TWO DIMENSIONAL AUCTIONS FOR EFFICIENT FRANCHISING OF PUBLIC MONOPOLIES

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Memorandum No. UCB/ERL/IGCT M93/41

10 April 1993

ELECTRONIC RESEARCH LABORATORY

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Two Dimensional Auctions for Efficient Franchising of Public Monopolies

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(Revised April 10, 1993)

Abstract

We consider the problem of a regulator who wishes to award an exclusive supply contract to one of several firms whose technologies are characterized by general, privately known, cost functions. The regulator's objective is to maximize social welfare. We investigate a bidding scheme in which the potential suppliers bid a fixed and a variable cost of supply and their payoffs are endogenously determined by a downward-slopping demand function. The winning bid is determined by a scoring rule combining both prices. We show that the only scoring rule which can lead to socially efficient outcomes is some monotone transformation of the consumer surplus. We further show that when such a scoring rule is used in a first price auction it is a dominant strategy for bidders to bid their marginal cost at the socially efficient production level. We also show that in a generalized version of second price auctions, the above scoring rule leads to dominant strategies which result in socially optimal selection and production. The second price auction described is in fact a two dimensional revelation mechanism.

1. Introduction

Multidimensional auctions in which bidders are required to specify several prices and nonprice attributes of their proposed contract are common practice in many arenas. Important applications of such auctions include logging rights (Wood [1989]), mineral leases (Rothkopf and Engelbart-Wiggins [1989]) and other natural resource auctions. Some of these auctions take the form of unit price bids where the competitors bid prices for the different types of resources (e.g. types of wood) while the total payments are determined ex-post based on the actual quantities. In the private sector many procurement and construction projects involve both price and nonprice factors. More recently, many electric utilities have been employing multidimensional auctions involving price and non price factors in contracting with non utility generators (NUG's) under the provisions of the Public Utility Regulatory Policy Act (PURPA). (Kahn, Rothkopf, Eto and Nataf (1990), Rothkopf, Kahn, Teisberg and Nataf [1990]). PURPA auctions have been studied in several recent articles by Bolle [1991], Stoft and Kahn [1991] and by Bushnell and Oren [1992]. The general approach in multidimensional auctions is to construct a one dimensional scoring rule which is known to the bidders and which is used to combine the various bid attributes in order to determine the winning bids. One of the issues addressed in the multidimensional bidding literature concerns the possibility of bidders "skewing" their bids (underbid one attribute and overbidding another) so as to take advantage of the scoring rule (Stark [1974]). This issue has been investigated in the context of bidding for logging rights by Wood [1989]) and is addressed as an incentive compatibility issue in PURPA auctions by Bushnell and Oren [1992].

In this paper we focus on two dimensional bidding schemes in which bids consist of two price components, a fixed payment and a variable price which endogenously determine the payoff to the winner. Auctions in which bidders specify variable prices while the payoff are endogenously determined have been discussed by Hansen [1988]. Mechanisms involving a fixed payment and a variable price have been studied with regard to monopoly regulation (Baron and Mayerson [1982], Loeb and Maget [1979]) and in the context of franchise bidding (Demset [1989], Riordan and Sappington [1987], Crew and Harstad [1992]) . The prevailing approach in this literature is to view the auction or incentive scheme as a one dimensional revelation mechanism and apply the standard machinery of revelation theory. Key assumptions in this treatment is the characterization of potential suppliers by a one dimensional (privately known) type parameter and further distributional assumption on types. In the paper by Riordan and Sappington [1987] for instance, the regulator's objective is to maximize consumer surplus which leads to production inefficiencies due to asymmetry of information. The restrictive characterization of types is essential to the demonstration of these results. Thus while the auction is two dimensional in terms of bid specification the assumption of one dimensional underlying type parameter reduces the problem to a one dimensional revelation problem.

Our work departs from the traditional approaches in several ways. First, we assume that the regulator's objective is to maximize social welfare rather than consumer surplus. This objective is motivated by the electric utility context mentioned above where the regulator, i.e., the Public Utility Commission represents the social interest which includes both consumers and private producers. Hence, our objective is to design contracts with incentive compatibility properties for achieving a first best solution. This allows us to relax some of the restrictive assumptions used in the revelation literature. In particular we allow for arbitrary privately known cost functions which may require multidimensional type characterization[1] and we do not make any assumptions on the distribution of types.

The mechanism introduced here bridges the multidimensional bidding literature and the revelation mechanisms literature by employing the concept of a scoring rule to rank multidimensional bids and characterizing scoring rules that are incentive compatible with social efficiency. It also offers a rare example of a truly multidimensional revelation scheme.

2. Problem

We consider the problem of regulating the supply of a good or service characterized by a downward-slopping demand function $D(p)$ (where p is the market price) when there are n possible suppliers, $i=1,...,n$, with corresponding cost functions $C_i(q)$, where q is the quantity produced. We assume that the cost functions are differentiable with possible "jumps" at the origin (to account for capital cost). We further assume that the cost functions are the private information of the suppliers. An exclusive supply license is to be awarded due to either technological constraint or efficiency considerations (e.g. economies of scale). The first best solution is thus to employ supply source i at a market price p, where i and p solve the total surplus maximization problem:

$$\underset{i}{Max}\ \underset{p}{Max}\{\int_p^\infty D(p)dp + pD(p) - C_i(D(p))\} \qquad (1)$$

The optimal solution to the above problem invokes two efficiency criteria:

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[1] As it will turn out, for a given demand function, the only relevant information regarding the marginal cost functions is their value and integral (total cost) at their respective efficient production levels. Hence a two dimensional distribution over these two characteristics constitutes sufficient statistics for this problem. Thus, from a revelation theory point of view the mechnism developed in this paper can be viewed as a two dimensional revelation mechanism. The results obtained here generalize the work reported in Bushnell and Oren [1992] which assumes two part cost functions described by a fixed and marginal cost while producers are characterized by a two dimensional type distribution on these two cost papramenters.

Production Efficiency: Corresponds to the inner maximization and is achieved by setting the market price so as to maximize total surplus for supply source i, i.e.

$$p_i^* = \underset{p}{Arg\,Max}\{\int_p^\infty D(p)dp + pD(p) - C_i(D(p))\} \qquad (2)$$

First order necessary conditions for (2) imply that $p_i^* = C'(D(p_i^*))$ i.e., price equal marginal cost. However, for a general cost function this condition may not necessarily yield a unique price.

Selection Efficiency: Corresponds to the outer maximization and is achieved by selecting the supplier who under production efficiency will attain the maximum social surplus, i.e.

$$i = \underset{i}{Arg\,Max}\{\int_{p_i^*}^\infty D(p)dp + p_i^* D(p_i^*) - C_i(D(p_i^*))\} \qquad (3)$$

In this paper we propose a first price auction mechanism which achieves production efficiency and a generalized second price auction mechanism which achieves both production and selection efficiency, thus, implementing the first best solution. In both cases the potential suppliers compete for an exclusive supply contract by submitting a two parameter bid *{p,F}* where *p* specifies the marginal price at which the supplier commits to supply the good and *F* is a lump sum payment which he requires for undertaking the supply commitment. A publicly known scoring formula *S(p,F)* is used to map the two dimensional bid into a one dimensional score and the lowest score bidder wins the exclusive supply licensee. In the case of a first price auction the winning bid is paid the lump sum specified in the bid while in the second price case the paid lump sum is adjusted on the basis of the best losing bid, as will be described later. I both case, however, the supplier is held to the marginal price specified in the bid.

3. Bidding Model

As indicated above we assume that bidders $i=1,\ldots,n$ have privately known cost functions $C_i(q)$ specifying their total cost of producing q units of the good to be supplied. The cost functions are continuous except for possible jumps at $q=0$.

The strategy of each bidder i consists of specifying a pair of "fixed" and "variable" prices $\{F_i, p_i\}$ $F_i \in [-K,K]$ and $p_i \in [0,L]$ for some sufficiently large values K and L

Each bid is scored using a publicly known scoring function:
$S(F,p):[-K,K]\times[0,L] \rightarrow \Re^1$ and the bid with the lowest score wins the exclusive supply license.

The payoff to the winning bidder i consists of two parts: sales revenues $p_i D(p_i)$ from selling the good at his bid price and a fixed payment (which could be positive or negative). The difference between a first price and second price auction will be manifested in that fixed portion of the payoff to the winning bid as described bellow.

First Price Auction
The fixed portion of the winning bid's payoff is the bid value F. Thus, the total payoff to bidder i, if he wins, is

$$\pi_i^I(F_i, p_i) = F_i + p_i D(p_i) - C_i(D(p_i)). \qquad (4)$$

Second Price Auction[2]

A natural extension of a Vickery [1961] second price auction to our two dimensional setup is to "adjust" the fixed portion of the payoff to the winning bid i so as to equate its score to the best losing score j. Thus, under this scheme, the payoff to winning bid i is

$$\pi_i^{II}(F_i,p_i)=\hat{F}_i+p_iD(p_i)-C_i(D(p_i)) \quad \text{where} \quad S(\hat{F}_i,p_i)=S(F_j,p_j) \text{ (5).}$$

The only degree of freedom available to the auction designer is the specification of the scoring function. Thus the remainder of this paper will focus on the characterization of this function and the implication of that characterization.

4. First Price Auction

Each bidder i forms a subjective probability of winning the auction as a function of score. This probability, which will be denoted as $P_i(S(F_i,p_i))$ may depend on the bidders' private information and is assumed to be once differentiable and decreasing in the score. Thus, the expected payoff of bidder i as a function of its bid is:

$$E\{\pi_i^I(F_i,p_i)\}=[F_i+p_iD(p_i)-C_i(D(p_i))]P_i(S(F_i,p_i)) \qquad (6)$$

We assume that all bidders are risk neutral and will hence choose their strategy so as to maximize their expected payoff. Our objective is to characterize a scoring function which will induce each bidder to bid a variable price that equals marginal cost at the corresponding demand quantity, the necessary condition for production efficiency of the auction outcome.

<u>Proposition 1:</u> *In the First Price Auction described above, a necessary condition for a Bayes-Nash equilibrium at which the bid variable prices equal marginal costs at the corresponding market demand is:*

$$\frac{\partial S/\partial p}{\partial S/\partial F}=D(p) \text{ for all } p \text{ s.t } p=C_i'(D(p)),\ i=1,\dots,n \qquad (7)$$

<u>Proof:</u>

First order necessary conditions for maximizing $E\{\pi_i(F_i,p_i)\}$ are:

$$\frac{\partial E\{\pi_i^I\}}{\partial p_i}=[p_iD'+D-C_i'(D)D']P_i+\pi_i^IP_i'\frac{\partial S}{\partial p_i}=0 \qquad (8)$$

$$\frac{\partial E\{\pi_i^I\}}{\partial F_i}=P_i+\pi_i^IP_i'\frac{\partial S}{\partial F_i}=0 \qquad (9)$$

Substituting (9) into (8) and setting $p_i=C_i'(D(p_i))$ yields (7) ■

The intuitive interpretation of condition (7) is that the marginal rate of substitution between the fixed and variable price in the score as to be the same as in the payoff at the efficient production prices corresponding to the potential suppliers. Since, however, the cost functions are private information, the only practical way to select a scoring rule that

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[2]This form of Second Price auction is obviously not unique. I is possible to define alternative forms by adjusting both values of the fixed and variable price in the payoff function so that the score corresponding to the adjusted values equals the best loosing score. We choose to adjust only the fixed portion since our objective is to elicit efficient marginal prices in the bids which should be subsequently used in the market place in order to achieve poduction efficiency.

meets condition (7) is to tighten it by requiring that it be satisfied for all values of p. This added restriction specifies the scoring rule uniquely up to a monotone transformation (which clearly will not affect the outcome of the auction) as shown bellow.

Proposition 2: *Let S(F, p) be a strictly monotonically increasing differentiable function in both arguments which satisfies the condition:*

$$\frac{\partial S/\partial p}{\partial S/\partial F} = D(p) \text{ for all } p \in [0, L] \text{ and } F \in [-K, K]$$

Then $S(\ )$ *is of the form*

$$S(F,p) = V\left(-F + \int_p^\infty D(p)dp\right) \tag{10}$$

where $V(\ )$ *is some strictly decreasing differentiable function.*

Proof:
For any level set of $S(\ )$ we have:

$$\left.\frac{dF}{dp}\right|_{S(F,p)=s} = -\frac{\partial S/\partial p}{\partial S/\partial F} = -D(p) \tag{11}$$

Integrating (11) yields:

$$F = \int_p^\infty D(p)dp - \Phi(s)$$

where $\Phi(s)$ represents an integration constant depending on the level s. The monotonicity condition on $S(\ )$ implies that $\Phi(s)$ is strictly decreasing. Thus,

$$S(F,p) = s = \Phi^{-1}\left(-F + \int_p^\infty D(p)dp\right)$$

which is equivalent to (10) ■

The argument of the function $V(\ )$ is the consumer surplus resulting from a fixed payment F and price p for the good. Furthermore, the outcome of the auction both for the supplier and the consumers is invariant to the specific form of the function $V(\ )$. Hence, without loss of generality we may take $V(\ )$ to be the negative identity so that the score is simply the negative of the consumer surplus. We show now that with the scoring characterized above, in fact guarantees production efficiency for the outcome of the first price auction. Evidently, using consumer surplus as a score produces the same effect as a subsidy equal to the realized consumer surplus, which according to Loeb and Maget [1979] will induce the producer to price at marginal cost.

Proposition 3: *In a two dimensional first price auction as described above with a scoring formula given by Eq. (10), it is a dominant strategy for each bidder to bid a variable price which maximizes total surplus under his cost function.*

Proof:
Let $CS_i = CS(F_i, p_i)$ denote the consumer surplus corresponding to bid $\{F_i, p_i\}$ i.e.,

$$CS(F,p) = -F + \int_p^\infty D(p)dp \tag{12}$$

Then, a strategy $\{F_i, p_i\}$ is equivalent to a strategy $\{CS_i, p_i\}$ and the expected payoff of bidder i given in (6) can be expressed as:

$$E\{\pi_i^I(CS_i, p_i)\} = [-CS_i + \int_{p_i}^\infty D(p)dp + p_i D(p_i) - C_i(D(p_i))]P_i(V(CS_i)) \tag{13}$$

It is clear, however that for any choice of CS_i, the right hand side of (13) is maximized when p_i maximizes the expression in the square bracket. But this is exactly p_i^* defined by Eq. (2) as the price that would yield production efficiency for bidder i . ■

5. Second Price Auction

We will consider now second price auctions in which the scoring rule is again a monotone function of the consumer surplus while the fixed payment to the winning bid is determined, as described earlier, based on the winning variable price and the best losing score. Based on Eq. (5) and (10) the fixed payment to winning bid i is

$$\hat{F}_i = F_j - \int_{p_j}^{\infty} D(p)dp + \int_{p_i}^{\infty} D(p)dp = F_j - \int_{p_j}^{p_i} D(p)dp \qquad (14)$$

where $\{F_j, p_j\}$ is the best losing bid.

Bidder i forms a subjective probability distribution over the score he needs to beat or equivalently over the consumer surplus CS_j corresponding to the best competing bid. Let $G_i(cs) = \Pr\{CS_j \le cs|i\}$ denote the subjective probability of bidder i that the consumer surplus corresponding to the best competing bid does not exceed cs. Substituting (14) in (5) and expressing the fixed payments in terms of the corresponding consumer surplus and the variable prices yields the expected payoff[3] for bidder i.

$$E\{\pi_i^{II}(CS_i, p_i)\} = \int_{-\infty}^{CS_i} [-cs + \int_{p_i}^{\infty} D(p) + p_i D(p_i) - C_i(D(p_i))] dG_i(cs) \qquad (15)$$

We will assume that the outside integral in (15) is a Stieljes integral to accommodate possible mass point in the subjective probability distribution.

<u>Proposition 4:</u> *In the second price auction described above, where the scoring formula is given by (10) and the fixed payment to the winning bid is as in (14), it is a dominant strategy for a bidder to bid a variable price that maximize total surplus under his cost function and a fixed price which equals the producers surplus under that variable price.*

<u>Proof:</u>
Clearly, selecting a strategy $\{F_i, p_i\}$ is equivalent to choosing the corresponding pair $\{CS_i, p_i\}$. Furthermore, regardless of the choice of CS_i , the right hand side of (15) is maximized pointwise when p_i is chosen so as to maximize the integrand in square bracket which differ by a constant from the total surplus. Thus the optimal variable price for bidder i is the production efficient price p_i^* defined in (2). Substituting that price into (15) and denoting the total surplus corresponding to that price as TS_i^* gives:

$$E\{\pi_i^{II}(CS_i, p_i^*)\} = \int_{-\infty}^{CS_i} [-cs + TS_i^*] dG_i(cs) = \int_{-\infty}^{TS_i^*} [-cs + TS_i^*] dG_i(cs) + \int_{TS_i^*}^{CS_i} [-cs + TS_i^*] dG_i(cs) \quad (16)$$

Note that Eq. (16) is analogous to the expected profit equation in a standard Vickery auction where $-CS_i$ represents the bid and $-TS_i{}^*$ the private cost. The last integral in (16) is clearly, nonpositive since the differential is positive and either the integrand is negative or the upper limit is smaller than the lower limit. Thus,

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[3]For simplicity we will exclude the possibility of ties altough those can be easily handled by a straight forward generalization.

$$E\{\pi_i^{II}(CS_i, p_i^*)\} \le \int_{-\infty}^{TS_i^*} [-cs + TS_i^*] dG_i(cs) \qquad (17)$$

and the maximum expected payoff is achieved when $CS_i = TS_i^*$ which implies:

$$-F_i + \int_{p_i^*}^{\infty} D(p)dp = \int_{p_i^*}^{\infty} D(p)dp + p_i^* D(p_i^*) - C_i(D(p_i^*))$$

Consequently,

$$F_i = -p_i^* D(p_i^*) + C_i(D(p_i^*)) \qquad (18)$$

which is exactly the net cost of supply at the production efficient price. ■

Corollary 1: *The outcome of the second price auction satisfies both production efficiency and selection efficiency criteria.*

Proof:
From Proposition 4. the winning bidder will bid a production efficient variable price given by (18). The consumer surplus corresponding to the winning bid, therefore equals it corresponding total surplus. Which implies that the winning bid is the one that achieves the highest total surplus at the production efficient variable price. ■

6. Conclusion

We have developed two bidding mechanisms based on a scoring rule which induces potential producers to truthfully reveal their relevant cost information which results in production and selection efficiency. The mechanisms do not depend on any distributional assumptions on types and can be viewed as two dimensional revelation mechanisms since the relevant private information is characterized by a general two dimensional distribution. In the case of the second price auction both parameters are truthfully revealed and the winning bidder retains the difference in social surplus between himself and the second best bid as information rent. In the one dimensional revelation problem it was demonstrated by Riordan and Sappington[1987] that a mechanism which distorts marginal cost can perform better from the consumers' point of view than one which achieves the socially optimal solution. This gain is achieved by employing two information componenets (transfer and price) to reveal a one dimensional type parameter. It is not clear however, whether this gain is still possible when a two dimensional type needs to be revealed with a two dimensional auction as in our model. Baron and Myerson [1982] examined a similar problem without any general conclusion. This remains an open question whose answer may depend on further assumptions about the information structure and type distribution.

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