},j_T}(T-1)\ \overline{G}_{j_T}\left(N\,D\left(\max\left\{a_t(i), a_{t+1}(j_{t+1}), \ldots, a_{T-1}(j_{T-1})\right\}\right)\right) \tag{4.3}$$

Given forecast $Z_t = i$ at time t, the best policy is to notify all customers with base costs less than $a_t(i)$, and leave unnotified all customers with base costs greater than $a_t(i)$. The cost of notifying the customer with base cost $a_t(i)$ at time t is then equal to the expected cost of notifying that customer not at time t but some later time.

However, notifications are irreversible. If all customers with base costs smaller than $x > a_t(i)$ have been notified previous to time t, then the best that can be done at time t is to notify no additional customers. Given state (x,i) at time t, the optimal decision is to notify all customers with base costs less than $x + u_t(x,i)$, where the open-loop feedback control $u_t(x,i)$ is given by (4.4).

$$u_t(x,i) = \left[a_t(i) - x\right]^+ \tag{4.4}$$

The control is convex decreasing in x, as illustrated by Figure 4.1.

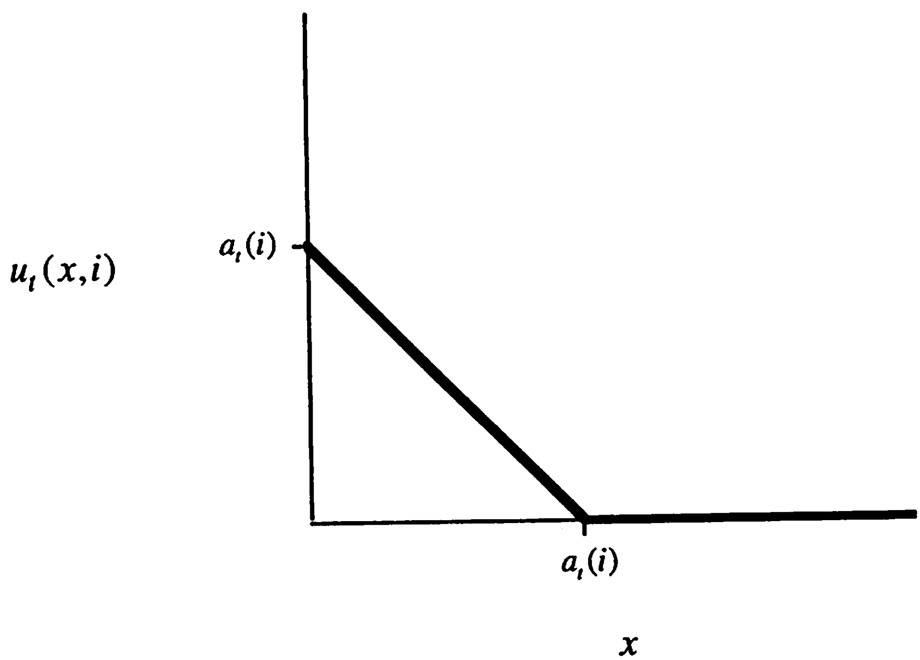

Figure 4.1: Control vs. State

The optimal value function is given by (4.5). Like the control, the optimal value function is convex decreasing in x.

$$V_t(x,i) = \sum_{j_{t+1}} \cdots \sum_{j_T} p_{i,j_{t+1}}(t) \cdots p_{j_{T-1},j_T}(T-1)$$
$$\int_{\max\{x, a_t(i), a_{t+1}(j_{t+1}), \ldots, a_{T-1}(j_{T-1})\}}^{N} \left(N \int_x^y c\, d(c)\, dc \right) dG_{j_T}(N\, D(y)) \tag{4.5}$$

Expression (4.3) implies that the target values are computed recursively. The targets at time t depend on the targets at future times $t+1,\ldots,T-1$. Since the optimal policy depends on both the revealed uncertainty in the shortfall magnitude and the future targets, constructing a closed-loop feedback control appears unlikely.

Of interest is the limiting case when the transition probabilities are given by (4.6).

$$p_{i,j}(t) = 1/K \quad \forall i,j \tag{4.6}$$

There is no information to be gained at times $0 < t < T$, so the allocation problem reduces to a two-period problem, as discussed in Strauss and Oren (forthcoming).

A numerical example is presented to illustrate the computation. Notifications may be issued at each of four times. The early notification benefit function h is given in Table 4.1. There are two types of shortfalls. The transition probabilities are given in Table 4.2. Both shortfall types have uniform distributions. Shortfalls of type 1 are relatively small, uniformly distributed between 0 and 100 MW. Shortfalls of type 2 are relatively large, uniformly distributed between 100 and 200 MW. Demand is also uniformly distributed, given by the density $d(c) = 1, 0 \le c \le 1$. The total quantity of demand, N, is 200 MW.

Table 4.1: Early Notification Benefit Function, Four Time Periods

t	$h(t)$
1	0.38
2	0.40
3	0.50
4	1.00

Table 4.2: Transition Probabilities, Two Shortfall Types

P(1)		P(2)		P(3)	
0.7	0.3	0.8	0.2	0.9	0.1
0.5	0.5	0.75	0.25	0.1	0.9

The optimal notification policy is described by the targets given in Table 4.3. These targets are plotted in Figure 4.2. While the targets for state 2 increase over time, the targets for state 1 decrease over time. The explanation is as follows. Early on, even when the forecast calls for a small shortfall (type 1), the optimal notification policy suggests hedging against the possibility of a large shortfall (type 2). As the time that the shortfall commences nears, the shortfall type becomes more apparent, so when the forecast is for a small shortfall, less hedging is required. Since at least 37 percent of the customers will be notified at time 1, additional customers will be notified at times 2 and 3 only if the forecast at those times is for a type 2 shortfall.

Table 4.3: Targets

State	Time 1	Time 2	Time 3
1	0.37	0.35	0.28
2	0.38	0.50	0.72

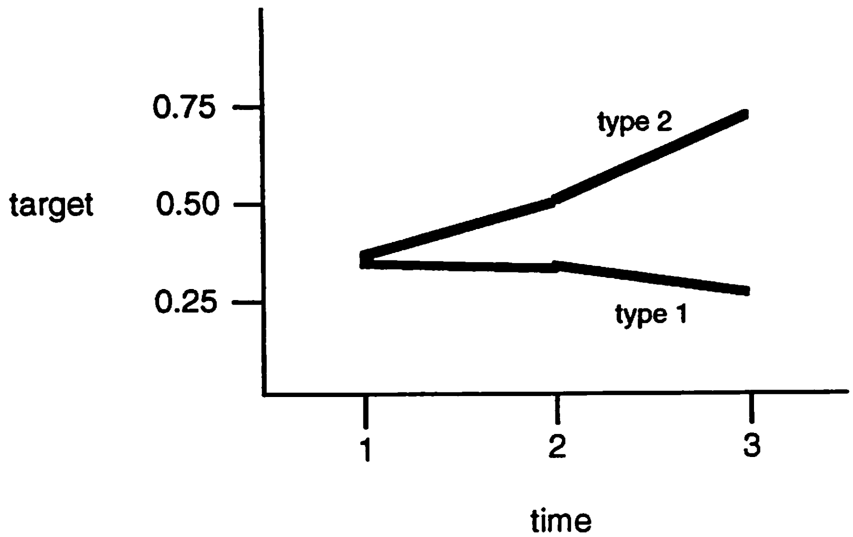

Figure 4.2: Targets

5. Conclusion

Total customer interruption lossess for shortages of electric power generation depend on utility notification policies. When customer notification costs are multiplicative or submodular, the optimal notification policy has been shown to be straightforward, since customers may be ordered by their notification costs. We have demonstrated that, in general, no such optimal priority ordering exists.

The detailed optimal notification policy, characterized as *whom to notify when*, further depends on the structure of the stochastic process that models the uncertainty in the shortfall magnitude. An optimal notification policy has been developed for Markov uncertainty and multiplicative notification costs. An open-loop feedback policy aims to notify customers according to targets that depend on both the early notification benefit function and the structure of the uncertainty in shortfall magnitude. Both the feedback control and optimal value function are convex decreasing in the state, indexed by the marginal customer.

Computing the target values may be too costly. Equation (4.3) implies that the targets are calculated recursively. Strategies such as solving the early notification problem for a two-period rolling horizon may provide good solutions at much less computational cost.

The model developed here stipulates that notifications irreversibly lead to interruptions. We can relax this provision by allowing notified customers to have positive marginal benefit from electric consumption. Relaxing this provision makes the model more realistic, since customers typically have buy-through options, allowing them to override notifications at some monetary penalty. When customers may be "unnotified" in effect, the number of interrupted customers exactly matches the magnitude of the supply shortfall, since any excess power at time T may be given to customers notified at earlier times. Since the total expected customer interruption loss can only decrease, efficiency is increased. The actual optimal notification policy may be more complicated, however. Heuristic notification policies that call for reversing notifications may not yield all of the possible benefits from relaxing the irreversibility of notifications, and in practice may not be significantly better than policies that do not call for reversing notifications.

Appendix:

Derivation of Optimal Notification Policy Under Multiplicative Notification Costs

The optimal notification policy under multiplicative notification costs, described in section 4, is derived below.

In the multiplicative case, (4.1) and (4.2) become (A.1) and (A.2), respectively.

$$V_t(x,i) = \min_{y \geq x}\left\{h(t)\,N\int_x^y c\,d(c)\,dc + \sum_j p_{ij}(t)\;V_{t+1}(y,j)\right\} \tag{A.1}$$

$$V_T(x,i) = \int_x^1 \left(N\int_x^y c\,d(c)\,dc\right) dG_i(N\,D(y)) \tag{A.2}$$

The optimal policy is first derived for time T–1. The optimal value function is rewritten in (A.3), where function φ is defined by (A.4). Derivatives of φ with respect to y are taken, and expressed in (A.5) and (A.6).

$$V_{T-1}(x,i) = \min_{y \geq x}\{\varphi(y;x,i)\} \tag{A.3}$$

$$\begin{aligned}\varphi(y;x,i) &= h(T-1)\,N\int_x^y c\,d(c)\,dc + \sum_j p_{ij}(T-1)\;V_T(y,j)\\ &= h(T-1)\,N\int_x^y c\,d(c)\,dc + \sum_j p_{ij}(T-1)\int_y^1\left(N\int_y^z c\,d(c)\,dc\right) dG_j(N\,D(z))\end{aligned} \tag{A.4}$$

$$\frac{d\varphi}{dy} = \left(h(T-1) - \sum_j p_{ij}(T-1)\;\overline{G}_j(N\,D(y))\right) N\,y\,d(y) \tag{A.5}$$

$$\frac{d^2\varphi}{dy^2} = \left(\sum_j p_{ij}(T-1)\, g_j(N\,D(y))\, d(y)\right) N\, y\, d(y)$$
$$+\left(h(T-1) - \sum_j p_{ij}(T-1)\, \overline{G}_j(N\,D(y))\right) N\left(d(y) + y\, d'(y)\right) \tag{A.6}$$

Any y satisfying (A.7) is a local minimum of φ, since $d\varphi/dy$ will equal zero and $d^2\varphi/dy^2$ will be positive. Implied are continuously positive densities for demand and shortfall magnitude, that is, $d(y) > 0$ and $g(z) > 0$ for all values in the domains; otherwise, satisfying (A.7) is necessary but not sufficient.

$$h(T-1) - \sum_j p_{ij}(T-1)\, \overline{G}_j(N\,D(y)) = 0 \tag{A.7}$$

Furthermore, since G_j is strictly increasing for every j, and D is also strictly increasing, the left side of (A.7) is strictly increasing in y. Hence, there is at most one value of y that satisfies (A.7).

Therefore, if $a_{T-1}(i)$ is the unique value satisfying (A.7), then the optimal value y^*—the y that satisfies (A.3)—is either $a_{T-1}(i)$ or x. The optimal value y^* equals x only when $a_{T-1}(i)$ is less than x, or equivalently, (A.8) is true. If there is no value y that satisfies (A.7), then $a_{T-1}(i)$ is defined to be zero, and hence is less than or equal to x.

$$h(T-1) - \sum_j p_{ij}(T-1)\, \overline{G}_j(N\,D(x)) > 0 \tag{A.8}$$

(A.7) may be interpreted as follows. Substituting $a_{T-1}(i)$ for y and multiplying through by $a_{T-1}(i)$, the condition (A.7) may be rewritten as (A.9). The term on the left side of (A.9) is the cost of notifying, at time T–1, a customer with base cost $a_{T-1}(i)$. The term on the right side is the expected cost of not notifying a customer with base cost $a_{T-1}(i)$ and, if needed at time T, interrupting that customer without early notification. Equation (A.9) indicates that the optimal policy is to notify customers in order of increasing base cost, such that the marginal customer at time $T-1$ is indifferent in expected value between being notified at time $T-1$ and being interrupted without early notification. This result has the flavor of the classical newsboy problem.

$$h(T-1)\,a_{T-1}(i) = \sum_j p_{ij}(T-1)\,\overline{G}_j\big(N\,D(a_{T-1}(i))\big)\,a_{T-1}(i) \tag{A.9}$$

However, notifications cannot be voided. Hence if $a_{T-1}(i)$ is less than x, y^* equals x, that is, it is best to notify no additional customers.

Of course, the optimal value y* depends on the state (x, i), as indicated by (A.10) and (A.11).

$$y^* = \max\{a_{T-1}(i), x\} = x + u_{T-1}(x,i) \tag{A.10}$$

$$u_{T-1}(x,i) = [a_{T-1}(i) - x]^+ \tag{A.11}$$

Substituting (A.10) for y in (A.4) yields the result that the optimal value function is convex decreasing in x.

These results are for time $T-1$. The results for general time t are stated in section 4 and derived through backward induction on time. While tedious, this is not terribly enlightening, and hence is omitted.

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