IMPLEMENTATION OF CHUA'S CIRCUIT WITH A CUBIC NONLINEARITY

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Guo-Qun Zhong

Memorandum No. UCB/ERL M94/42

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Guo-Qun Zhong

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Electronics Research Laboratory and
Department of Electrical Engineering and Computer Sciences
University of California, Berkeley
CA 94720, U. S. A.

Abstract

This paper reports an implementation of Chua's circuit with a smooth nonlinearity, described by a cubic polynomial. Some bifurcation phenomena and chaotic attractors observed experimentally from the laboratory model and simulated by computer for the model are also presented. Comparing both the observations and simulations, the results are satisfactory.

I Introduction

The well-known Chua's circuit shown in Fig. 1, in which the nonlinearity of the Chua's diode is described by a piecewise-linear function, has been studied worldwide since it was invented by Chua in 1983 and confirmed by computer simulation and experimental observation, respectively [1–4].

The state equations describing the circuit are as follows:

$$\left.\begin{aligned} \frac{dv_{c1}}{dt} &= \frac{1}{C_1}\left[\frac{1}{R}(v_{c2}-v_{c1})-g(v_{c1})\right] \\ \frac{dv_{c2}}{dt} &= \frac{1}{C_2}\left[\frac{1}{R}(v_{c1}-v_{c2})+i_L\right] \\ \frac{di_L}{dt} &= \frac{1}{L}\left[-v_{c2}-R_0 i_L\right] \end{aligned}\right\} \tag{1}$$

where $g(v_R)$ is a piecewise-linear function defined by

$$g(v_R) = G_b v_R + \frac{1}{2}(G_a - G_b)[|v_R + E| - |v_R - E|] \tag{2}$$

and R_0 denotes the small positive resistance of the inductor [1]. Most interesting chaotic phenomena and chaotic dynamics can be described by this piecewise-linear Chua's equation.

[1] In the recent global unfolding of Chua's circuit [5], R_0 may assume any positive or negative value. This generalization is now called Chua's oscillator [2].

Recent numerical simulations reveal, however, that not all features of a real circuit are captured correctly by this piecewise-linear circuit [6]. It is therefore desirable to realize a smooth nonlinearity described by the following cubic polynomial for Chua's circuit:

$$g(v_R) = a_0 + av_R + bv_R^2 + cv_R^3 \tag{3}$$

In Section II we present a practical implementation of this cubic nonlinearity. Some bifurcation sequences and chaotic attractors observed experimentally and simulated via the software INSITE are presented in Section III.

II Practical implementation of a cubic polynomial

The basic circuit we use to realize the cubic polynomial (3) is a multiplier circuit with a feedback loop, as shown in Fig. 2(a). The equivalent circuit of the multiplier circuit is shown in Fig. 2(b). By applying Kirchhoff's Voltage Law to the equivalent circuit, we have

$$i = [v_1 - \frac{v_1 v_2}{10V} - v_0]\frac{1}{R} \tag{4}$$

where the factor $10V$ is an inherent scaling voltage in the multiplier, and v_0 is a dc voltage.

Obviously, when $v_2 = v_1$ and $v_2 = v_1^2$, we obtain

$$i_1 = [v_1 - \frac{v_1^2}{10V} - v_0]\frac{1}{R} \tag{5}$$

and

$$i_2 = [v_1 - \frac{v_1^3}{10V} - v_0]\frac{1}{R} \tag{6}$$

respectively.

By adding (5) and (6), we obtain the following desired cubic polynomial:

$$i = a_0 + av_1 + bv_1^2 + cv_1^3 \tag{7}$$

where $i = i_1 + i_2, a_0 = -\frac{2v_0}{R}, a = \frac{2}{R}, b = -\frac{1}{R}\frac{1}{10V}, c = -\frac{1}{R}\frac{1}{10V}$.

The circuit implementation for the cubic polynomial (7) is shown in Fig. 3.

Note from the procedure above that any polynomial with higher-order terms and real coefficients can be realized in the same way. By choosing the signs of the resistors R_1 and R_2, the signs of the coefficients a_0, a, b, and c can be changed, respectively.

III Bifurcation and chaos in Chua's circuit with a cubic nonlinearity

1 Practical implementation of Chua's circuit with a cubic nonlinearity

Since the desired $v-i$ characteristic of the nonlinear resistor N_R in Chua's circuit is an odd-symmetric function with respect to origin, here we use the cubic polynomial (7) with the coefficients $a_0 = 0, a < 0, b = 0$, and $c > 0$ for the nonlinearity of Chua's circuit in Fig. 1, i.e.,

$$i_R = g(v_R) = av_R + cv_R^3 \tag{8}$$

where $a < 0$ and $c > 0$.

The practical circuit for realizing the cubic polynomial (8) is shown in Fig. 4(a). The two-terminal nonlinear resistor N_R consists of one Op Amp, two multipliers and 5 resistors. In the circuit we utilize two analog multipliers AD633JN and an Op Amp AD711kN, both manufactured by Texas Instruments. The connections of the Op Amp AD711kN and the resistors R_1, R_2 and R_3 form an equivalent negative resistance R_e since we have $R_e = -R_3$ when $R_1 = R_2$ and the Op Amp operates in its linear region, in order to obtain the desired coefficients $a < 0$ and $c > 0$ in (8). Inversely, in the case where R_e is a positive resistance, we will obtain $a > 0$ and $c < 0$ in (8). The driving-point $v-i$ characteristic of N_R is as below:

$$i_R = g(v_R) = -\frac{1}{R_3}v_R + \frac{R_4+R_5}{R_3R_4}\frac{1}{10V}\frac{1}{10V}v_R^3 = av_R + cv_R^3 \tag{9}$$

where $a = -\frac{1}{R_3}$, $c = \frac{R_4+R_5}{R_3R_4}\frac{1}{10V}\frac{1}{10V}$. The factor $10V$ is an inherent scaling voltage in the multiplier, as mentioned above. The network connected by the resistors R_4 and R_5 increases the gain of the system by the ratio $\frac{R_4+R_5}{R_4}$ in order to obtain a variable scale factor $\frac{R_4+R_5}{R_4}$. This ratio is limited to 100 in practical applications[2]. Usually, choose $R_4 \geq 1k\Omega$, and $R_5 \leq 100k\Omega$. Note that the coefficients a and c can be adjusted by tuning the resistance R_3, and c can independently be adjusted by tuning the resistance R_5.

In our experimental model, we choose $R_1 = R_2 = 2k\Omega, R_3 = 1.668k\Omega, R_4 = 3.01k\Omega$, and $R_5 = 7.91k\Omega$. The $v-i$ characteristics of the Chua's diode N_R calculated according to the polynomial (9) and measured experimentally, based on the parameter values listed above, are shown in Fig. 4(b) and (c), respectively, where $a = -0.599mS, c = 0.0218mS/V^2$. Note that there is a very good agreement between the two curves.

2 Bifurcation and chaos from Chua's circuit with a cubic nonlinearity

The state equations for Chua's circuit in Fig. 1 with a cubic nonlinearity are as follows:

[2] Refer to Data Converter Reference Manual by Analog Devices.

$$\left.\begin{aligned}\frac{dv_{c1}}{dt} &= \frac{1}{C_1}[\frac{1}{R}(v_{c2}-v_{c1})-g(v_{c1})] \\ \frac{dv_{c2}}{dt} &= \frac{1}{C_2}[\frac{1}{R}(v_{c1}-v_{c2})+i_L] \\ \frac{di_L}{dt} &= \frac{1}{L}[-v_{c2}-R_0 i_L]\end{aligned}\right\} \quad (10)$$

where

$$g(v_{c1}) = av_{c1} + cv_{c1}^3 \quad (11)$$

Figs. 5(a)-(r) show the bifurcation sequence with respect to R and the chaotic attractors observed experimentally from our experimental setup , including the time waveforms and the spectra of voltages v_{c1} relative to these phase portraits. Note from these oscilloscope pictures that there is a period-doubling route to chaos similar to that observed from Chua's circuit with a piecewise-linear function.

By adjusting parameters C_1, C_2, and L, a similar bifurcation phenomenum can also be observed, respectively. As an example, we present the bifurcation sequence with respect to capacitor C_2 in Fig. 6(a)-(h). It can be noted from these observations that there is a much wider range of the bifurcation with respect to C_2, e.g., a Double Scroll Chua's attractor can still be observed when C_2 increases up to $600nF$, as shown in Fig. 6(g). In this case the components of high frequencies in Fig. 6(h) are reduced, as expected. This feature will probably be of interest in sound synthesis.

In addition, we do some simulations of this smooth model using the software INSITE. The periodic orbits and chaotic attractors are presented in Figs. 7(a)-(f). Note that the simulations confirm completely our experimental observations.

IV Concluding remarks

It is well known that Chua's circuit can exhibit a wide variety of nonlinear behaviors, it has become an attractive paradigm for experimental investigation of chaotic dynamical systems. Though most of the interesting chaotic phenomena can be described by Chua's circuit with a piecewise-linear Chua's diode, some subtle features of the real circuit may be missed by the piecewise-linear approximation. The implementation of a smooth nonlinearity with a cubic polynomial (with even higher order terms) presented in this paper contributes a robust model, with which a more complete experimental model of Chua's circuit can be used for experimental investigations. This model is robust and can be easily integrated in a chip. Furthermore, this method can also be used to design Chua's diodes with almost any smooth nonlinearity.

Acknowledgement

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References

[1] L. O. Chua, "The genesis of Chua's circuit," *Archiv Elektronik Ubertragungstechnik*, vol. 46, no. 4, pp. 250–257, 1992.

[2] R. Madan (Guest Editor), "Special issue on Chua's circuit: A paradigm for chaos, part I: Introduction and Applications," *J. Circuit Syst. Comput.*, vol. 3, March 1993.

[3] T. Matsumoto, "A chaotic attractor from Chua's circuit," *IEEE Trans. Circuits Syst.*, vol. CAS–31, no. 12, pp. 1055–1058, 1984.

[4] G. Q. Zhong and F. Ayrom, "Experimental confirmation of chaos from Chua's circuit," *Int. J. Circuit Theory Applications*, vol. 13, no. 1, pp. 93–98, 1985.

[5] L. O. Chua, "Global unfolding of Chua's circuit," *IEICE TRANS. FUNDAMENTALS*, vol. E76–A, no. 5, pp. 704–734, May 1993.

[6] A. I. Khibnik, D. Roose, and L. O. Chua, "On periodic orbits and homoclinic bifurcations in Chua's circuit with a smooth nonlinearity," *Int. J. Bifurcation and Chaos*, vol. 3, no. 2, pp. 363–384, 1993.

Figure Captions

Fig. 1 Chua's circuit.

Fig. 2 (a) The multiplier circuit with a feedback loop,
(b) The equivalent circuit of the multiplier circuit in Fig. 2(a).

Fig. 3 A circuit implementation for a cubic-polynomial Chua's diode.

Fig. 4 (a) Practical circuit for realizing a cubic polynomial $v - i$ characteristic.
(b) The calculated $v - i$ characteristic of Chua's diode N_R with a cubic nonlinearity $i = av + cv^3$.
(c) The measured $v - i$ characteristic of Chua's diode N_R with a cubic nonlinearity $i = av + cv^3$, Horizontal axis v, scale: $1V$/div, Vertical axis i, scale: $0.5mA$/div, $a = -0.59mS, c = 0.02mS/V^2$.

Fig. 5 Bifurcation sequence with respect to the parameter R
(a), (d), (g), (j), (m), and (p) Phase portraits in the $v_{c1} - v_{c2}$ plane, Horizontal axis v_{c1}, scale: $1V$/div; Vertical axis v_{c2}, scale: $0.2V$/div,
(b), (e), (h), (k), (n), and (q) Time waveforms, Horizontal axis t, scale: (b), (e), and (h) $500\mu S$/div, (k), (n), and (q) $1mS$/div; Vertical axis v_{c2} (top) and v_{c1} (bottom), scale: $0.5V$/div for v_{c2}, $2V$/div for v_{c1},
(c), (f), (i), (l), (o), and (r) Spectra of voltages v_{c1}, scale: $10db$/div.
Parameter values: $C_1 = 7nF, C_2 = 78nF, L = 18.91mH, R_0 = 14.99\Omega$ (the internal

resistance of the inductor L), $a = -0.59mS, c = 0.02mS/V^2$.
(a)–(c) $R = 2200\Omega$, (a) period–1 limit cycle;
(d)–(f) $R = 2103\Omega$, (d) period–2 limit cycle;
(g)–(i) $R = 2090\Omega$, (g) period–4 limit cycle;
(j)–(l) $R = 2083\Omega$, (j) intermittency of type I;
(m)–(o) $R = 2033\Omega$, (m) spiral Chua's attractor;
(p)–(r) $R = 1964\Omega$, (p) Double-Scroll Chua's attractor .

Fig. 6 Bifurcation sequence with respect to the parameter C_2
(a)–(g) Phase portraits in $v_{c1} - v_{c2}$ plane, Horizontal axis v_{c1}, scale: $1V$/div; Vertical axis v_{c2}, scale: $0.2V$/div, (h) Spectrum of voltage v_{c1}, scale: $10db$/div.
Parameter values: $C_1 = 7nF, L = 18.91mH, R = 1964\Omega, R_0 = 14.99\Omega$ (the internal resistance of the inductor L), $a = -0.59mS, c = 0.02mS/V^2$.
(a) $C_2 = 30nF$, dc equilibrium point;
(b) $C_2 = 32nF$, period–1 limit cycle;
(c) $C_2 = 54nF$, period–2 limit cycle;
(d) $C_2 = 57nF$, intermittency of type I;
(e) $C_2 = 64nF$, spiral Chua's attractor;
(f) $C_2 = 78nF$, Double-Scroll Chua's attractor;
(g) $C_2 = 600nF$, Double-Scroll Chua's attractor having a much lower frequency spectrum;
(h) $C_2 = 600nF$, spectrum of voltage v_{c1}.

Fig. 7 Simulated bifurcation sequence with respect to the parameter R
Phase portraits in $v_{c1} - v_{c2}$ plane, Horizontal axis v_{c1}, Vertical axis v_{c2}.
Parameter values: $C_1 = 7nF, C_2 = 78nF, L = 18.91mH, R_0 = 14.99\Omega$ (the internal resistance of the inductor L) $a = -0.59mS, c = 0.02mS/V^2$.
(a) $R = 2200\Omega$, period–1 limit cycle;
(b) $R = 2140\Omega$, period–2 limit cycle;
(c) $R = 2134\Omega$, period–4 limit cycle;
(d) $R = 2131\Omega$, period–8 limit cycle;
(e) $R = 2083\Omega$, spiral Chua's attractor;
(f) $R = 1964\Omega$, Double-Scroll Chua's attractor.

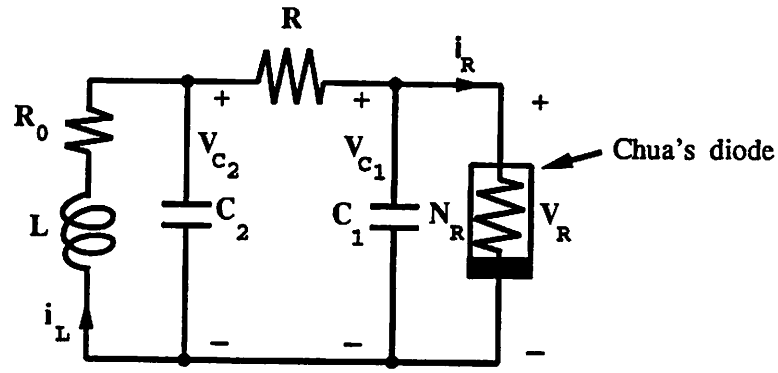

R
i_R
R_0
+
+
+
V_{C_2}
V_{C_1}
Chua's diode
L
C_2
C_1
N_R
V_R
i_L
–
–
–

Fig. 1

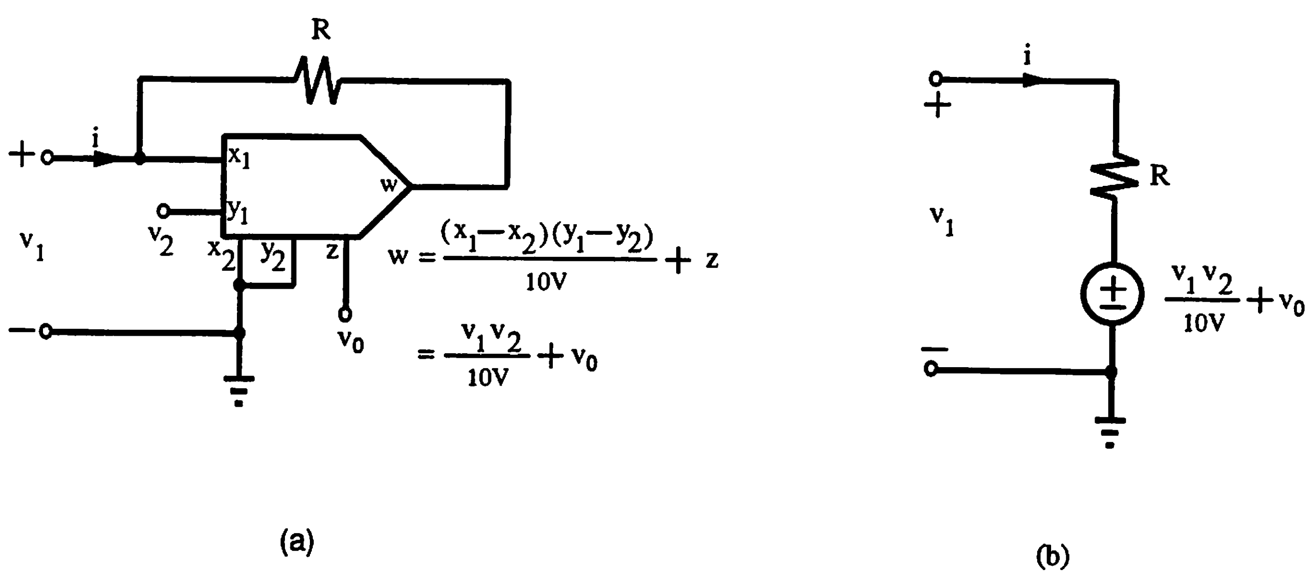

R
i
+
x_1
y_1
w
v_1
v_2
x_2
y_2
z
$w = \frac{(x_1-x_2)(y_1-y_2)}{10V} + z$
–
v_0
$= \frac{v_1 v_2}{10V} + v_0$
(a)
i
+
R
v_1
±
$\frac{v_1 v_2}{10V} + v_0$
–
(b)

Fig. 2

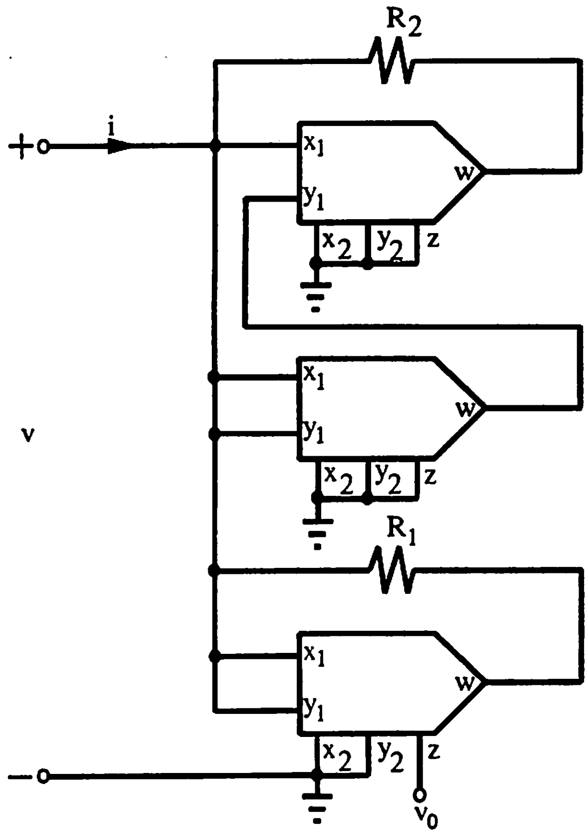

R2
+
i
x1
w
y1
x2
y2
z
v
x1
y1
w
x2
y2
z
R1
x1
w
y1
x2
y2
z
−
v0

Fig. 3

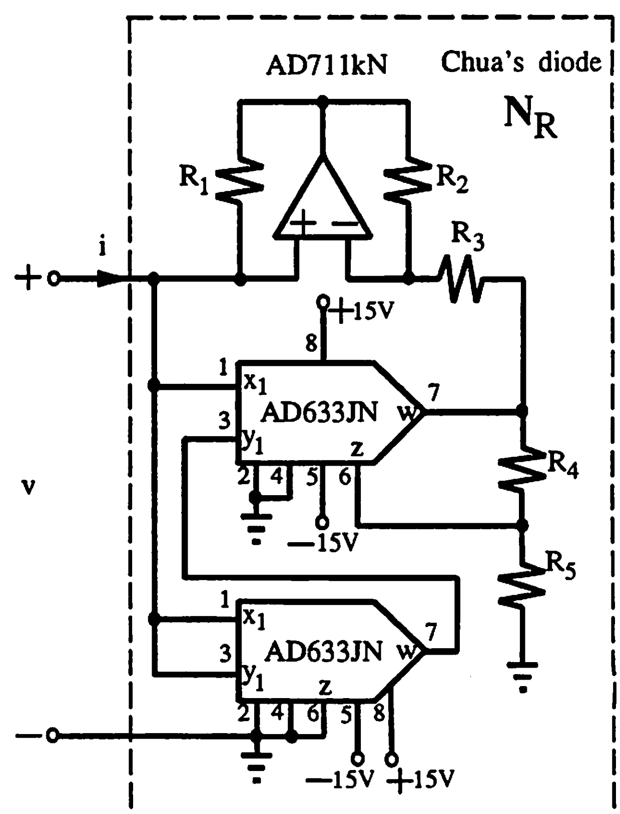
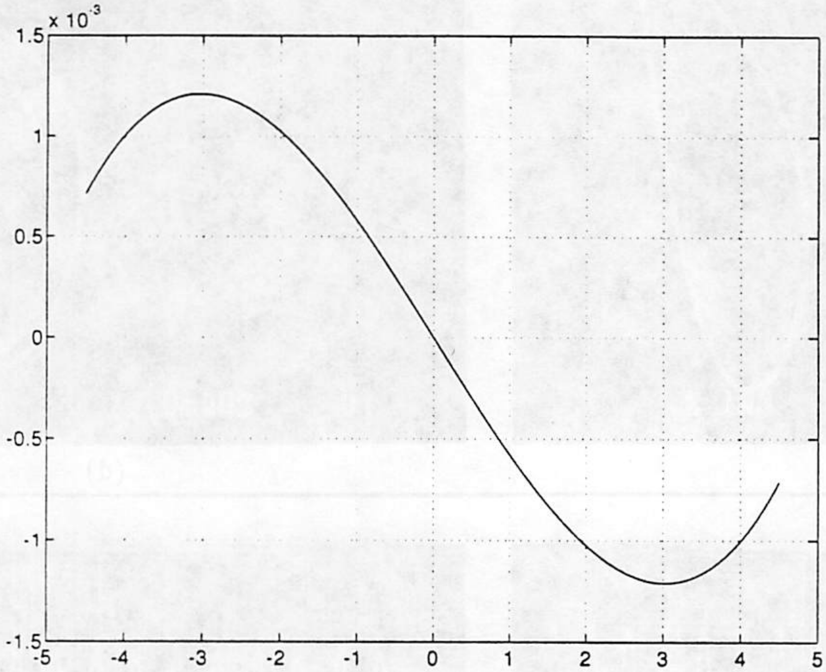
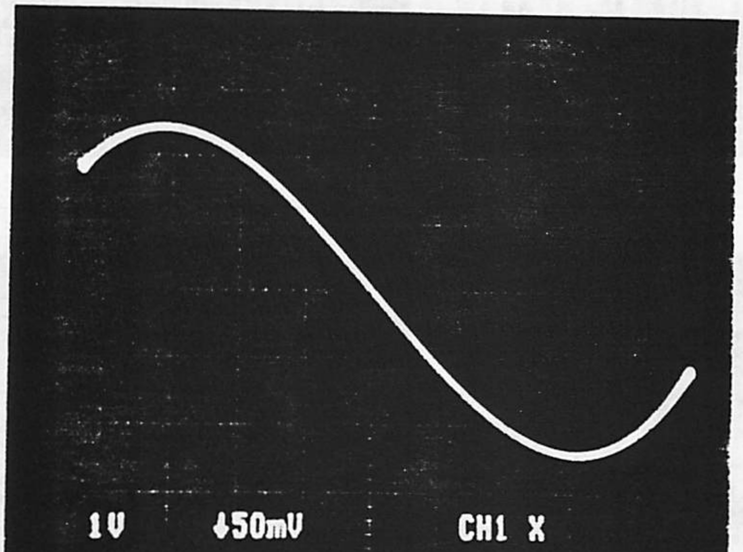

AD711kN
Chua's diode
NR
R1
R2
i
+
R3
+15V
8
1
x1
7
AD633JN
w
3
y1
z
R4
v
2
4
5
6
−15V
R5
1
x1
7
AD633JN
w
3
y1
z
−
2
4
6
5
8
−15V
+15V

Fig. 4 (a)

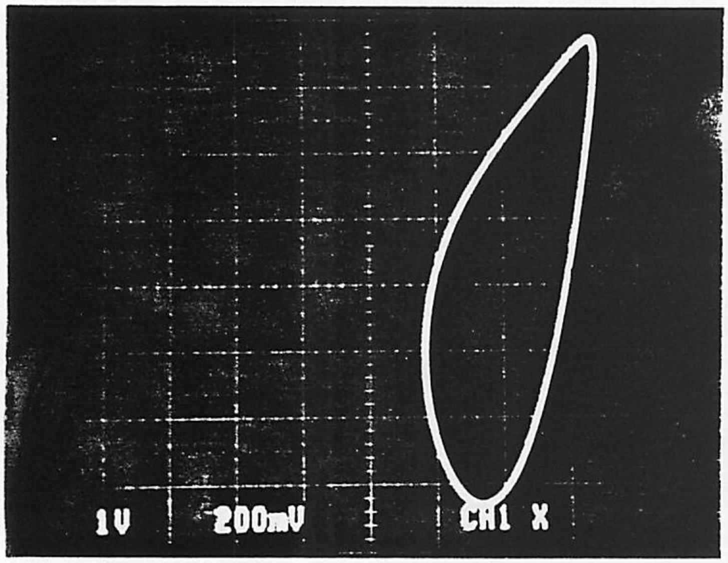


(b)

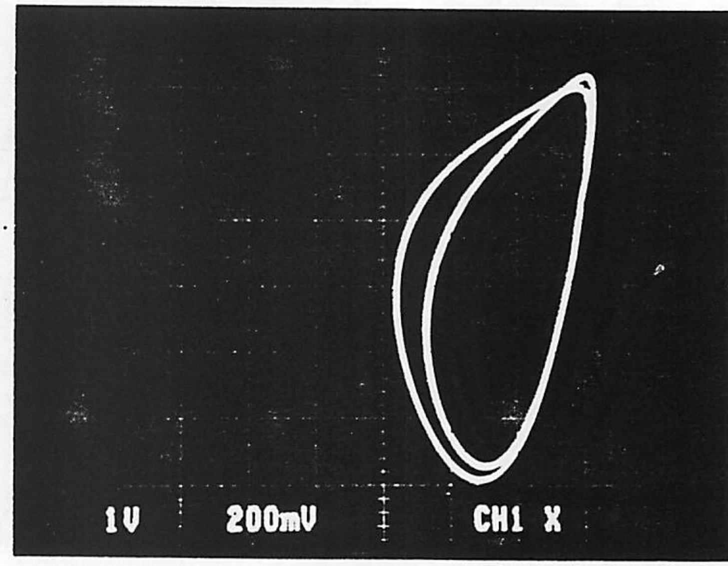


(c)

Fig. 5


1V 200mV CH1 X

(a)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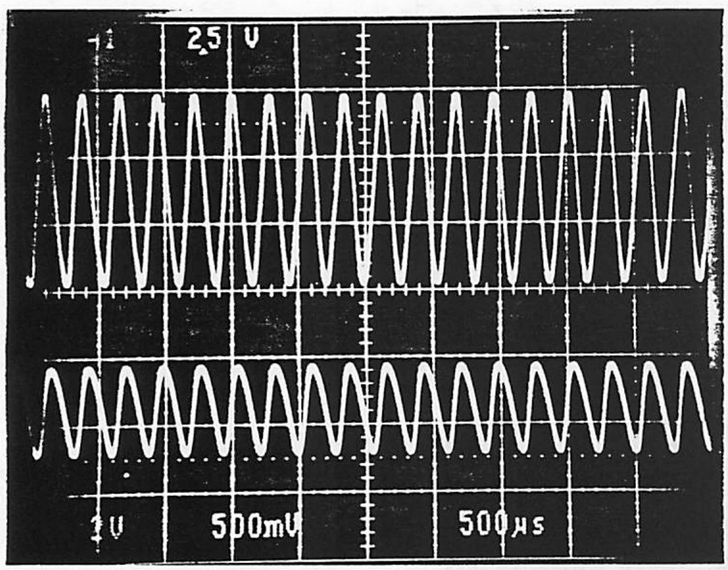
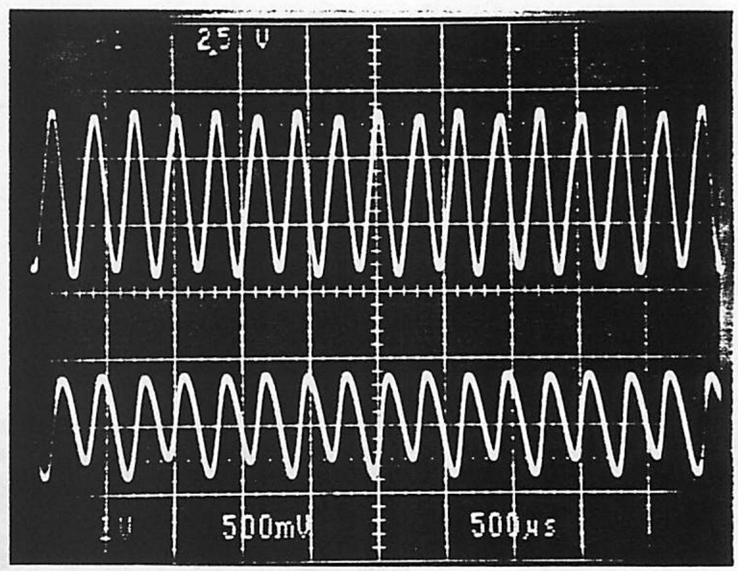
(d)


2.5 V
1V 500mV 500µs

(b)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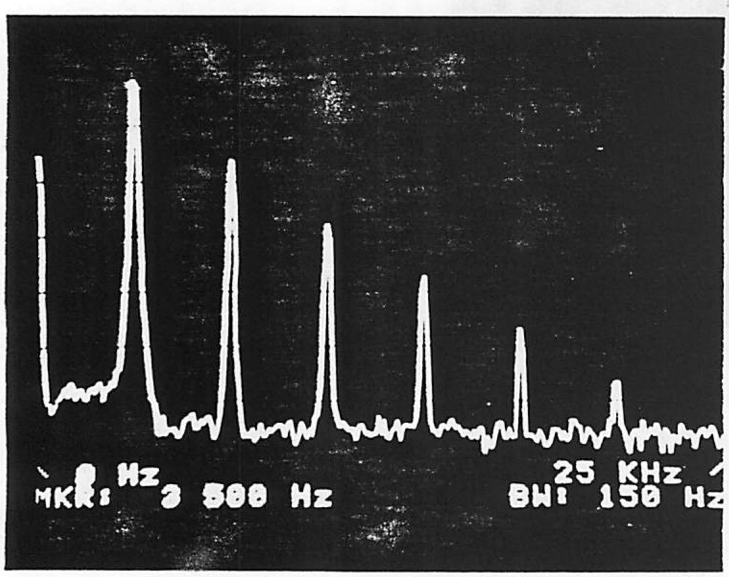
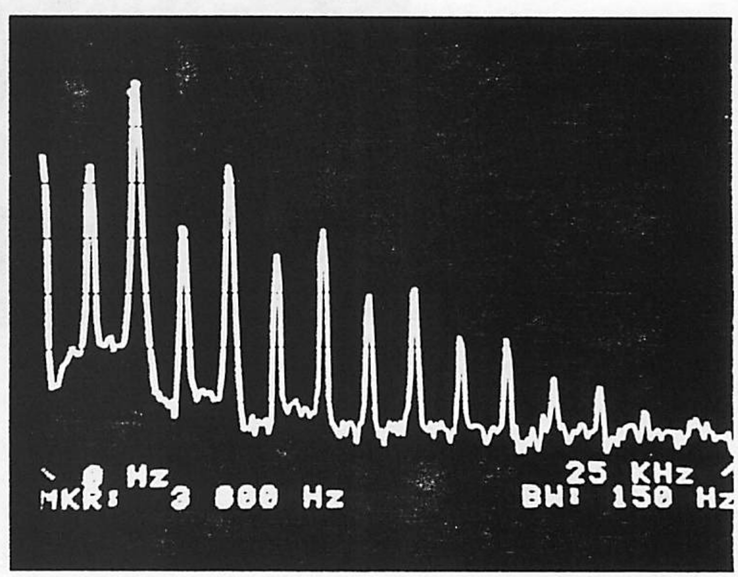
1V 500mV 500µs

(e)


0 Hz 25 KHz
MKR: 3 500 Hz BW: 150 Hz

(c)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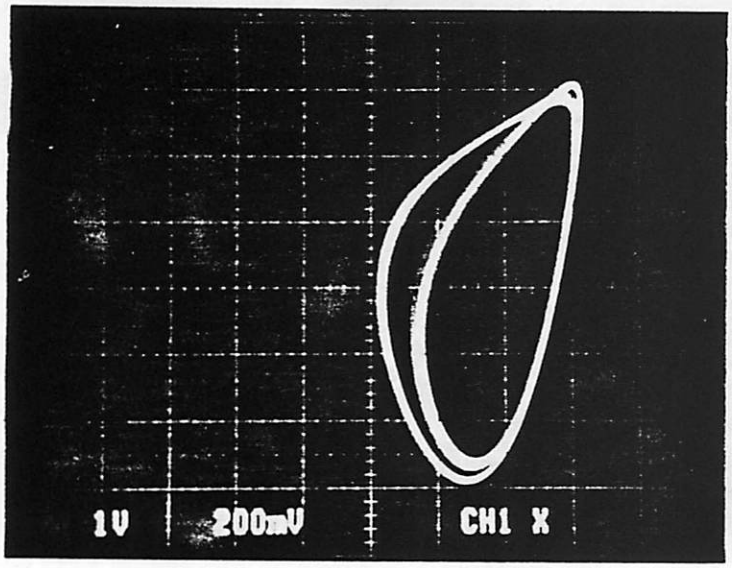
MKR: 3 800 Hz BW: 150 Hz

(f)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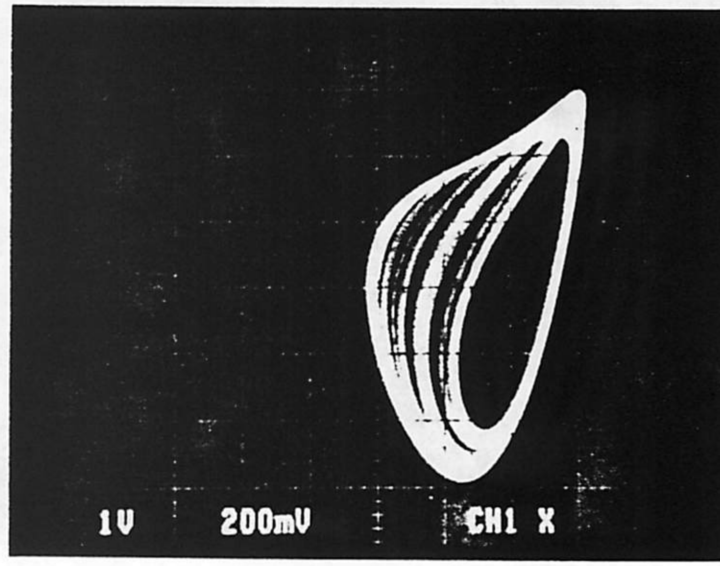
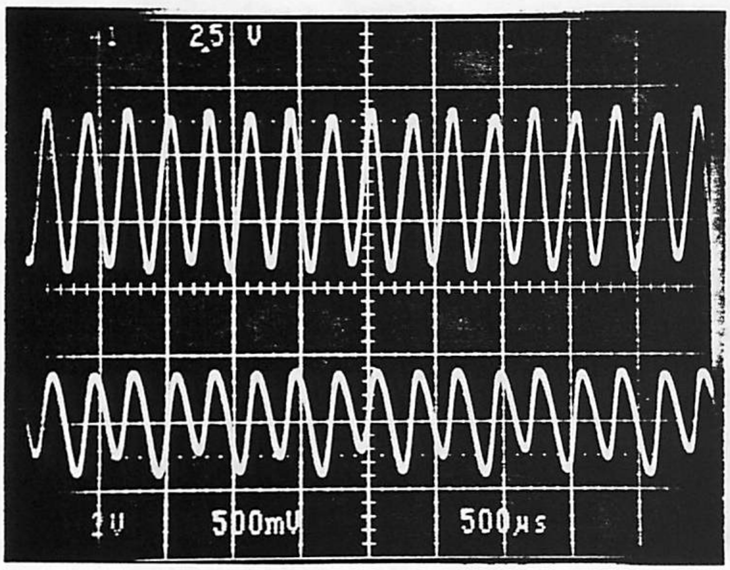
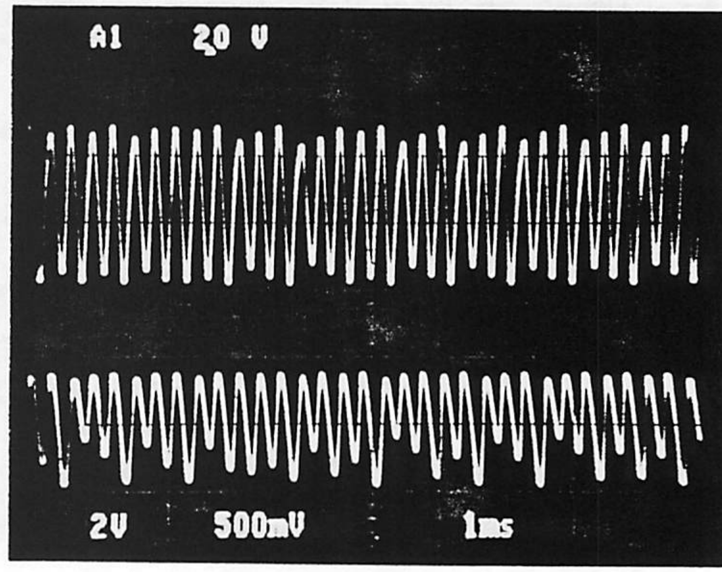
(g)


1V 200mV CH1 X

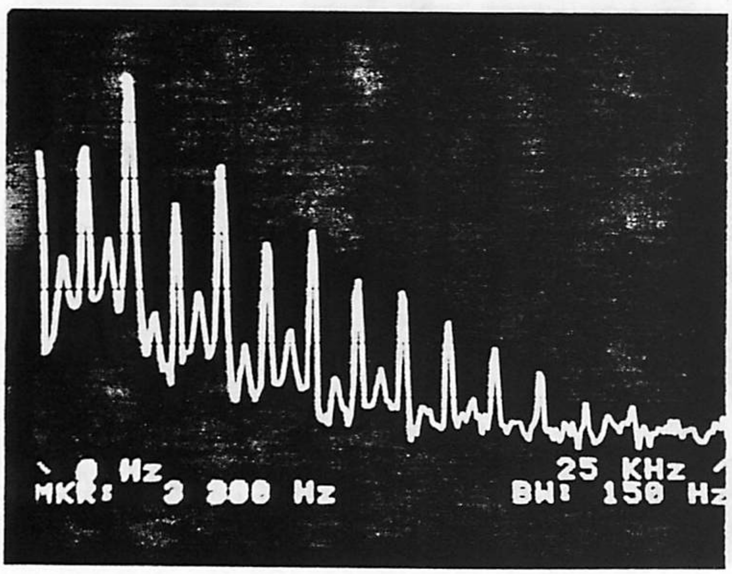
(j)


2.5 V
1V 500mV 500μs

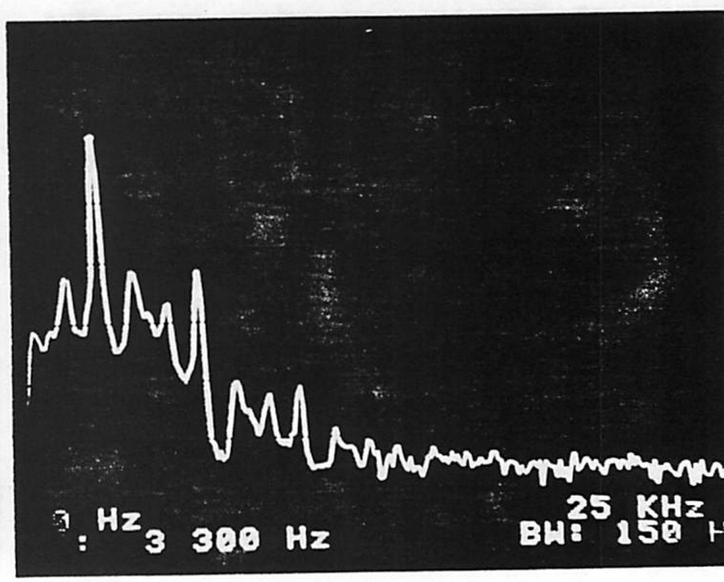
(h)


A1 2.0 V
2V 500mV 1ms

(k)


25 KHz
MKR: 3 300 Hz
BW: 150 Hz

(i)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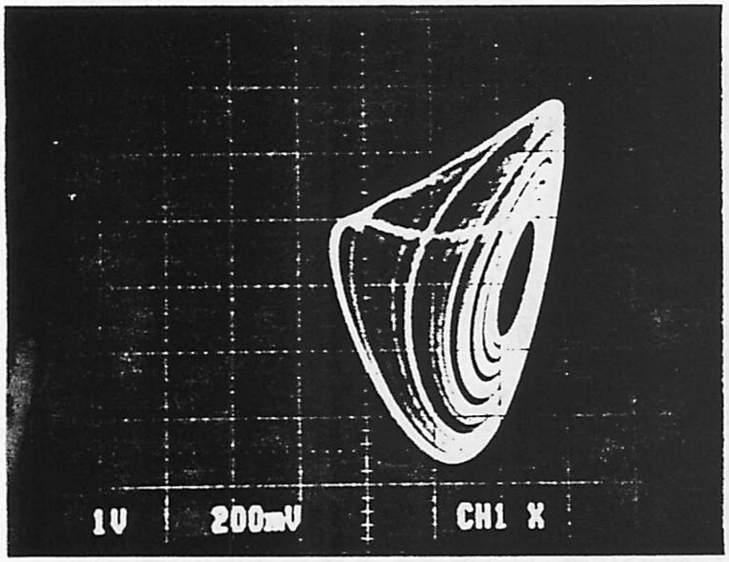
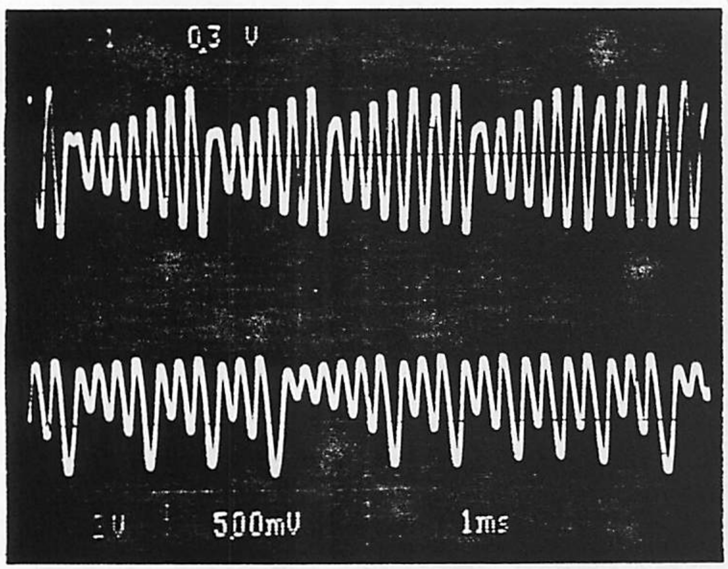
3 300 Hz
BW: 150

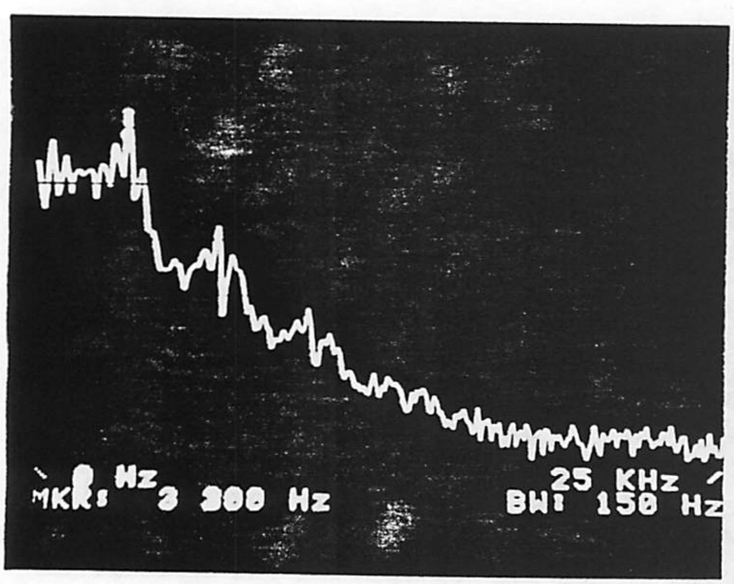
(l)


1V
200mV
CH1 X

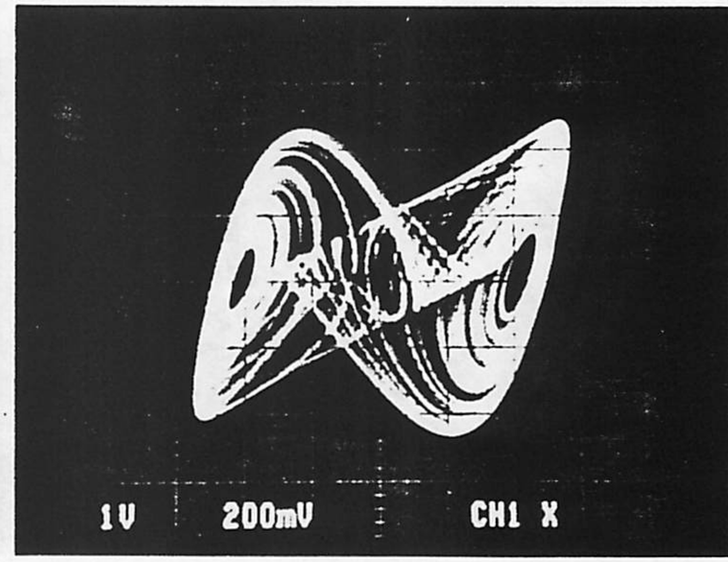
(m)


0.3 V
1V
500mV
1ms

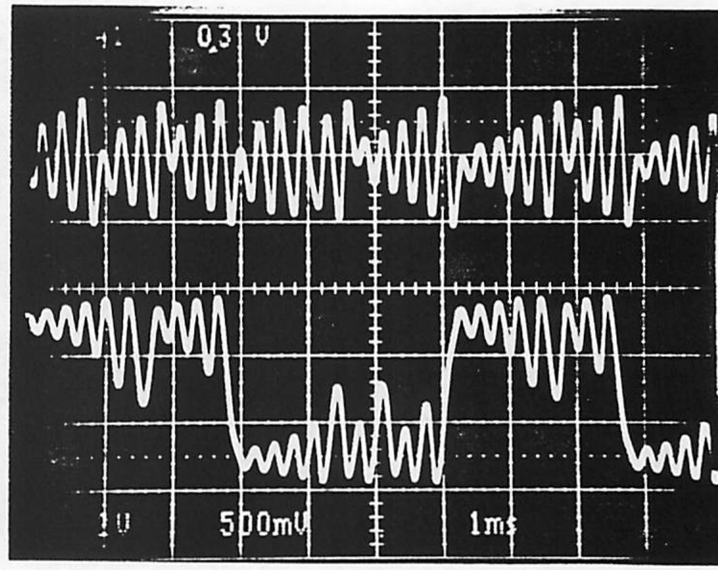
(n)


0 Hz
MKR: 3 200 Hz
25 KHz
BW: 150 Hz

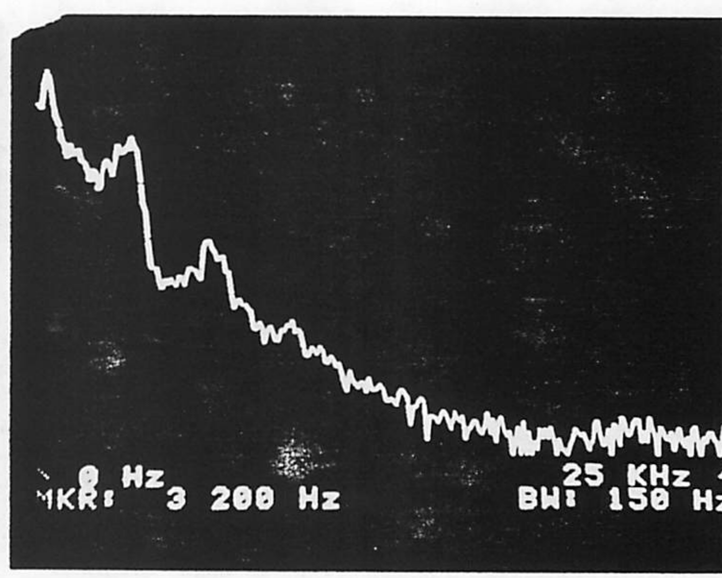
(o)


1V
200mV
CH1 X

(p)

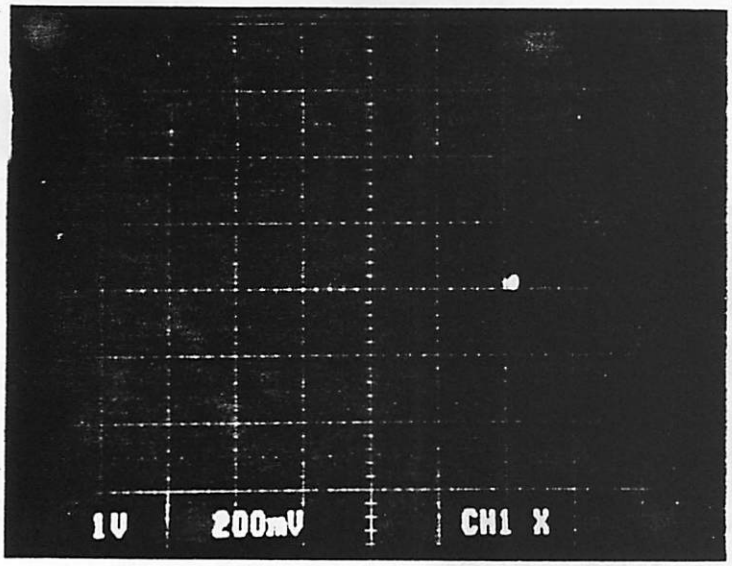

0.3 V
1V
500mV
1ms

(q)

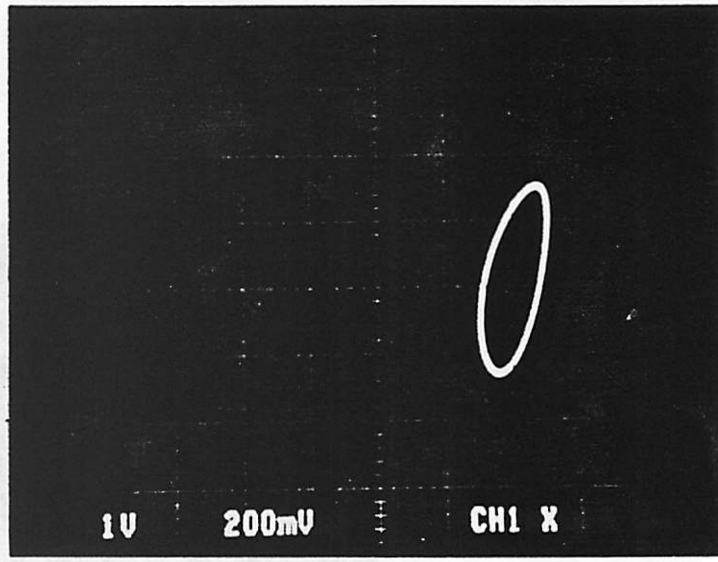

0 Hz
MKR: 3 200 Hz
25 KHz
BW: 150 Hz

(r)

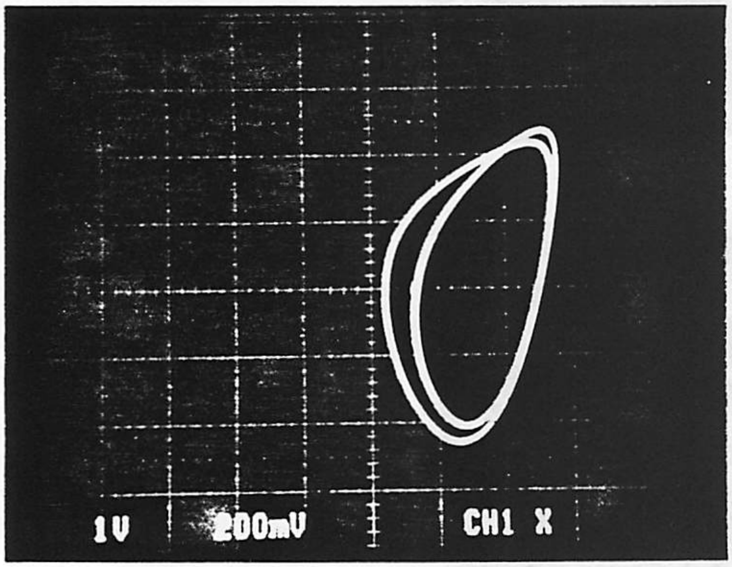
Fig. 6


1V
200mV
CH1 X

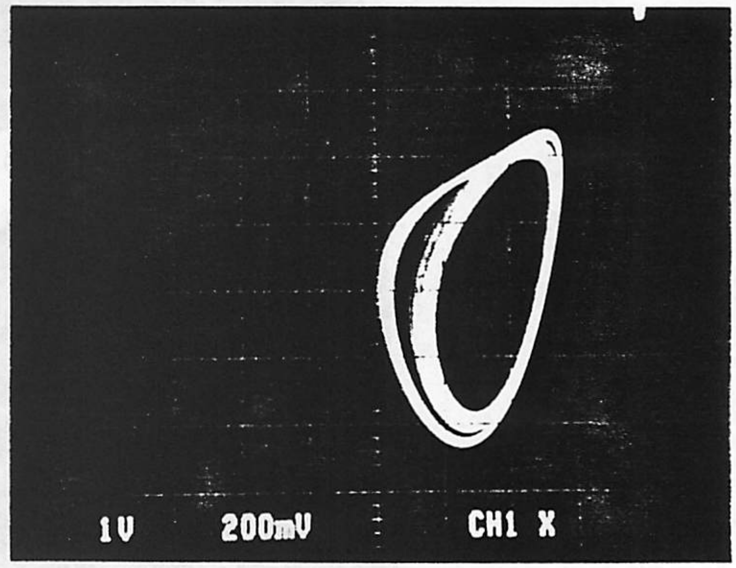
(a)


1V
200mV
CH1 X

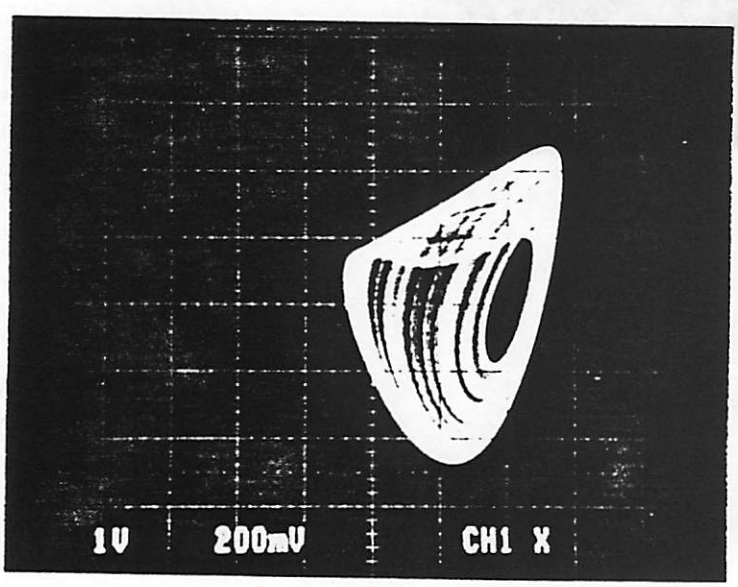
(b)


1V
200mV
CH1 X

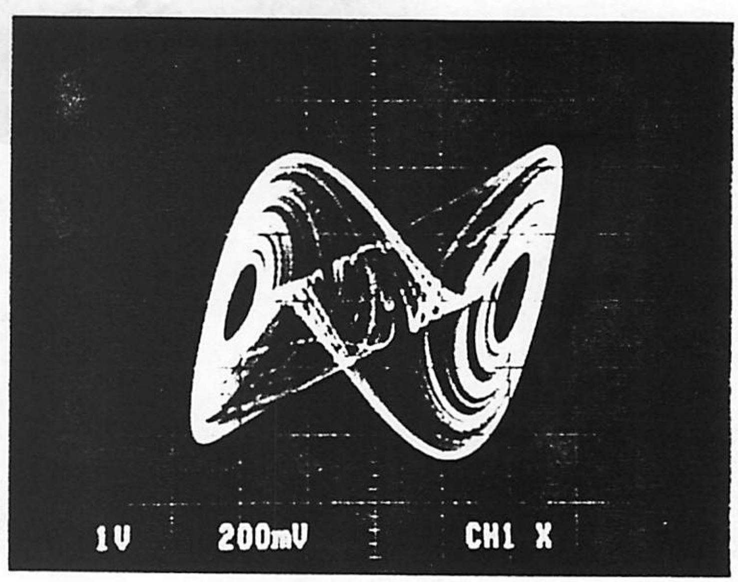
(c)


1V
200mV
CH1 X

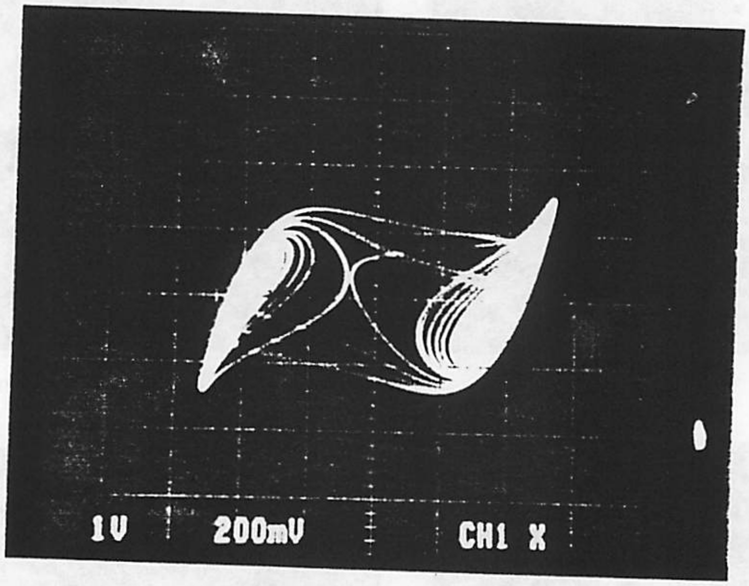
(d)


1V
200mV
CH1 X

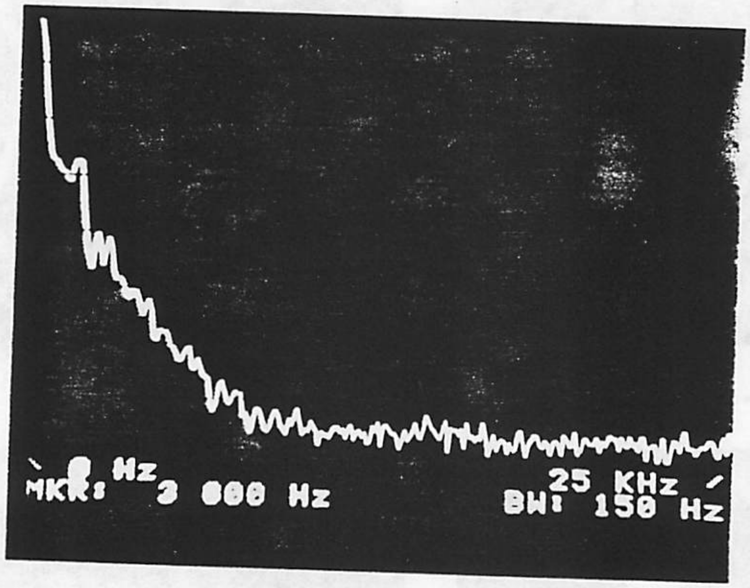
(e)


1V
200mV
CH1 X

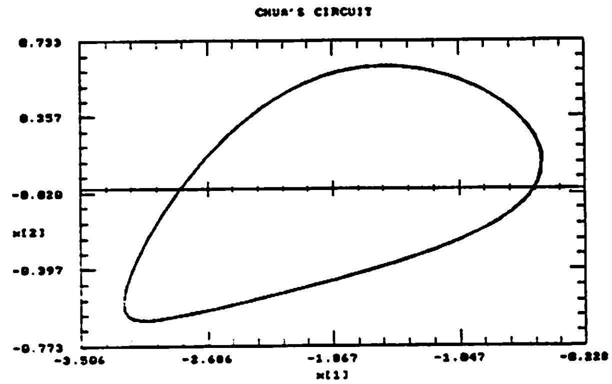
(f)



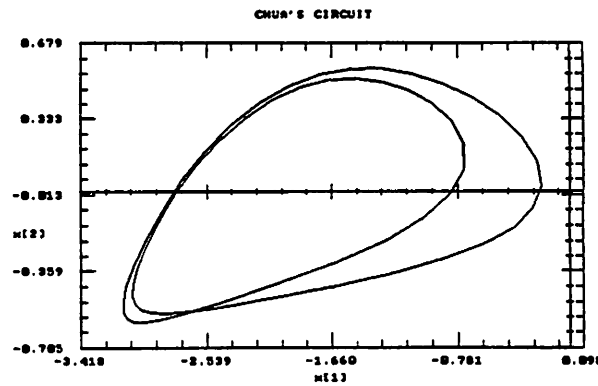
(g)



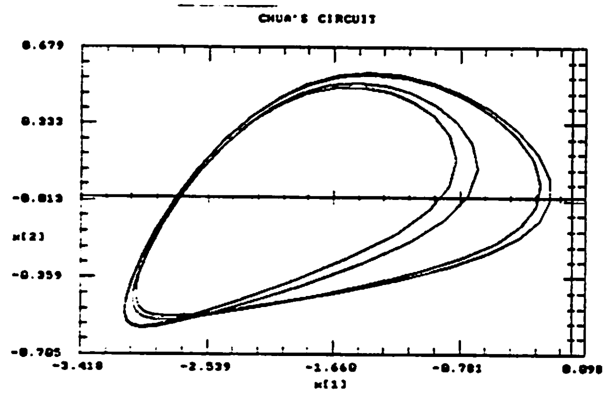
(h)


CHUA'S CIRCUIT
0.733
0.357
-0.020
x[2]
-0.397
-0.773
-3.506
-2.686
-1.867
-1.047
-0.228
x[1]

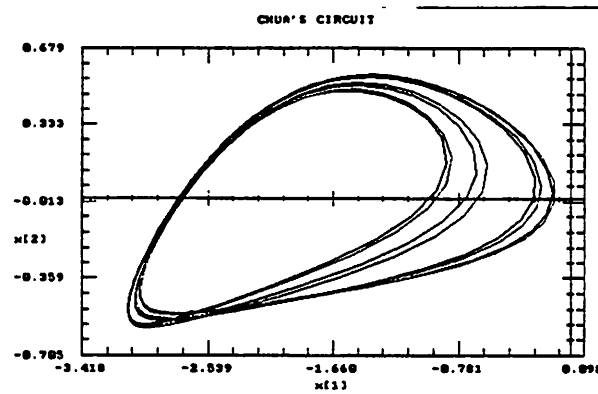
(a)


CHUA'S CIRCUIT
0.679
0.333
-0.013
x[2]
-0.359
-0.705
-3.418
-2.539
-1.660
-0.781
0.098
x[1]

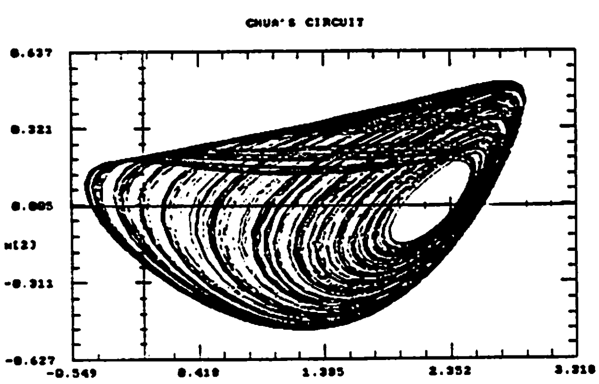
(b)


CHUA'S CIRCUIT
0.679
0.333
-0.013
x[2]
-0.359
-0.705
-3.418
-2.539
-1.660
-0.781
0.098
x[1]

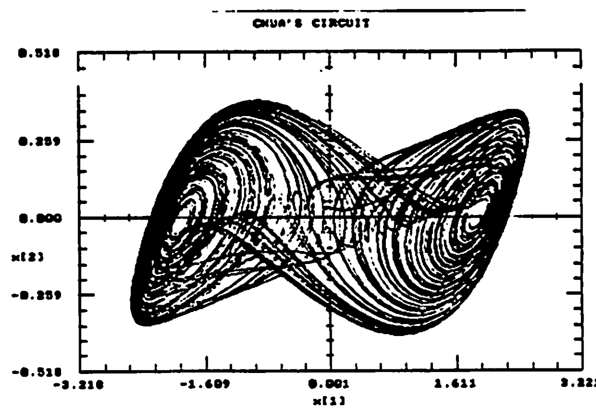
(c)


CHUA'S CIRCUIT
0.679
0.333
-0.013
x[2]
-0.359
-0.705
-3.418
-2.539
-1.660
-0.781
0.098
x[1]

(d)


CHUA'S CIRCUIT
0.637
0.321
0.005
x[2]
-0.311
-0.627
-0.549
0.418
1.385
2.352
3.318
x[1]

(e)


CHUA'S CIRCUIT
0.518
0.259
0.000
x[2]
-0.259
-0.518
-3.218
-1.609
0.001
1.611
3.221
x[1]

(f)

Fig. 7