CONTINUOUS PLATOONING: A NEW EVOLUTIONARY AND OPERATING CONCEPT FOR AUTOMATED HIGHWAY SYSTEMS

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Abstract

In this paper we propose a new strategy for AHS called continuous platooning. Continuous platooning (CP) combines the simplicity and flexibility of Autonomous Intelligent Cruise Control (AICC) and the performance advantages of platooning by combining the decentralized and autonomous feature of the former and the preview property of the latter. CP easily accommodates mixed traffic and is particularly amenable to seamless evolution. We conduct a system level analysis and optimal design for the scheme. A necessary and sufficient condition is derived for vehicle chain stability when control is used on the feedback linearized models. Semi-infinite constrained parametric optimization is used to design the control system. Simulations show that the proposed scheme has transient response similar to or better than AICC and platooning and its performance improves with the increase of the preview horizon.

I. Introduction

As traffic becomes increasingly congested in many parts of the world, increasing highway throughput and improving traffic safety become more and more pressing challenges for our society. It has been generally recognized that driver behavior and slow human reaction time (from 0.25-1.25 seconds) are the bottlenecks for improvements in these regards (see [4] and the references therein). Developing and implementing automatic vehicle control and relieving the human driver of this task shows great promise toward achieving these improvements [1], [2], [3], [4].

Of a particularly detrimental nature to an efficient traffic flow is the slow reaction phenomenon mentioned above, which often rears itself when a driver fails to respond quickly enough to a sudden change in the velocity of the preceding vehicle— resulting in a collision. But even if the

† This research is supported by California PATH under MOU-34.

maneuver of a driver is gentle enough such that there is no collision between two following vehicles, the subsequent transient behavior of the succeeding vehicles is far from ideal. Observations of human drivers documented in [9] show that in a group of closely spaced vehicles, the disturbance in the inter-vehicle spacings due to a small velocity change is magnified as succeeding cars in the chain react to the disturbance— what we call the *magnification effect* (also known as the *slinky effect* [7], [8]). Subsequently, in the process of recovery, the velocities of the vehicles in the chain exhibit an oscillatory behavior as the drivers bring their vehicles back to their steady-state cruising speed. These negative consequences of the long reaction time can be reduced some if drivers keep a substantial distance from the preceding vehicle. Doing so, however, restricts the throughput on our highways. It is reasonable to expect that a properly designed automatic controller with a substantially smaller reaction time could be applied to the longitudinal control of each car in a chain of vehicles to eliminate the undesirable qualities of human following control, thereby enhancing both safety and throughput. This is the objective of the program on Automatic Vehicle Control Systems (AVCS) within the Intelligent Vehicles and Highway Systems (IVHS) initiative [6].

A reasonable first proposal for an automatic longitudinal control scheme is depicted in Figure 1a, where each vehicle has sensors which measures its relative position (and its derivatives) with respect to the preceding vehicle. The controller in each vehicle would use the sensor information to try to match the velocity and acceleration of the preceding vehicle, while maintaining a specified constant spacing, a control scheme we refer to as *pure vehicle following*. However, analysis and simulation shows that under this simple law, the chain of vehicles is prone to same deficiencies of human driver control (primarily, the magnification effect and oscillations), albeit to a lesser extent.

There have been two notable alternative control schemes proposed that attempt to correct these problems. In [7], Sheikholeslam considers the *platooning* scheme of [5], where through vehicle-to-vehicle communication, the traffic flow is organized into tightly spaced groups of vehicles called platoons, with large spaces maintained between platoons. Each platoon consists of a lead vehicle and a number of followers. The key idea is that each vehicle in this platoon has access to the state information of the preceding vehicle (relative position, velocity, and acceleration) along with similar information from the lead vehicle in the platoon. This information flow is depicted in Figure 1b. In [8], Chien and Ioannou propose their *autonomous intelligent cruise control* (AICC) scheme where the longitudinal controller has access only to the relative state information from the previous vehicle only (as with pure vehicle following). In this case, the spacing regulation problem is relaxed by instituting a velocity-proportional steady-state spacing requirement. It is shown in [7] and [8] that in the simplified case where ideal vehicles are assumed and sensor noise and communication delays are neglected, controllers can be designed to avoid the magnification effect under each of the AICC and platooning schemes.

In this paper we propose a new control strategy for longitudinal vehicle control called Continuous Platooning (CP), which combines the simplicity and flexibility of AICC and the performance advantages of platooning by combining the decentralized and autonomous feature of the former and the preview property of the latter. In the vein of [7] and [8], we assume that we are trying to regulate the spacing of a chain of vehicles (not necessarily identical) subject to velocity and acceleration disturbances from within the group of vehicles. The paper is organized as follows. In the next section, we describe the scheme of continuous platooning and highlight its potential advantages. In section III, we present the feedback linearized vehicle model proposed in [7], which we use in our subsequent analysis. In section IV, we first review some of the previous work in the design of longitudinal vehicle controllers, and then in the subsequent section, we perform an analysis of the vehicle chain dynamics and an optimal design for our continuous platooning scheme. A necessary and sufficient condition for vehicle chain stability is derived. In section VI, we present some of our simulation results in which we compare the performance of continuous platooning with the previously presented schemes. Finally, in section VII, we draw some conclusions and point out future works.

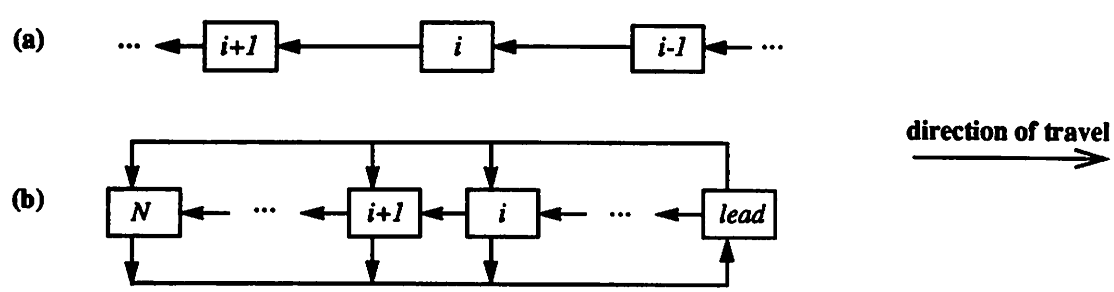


FIGURE 1. Information flow in (a) pure vehicle following and (b) platooning.

II. Continuous platooning: a new evolutionary and operating concept for AVCS

The heart of continuous platooning is a "relay" system for conveying the information through the chain of vehicles so that each vehicle has state information (relative position, velocity, acceleration, and intended maneuvers) from a number of vehicles in front of it. Based on this "preview" information, each vehicle is to make autonomous decisions about its control and spacing from the vehicle in front.

In a prototypical scenario, the ith vehicle in the chain receives the vehicle state information from the L preceding vehicles $(i-1, i-2, \ldots, i-L)$ and any other pertinent information. This bundle of information comes from the $(i-1)$th vehicle via direct line-of-sight communication (e.g. infrared). The ith vehicle in turn continues the relay process by stripping off the information associated with the Lth car in front of it (remembering for its own use however) and appending its own information, thus passing the analogous block of data onto the $(i+1)$th vehicle. This information flow is demonstrated in Figure 2. In this way, each vehicle in the chain has information not just from the vehicle immediately ahead of it, but from the L preceding vehicles. The preview parameter, L, is an important design parameter to be chosen, and could change dynamically as traffic conditions change. When L=1, the scheme is the same as AICC in terms of the information flow. For larger values of L, CP resembles platooning in that a vehicle has advance information from vehicles beyond the one immediately preceding it. Unlike platooning, however, we assume no arbitrary grouping of vehicles, and thus no artificial "leader" classification. All vehicles are treated equivalently and there is only one direction of information transmission, i.e.- backward; and each vehicle is a "free agent" and makes its own decisions.

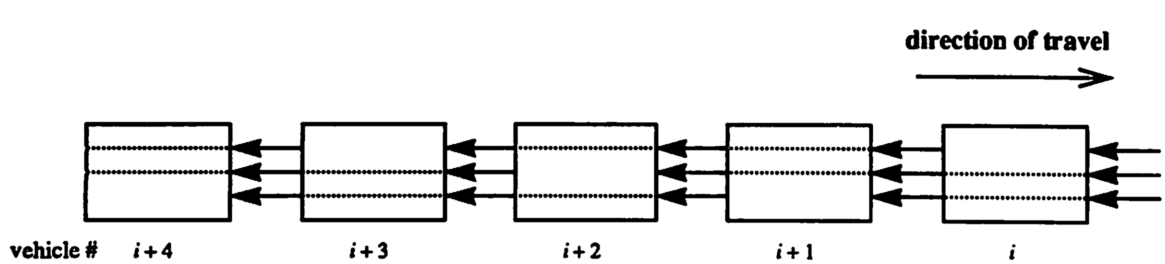


FIGURE 2. Information flow in a chain of vehicles operating under L=3 continuous platooning

With the concept of a lead vehicle having been removed and because of the unidirectional data flow, continuous platooning needs only the capability to transmit information in a single direction, thus reducing the complexity of communications hardware and protocols†. It has been argued that the grouping of vehicles with large inter-group spacings (as in platooning) gives rise to better safety and smoother traffic (allowing easier lane changing). It is not difficult to see that such vehicle distributions not achievable by AICC, can be achieved by CP with the unidirectional communication and the individual vehicle-based decision making of CP. And with this one-way data pipe in place, other possibilities for increased safety and efficiency come about. For example, internal signals of vehicles (such as brake and malfunction signals) can then be transmitted to the succeeding vehicles. There are usually non-negligible time delays from the onset of these signals to the manifested changes in the vehicle dynamics. Timely transmission of such signals to the following vehicles certainly hold promise for improving safety. Also, transmitted information can provide certain redundancy to sensor information, thus improving reliability.

In addition to the increased simplicity afforded by the unidirectional information flow, we believe that continuous platooning has certain advantages to offer arising from the use of preview information. Where in both platooning and AICC, a vehicle learns only indirectly about a disturbance, say, three cars ahead, under CP the state information of L different vehicles "immediately" (assuming the communication delays are negligeble compared with mechanical delays) affects a given vehicle, with substantial safety benefits. While this point is self-evident for AICC, the platooning case may require some explanation. In the current platoon control scheme, the ith vehicle knows the information from the $(i-1)$th vehicle and the lead vehicle but not the information from the vehicles in between. Therefore, while platooning may have a superior response to lead vehicle disturbances, not using the information from other in-platoon vehicles makes the system susceptible to intra-platoon malfunctions and collisions. In all of this, we are making the reasonable assumptions that with high frequency (infrared) vehicle-to-vehicle communication links, communication delays are much smaller than delays of vehicle mechanical systems such as throt-

† Due to the complexity of protocols, it has been suggested [4] that only one platoon maneuver including split, merge, and lane change can be executed at any given time. Therefore given the complexity of urban multi-lane traffic, vehicles in platooning may be constantly engaged in platoon maneuvers, resulting in passengers discomfort and reduced throughput.

tles and brakes, and that communication capacities are more than adequate to relay the state information of dozens of vehicles [14], [15], [16].

As we demonstrate later, the obvious benefit of longer preview distances are shorter allowable spacings and the accordingly larger throughputs. It should be noted that other less obvious benefits come from these shorter spacings. First, distance sensing and inter-vehicle communication are more reliable and accurate with less interference, attenuation and diffusion, and less severe multipath effects, which contribute to offset possible safety problems due to smaller spacing†. Also, under the assumption of driver-directed lane changes (addressed below), smaller spacings discourage other vehicles from cutting into a tight space, a potentially dangerous situation. Finally, it has been shown in recent wind tunnel tests conducted at the University of Southern California that smaller spacing substantially improve vehicle fuel efficiency. The last two facts may also provide incentives for drivers to choose a small spacing when preview is used to improve safety.

While it may be that platooning will achieve the ultimate throughput improvement, continuous platooning holds significant advantages over platooning with regard to implementation and evolutionary development of an AHS. It is easy to envision that vehicles will gradually become equipped with automatic control capabilities and then later acquire inter-vehicle communication capabilities. While platooning requires a group of vehicles all equipped with both of these capabilities, CP can adapt the information relaying dynamically to allow arbitrary mixed operation of vehicles ranging from those with manual control only to those with full automation and communications capabilities. As has been argued in [13], a seamless evolution allowing mixed traffic appears to be more feasible than reserving exclusive lanes for automatic vehicles.

It can be expected that traffic efficiency and safety will increase with increasing percentages of vehicles in traffic equipped with automatic control and information relaying capabilities. Such a close correlation between costs and benefits is extremely important for market penetration. It has been argued in [13] that one of key hindrances for introducing platooning may be that there is lit-

† Even without considering the communication aspects, smaller spacings do not necessarily translate to poorer safety. It is not too difficult to see that with very small spacing, while the frequency of collisions may increase, the severity of the impact will decrease.

tle driver-perceived benefits for upgrading from AICC to platooning. This is clearly not so with continuous platooning which offer substantial safety benefits through preview.

Information relay inherent in CP also holds promise to allow some vehicle coordination to improve traffic homogeneity, particularly compared with AICC. For instance, in CP, it is possible to transmit to following vehicles the intended maneuver of a vehicle (such as slowing down to exit or changing lanes). This has the advantage in that it allows the following vehicles to coordinate and react accordingly. For example, slowing down to exit, a transient disturbance, will induce less drastic reaction than does slowing down due to adverse traffic conditions ahead. Also, in CP, a common target speed can easily be propagated down the traffic flow without relying on roadside beacons along the way. The relay mechanism can easily be altered to allow emergency signalling to be sent beyond the normal L vehicle range during emergency situations. Such a mechanism can provide advance warning to adverse road surface conditions or incidents ahead and thus prevent a disastrous pile-up of vehicles.

In this paper we are mainly concerned with automation of longitudinal control. Incorporating lane-keeping lateral control does not present much conceptual difficulty. As for lane changes, due to the difficulty with lane-to-lane communication, a pragmatic approach, at least as an evolutionary step, may be to simply have lane changes performed by human drivers, or alternately, have them initiated by human drivers, but executed automatically.

In the remainder of the paper, we conduct a system level analysis and design of a continuous platooning control strategy under the simplifying assumptions that the vehicles are driving on a straight lane of highway, that every vehicle has automatic control and information relay capabilities, that the control system has access to both the brake and throttle and that there is no communication or control delay. It should be emphasized that this is only a preliminary design. Full potentials of CP can only be realized with intelligent controllers which are nonlinear, account for more realistic models and scenarios, and possess emergency diagnosis and handling functions among other things.

III. Vehicle model

For our design, we use the system model and design requirements first set forth in [7] and used again in [8]. Consider the following three-state simplified nonlinear vehicle model for the ith vehicle, where the faster engine dynamics have been neglected. We have suppressed the indexing with respect to i.

$$\dot{x} = v$$
$$\dot{v} = -\frac{1}{m}(F_w - F_e + F_{dist} + F_d) \tag{1}$$

where x is the longitudinal position of the ith vehicle,

v is the velocity of the ith vehicle,

m is the mass of the ith vehicle,

$F_w = -K_w v^2$ is the force due to wind resistance, where K_w is the aerodynamic drag coefficient

F_{drag} is the force due to mechanical drag,

F_{dist} is any arbitrary unmodeled disturbance force,

and F_e is the engine traction force, which we assume to evolve under the dynamics:

$$\dot{F}_e = -\frac{F_e}{\tau} + \frac{u}{\tau}, \tag{2}$$

where τ is the engine time constant and u is the throttle/brake input. Defining $a = \dot{v}$, we find that

$$\dot{a} = -\frac{1}{m}(-2K_w v\dot{v} + \frac{F_e}{\tau} + \frac{\dot{F}_{dist}}{\tau} - \frac{u}{\tau})$$

Rewriting F_e using equation (1) above,

$$F_e = ma + K_w v^2 + F_{dist} + F_{drag}$$

we find

$$\dot{a} = \frac{-1}{m\tau}[-2\tau K_w v\dot{v} + ma + K_w v^2 + F_{dist} + F_{drag} + \dot{F}_{dist} - u] \text{ p}$$

If we assume that we have access to both the velocity and acceleration of the vehicle and the other vehicle parameters, then the model can be linearized by state feedback. Following [7, 11], setting

$u = (-m\tau)\,(c + 2\tau v a K_w - ma - K_w v^2 - F_{drag})$, we find that the new linearized vehicle model is given by $\dot{a} = c - \frac{\dot{F}_{dist}}{m\tau} - \frac{F_{dist}}{m\tau}$, where c is an exogenous control input. Without loss of generality, we can group the F_{dist} disturbance terms into one by defining $d = -\frac{\dot{F}_{dist}}{m\tau} - \frac{F_{dist}}{m\tau}$. Now the dynamics of the linearized vehicle are summarized by the three simple state equations:

$$\dot{x} = v \qquad \dot{v} = a \qquad \dot{a} = c + d \tag{3}$$

It is worth pointing out that while the vehicles may not be identical, the above linearized models are identical. This considerably simplifies the design. Our controller design as well as the designs of [7] and [8] use this linearized model (with the disturbance term neglected).

IV. Analysis and Design

IV.A: Previous work- AICC and platooning

Using this linearized vehicle model, in [7], Sheikholeslam incorporates the lead vehicle information into the control law of the ith vehicle in the following manner (we now index state information with respect to the vehicle number i):

$$c_i(t) = K_{p_1}\delta_i(t) + K_{v_1}\dot{\delta}_i(t) + K_{a_1}\ddot{\delta}(t) + K_{v_{lead}}(v_{lead}(t) - v_i(t)) + K_{a_{lead}}(a_{lead}(t) - a_i(t))$$

where

$\delta_i(t) = x_{i-1}(t) - x_i(t) - l_i - H$,

v_{lead} and a_{lead} are the velocity and acceleration of the lead (1st) vehicle of the platoon,

l_i is the length of the ith vehicle,

H is the constant desired vehicle spacing when the platoon is at steady state,

and $K_{p_1}, K_{v_1}, K_{a_1}, K_{v_{lead}}$ and $K_{a_{lead}}$ are the controller parameters. That is, added to a pure vehicle following controller are two terms to penalize deviation from the velocity and acceleration of the lead vehicle in the platoon.

Considering the dynamics of the ith and the $(i-1)$th vehicle, he computes the transfer function from δ_{i-1} to δ_i:

$$\frac{\delta_i(s)}{\delta_{i-1}(s)} \triangleq T(s) = \frac{K_{a_1}s^2 + K_{v_1}s + K_{p_1}}{s^3 + (K_{a_1} + K_{a_{load}})s^2 + (K_{v_1} + K_{v_{load}})s + K_{p_1}} \qquad (4)$$

This transfer function characterizes how a spacing disturbance in front of vehicle $i-1$ affects the spacing in front of vehicle i. Next, we consider the relationship between $\delta_i(s)$ and $\delta_{i+n}(s)$ for integers $n \geq 1$, and we denote the corresponding transfer function $P_n(s)$. For the case of platooning, we can show that $P_n(s) = T^n(s)$.

In the case of platooning, Sheikholeslam observes that eliminating the undesirable qualities of longitudinal vehicular motion can be accomplished by placing design requirements on $T(s)$ and choosing the design constants in accordance with them. Specifically, he requires:

- P1) $T(s)$ should be stable.
- P2) $|T(j\omega)| < 1$ for all $\omega > 0$, to eliminate magnification.
- P3) The impulse response corresponding to $T(s)$, should be positive for all $t > 0$. to prevent oscillatory behavior [7], [11].

These requirements are shown to be satisfied under a suitable choice of constants [7], [11].

Chien and Ioannou use this same linearized vehicle model (3) for their design in [8]. Their AICC controller resembles pure vehicle following with two modifications. First, they include an additional term, $C_a a_i(t)$, to increase their freedom in placing the zeros of their chain dynamics, which has the effect of smoothing the traffic flow by reducing the jerk. Also, mimicking human drivers, a velocity dependent spacing requirement has been introduced. That is, $\delta_i(t)$ is now defined as $\delta_i(t) = x_{i-1}(t) - x_i(t) - l_i - (H + \lambda v_i(t))$.

Here, the final term $H + \lambda v_i(t)$ in the above expression can be seen to be the inter-vehicle spacing requirement, where H is a base steady-state spacing and $\lambda v_i(t)$ imposes an additional velocity-proportional spacing requirement. Fully stated the AICC control law of Chien and Ioannou is

$$c_i(t) = K_{p_1}\delta_i(t) + K_{v_1}\dot{\delta}_i(t) + K_{a_1}\ddot{\delta}_i(t) + C_a a_i(t) \qquad (5)$$

Again, considering the closed-loop dynamics of the ith and $(i+1)$th vehicle, we can compute a transfer function, $T(s)$, from δ_{i-1} to δ_i:

$$\frac{\delta_i(s)}{\delta_{i-1}(s)} \triangleq T(s) = \frac{C_a s^2 + C_p}{(1+\lambda C_a)s^3 + (C_a - K_a)s^2 + (\lambda C_p - K_v)s + C_p} \tag{6}$$

If we again define $P_n(s)$ as above, we can write $P_n(s) = T^n(s)$. And again, satisfying the design requirements (P1)-(P3) above will again guarantee a "good" response to a disturbance in the chain. With the available parameters, Chien and Ioannou are able to choose design constants such that $T(s)$ satisfies these requirements.

IV.B: Continuous platooning

In contrast to the above schemes, continuous platooning makes use of the state information from the L immediately preceding vehicles. In this section, we consider the following controller which incorporates this information:

$$\begin{aligned} c_i(t) = {} & K_{p_1}\delta_i(t) + K_{v_1}\dot{\delta}_i(t) + K_{a_1}\ddot{\delta}_i(t) + \\ & K_{p_2}\delta_{i-1}(t) + K_{v_2}\dot{\delta}_{i-1}(t) + K_{a_2}\ddot{\delta}_{i-1}(t) + \dots \\ & \dots + K_{p_L}\delta_{i-(L-1)}(t) + K_{v_L}\dot{\delta}_{i-(L-1)}(t) + K_{a_L}\ddot{\delta}_{i-(L-1)}(t) \end{aligned} \tag{7}$$

where $\delta_i(t) = x_{i-1}(t) - x_i(t) - l_i - (H + \lambda v_i(t))$. Note that in the above, the control laws are assumed to be the same for each vehicle, and when a vehicle has fewer than L vehicles in front of it, a value of zero is assumed in place of any missing information.

In analyzing a system of vehicles operating under continuous platooning, we observe that $\delta_i(t)$ is now affected directly not only by variations in $\delta_{i-1}(t)$, but also by $\delta_{i-2}(t), \dots, \delta_{i-L}(t)$. Hence, no longer does a single transfer function $T(s)$ completely describe how a vehicle is affected by all of the vehicles from which it receives information. Now there are L such transfer functions, one characterizing the effects from each of the L preceding vehicles. We denote these transfer functions below as $T_m(s)$, where for preview-L continuous platooning, $m = 1, \dots, L$.

To derive these transfer functions, we assume that a chain of vehicles are operating under a preview-L continuous platooning control law and consider the quantity $\dddot{\delta}_i(t)$:

$$\dddot{\delta}_i(t) = \dot{a}_{i-1}(t) - \dot{a}_i(t) - \lambda \ddot{a}_i(t)$$
$$= c_{i-1}(t) - c_i(t) - \lambda \dot{c}_i(t)$$

Assuming the vehicles are initially at equilibrium, i.e. $\delta_i(0) = \dot{\delta}_i(0) = \ddot{\delta}_i(0) = \dddot{\delta}_i(0) = 0$, we find

$$s^3\delta_i(s) = [-(\lambda K_{a_1})s^3 - (K_{a_1} + \lambda K_{v_1})s^2 - (K_{v_1} + \lambda K_{p_1})s - (K_{p_1})]\,\delta_i(s) +$$
$$[-\lambda K_{a_2}s^3 + (K_{a_1} - K_{a_2} - \lambda K_{v_2})s^2 + (K_{v_1} - K_{v_2} - \lambda K_{p_2})s + (K_{p_1} - K_{p_2})]\,\delta_{i-1}(s) + \dots$$
$$[-\lambda K_{a_2}s^3 + (K_{a_{L-1}} - K_{a_L} - \lambda K_{v_L})s^2 + (K_{v_{L-1}} - K_{v_L} - \lambda K_{p_L})s + (K_{p_{L-1}} - K_{p_L})]\,\delta_{i-(L-1)}(s) +$$
$$[K_{a_L}s^2 + K_{v_L}s + K_{p_L}]\,\delta_{i-L}(s)$$

Rearranging terms and dividing, we find that

$$\delta_i(s) = \left[\frac{-\lambda K_{a_2}s^3 + (K_{a_1} - K_{a_2} - \lambda K_{v_2})s^2 + (K_{v_1} - K_{v_2} - \lambda K_{p_2})s + (K_{p_1} - K_{p_2})}{F(s)}\right]\delta_{i-1}(s) + \dots$$
$$\left[\frac{-\lambda K_{a_L}s^3 + (K_{a_{L-1}} - K_{a_L} - \lambda K_{v_L})s^2 + (K_{v_{L-1}} - K_{v_L} - \lambda K_{p_L})s + (K_{p_{L-1}} - K_{p_L})}{F(s)}\right]\delta_{i-(L-1)}(s) +$$
$$\left[\frac{K_{a_L}s^2 + K_{v_L}s + K_{p_L}}{F(s)}\right]\delta_{i-L}(s) \tag{8}$$

where $F(s) = (1 + \lambda K_a)s^3 + (K_{a_1} + \lambda K_{v_1})s^2 + s(K_{v_1} + \lambda K_{p_1}) + K_{p_1}$.

From this expression, we define

$$T_m(s) = \begin{cases} \dfrac{-\lambda K_{a_{m+1}} s^3 + (K_{a_m} - K_{a_{m+1}} - \lambda K_{v_{m+1}}) s^2 + (K_{v_m} - K_{v_{m+1}} - \lambda K_{p_{m+1}}) s + (K_{p_m} - K_{p_{m+1}})}{F(s)} & 1 \le m \le L \\ \dfrac{K_{a_m} s^2 + K_{v_m} s + K_{p_m}}{F(s)} & m = L \end{cases}$$

which allows us to write the compact expression,

$$\delta_i(s) = \sum_{m=1}^{L} T_m(s) \delta_{i-m}(s) \tag{9}$$

We now use (9) to calculate $P_n(s)$, again, the effect of an external disturbance in the ith inter-vehicle spacing $\delta_i(s)$ on the subsequent vehicle spacings in the chain, $\delta_{i+1}(s), \ldots, \delta_{i+n}(s), \ldots$. To do so, we first assume that the vehicles are in steady state with all δ_i's and all derivatives of δ_i equal to zero. Additionally, we make the important assumption that δ_i is the only spacing externally disturbed in the chain of vehicles. Under these conditions, all of the preceding spacings are zero for all time, allowing us to delete the $\delta_j(s), j < i$ terms in the following derivation.

From (9), a disturbance in $\delta_i(s)$ affects $\delta_{i+1}(s)$ according to

$$\delta_{i+1}(s) = T_1(s) \delta_i(s) \tag{10}$$

with the terms $\{\delta_j(s), i - (L-1) \le j \le i-1\}$ having been deleted as justified above. Similarly, for $\delta_{i+2}(s)$, we have

$$\begin{aligned} \delta_{i+2}(s) &= T_1(s)\delta_{i+1}(s) + T_2(s)\delta_i(s) \\ &= T_1(s)\,[T_1(s)\delta_i(s)] + T_2(s)\delta_i(s) \\ &= [T_1^2(s) + T_2(s)]\,\delta_i(s) \end{aligned} \tag{11}$$

Continuing the pattern of (10) and (11), we can compute P_n through the following recursion,

$$P_1(s) = T_1(s)$$

$$P_n(s) = \begin{cases} \sum_{k=1}^{n-1} [T_k(s)P_{n-k}(s)] + T_n(s), & 1 \le n \le L \\ \sum_{k=1}^{L} T_k(s)P_{n-k}(s), & n \ge L+1 \end{cases} \tag{12}$$

then we can summarize the effect of a disturbance in δ_j using these P_n's:

$$\delta_{j+n}(s) = P_n(s)\delta_j(s), \qquad n = 1, 2, \ldots \tag{13}$$

Because of the more complicated interrelations of vehicles in continuous platooning, previous definitions of the stability of a chain of vehicles are not appropriate. To address this, we make the following definition:

Definition: Let I denote the set of vehicle indices for a chain of vehicles. A longitudinal control law for the set of vehicles is said to yield *chain stability* if $\sup_{i \in I} \sup_{t > 0} |\delta_i(t)| < \infty$ for all bounded initial conditions and all bounded disturbances.

Theorem 1: For continuous platooning control laws of the form of (7), a necessary and sufficient condition for chain stability of an infinite vehicle chain is that $F(s)$ is stable, $1 - T_1(j\omega)z - T_2(j\omega)z^2 - \ldots - T_L(j\omega)z^L \neq 0$, $\forall\omega > 0$, $\forall|z| > 1$, and that there is no repeated root on the unit circle.

Proof: The theorem follows upon noticing that (9) can be regarded as an autonomous difference equation indexed by i and parameterized by ω.

Before proceeding with our controller synthesis, we make an observation about the case of pure vehicle following with constant spacing that supports our decision to try longer preview policies. Based on the above analysis, we know that when $L = 1$,

$$T_1(s) = P_1(s) = \frac{K_{a_1}s^2 + K_{v_1}s + K_{p_1}}{s^3 + K_{a_1}s^2 + K_{v_1}s + K_{p_1}}. \tag{14}$$

If we want to compute $|P_1(j\omega)|$, we first observe that the real parts of both the numerator and denominator are the same for any value of $j\omega$. However, if we evaluate $P_i(j\omega)$ at a small enough value of ω, say $\omega < \frac{\sqrt{K_{v_L}}}{2}$, then we have that the imaginary part of the denominator is smaller than the imaginary part of the numerator. So we are guaranteed that there exists a value of ω such that $|P_1(j\omega)| > 1$, and thus, that the magnification effect is inescapable in pure vehicle following.

V. Controller Synthesis by Optimization

The conditions stated in Theorem 1 are analogous to (P1) and (P2) above and establish the bare minimum design constraint of chain stability. Our method for finding controller parameter vectors that satisfy these requirements will be a constrained optimization over the parameter space, where the conditions of Theorem 1 will be cast in the appropriate form for optimization. For practical reasons, we will place two further constraints on our optimization. First, since we anticipate an increased degree of safety in a design that uses information from multiple vehicles, we fix our constant time headway requirement, λ, at a (somewhat arbitrary) value of 0.1 seconds, half of its value in [8]. This condition is motivated by the fact that smaller values of λ have the obvious advantage of reduced inter-vehicle spacing and thus improved throughput. Also, as the cost function we will use to refine our choice of parameters is the peak magnitude of a time response of a regulation problem, it is likely that the some of the controller gains might tend to large values in an optimization, making the source of any performance gains suspect. Therefore, we place upper bounds on the controller gains, requiring $|K_{p_m}| < 250$, $|K_{v_m}| < 250$ and $|K_{v_m}| < 100$. For the purpose of our optimization, we say that a controller parameter vector for CP is *feasible* if all of the above constraints are satisfied.

As stated above, we use a cost function to locate the "best" design vector among those that are feasible. Our choice of cost function is motivated our desire that the effect of disturbances on the vehicle spacings be minimal. Thus, for a given vector $\vec{K} = \{K_{p_1}, K_{v_1}, K_{a_1}, K_{p_2}, K_{v_2}, K_{a_2}, \ldots, K_{p_L}, K_{v_L}, K_{a_L}, \lambda\}$ of control parameters for a preview-L CP system, we look at the time response of the corresponding CP system to the typical maneuver shown in Figure 4a, which we assume to occur in the "0th" vehicle at $t = 0$. From the response of the subsequent vehicles in the chain to this maneuver, we define the cost function $J(\vec{K})$, to be

$$J(\vec{K}) = \max_{n \geq L+1} \{ \max_{t \in (0,\infty)} \{ |\delta_n(t, \vec{K})| \} \} . \quad (15)$$

Note that we specify $n \geq L + 1$ because for a preview-L system, the $L + 1$th vehicle in the chain is the foremost one which has a full volume of information available to it.

We note that disturbance rejection capability is but one facet of the transient response we can choose to optimize. The optimization routines are general enough as to allow constraints or a cost function based other aspects of either time or frequency response waveforms.

Fully stated, our optimization problem is to minimize our cost function $J(\vec{K})$ subject to the constraints:

- The chain stability conditions of Theorem 1 are satisfied
- $|K_{p_m}| < 250, |K_{v_m}| < 250, |K_{a_m}| < 100$
- λ=0.1

This type of problem is well-suited for use with Matlab's constrained optimization routine, *constr()*, an implementation of Sequential Quadratic Programming. All of the above constraints are easily cast as inequality constraints (as required by *constr()*), except for the semi-infinite polynomial constraint in Theorem 1. In this case, we check that the condition is true on a fine discretization of the low frequency region (to 1000 rad/s) and verify a posteriori that the constraint holds for all $\omega > 0$ to speed up the computations. The time domain simulation for the cost function is implemented in C and interfaced to Matlab using the *cmex* interface within Matlab.

We perform designs for various values of L to study the merits of increasing preview policies. In lieu of a 3L+1 parameter optimization for a large value of L, we also consider an *incremental* design, solving instead a sequence of optimization problems along the lines of the following.

Start with an L=1 optimization:

O1) Solve optimization problem for L=1. (All 3 parameters are free to vary.)

Then, solve the problem for the L=2 case:

O2A) *(incremental design)* Solve optimization problem for L=2, additionally constraining parameters $K_{(p,v,a)_1}$ to their optimal values as determined in O1). (Only 3 parameters vary.)

O2B) *(total design)* Solve optimization problem for L=2, using the parameters from O2A) as the starting point of the optimization. (3*2 parameters are free to vary.)

Next, we consider the L=3 case:

O3A)*(incremental design)* Solve optimization problem for L=3, additionally constraining parameters $K_{(p,v,a)_1}$ and $K_{(p,v,a)_2}$ to their optimal values as determined in O2A). (Only 3 parameters vary.)

O3B) *(total design)* Solve optimization problem for L=3, using the parameters from O3A) as the starting point for the optimization. (3*3 parameters free to vary.)

This sequence can be continued to arrive at a design for any desired value of L. As our cost function is of relatively complex, we carry out multiple optimizations with different starting points to help ensure convergence to the best possible parameter vector.

We also consider the same sequence of designs for the constant spacing case, i.e., where λ is constrained to be 0. For this case, however, optimization using criterion the above constraints fails to yield a parameter vector satisfying the roots constraint, so we instead relax the roots constraint of the chain stability theorem and consider the alternate cost function:

$$\tilde{J}(\vec{K}) = J(\vec{K}) + a\max\{\left|\text{roots}\{1 - T_1(j\omega)z - T_2(j\omega)z^2 - \ldots - T_L(j\omega)z^L\}\right|\} \quad (16)$$

where we empirically choose $a = 0.01$ so as to weight chain stability roughly equal to a good time response in our cost function. Our motivation here, is that if we cannot achieve the lack of magnification of disturbances implied by satisfying the roots constraint, then we hope to minimize the growth rate of the deviations along the chain of vehicles (which is related to the magnitude of the roots of this polynomial). For the more realistic case of finite chains of vehicles, a small growth rate can still yield satisfactory performance, as we demonstrate in the simulations below.

VI. Design and Simulation Results

In this section, we present and compare design and simulation results for each of twelve systems of vehicles. Specifically, we present results from the cases:

a) Platooning of Sheikholeslam and Desoer, [7]
b) AICC of Chien and Ioannou, [8]
c) L=1 continuous platooning with $\lambda = 0.1$ (optimization O1).
d) L=2 continuous platooning with $\lambda = 0.1$ (optimization O2A).
e) L=2 continuous platooning with $\lambda = 0.1$ (optimization O2B).
f) L=3 continuous platooning with $\lambda = 0.1$ (optimization O3A).
g) L=3 continuous platooning with $\lambda = 0.1$ (optimization O3B).
h) L=1 continuous platooning with $\lambda = 0$ (optimization O1).
i) L=2 continuous platooning with $\lambda = 0$ (optimization O2A).
j) L=2 continuous platooning with $\lambda = 0$ (optimization O2B).
k) L=3 continuous platooning with $\lambda = 0$ (optimization O3A).
l) L=3 continuous platooning with $\lambda = 0$ (optimization O3B).

The following table shows the control parameters corresponding to each of the above designs:

Controller	K_{p_1}	K_{v_1}	K_{a_1}	K_{p_2}	K_{v_2}	K_{a_2}	K_{p_3}	K_{v_3}	K_{a_3}	$K_{v_{lead}}$	$K_{a_{lead}}$	C_a	λ
a)	120	49	5	-	-	-	-	-	-	25	10	-	-
b)	224	127.2	5	-	-	-	-	-	-	-	-	3.56	0.2
c)	205.1	250.0	21.5	-	-	-	-	-	-	-	-	-	0.1
d)	205.1	250.0	21.5	203.5	230.3	-0.65	-	-	-	-	-	-	0.1
e)	250.0	250.0	18.2	212.6	208.5	-9.43	-	-	-	-	-	-	0.1
f)	250.0	250.0	18.2	212.6	208.5	-9.43	115.0	47.1	1.45	-	-	-	0.1
g)	208.6	250.0	20.9	204.3	264.2	1.57	97.4	119.4	0.34	-	-	-	0.1
h)	250	250	94.9	-	-	-	-	-	-	-	-	-	-
i)	250	250	94.9	248.6	244.2	94.0	-	-	-	-	-	-	-
j)	250	250	94.9	248.6	244.2	94.0	-	-	-	-	-	-	-
k)	250	250	94.9	248.6	244.2	94.0	250.0	249.9	100	-	-	-	-
l)	249.8	249.8	99.9	247.6	250.0	99.9	249.8	247.3	98.7	-	-	-	-

As required, all systems c)-l) satisfy the constraint on $F(s)$ imposed by the stability theorem. The following table shows the eigenvalues for each CP design:

Controller	Eigenvalues of $F(s)$
c)	-6.9421 + 5.0523i, -6.9421 - 5.0523i, -0.8846
d)	-6.9421 + 5.0523i, -6.9421 - 5.0523i, -0.8846
e)	-7.1177 + 5.6044i, -7.1177 - 5.6044i, -1.0793
f)	-7.1177 + 5.6044i, -7.1177 - 5.6044i, -1.0793
g)	-6.9776 + 5.1402i, -6.9776 - 5.1402i, -0.8989
h)	-92.1824, -1.3413 + 0.9555i, -1.3413 - 0.9555i
i)	-92.1824, -1.3413 + 0.9555i, -1.3413 - 0.9555i
j)	-92.1824, -1.3413 + 0.9555i, -1.3413 - 0.9555i
k)	-92.1824, -1.3413 + 0.9555i, -1.3413 - 0.9555i
l)	-97.3842, -1.2693 + 0.9768i, -1.2693 - 0.9768i

Next we check that our synthesis has produced controllers that satisfy the chain stability constraint on polynomial roots. The plots in Figure 3 show the roots of the pertinent polynomial as a function of the parameter ω. We see in these plots that for cases (c) through (g), the roots all have less than unity magnitude for $\omega > 0$, thus satisfying the chain stability condition. It is not as apparent from the small figures, but in each of cases (h) thorough (l), the plot of the roots peaks at a value slightly greater than 1. We should also point out the optimal controller gains for $\lambda = 0$ are larger than for $\lambda = 0.1$. The poles with large magnitudes and larger bandwidth in Figures 3(h-l) also indicate that the control bandwidth is much larger for $\lambda = 0$, hence the system is less robust. However, the transient performance for $\lambda = 0$ remains satisfactory in the ideal case as shown below. Therefore whether CP with constant spacing is practical or not remains to be analyzed in a more realistic scenario.

What remains to be evaluated is the relative performance of each of these vehicle control systems by studying the time response of each system to test maneuver. In the remainder of this section we present simulation results allowing us to make comparisons both between previous architectures and those we have developed, as well as between various CP arrangements. For each controller design, we consider the time response of a 20 vehicle chain initially traveling at 25 km/s with the correct steady state spacing. Specifically, the lead vehicle undergoes each of the disturbance maneuvers depicted in Figure 5 starting at $t = 0$. The maneuver of Figure 5a is a simple deceleration maneuver, and you may recall that it is the same one used in our optimal design in section V. Using the richer scenario of Figure 5b, which is characterized by both a drastic acceleration and deceleration, we can test the robustness of our continuous platooning designs to more severe maneuvers.

Each of figures 5-9 show a particular aspect of the time response of the vehicle chain to one of the given disturbance scenarios. Obviously, in many regards, we cannot fairly compare certain results for the optimized CP controllers to those for the unoptimized controllers of [7] and [8], In such cases, however, we can reasonably assert that the performance of the CP controller is at least as good.

We first examine Figures 5 and 10, the responses of the δ_i's (the deviation from the desired inter-vehicle spacing) of each vehicle chain to the two lead vehicle disturbances of Figures 4. Our first observation, based on the delightfully small effect each lead vehicle disturbance has on the δ_i's in cases (c) through (l), is that our CP scheme is as capable of good spacing regulation as are the platooning and AICC schemes. Additionally, we note a correlation between increasing the preview horizon, L, and the ability of continuous platooning schemes to reject the disturbances. This follows our intuition that the more vehicles from which a vehicle has information, the better it is able to react to disturbances in preceding vehicles. Next, by comparing the δ_i's of incremental and unconstrained designs for both $L = 2$ and $L = 3$, we observe that the reduced "design energy" of the incremental designs offer significant gains over the unconstrained designs of the next smaller size (e.g., compare Figures 5f and 5e). In fact, in many cases, incremental controllers approach the corresponding unconstrained design in their performance (e.g., compare Figures 5f and 5g). This would suggest that near-optimal controllers could be achieved for large values of L by a sequence of L low dimensional optimizations. Finally we compare the desired spacing responses for the cases $\lambda = 0$ and $\lambda = 0.1$. From Figures 5(c-g) and 5(h-l), the lack of chain stability is apparent from the lack of attenuation seen as the disturbance propagates unaffected down the chain of vehicles. However, given our efforts during the optimization towards minimizing this magnification, our CP controller averts any catastrophic magnification of the δ_i's for this reasonably sized chain of 20 vehicles. This would suggest that under reasonable restrictions on maximum chain length, it is plausible that constant spacing continuous platooning would perform well under normal traffic situations. (However, see the comments earlier.)

The total inter-vehicle spacing responses of Figure 6, velocity responses of Figure 7, and acceleration responses of Figure 8 are included given primarily for the sake of completeness, although a few interesting observations can be made in each case†. For example, given the relatively ideal performance of all the controllers in maintaining the desired spacings, the plots of the total inter-vehicle spacing in Figure 6 serves to highlight expected throughput under ideal conditions, with platooning and $\lambda = 0$ CP exhibiting the smallest intervehicle spacings and thus the largest throughputs. Figure 7 highlights the inherent time lag introduced into the chain for nonzero values of λ. In the acceleration plots Figure 8, we begin to observe that increasing the preview parameter L results in an increased "smoothness" in a given vehicle's response to disturbances.

This "smoothness" trend is also apparent in the plots of the linearized control effort by examining the trends in Figures 9(c-g) as well as those in Figures 9(h-l). With regard to the magnitude of the control effort required to respond to the disturbances, we note that continuous platooning achieves its high performance without any added control effort over AICC and Platooning. And based on these plots, increasing the preview parameter L can has the effect of reducing the necessary control effort needed to adapt to a disturbance. This effect is especially notable for the case where $\lambda = 0$.

The data in these figures support our contention that continuous platooning systems are well capable of matching the performance of platooning and AICC for these demanding maneuvers.

† We have omitted the corresponding plots for the disturbance scenario in Figure 4b because all useful observations can be made based on the responses to disturbance 4a.

VII. Conclusions and future works

Continuous platooning is proposed for AHS and is shown to have many salient advantages. However, the above system level analysis is only a first step in establishing the true merit of continuous platooning. Controller design and analysis using a more detailed vehicle model as in [12] should be carried out. The effects of communication and control delays and sensor noise need to be analyzed and accounted for. An intelligent controller which handles various traffic conditions and possesses emergency handling and fault diagnosis functions needs to be developed. Lane change maneuvers need to be studied. The impacts of various AHS schemes on traffic need to evaluated through comprehensive computer simulations.

References

[1] Special Issue on IVHS, IEEE Trans. Vehicular Technology, vol. 40, Feb, 1991.

[2] IEE Colloquium on "The Car and its Environment-What Drive and Prometheus Have to Offer," London, UK, Jan 25, 1990.

[3] S. Tsugawa, et al, "Super smart vehicle system-its concept and preliminary works," in Proc. Vehicle Navigation Inf. Syst. Conf., Dearbon, MI, Oct. 20-23, 1991, 269-277.

[4] P. Varaiya, "Smart Cars on Smart Roads: Problems for Control," IEEE Trans. AC, vol. 38, 195-207, 1993

[5] S. E. Shladover, "Longitudinal control of automated guideway transit vehicles within platoons," J. Dynamic Syst., Meas., and Contr., 1978, 100():302-310.

[6] S. Shladover, et al, "Automatic Vehicle Control Developments in the PATH Program," IEEE Trans. Vehicular Technology, vol. 40, 114-130, Feb, 1991.

[7] Shahab Sheikholeslam, "Control of a class of interconnected nonlinear dynamical systems: the platoon problem," PhD. Dissertation, University of California, Berkeley, 1991.

[8] C. C. Chien and P. Ioannou, "Automatic Vehicle-Following," American Control Conf., pp. 1748-52, June, 1992, Boston.

[9] George A. Bekey, Gerald O. Burnham, and Jinbom Seo, "Control Theoretic Models of Human Drivers in Car Following," Human Factors, 1977, 19(4):399-413.

[10] Shahab Sheikholeslam and Charles A. Desoer, "Longitudinal control of a platoon of vehicles with no communication of lead vehicle information," Proc. of the 1991 American Control Conf., June 1991, Evanston, IL., vol. 3, pp. 3102-6.

[11] Shahab Sheikholeslam and Charles A. Desoer, "A system level study of the longitudinal control of a platoon of vehicles," Transactions of the ASME, Journal of Dynamic Systems, Measurement and Control, June 1992, vol. 114, pp.286-92.

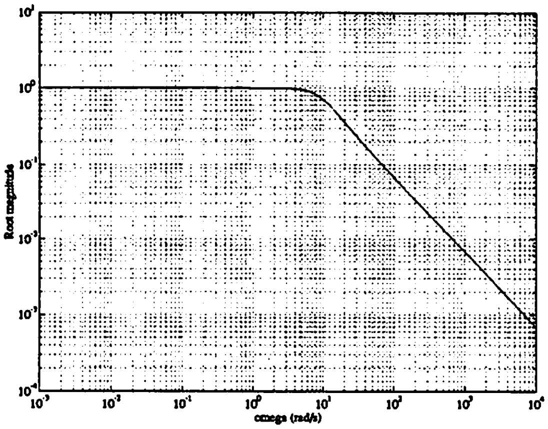
[12] D.H. McMahon, D.H., Hedrick, J.K., and Shladover, S.E., "Vehicle modelling and control for automated highway systems," Proc. of the 1990 American Control Conf., May 1990, San Diego, CA., vol. 1, pp.297-303.

[13] J. D. Ward, "An Hypothesized Evolution of an Automated Highway System," working paper, Jan. 1994.

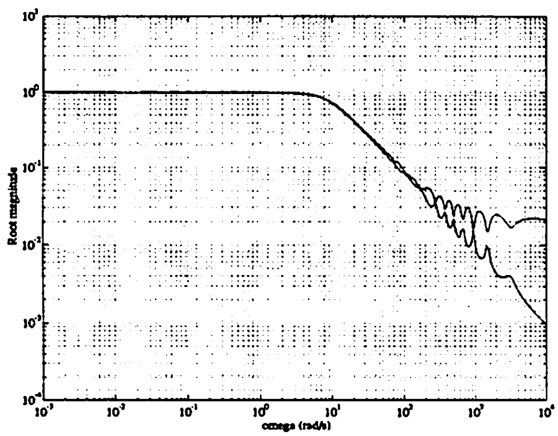
[14] Patarasen, S., Georghiades, C.N., "Infrared road-automobile communication," Proc. of the 1991 American Control Conf., June 1991, Evanston, IL, vol. 3, pp. 2551-6.

[15] Kahn, J. M., Barry, J. R., Krause, W. J., Audeh, M. D., et. al., "High-speed infrared communication for wireless local-area networks," Conference Record of The Twenty-Sixth Asilomar Conference on Signals, Systems and Computers, October, 1992, Pacific Grove, CA, vol. 1, pp. 83-7.

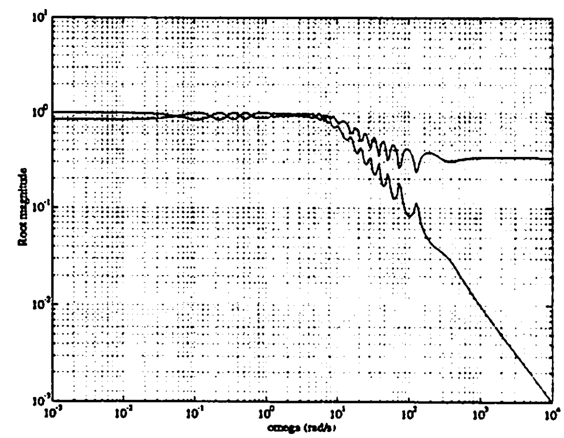
[16] McKeown, N., J. Walrand, "An Infrared Communication Link for the PATH Project," PATH Research Report draft no. 91-0703-1.



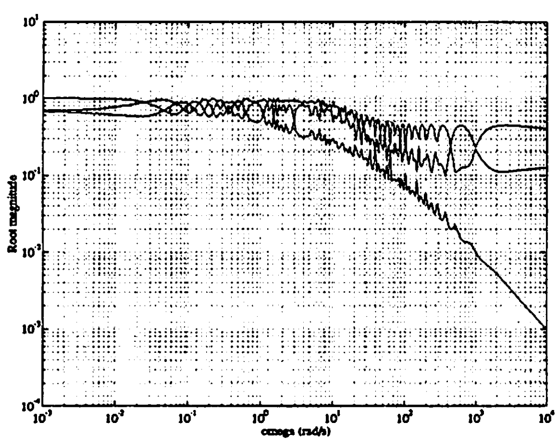
(c) Unconstrained L=1 CP, λ = 0.1



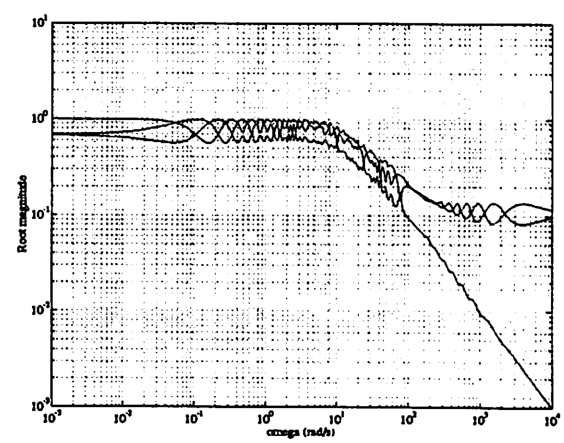
(d) Incremental L=2 CP, λ = 0.1



(e) Unconstrained L=2 CP, λ = 0.1

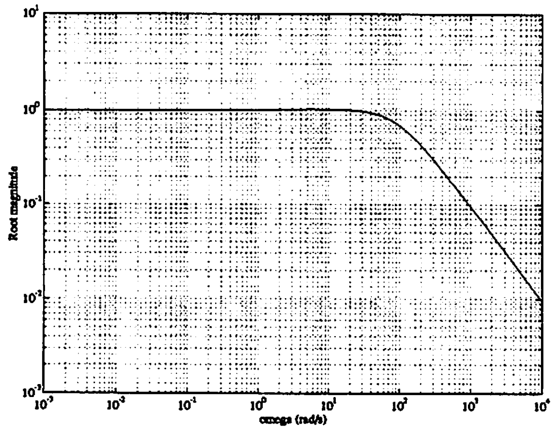


(f) Incremental L=3 CP, λ = 0.1

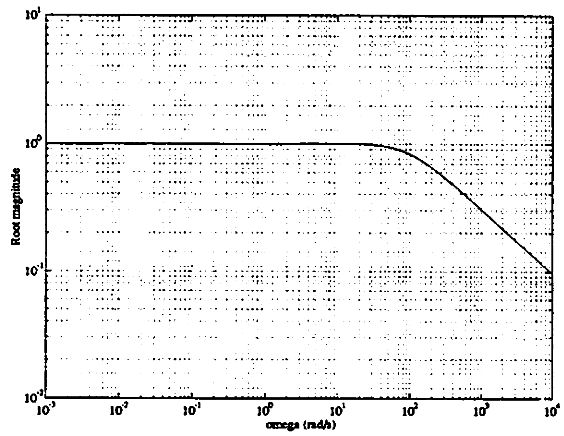


(g) Unconstrained L=3 CP, λ = 0.1

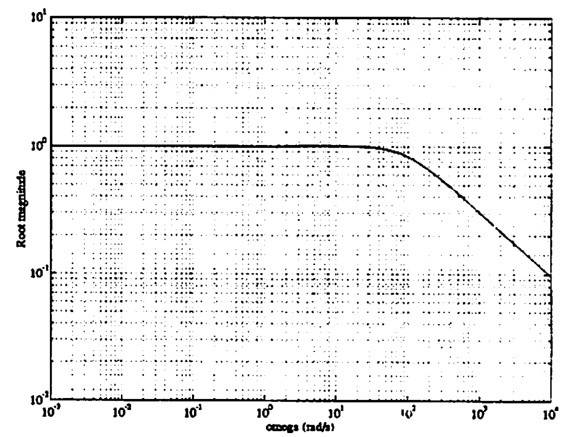
FIGURE 3. Roots of chain stability polynomial for CP controllers



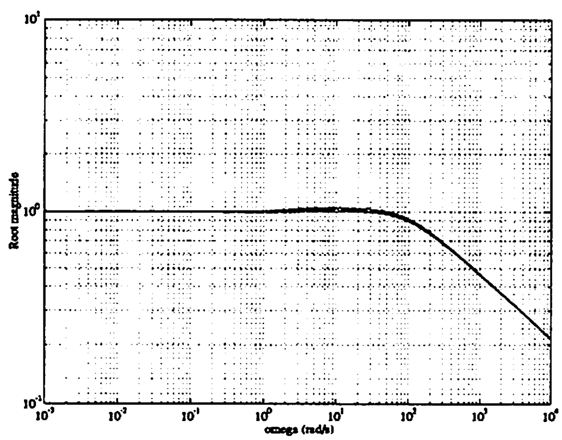
(h) Unconstrained L=1 CP, λ = 0



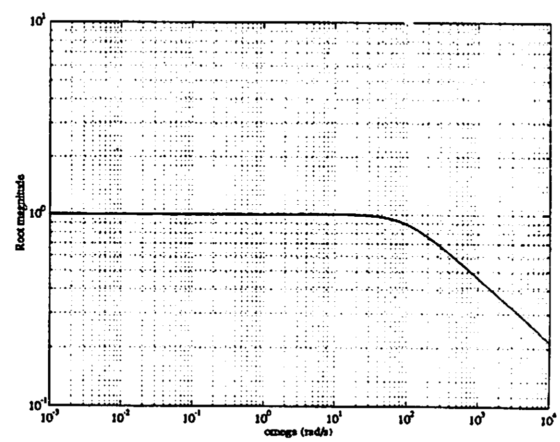
(i) Incremental L=2 CP, λ = 0



(j) Unconstrained L=2 CP. λ = 0

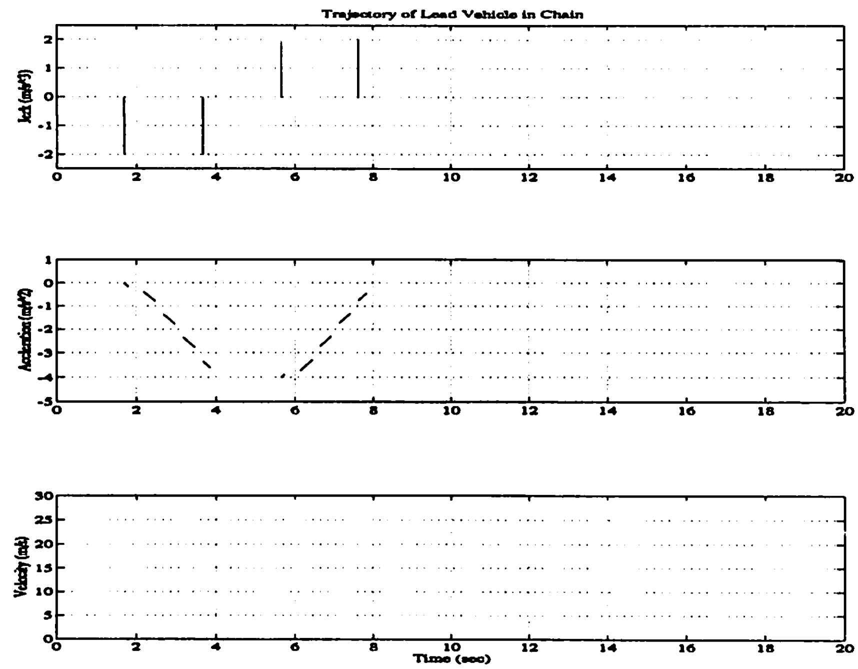


(k) Incremental L=3 CP, λ = 0

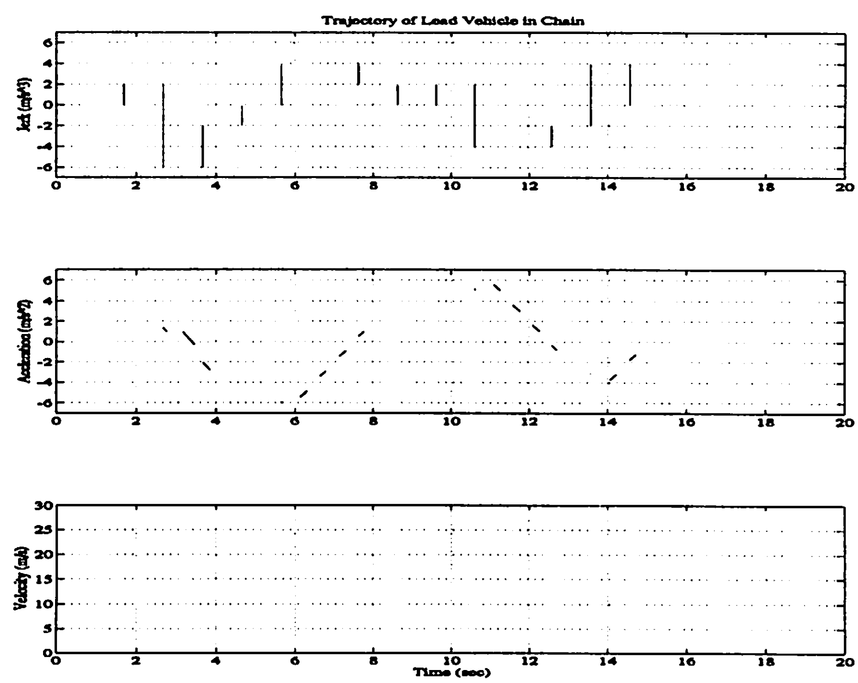


(l) Unconstrained L=3 CP, λ = 0

FIGURE 3. (cont.) Roots of chain stability polynomial for CP controllers



(a) Simple deceleration maneuver



(b) Sudden deceleration and acceleration maneuver

FIGURE 4. Lead vehicle disturbance trajectory in test scenarios

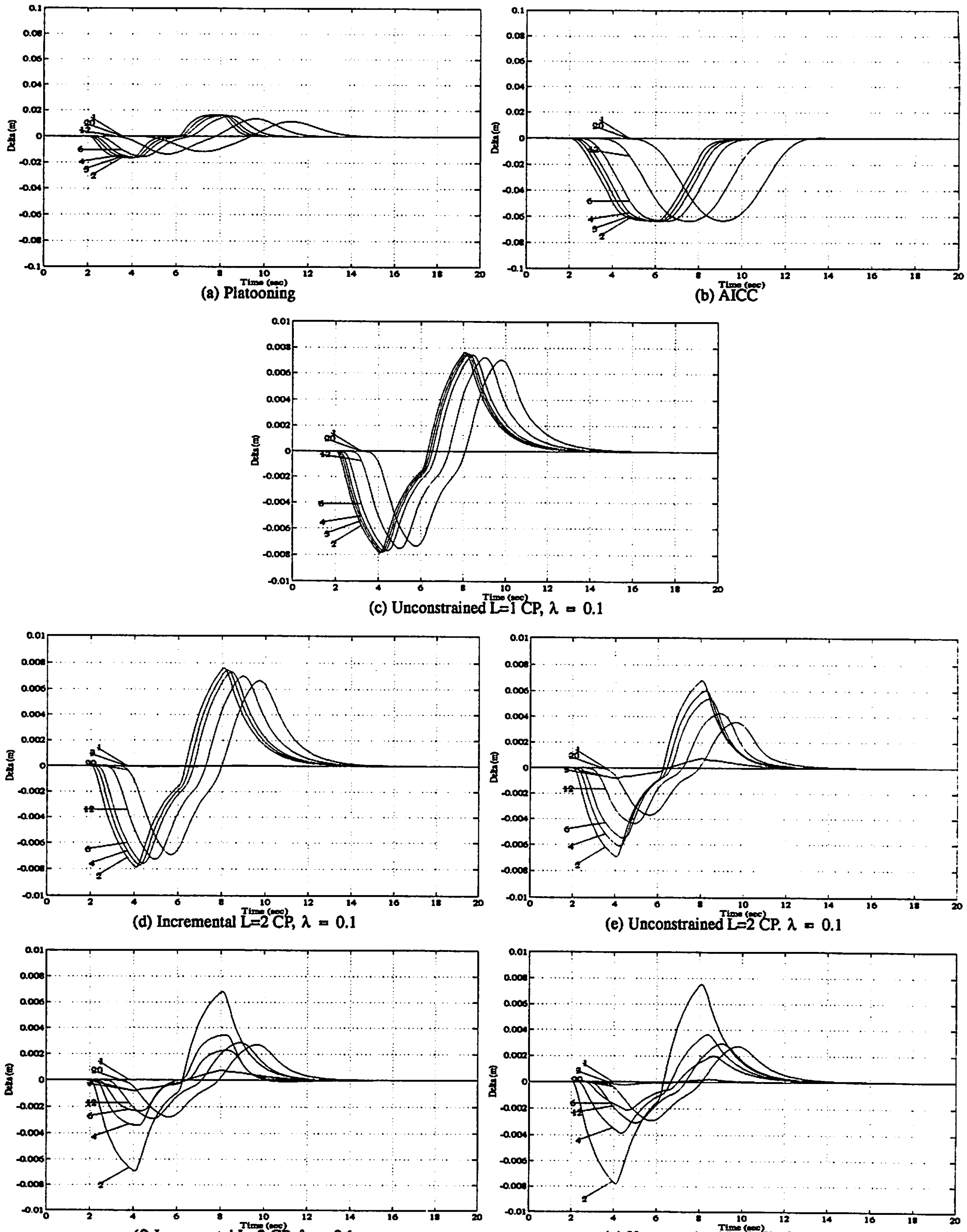
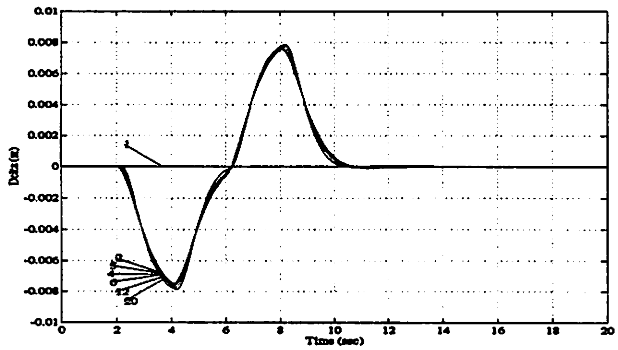
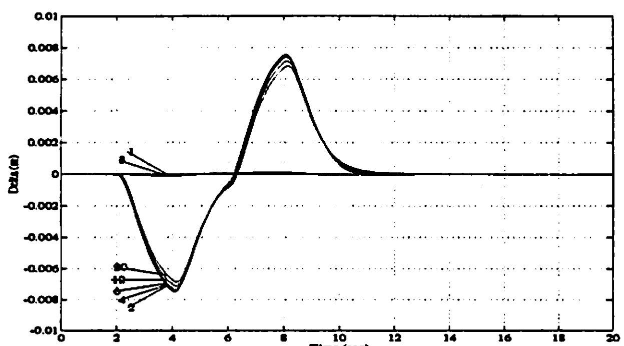


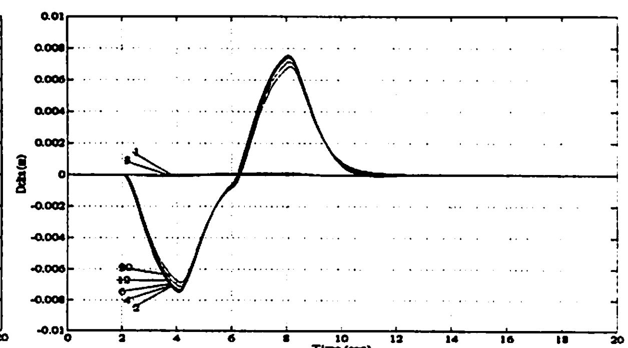
FIGURE 5. Deviation from desired vehicle spacing (delta) in response to disturbance in Figure 4a



(h) Unconstrained L=1 CP, λ = 0



(i) Incremental L=2 CP, λ = 0



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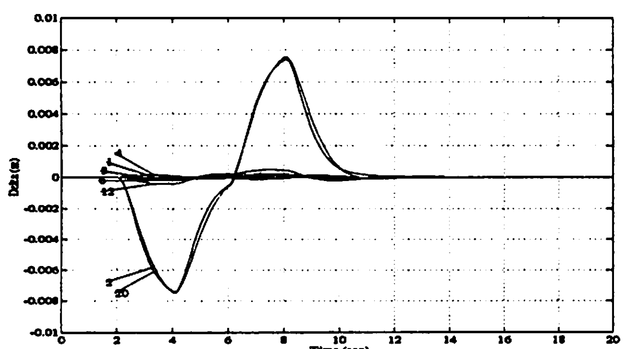
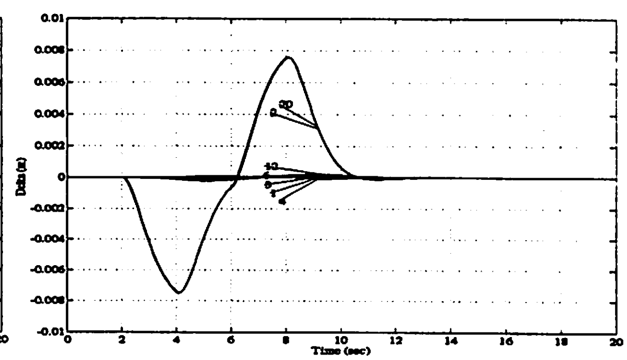
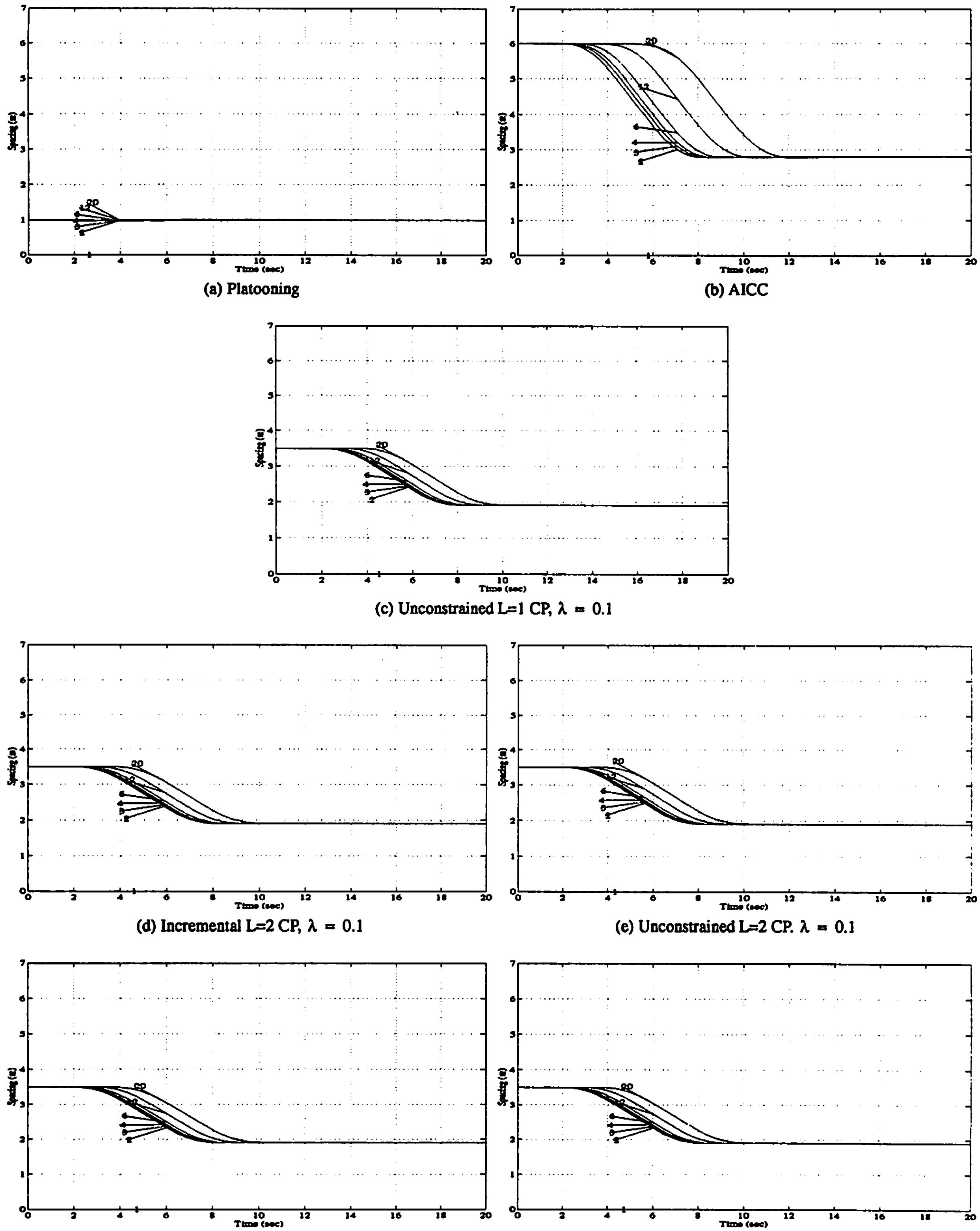


(k) Incremental L=3 CP, λ = 0



(l) Unconstrained L=3 CP, λ = 0

FIGURE 5. (cont.) Deviation from desired vehicle spacing (delta) in response to disturbance in Figure 4a



(a) Platooning

(b) AICC

(c) Unconstrained L=1 CP, λ = 0.1

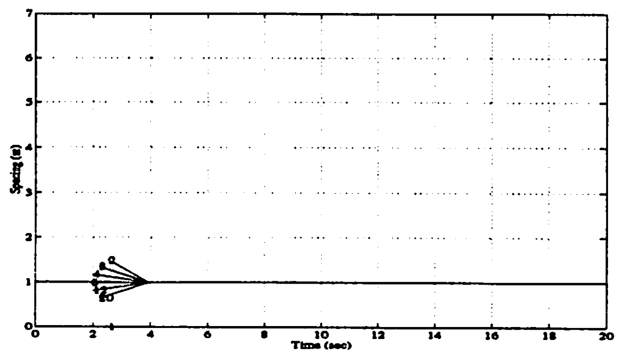
(d) Incremental L=2 CP, λ = 0.1

(e) Unconstrained L=2 CP, λ = 0.1

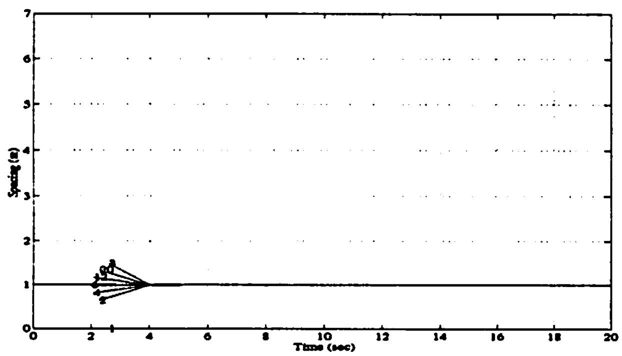
(f) Incremental L=3 CP, λ = 0.1

(g) Unconstrained L=3 CP, λ = 0.1

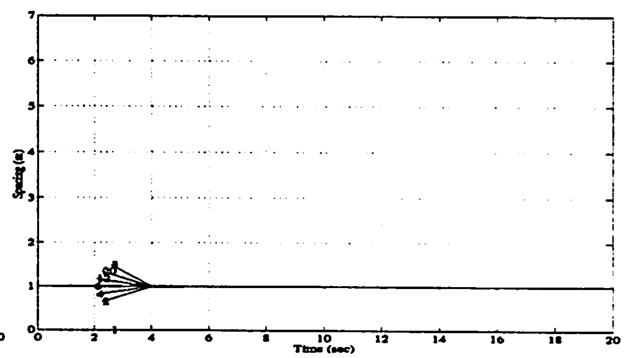
FIGURE 6. Inter-vehicle spacings in response to disturbance in Figure 4a



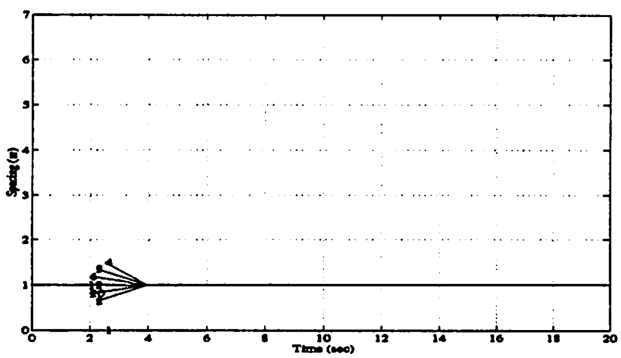
(h) Unconstrained L=1 CP, λ = 0



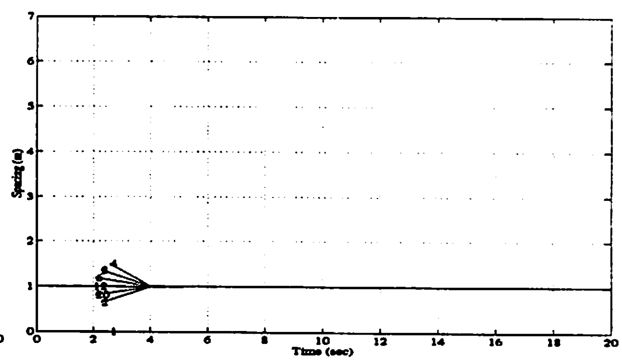
(i) Incremental L=2 CP, λ = 0



(j) Unconstrained L=2 CP. λ = 0



(k) Incremental L=3 CP, λ = 0



(l) Unconstrained L=3 CP, λ = 0

FIGURE 6. (cont.) Inter-vehicle spacings in response to disturbance in Figure 4a

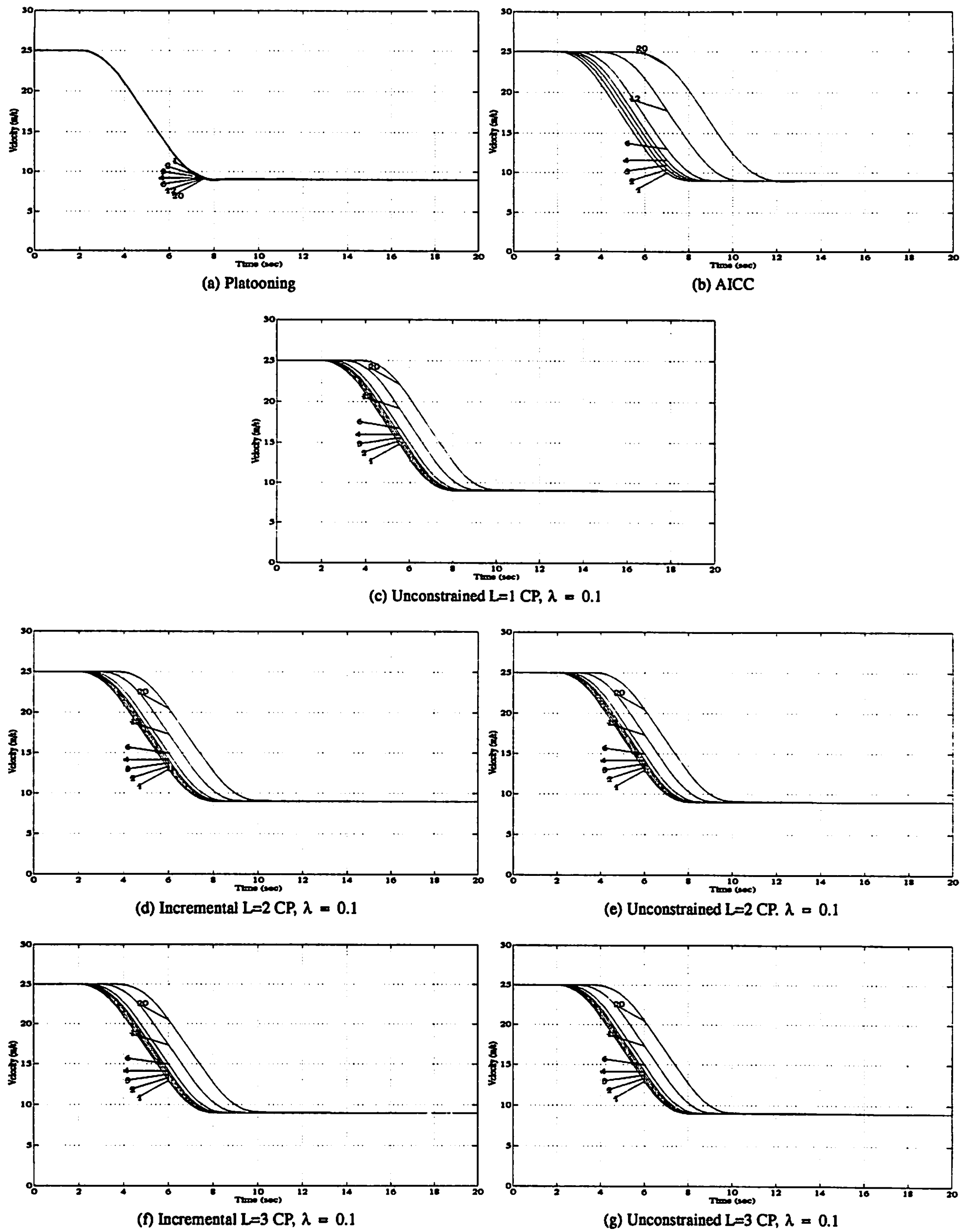
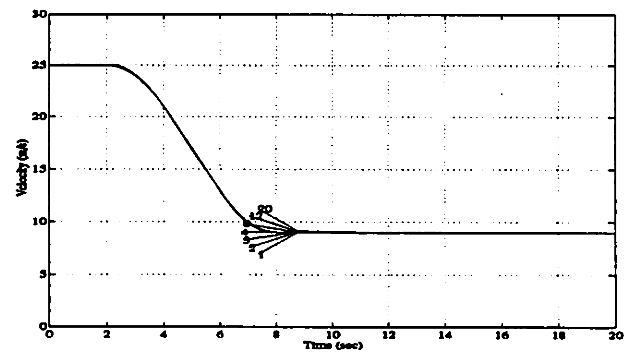
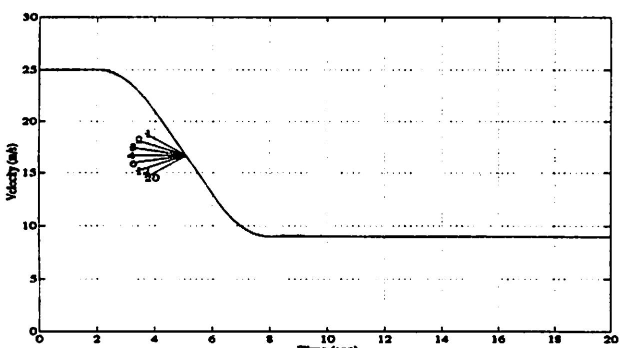


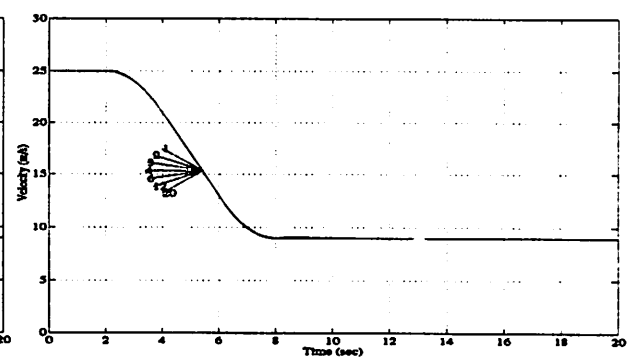
FIGURE 7. Vehicle velocities in response to disturbance in Figure 4a



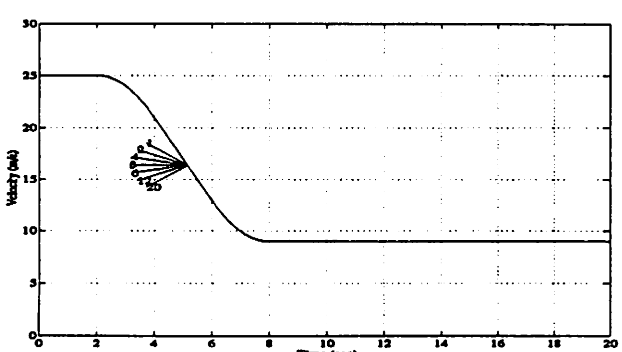
(h) Unconstrained L=1 CP, λ = 0



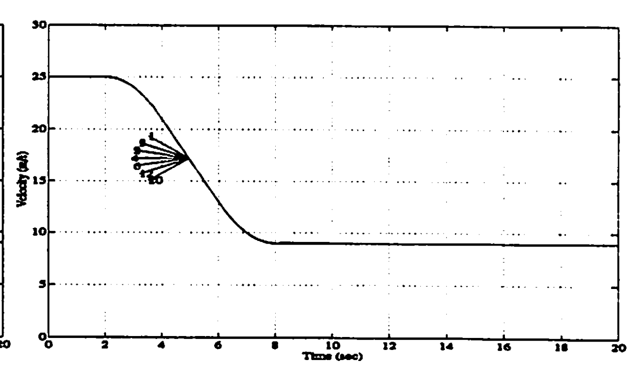
(i) Incremental L=2 CP, λ = 0



(j) Unconstrained L=2 CP. λ = 0



(k) Incremental L=3 CP, λ = 0



(l) Unconstrained L=3 CP, λ = 0

FIGURE 7. (cont.) Vehicle velocities in response to disturbance in Figure 4a

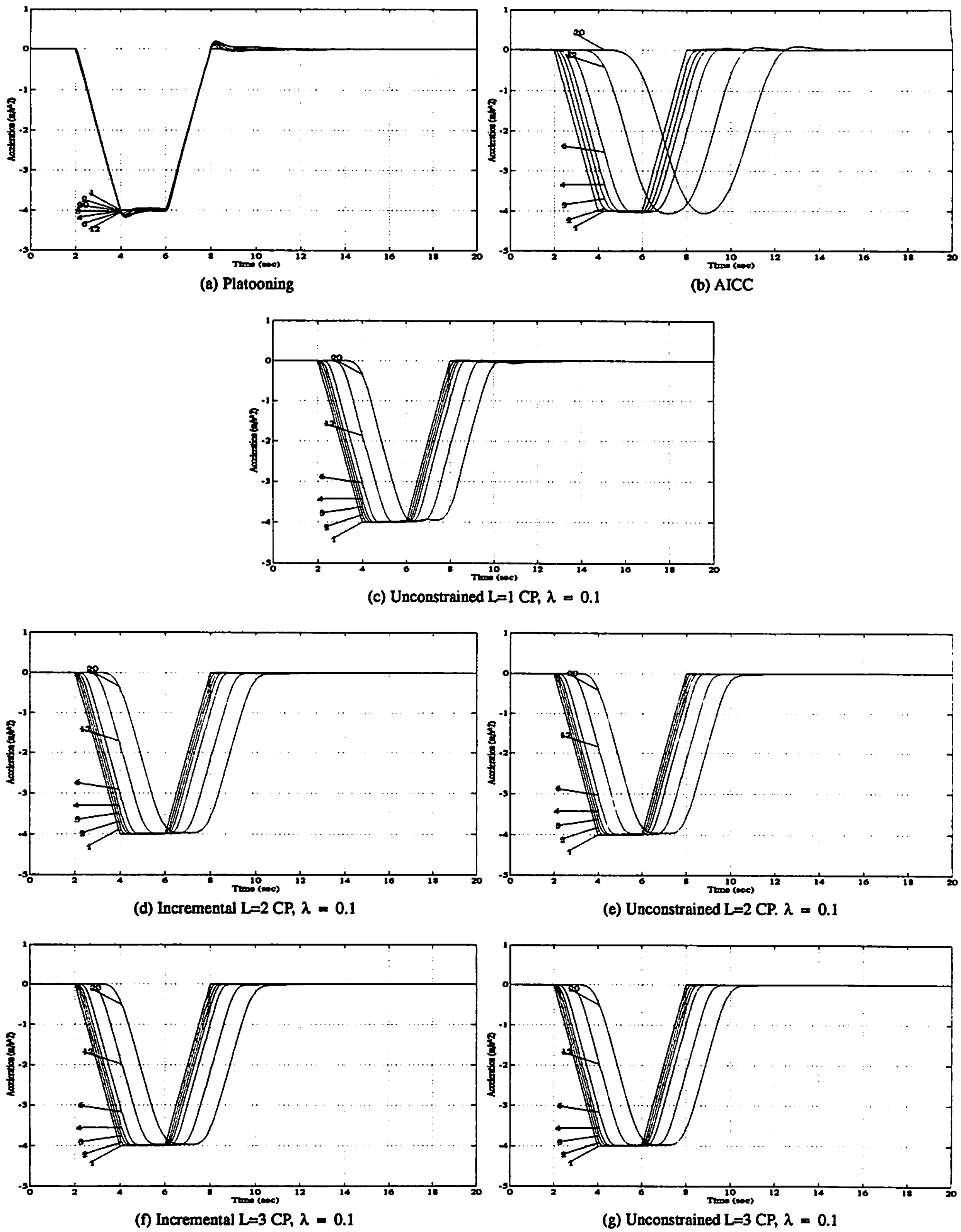
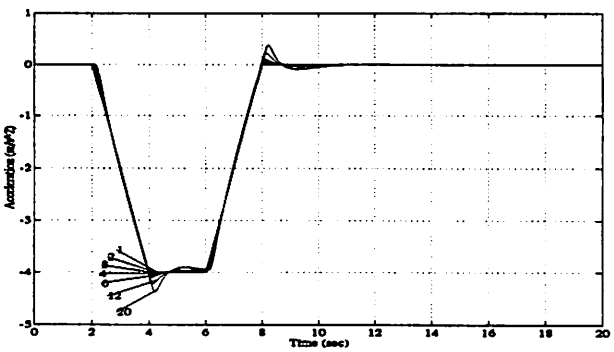
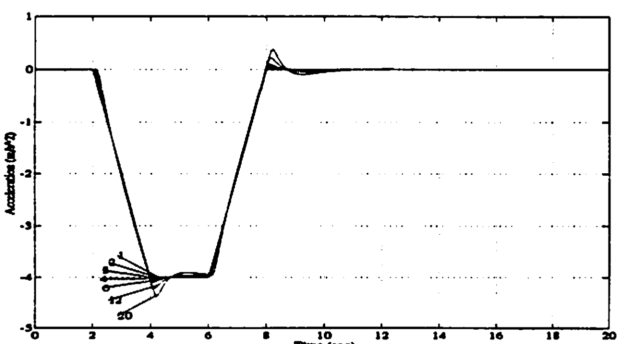


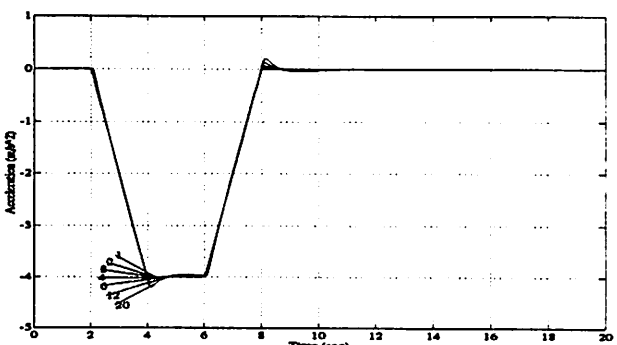
FIGURE 8. Vehicle accelerations in response to disturbance in Figure 4a



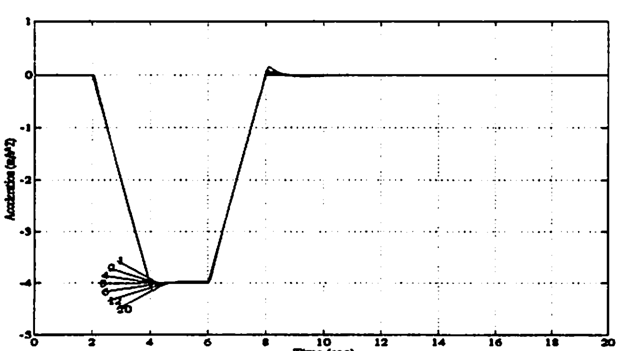
(h) Unconstrained L=1 CP, λ = 0



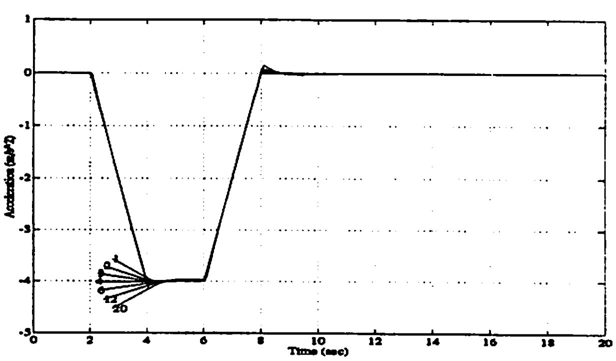
(i) Incremental L=2 CP, λ = 0



(j) Unconstrained L=2 CP. λ = 0

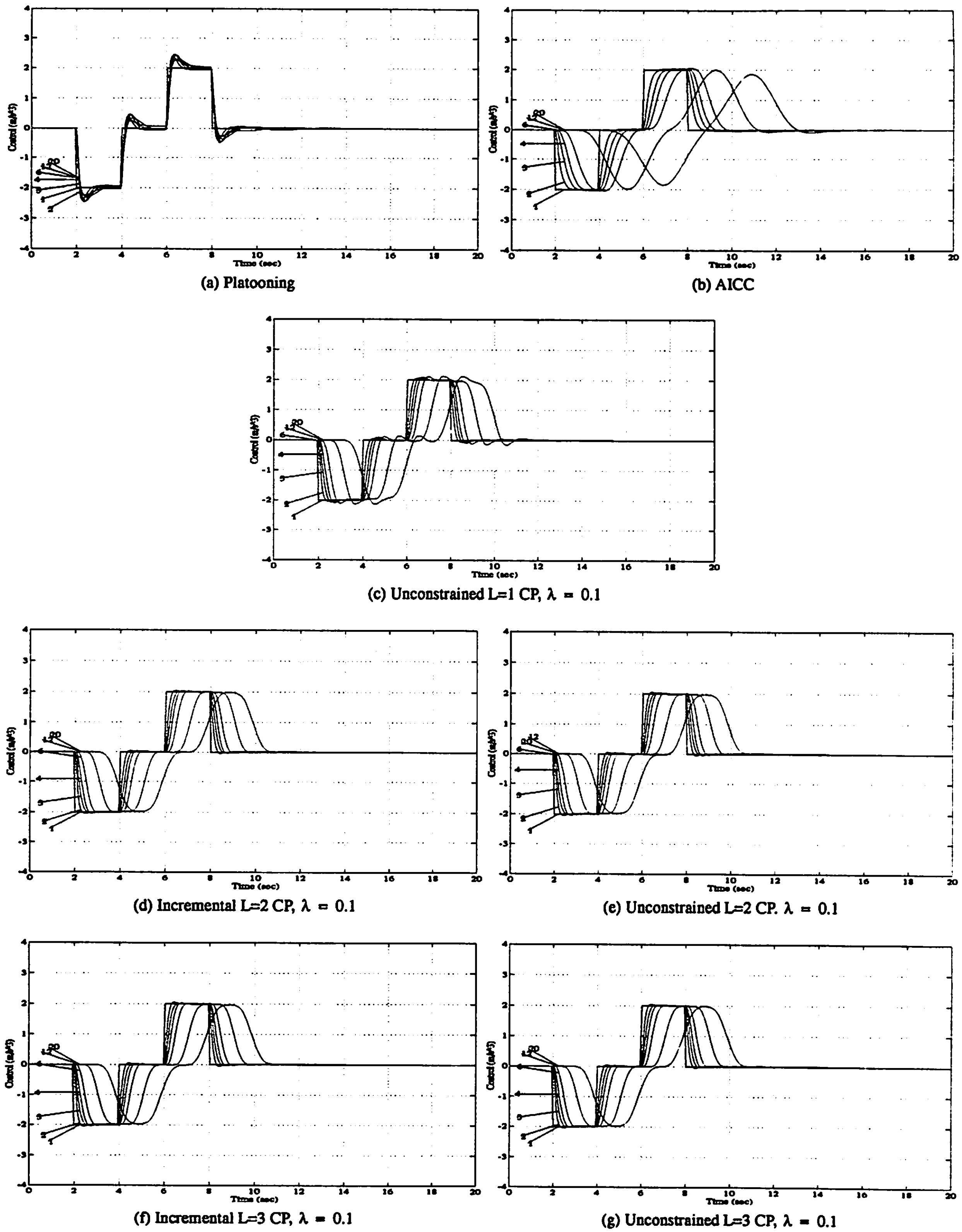


(k) Incremental L=3 CP, λ = 0



(l) Unconstrained L=3 CP, λ = 0

FIGURE 8. (cont.) Vehicle accelerations in response to disturbance in Figure 4a



(a) Platooning

(b) AICC

(c) Unconstrained L=1 CP, λ = 0.1

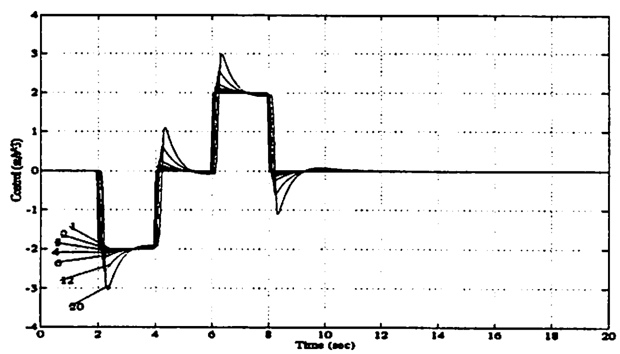
(d) Incremental L=2 CP, λ = 0.1

(e) Unconstrained L=2 CP, λ = 0.1

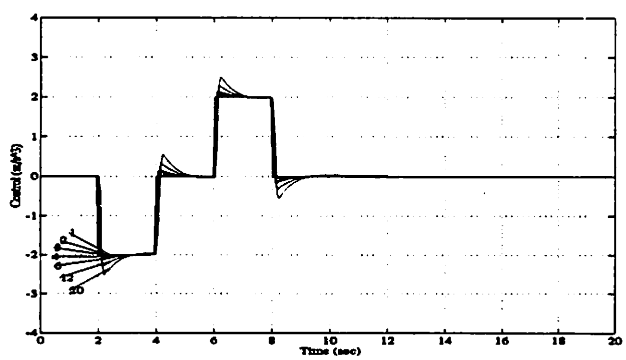
(f) Incremental L=3 CP, λ = 0.1

(g) Unconstrained L=3 CP, λ = 0.1

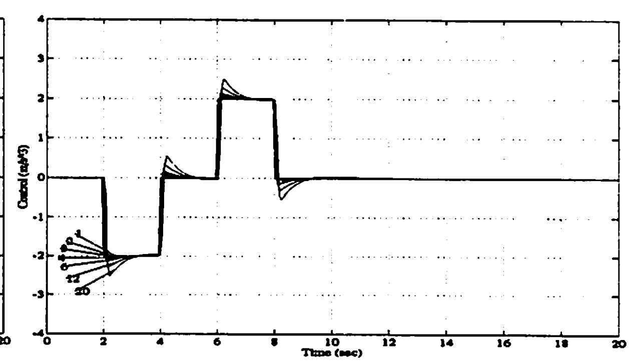
FIGURE 9. Linearized vehicle control effort in response to disturbance in Figure 4a



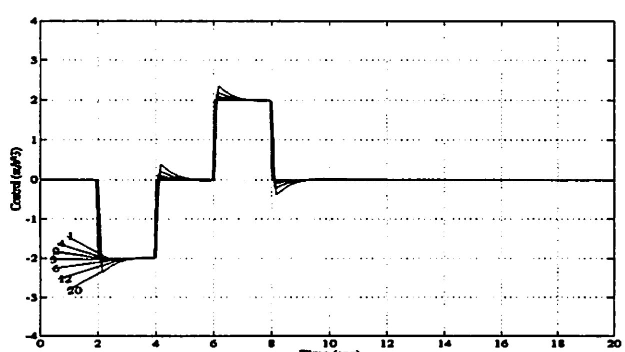
(h) Unconstrained L=1 CP, $\lambda = 0$



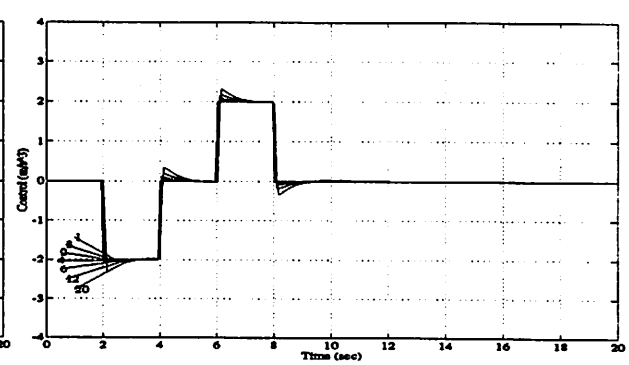
(i) Incremental L=2 CP, $\lambda = 0$



(j) Unconstrained L=2 CP, $\lambda = 0$

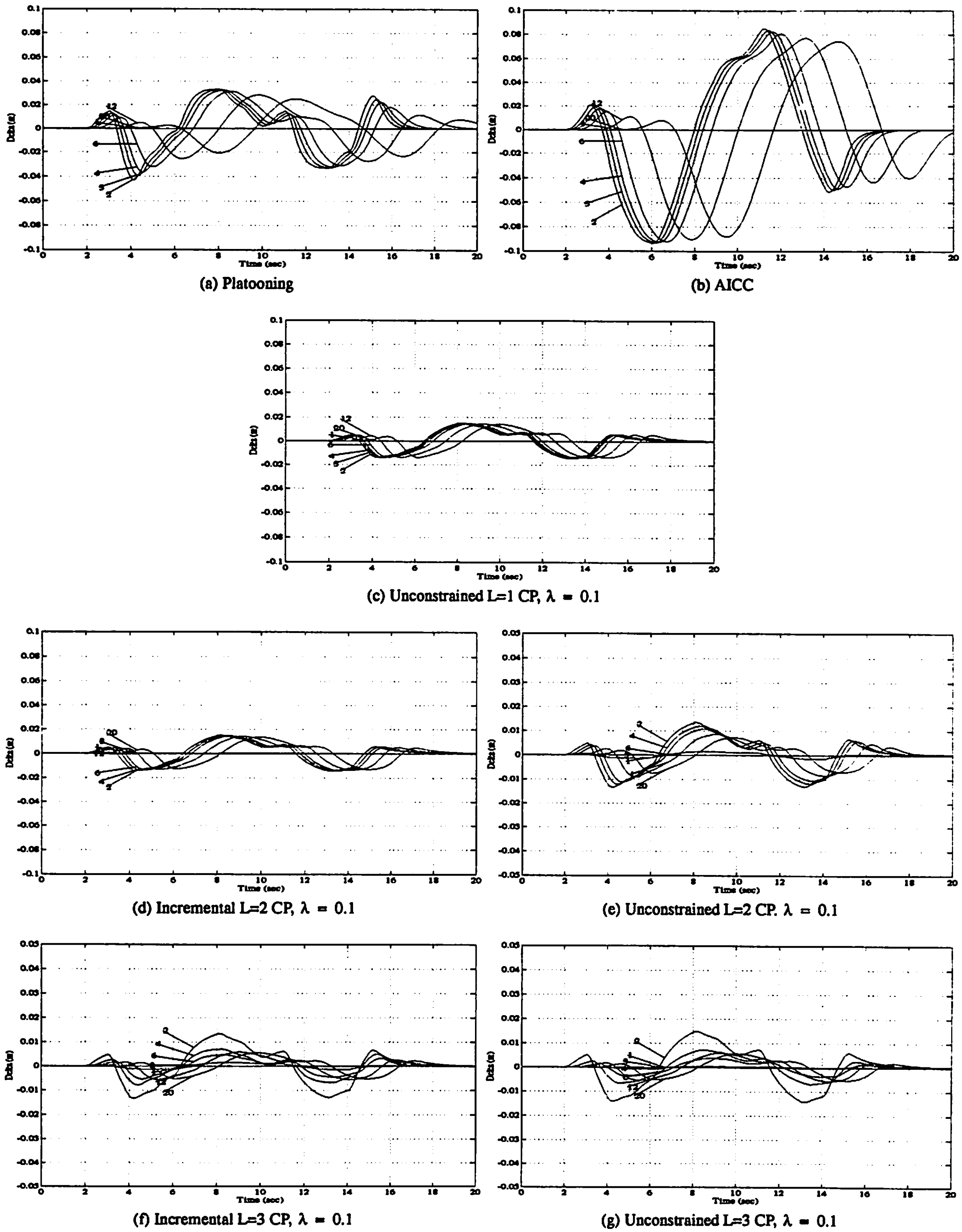


(k) Incremental L=3 CP, $\lambda = 0$



(l) Unconstrained L=3 CP, $\lambda = 0$

FIGURE 9. (cont.) Linearized vehicle control effort in response to disturbance in Figure 4a



(a) Platooning

(b) AICC

(c) Unconstrained L=1 CP, λ = 0.1

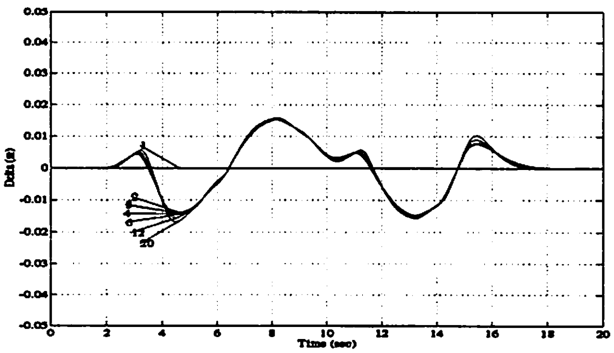
(d) Incremental L=2 CP, λ = 0.1

(e) Unconstrained L=2 CP. λ = 0.1

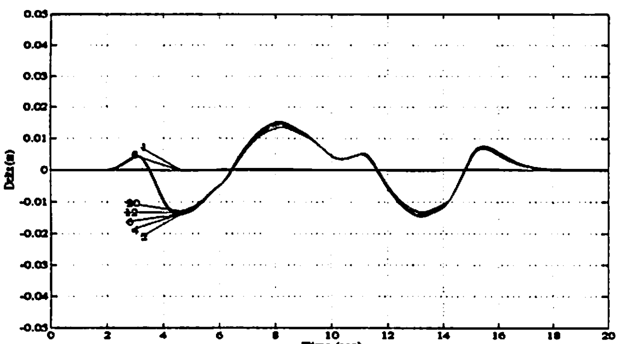
(f) Incremental L=3 CP, λ = 0.1

(g) Unconstrained L=3 CP, λ = 0.1

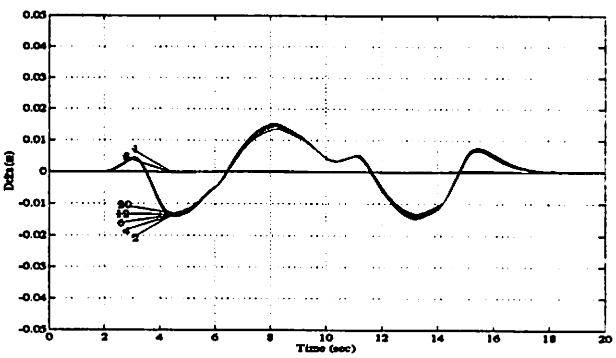
FIGURE 10. Deviation from desired vehicle spacing (delta) in response to disturbance in Figure 4b



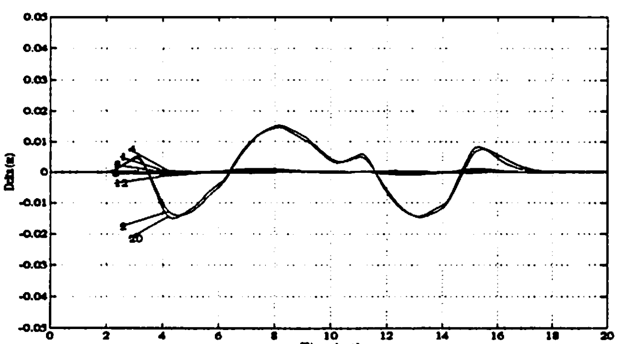
(h) Unconstrained L=1 CP, λ = 0



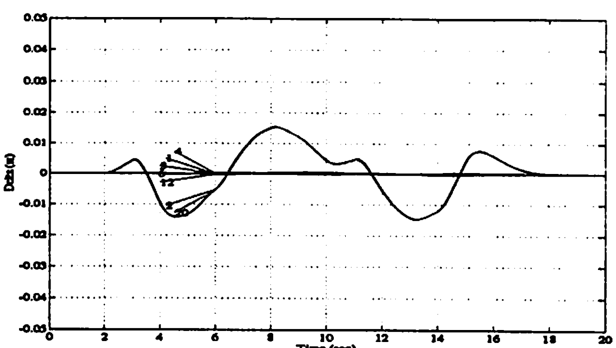
(i) Incremental L=2 CP, λ = 0



(j) Unconstrained L=2 CP, λ = 0



(k) Incremental L=3 CP, λ = 0



(l) Unconstrained L=3 CP, λ = 0

FIGURE 10. (cont.) Deviation from desired vehicle spacing (delta) in response to disturbance in Figure 4b