CHAOTIC DIGITAL CODE-DIVISION MULTIPLE ACCESS (CDMA) COMMUNICATION SYSTEMS

by

Tao Yang and Leon O. Chua

Memorandum No. UCB/ERL M97/58

3 July 1997

CHAOTIC DIGITAL CODE-DIVISION MULTIPLE ACCESS (CDMA) COMMUNICATION SYSTEMS

by

Tao Yang and Leon O. Chua

Memorandum No. UCB/ERL M97/58

3 July 1997

ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

Chaotic Digital Code-Division Multiple Access (CDMA) Communication Systems

Tao Yang and *Leon O. Chua*

Electronics Research Laboratory and

Department of Electrical Engineering and Computer Sciences,

University of California at Berkeley,

Berkeley, CA 94720, U.S.A.

July 3, 1997

Abstract

In this paper, the structure, principle and framework of chaotic digital code-division multiple access ($(CD)^2$MA) communication systems are presented. Unlike the existing CDMA systems, $(CD)^2$MA systems use continuous pseudo-random time series to spread the spectrum of message signal and the spreaded signal is then directly sent through channel to the receiver. In this sense, the carrier used in $(CD)^2$MA is a continuous pseudo-random signal instead of a single tone as used in CDMA. We give the statistical properties of the noise-like carriers. In a $(CD)^2$MA system, every mobile station has the same structure and parameters, only different initial conditions are assigned to different mobile stations. Instead of synchronizing two binary pseudo-random sequences as in CDMA systems, we use an impulsive control scheme to synchronize two chaotic systems in $(CD)^2$MA. The simulation results show that the channel capacity of $(CD)^2$MA is twice as large than that of CDMA.

1 Introduction

In the 1980s, many analog cellular communication networks were implemented over the world. These networks are already reaching their capacity limits in several service areas. This wireless communication technology has evolved form simple first-generation analog systems for business applications to second-generation digital systems with rich features and service for residential and business environments. There are several reasons for the transition from wireless analog to digital technology: increasing traffic, which requires greater cell capacity, speech privacy, new services and greater radio link robustness.

During the late 1980s and early 1990s, the rapid growth in mobile communications put a high demand on system capacity and the availability of the technology for low-cost implementation of cellular and personal communication services(PCS)[1]. CDMA has a larger system capacity than the existing analog systems. The increased system capacity is due to improved coding gain/modulation density, voice activity, three-sector sectorization, and reuse of the same spectrum in every cell. CDMA is a cost-effective technology that requires fewer, less-expensive cells and no costly frequency reuse pattern. The average power transmitted by the CDMA mobile stations averages about 6-7mW, which is less than one tenth of the average power typically requires by FM and TDMA phones. Transmitting less power means longer battery life. CDMA can improve the quality-of-service by providing both robust operation in fading environments and transparent(soft) hand-off. CDMA takes advantage of multi-path fading to enhance communications and voice quality. In narrow-band systems, fading causes a substantial degradation of signal quality.

Since some new services, such as wide-band data and video, are much more spectrum-intensive than voice service, even the channel capacity improvement provided by CDMA will be depleted in the near future. This motivates some advanced wireless communication schemes, which can provide a bigger capacity. In this paper, we present a chaotic digital CDMA scheme. Before we can design a chaotic digital CDMA system, we should solve the following problems.

1.1 Spreading carriers

In CDMA systems, pseudo-random signals are used to 1) spread the bandwidth of the modulated signal to the larger transmission bandwidth and 2) distinguish among the different user signals which are using the same transmission bandwidth in the multiple-access scheme.

Ideally, these pseudo-random signals should be samples of a sequence of independent random variables, uniformly distributed on an available alphabet or range. In this case, the CDMA system is equivalent to a one-time pad used in cryptographic system requiring the highest level of security. Since the key signal in a one-time pad should be as long as the message signal, it is not feasible to use it in CDMA[11].

We must figure out a way to store/generate good pseudo-random signals in both the transmitter and the receiver, despite the finite storage capacity/generating capacity of physical processing systems. It would be very expensive and energy efficient to generate such a sequence by using some chaotic circuits, e.g., Chua's circuits. In fact, some methods to generate good pseudo-random signals for cryptographic purposes by using Chua's circuits[17] have already been developed.

1.2 Orthogonal functions

Orthogonal functions are used to improve the bandwidth efficiency of a spread spectrum system. In CDMA, each mobile station uses one of a set of orthogonal functions representing the set of symbols used for transmission. Usually, the Walsh and Hadamard sequences are used to generate these kind of orthogonal functions for CDMA. In $(CD)^2$MA, it is very hard to find a theory to guarantee that the spectrum-spreading carriers are orthogonal. However, from simulations we find that there exist many methods to generate signals, which have very small cross correlations, by using chaotic signals. We can therefore choose good spectrum-spreading carriers from many promising candidates.

In CDMA, there exist two different methods of modulating the orthogonal functions into the information stream of the CDMA signal. The orthogonal set of functions can be used as the spreading code or can be used to form modulation symbols that are orthogonal. In $(CD)^2$MA, however, the "orthogonal function" itself serves as the carrier.

1.3 Synchronization considerations

In a CDMA system, the heart of the receiver is its synchronization circuitry, and the heartbeats are the clock pulses which control almost every step in forming the desired output. There exist three levels of synchronization in a CDMA system: 1) correlation interval synchronization, 2) spread-spectrum generator synchronization and 3) carrier synchronization.

To correlate the Walsh codes at the receiver requires that the receiver be synchronized with the transmitter. In the forward direction, the base station can transmit a pilot signal to enable the receiver to recover synchronization. Just as the designers of the IS-665 wide-band CDMA system believed, with a wider bandwidth the base station can also recover the pilot signal sent by mobile stations. In $(CD)^2$MA, we also use this symmetric system between base station and mobile station.

In the $(CD)^2$MA system, we need to synchronize two chaotic systems. If we use the continuous synchronization scheme, we need a channel to transmit the chaotic signal. Even though we can embed a message signal into a chaotic carrier[3], we can not achieve a chaotic CDMA system. A promising method to improve this is the framework of impulsive synchronization[15, 16]. In this paper, we will show that the $(CD)^2$MA system does not need the correlation interval synchronization and carrier synchronization. This make the receiver in $(CD)^2$MA simple and low-power.

The $(CD)^2$MA system can increase the capacity of a radio channel. For mobile subscribers, this increased capacity translates to better service at a lower price. On the other hand, $(CD)^2$MA is also a promising technology for low-cost implementation of cellular and PCS. The organization of this paper is as follows. In Sec. 2, we present the structure of $(CD)^2$MA system. In Sec. 3, we study the randomness of spreading carriers. In Sec. 4, we give the concept of impulsive synchronization. In Sec. 5, we show that a $(CD)^2$MA system has a larger channel capacity than CDMA. Sec. 6 contains conclusions.

2 Structures of CDMA and $(CD)^2$MA systems

Information can be modulated into the spread-spectrum signal by several methods. The most common method is to add the information into the spectrum-spreading code before it is used for modulating the carrier frequency. The corresponding CDMA system is shown in Fig.1. In Fig.1, $d(t)$ and $c(t)$ are respectively called "message signal" and "spreading signal". The signal $cos(\omega t)$ is called the carrier, and $x(t)$ is the encoded signal. At the receiver end, $I(t)$ is interference signal which consists of all channel noise, interference and/or jamming, and $r(t)$ is the recovered signal. $d(t)$ is a low frequency digital message signal with a data rate $9.6kbps$ for IS-95 CDMA system. $c(t)$ is a high frequency spreading signal with a chip rate $1.2288Mcps$ for IS-95 CDMA system. We can see that the chip rate is much higher than data rate. This is the method by which the bandwidth of the message signal is spread.

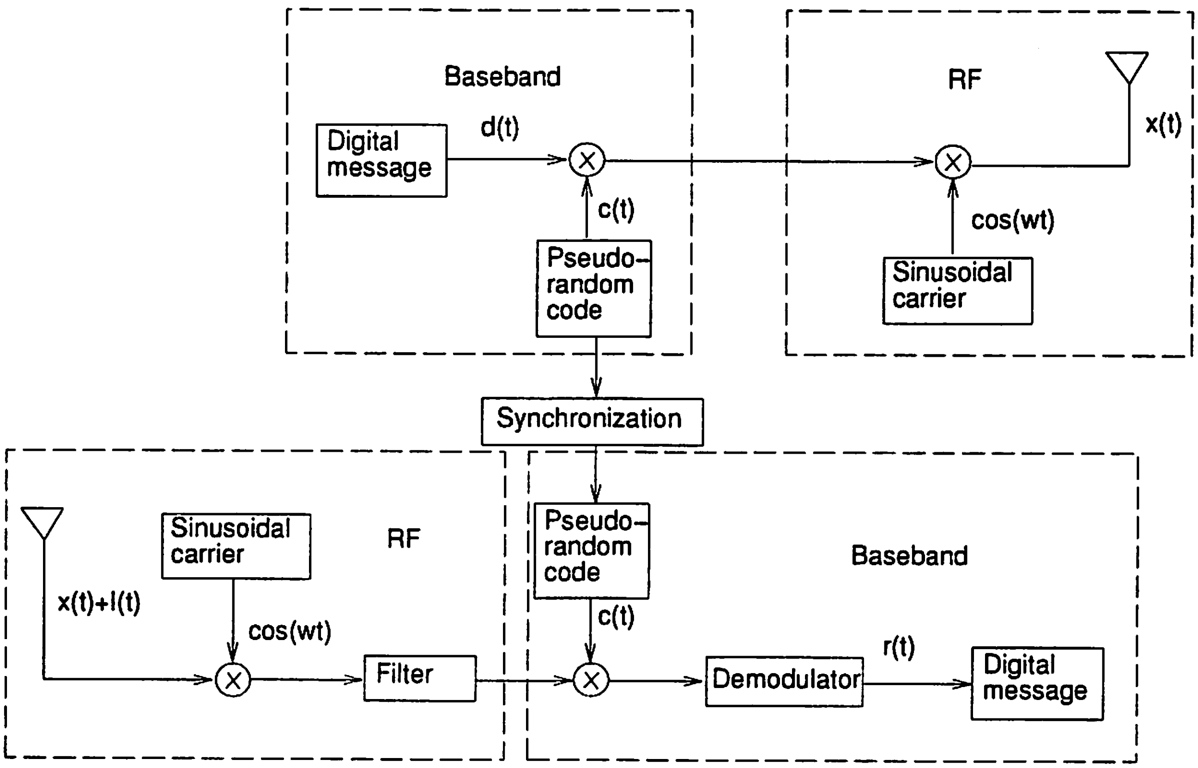


Figure 1: The Block diagram of the CDMA system.

On the other hand, the message signal can also be used to modulate the carrier directly given the carrier also functions as spectrum-spreading signal. In $(CD)^2$MA system, we use this kind of modulation scheme. The corresponding $(CD)^2$MA system is shown in Fig.2.

Comparing the $(CD)^2$MA system in Fig.2 with the CDMA system in Fig.1, we can see that both schemes use the synchronization of two identical spreading carrier generators. Instead of modulating a single tone (an RF sinusoidal signal) as in the CDMA system, a $(CD)^2$MA system transmits a pseudo-random RF spreading carrier directly. We give the details of every

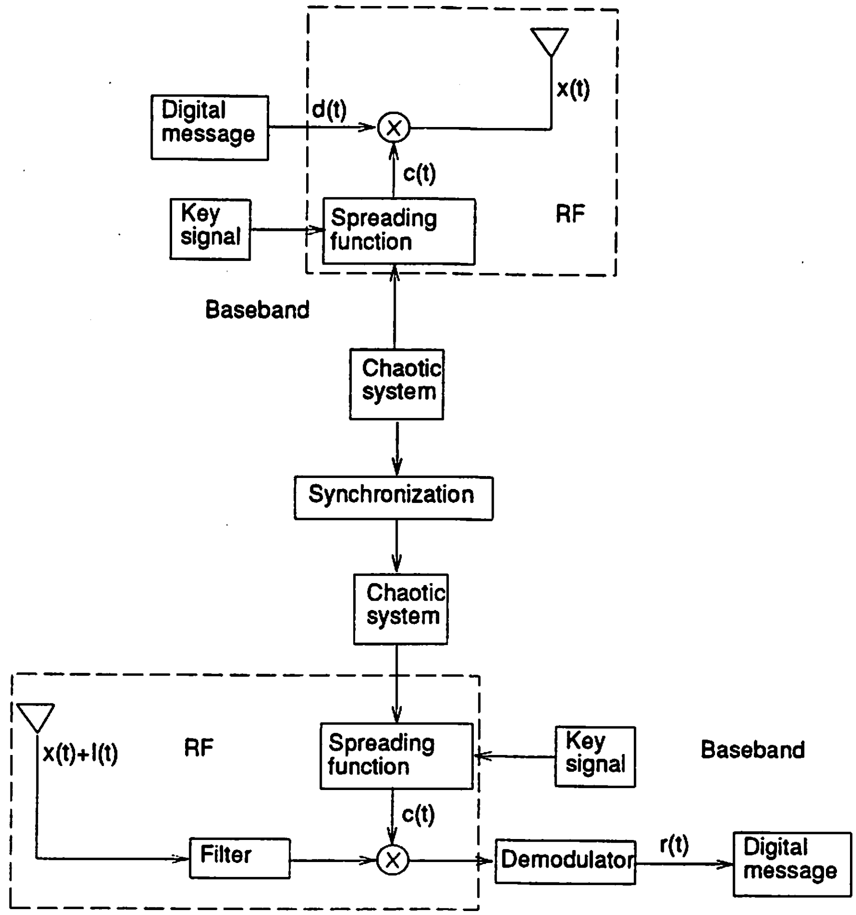


Figure 2: The Block diagram of the $(CD)^2$MA system.

block in Fig.2. To enhance the security of the $(CD)^2$MA system, we can also use two key signals to scramble the spreading carrier. The key signal can be assigned to each transmitter and receiver pair by the base station(cell). Since each mobile station can function either as a transmitter or as a receiver, we suppose that the key signals are set by both the receiver and the transmitter and can be refreshed during conversations. The key signals may only be used in cases where a very high security should be taken into account(e.g., military applications). In commercial applications, the spreading signal along is secure enough.

In the $(CD)^2$MA paradigm, every mobile station has the same chaotic circuit, e.g., Chua's circuit. Whenever two users are connected by the base station, they are assigned the same set of initial conditions to their chaotic circuits. Then, the impulsive synchronization scheme is used to maintain the synchronization between the two chaotic circuits.

Since the chaotic circuit works in a low frequency range, we need a spreading function to spread it into bandwidths in the MHz range. In principle, there are many nonlinear functions which can be used as spreading functions. In this paper, the spreading function is chosen according to that used in [17]. That is, we use an n-shift scheme to spread the spectrum of the chaotic signal. The chaotic systems in both transmitter and receiver are identical and the synchronization between them are achieved by a new chaotic synchronization scheme called *impulsive synchronization*. The demodulator consists of some low-pass filtering and thresholding blocks.

3 Statistical Properties of Chaotic Spreading Carriers

We now study the properties of the spreading signal, which should have a narrow autocorrelation function for achieving a big channel capacity and small cross-relations. One of the ideal candidates for this kind of signal is white noise which has an autocorrelation function of a Dirac Delta function at the origin. Although from a deterministic model we can not generate a true white noise signal, we still have the chance to generate its approximation which we call a pseudo-random signal. Since the cryptographic community has spent half a century to try to find a good binary psudo-random signal for purpose of high level of security of cryptographic algorithms, we borrow some methods used by them.

We first use a chaotic system to generate a good seed signal(low frequency in the range of KHz) and then use some spreading function to spread the spectrum into the range of MHz. In this paper, we choose the Chua's oscillator[8] as our chaotic system which has dynamics given by

$$\begin{cases} \frac{dv_1}{dt} = \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)] \\ \frac{dv_2}{dt} = \frac{1}{C_2}[G(v_1 - v_2) + i_3] \\ \frac{di_3}{dt} = -\frac{1}{L}[v_2 + R_0 i_3] \end{cases} \tag{1}$$

where $f(\cdot)$ is the nonlinear characteristics of Chua's diode given by

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + E| - |v_1 - E|) \tag{2}$$

and E is the breakpoint voltage of Chua's diode. The corresponding circuit is shown in Fig.3.

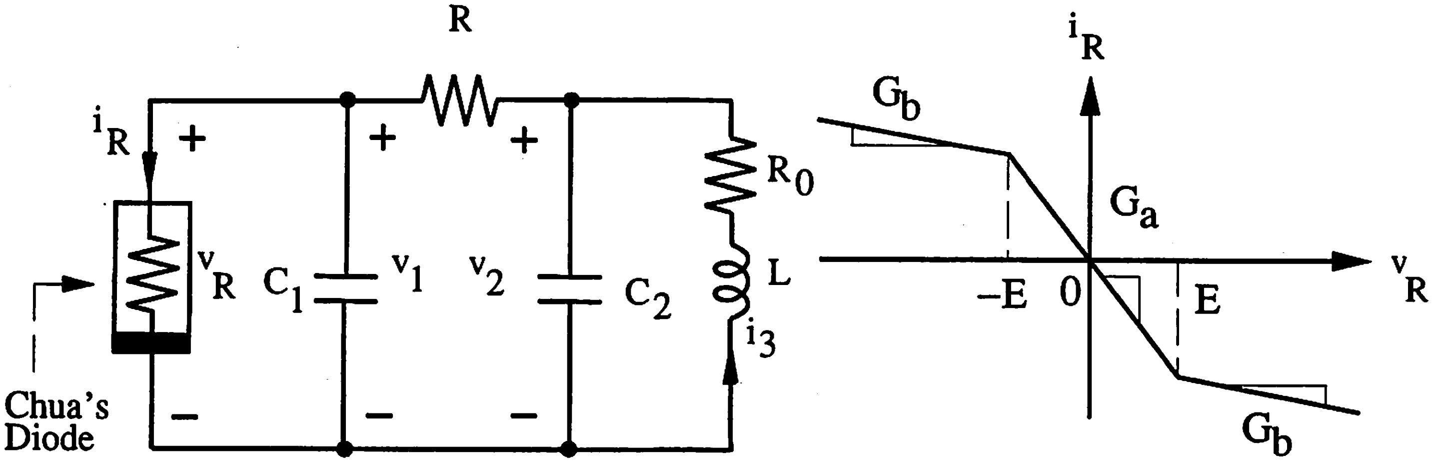


Figure 3: (a) Chua's oscillator. (b) Chua's diode.

We use a continuous n-shift cipher to spread the chaotic signal generated by a Chua's circuit. The n-shift cipher is given by

$$x(t) = e(p(t)) = \underbrace{f_1\Big(\ldots f_1\Big(f_1}_{n}\Big(p(t), \underbrace{p(t)\Big), p(t)\Big), \ldots, p(t)\Big)}_{n} \tag{3}$$

where h is chosen such that $p(t)$ lies within $(-h, h)$. $f_1(\cdot, \cdot)$ is a nonlinear function

$$f_1(x, k) = \begin{cases} (x+k)+2h, & -2h \le (x+k) \le -h \\ (x+k), & -h < (x+k) < h \\ (x+k)-2h, & h \le (x+k) \le 2h \end{cases} \tag{4}$$

This function is shown in Fig.4(a). Since the "jump-type" break points in Fig.4(a) can not be implemented in a practical circuit, what we need is the continuous version of this characteristics shown in Fig.4(b). In this case, $f_1(\cdot,\cdot)$ is given by

$$f_1(x,k) = \begin{cases} \frac{h}{h-\Delta}(x+k)+2h, & -2h \leq (x+k) \leq -h-\Delta \\ -\frac{h}{\Delta}(x+k+h), & -h-\Delta \leq (x+k) < -h+\Delta \\ \frac{h}{h-\Delta}(x+k), & -h+\Delta \leq (x+k) < h-\Delta \\ -\frac{h}{\Delta}(x+k-h), & h-\Delta \leq (x+k) < h+\Delta \\ \frac{h}{h-\Delta}(x+k)-2h, & h+\Delta \leq (x+k) \leq 2h \end{cases} \quad (5)$$

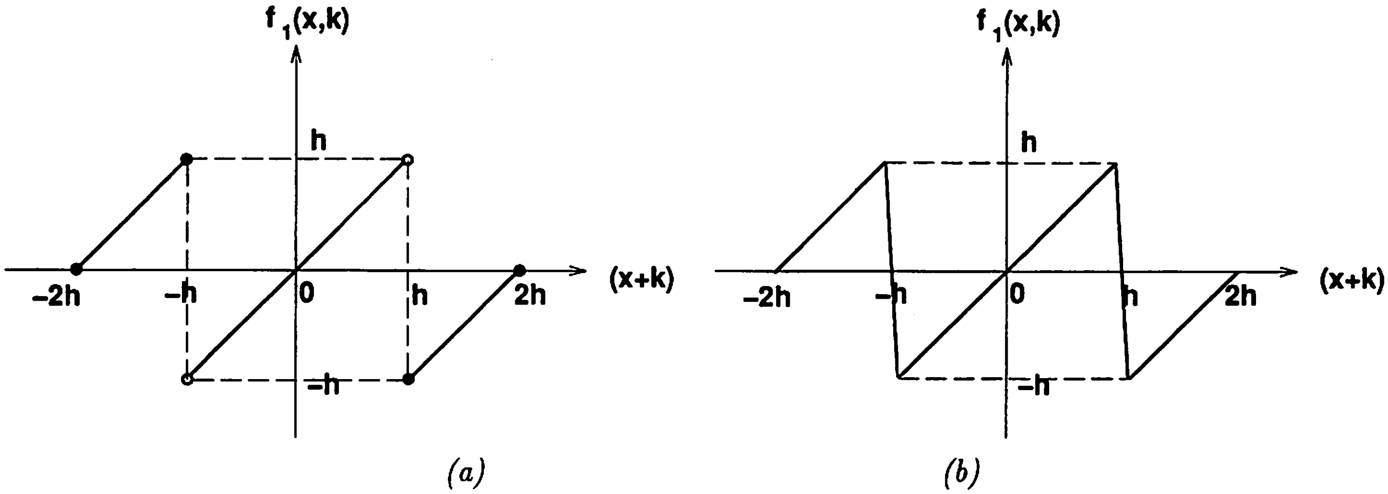


Figure 4: Nonlinear function used in continuous shift cipher. (a) The ideal model. (b) The practical implementation.

In our $(CD)^2$MA systems, we choose the seed signal $p(t)$ as the voltage $v_1(t)$ of Chua's circuit. Since we choose $h = 1V$, $p(t)$ is a normalized version of $v_1(t)$. Before we can use the output of the n-shift function as the spreading carrier, we have to test its statistical properties. In our $(CD)^2$MA systems, the following parameters are used: $C_1 = 5.56nF$, $C_2 = 50nF$, $G = 0.7mS$,$L = 7.14mH$, $G_a = -0.8mS$, $G_b = -0.5mS$, $E = 1v$, $R_0 = 0\Omega$ and $\Delta = 0.01$. Given this set of parameters, Chua's circuit has a double-scroll attractor.

There are many tests that can be used to help establish the random characteristics of a signal[5]. In this paper, we use two frequency tests: the χ^2 test and the Kolmogorov-Smirnoff(K-S) test. See [5] for details about both of the tests. They are used to verify that the signal is uniformly distributed.

For the χ^2 test, the range of $x(t)$ is divided into 51 intervals. Our expectation is an equal number of observations in each interval. Thus the χ^2 values computed on each sample will be expected to follow a χ^2 distribution with 50 degrees of freedom.

For the K-S test, K^+ and K^- are computed for each of the samples of $x(t)$. The values are expected to be K_n-distributed, where n is the number of samples in sampled $x(t)$. In our simulations, $n = 10^5$ and the sampling interval is $10^{-7}s$.

The following tests which based on χ^2-tests are also used.

Serial test: In the sequences of sampled $x(t)$, we want pairs of successive numbers to be uniformly distributed in an independent manner. To carry out the serial test, we simply count the number of times that the sampled pair $(x(2j), x(2j+1))$ fall in different regions of the plane $[-1,1] \times [-1,1]$. We split the interval $[-1,1]$ into d subintervals and code each of them by an integer from 1 to d. When $x(2j)$ falls into a subinterval, we use the code of this subinterval, an integer Y_{2j}, to represent it. This kind of coding system is also used in the other tests. To carry out the serial test, we simply count the number of times that $(Y_{2j}, Y_{2j+1}) = (q, r)$ occurs, for $1 \leq q, r \leq d$. And the χ^2-test is applied to these $k = d^2$ categories with probability $\frac{1}{d^2}$ for each category. We chose $d = 8$. The sequence of block should have a length $\geq 5d^2 = 1280$.

Gap test: This test is used to test the length of "gaps" between occurrences of Y_j in a certain range. If α and β are two real numbers with $0 \leq \alpha < \beta < d$, we want to consider the lengths of consecutive subsequences $Y_j, Y_{j+1}, ..., Y_{j+r}$ in which Y_{j+r} lies between α and β but the other Y's do not. In this test, we chose $d = 9$.

Poker test: The classical poker test considers n group of five successive integers, $\big(Y_{5j}, Y_{5j+1}, Y_{5j+2}, Y_{5j+3}, Y_{5j+4}\big)$ for $0 \leq j < n$, and observes which of the following five categories is matched by each quintuple: 5 different, 4 different, 3 different, 2 different and 1 different.

In general we can consider n groups of k successive numbers, and we can count the number of k-tuples with r different values. A χ^2-test is then made, using the probability

$$p_r = \frac{d(d-1)...(d-r+1)}{d^k} \left\{ \begin{matrix} k \\ r \end{matrix} \right\} \tag{6}$$

that there are r different. $\left\{ \begin{matrix} k \\ r \end{matrix} \right\}$ is a Stirling number[3]. In this case, we chose $d = 9$.

Table 1 lists the test results of $x(t)$ with different initial conditions with 10-shift cipher function.

Table 1: Results of different statistical tests of chaotic spreading carriers when different initial conditions are used.

	$v_1(0) = -0.204$v $v_2(0) = 0.045$v $i_3(0) = 1.561$mA	$v_1(0) = 1.568$v $v_2(0) = -0.741$v $i_3(0) = 2.301$mA	$v_1(0) = 0.1$v $v_2(0) = 2.515$v $i_3(0) = -1.901$mA	$v_1 = -2.56$v $v_2 = -1.349$v $i_3 = -2.002$mA
χ^2 Test	0.021	0.013	0.054	0.07
K-S Test, K^+	0.011	0.044	0.034	**0.009**
K-S Test,K^-	0.034	0.025	0.029	0.012
Serial Test	**0.008**	0.067	0.045	0.074
Gap Test	0.055	0.054	0.082	0.03
Poker Test	0.063	0.019	0.018	0.06

In Table 1, when a test result is between 0.01 and 0.99, it denotes that $x(t)$ can be viewed as a good pseudo-random sequence. The bold face numbers in Table 1 denote the test results which don't belong to good pseudo-random sequences. Since there are only two test results belonging to the "bad" pseudo-random cases, we conclude that the randomness of $x(t)$ is good enough for designing spreading carriers. And we can see that this randomness is initial condition independent. This initial condition independent property can be used to simplify the hardware of mobile stations because we only need to give different initial conditions to each mobile station for generating its spreading carrier. Since these tests are used in testing the randomness of sequences for cryptographic purposes, we can conclude that a good pseudo-random sequence should have a very narrow autocorrelation function.

4 Impulsive synchronization of Chua's circuits in $(CD)^2$MA

The idea of applying of impulsive synchronization to $(CD)^2$MA is inspired by the fact that every mobile station of a CDMA system has a clock signal to make the receiver work autonomously once the clock signal is synchronized with that of the transmitter. The difference between impulsive synchronization and continuous synchronization is that, in the former, once the synchronization achieved the receiver can work autonomously for a given time duration. This is achieved by sending synchronizing impulses to the receiver. We should send synchronizing impulses to the receiver at a given frequency because the noise and parameter mismatches between the chaotic systems in the transmitter and the receiver will soon desynchronize both. For this reason, in the $(CD)^2$MA system we need an overhead channel to transmit synchronizing impulses.

In this section, we study the impulsive synchronization of two Chua's oscillators. One of the Chua's oscillators is called the *driving system* and the other is called the *driven system.* In an impulsive synchronization configuration, the driving system is given by Eq.(1). Let $\mathbf{x}^T = (v_1\ v_2\ i_3)$, then we can rewrite the driving system in Eq.(1) into the form

$$\dot{\mathbf{x}} = A\mathbf{x} + \Phi(\mathbf{x}) \tag{7}$$

where

$$A = \begin{pmatrix} -G/C_1 & G/C_1 & 0 \\ G/C_2 & -G/C_2 & 1/C_2 \\ 0 & -1/L & -R_0/L \end{pmatrix}, \Phi(\mathbf{x}) = \begin{pmatrix} -f(v_1)/C_1 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

Then the driven system is given by

$$\dot{\tilde{\mathbf{x}}} = A\tilde{\mathbf{x}} + \Phi(\tilde{\mathbf{x}}) \tag{9}$$

where $\tilde{\mathbf{x}}^T = (\tilde{v}_1, \tilde{v}_2, \tilde{i}_3)$ is the state variables of the driven system.

At discrete instants, $\tau_i, i = 1, 2, ...$, the state variables of the driving system are transmitted to the driven system and then the state variables of the driven system are subject to jumps at these instants. In this sense, the driven system is described by an impulsive differential equation

$$\begin{cases} \dot{\tilde{\mathbf{x}}} = A\tilde{\mathbf{x}} + \Phi(\tilde{\mathbf{x}}),\ t \neq \tau_i \\ \Delta\tilde{\mathbf{x}}|_{t=\tau_i} = -B\mathbf{e}, i = 1, 2, ... \end{cases} \tag{10}$$

where B is a 3×3 matrix, and $\mathbf{e}^T = (e_1, e_2, e_3) = (v_1 - \tilde{v}_1, v_2 - \tilde{v}_2, i_3 - \tilde{i}_3)$ is the *synchronization error*. If we define

$$\Psi(\mathbf{x}, \tilde{\mathbf{x}}) = \Phi(\mathbf{x}) - \Phi(\tilde{\mathbf{x}}) = \begin{pmatrix} -f(v_1)/C_1 + f(\tilde{v}_1)/C_1 \\ 0 \\ 0 \end{pmatrix} \tag{11}$$

then the error system of the impulsive synchronization is given by

$$\begin{cases} \dot{\mathbf{e}} = A\mathbf{e} + \Psi(\mathbf{x}, \tilde{\mathbf{x}}), \ t \neq \tau_i \\ \Delta \mathbf{e}|_{t=\tau_i} = B\mathbf{e}, i = 1, 2, ... \end{cases} \tag{12}$$

The conditions for the asymptotic stability of impulsive synchronization can be found in [16]. The results in [16] also show that the impulsive synchronization is robust enough to additive channel noise and the parameter mismatch between the driving and driven systems.

5 Considerations for investigating the capacities of CDMA and $(CD)^2MA$

Several approaches to estimate cellular CDMA capacity have been developed[2, 10, 7, 9]. In this paper, we use the method presented in [2] to estimate the capacity of $(CD)^2$MA. Due to the structural differences between CDMA and $(CD)^2$MA, we revise the method presented in [2], which was originally proposed for CDMA, to cope with our $(CD)^2$MA schemes.

As in CDMA, we focus on the reverse link capacity because the forward link uses coherent demodulation by the pilot carrier, which is being tracked, and since its multiple transmitted signals are synchronously combined, its performance will be superior to that of the reverse link.

The estimate of the capacity of a CDMA system depends on the model of the whole CDMA system. To model a CDMA system, the following factors should be considered.

1. Interference

We usually approximate the interference to a given user from all other multiple access users by a Gaussian process[12]. As with any digital communication system, spread spectrum or not, there are four components of the demodulator output:

- the desired output.
- the inter-chip interference components, which is usually called inter-symbol interference for non-spread digital demodulation.
- the component due to background noise.
- the other-user interference components.

In a CDMA system, the interference from the other users is much stronger than that from noise.

2. Power control

It is well-known that one of the most serious problems faced by a DS CDMA system is the multi-user interference. Because all users are transmitting in the same frequency band and the crosscorrelatons of the codes are rarely zero, the signal-to-inference ratio, and hence the performance, decreases as the number of users increases, which shows that DS CDMA is an interference-limited, rather than a noise-limited, system.

An effect known as the "near-far" effect plays an especially important role when considering multi-user interference. The near-far effect can be explained by considering the reverse link. Due to the path-loss law(which implies that the received power decreases as the transmitter-receiver distance increases), a close user will dominate over a user located at the boundary. In order to overcome the near-far effect, power control can be used.

The propagating loss is generally modeled as the product of the mth power of distance r and log-normal component representing shadowing losses. This model represents slowly varying losses, even for users in motion, and applies to both reverse and forward links. The more rapidly varying Rayleigh fading losses are not included here. Thus, for a user at a distance r from a base station, attenuation is proportional to

$$\alpha(r, \epsilon) = r^{m} 10^{\epsilon/10} \tag{13}$$

where ϵ is the decibel attenuation due to shadowing, with zero mean and standard deviation σ. Experimental data[6] suggest the choices of $m = 4$ for the power law and $\sigma = 8dB$ for the standard deviation of ϵ.

Power control can be established by letting the base station continuously transmit a pilot signal that is monitored by all mobile stations. According to the power level detected by the mobile station, the mobile station adjusts its transmission power. In a practical power control system, power control errors occur[4], implying that the average received power at the base station may not be the same for each user signal.

3. Multi-path propagation

In terrestrial communication, the transmitted signal is reflected and refracted by different smooth or rough surfaces and different objects, so that it is replicated at the mobile station with different time delays. This is called *multi-path propagation*. It can be quite severe in urban areas or within a building. Different paths arrive at different amplitudes and carrier phases. The path amplitude depend on the relative propagation distances and the reflective or refractive properties of the terrain or buildings. In many cases, particularly in a confined area, each of the distinguishable multi-path components will actually be itself the linear combination of several indistinguishable paths of varying amplitudes. Since these will add as random vectors, the amplitude of each term will appear to be Rayleigh-distributed, and the phase uniformly distributed. This is the most commonly accepted model[13].

Since two code sequences with a relative delay of more than two chip durations usually have a low correlation value compared to the fully synchronized situation, DS CDMA offers the possibility to distinguish between paths with a relative delay of more than two chip durations. This is called *the inherent diversity* of DS CDMA, implying that it is possible to resolve a number of paths separately using only one receiver. This property makes DS CDMA suitable for applications in mobile radio environments, which are usually corrupted with severe multi-path effects.

The multi-path fading channel for the kth mobile station is characterized by a set of its low-pass equivalent complex-values impulse responses

$$\{h_k(t) = \sum_{l=1}^{L} a_{kl}\delta(t - \tau_{kl})exp(-j\phi_{kl})\}_{k=1}^{K} \qquad (14)$$

where K denote the number of the active users. Here we assume that every link has a fixed number L of resolvable paths. The path gains $\{a_{kl}\}_{l=1}^{L}$, path delays $\{\tau_{kl}\}_{l=1}^{L}$, and path phases $\{\phi_{kl}\}_{l=1}^{L}$ are three random variables. For a given k, $\{a_{kl}\}_{l=1}^{L}$ is modeled as the set of independent Rayleigh random variables whose probability density functions are given by:

$$p(\alpha_k) = \frac{2\alpha_k e^{-\alpha_k^2/\sigma_{kr}^2}}{\sigma_{kr}^2}, \alpha > 0, k = 1, 2, ..., L \tag{15}$$

We suppose that the L multipath components are all Rayleigh of equal average strength, so that

$$\sigma_{kr}^2 = \sigma_r^2, \text{for all } k = 1, 2, ..., L \tag{16}$$

According to [14], we choose $\sigma_r = 4dB$. The $\{\tau_{kl}\}_{l=1}^{L}$ are mutually independent and uniformly distributed over $[\Delta_1, \Delta_2]$, and the $\{\phi_{kl}\}_{l=1}^{L}$ are independent uniform random variables over $[0, 2\pi)$, all of which are also statistically independent of each other. In this paper, we choose $L = 5$, and Δ_1 and Δ_2 are chosen in the corresponding simulation which will be presented later.

6 Channel capacity of $(CD)^2$MA

For simplicity, in this paper we only study the unsectorized cases. For the sectorized cases all the arguments of the relations between CDMA and $(CD)^2$MA are also valid.

For the Shannon limit, the number of users that we can have in a cell is

$$M = \frac{G_p}{E_b/N_0} \leq 1.45 G_p \tag{17}$$

where E_b is the energy per bit and N_0 is the noise power spectral density. G_p is the *system processing gain* which is given by

$$G_p = \frac{B_w}{R} \tag{18}$$

where B_w is the bandwidth of the channel and R is the *information rate.*

In an actual system, the CDMA cell capacity is much lower than the theoretical upper-bound value given in Eq. (17). The CDMA cell capacity is affected by the receiver modulation performance, power control accuracy, interference from other non-CDMA system sharing the same frequency band, and other effects.

Every cell in a CDMA system shares the same bandwidth therefore causing the inter-cell interference, which we account for by introducing a factor β. The practical range of β is $0.5 \sim 0.55$. The interference from users in other cells reduces the number of users in a cell. The power control accuracy is represented by a factor α. The practical range for α is $0.5 \sim 0.9$. The reduction in the interference due to voice activity is represented by ν which has a practical range of $0.45 \sim 1$. Then Eq.(17) becomes

$$M = \frac{G_p}{E_b/N_0} \times \frac{1}{1+\beta} \times \alpha \times \frac{1}{\nu} \tag{19}$$

In the rest of this section, we give a design example to show how $(CD)^2$MA can have a larger capacity than CDMA. In CDMA we use phase-shift keying (BPSK) for the data modulation and quadrature phase-shift keying (QPSK) for the spreading modulation. However, just for the purpose of demonstrating the improvement of the $(CD)^2$MA to the channel capacity, we suppose that the coherent BPSK is employed for both the data modulation and the spreading modulation. Then the encoded signal in CDMA system is given by

$$x(t) = c(t)d(t)\cos\omega t \tag{20}$$

and for $(CD)^2$MA it is given by

$$x(t) = c(t)d(t) \tag{21}$$

The CDMA receiver multiplies $x(t)$ by the PN waveform to obtain the signal

$$r_1(t) = c(t)(x(t) + I(t)) = c^2(t)d(t)\cos\omega t + c(t)I(t) = d(t)\cos\omega t + c(t)I(t) \tag{22}$$

where $I(t)$ denotes the sum of noise and interference. For the $(CD)^2$MA receiver, we have

$$r_2(t) = c(t)(x(t) + I(t)) = d(t) + c(t)I(t) \tag{23}$$

Since the frequency of $d(t)$ is much less than $c(t)I(t)$, from Eq.(23) we know that the SNR of $r_2(t)$ can be significantly enhanced by using a low-pass filter before we do further processing. This is not the case when we inspect Eq.(22), in which $d(t)\cos\omega t$ and $c(t)I(t)$ have similar frequency range. We only need a small E_b/N_0 in $(CD)^2$MA. Furthermore, the interference from other users is also reduced.

We then used simulation results to show that $(CD)^2$MA has a bigger capacity than CDMA. The following conditions are used. The RF bandwidth $B_w = 1.25MHz$. The chip rate is $1.2288Mcps$, and the data rate is $R = 9.6Kbps$. For a CDMA system, assume that $E_b/E_0 = 6dB$, the interference from neighboring cells $\beta = 60\%$, the voice activity factor $\nu = 50\%$, and the power control accuracy factor $\alpha = 0.8$. Then for a CDMA system, the channel capacity is 33 mobile users per cell[1].

The most important benchmark for evaluating the service quality of a digital communication system is the bit-error-rate(BER). In this paper, the desired performance of the $(CD)^2$MA system is chosen to be $BER \leq 10^{-3}$, which is the same as that used for a CDMA systems. The condition for the evaluation is as follows. Suppose that every mobile station has a perfect power control performance then the base station receive equal power from each mobile station. We suppose that the delays due to the multi-paths are distributed uniformly in $(0.4\mu s, 1.2\mu s)$, i.e. $\Delta_1 = 0.4\mu s$ and $\Delta_2 = 1.2\mu s$, which corresponds to the range from 0.5 chip duration to 1.5 chip duration used in CDMA. The simulation results for 60, 90 and 110 users/cell are summarized in Table 2, 3 and 4 respectively.

Table 2: The relation between BER and E_b/N_0 with 60 users/cell.

E_b/N_0	3dB	4dB	5dB	6dB
BER	3.4×10^{-3}	6.8×10^{-4}	1.3×10^{-4}	3.7×10^{-5}

[1]Notice that we only study the unsectorized cell here.

Table 3: The relation between BER and E_b/N_0 with 90 users/cell.

E_b/N_0	3dB	4dB	5dB	6dB
BER	7.5×10^{-3}	2.3×10^{-3}	7.3×10^{-4}	2.9×10^{-4}

Table 4: The relation between BER and E_b/N_0 with 110 users/cell.

E_b/N_0	3dB	4dB	5dB	6dB
BER	4.4×10^{-2}	3.8×10^{-2}	3.1×10^{-2}	2.7×10^{-2}

From Table 2 we find that for $(CD)^2$MA system, $E_b/N_0 = 4dB$ is enough to give a good results under the condition of 60 users/cell. We use simulation to show this result. In our simulation, the RF bandwidth $B_w = 1.25MHz$. A 10-shift cipher is used to generate the spread carriers. The data rate is $R = 9.6Kbps$. An overhead channel is used to transmit synchronizing impulses for all users. The synchronizing impulses are digitalized into 32-bit floating point numbers. Every user needs to refresh their synchronizing impulses once a second. The bit rate of the overhead channel should be greater than $5.76Kbps$ for all the 60 users.

The simulation results are shown in Fig.5. Figure 5(a) shows the mixed signal in the channel which is a mixture of 60 spreading carriers. Figure 5(b), (c) and (d) show the spreading carriers of the users A, B and C, respectively. Figure 5(e), (f) and (g) show the message signals(green waveforms) and the recovered signals(red waveforms) for the users A, B and C, respectively. We can see that the digital signals are recovered from the $(CD)^2$MA schemes.

For comparison, we also show the case of $E_b/N_0 = 4dB$ and 110 users/cell. From Table 4 we can see that the interference become so strong that the increase of E_b/N_0 can not decrease the BER significantly. The simulation results are shown in Fig.6. Figure 6(a), (b) and (c) show the recovered signals and the message signals of users A, B and C, respectively. We can see that for user C, there exists very serious bit errors. This is a big bit error probability which may degrade the speech quality very much.

Between 60 users/cell and 110 users/cell, we show the case $E_b/N_0 = 4dB$ and 90 users/cell. From Table 3 we can see that the service quality is decreased but still usable. We also presented the simulation results in Fig.7. Figure 7(a), (b) and (c) shows the recovered signals and the message signals of users A, B and C, respectively. We can see that for user C, there exist a few bit errors.

7 Conclusions

In this paper, we present a high capacity chaotic digital CDMA scheme which has twice the channel capacity[2] of CDMA. The improvement makes us use almost one-third of the Shannon limit of an ideal channel. We find that $(CD)^2$MA is a very promising scheme to reduce the subscriber fee of PCS by one-half, compared to the CDMA system. One of the critical problems in a chaotic CDMA scheme is to maintain the synchronization between two chaotic systems, since the synchronization signals would take a bandwidth on the order of the date rate if the continuous synchronization scheme is used. To overcome this problem we use an impulsive synchronization scheme to synchronize the two chaotic circuits in the linked pair. Thus, we only need a very small bandwidth for transmitting synchronization impulses.

Acknowledgment

This work is supported by the Office of Naval Research under grant No. N00014-96-1-0753.

References

[1] Vijay K. Garg, Kenneth F. Smolik, and Joseph E.Wilkes. *Applications of CDMA in wireless/personal communications.* Feher/Prentice Hall digital and wireless communication series. Prentice Hall PTR, Upper Saddle River, NJ, 1997.

[2] K.S. Gilhousen, I.M. Jacobs, R. Padovani, A.J. Viterbi, and C.E. Wheatley. On the capacity of a cellular cdma system. *IEEE Transactions on Vehicular Technology*, 40(2):303–12, May 1991.

[2] If the E_b/N_0 is increased to $6dB$ as that used in CDMA systems, the $(CD)^2$MA system may even triple the channel capacity as shown in Table 3.

[3] K.S Halle, Chai Wah Wu, M. Itoh, and L.O. Chua. Spread spectrum communication through modulation of chaos. *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, 3(2):469–77, April 1993.

[4] M.G. Jansen and R. Prasad. Capacity, throughput, and delay analysis of a cellular ds cdma system with imperfect power control and imperfect sectorization. *IEEE Transactions on Vehicular Technology*, 44(1):67–75, Feb. 1995.

[5] D.Z. Knuth. *The art of computer programming(vol.2).* Assison-Wesley, Canada, 1981.

[6] W.C.Y Lee. *Mobile Cellular Telecommunications Systems.* McGraw-Hill, New York, 1989.

[7] W.C.Y. Lee. Overview of cellular cdma. *IEEE Trans. Vehicular Technol.*, 40(2):291–302, May 1991.

[8] L.O.Chua. Chua's circuit— an overview ten years later. *Journal of Circuit, Systems, and Computers*, 4(2):117–159, Jan. 1994.

[9] R.L. Pickholtz, L.B. Milstein, and D.L. Schilling. Spread spectrum for mobile communication. *IEEE Trans. Vehicular Technol.*, 40(2):313–322, May 1991.

[10] A. Salmasi. Cellular and personal communicarion networks based on the application of code division multiple acess(cdma). In *Proc. Virginia Tech's 1st symp. Wireless Personal Commun.*, pages 10.1–10.9, June 3-5 1991.

[11] M. K. Simon, J.K. Omura, R.A. Scholtz, and B.K. Levitt. *Spread spectrum communications handbook.* McGraw-Hill, New York, 1994.

[12] J.A. Ulloa, D.P. Taylor, and W.F.S. Poehlman. An expert system approach for cellular cdma. *IEEE Transactions on Vehicular Technology*, 44(1):146–54, Feb. 1995.

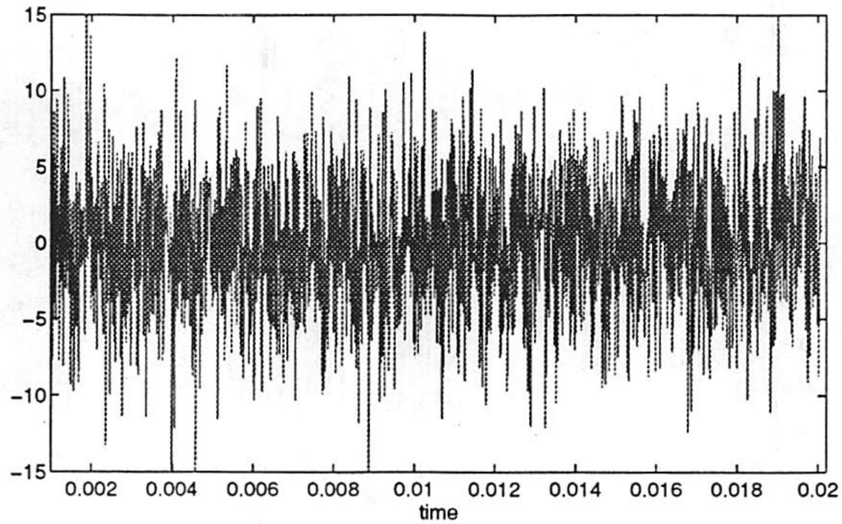
[13] A.J. Viterbi. *CDMA : principles of spread spectrum communication.* Addison-Wesley wireless communications series. Addison-Wesley Pub. Co., Reading, Mass., 1995.

[14] Geng Wu, A. Jalali, and P. Mermelstein. On channel model parameters for microcellular cdma systems. *IEEE Transactions on Vehicular Technology,*, 44(3):706–11, Aug. 1995.

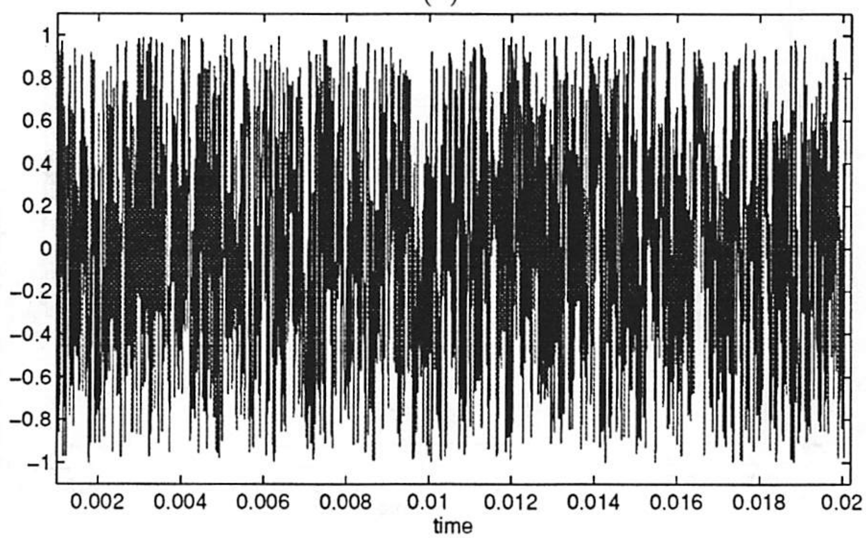
[15] Tao Yang and L.O.Chua. Impulsive control and synchronization of chaotic systems and secure communication. Memorandum UCB/ERL M97/12, Electronics Research Laboratory, College of Engineering, University of California, Berkeley, CA 94720, 29 Jan. 1997.

[16] Tao Yang and L.O.Chua. Impulsive control and synchronization of nonlinear dynamical systems and application to secure communication. *International Journal of Bifurcation and Chaos*, 7(3):In press, 1997.

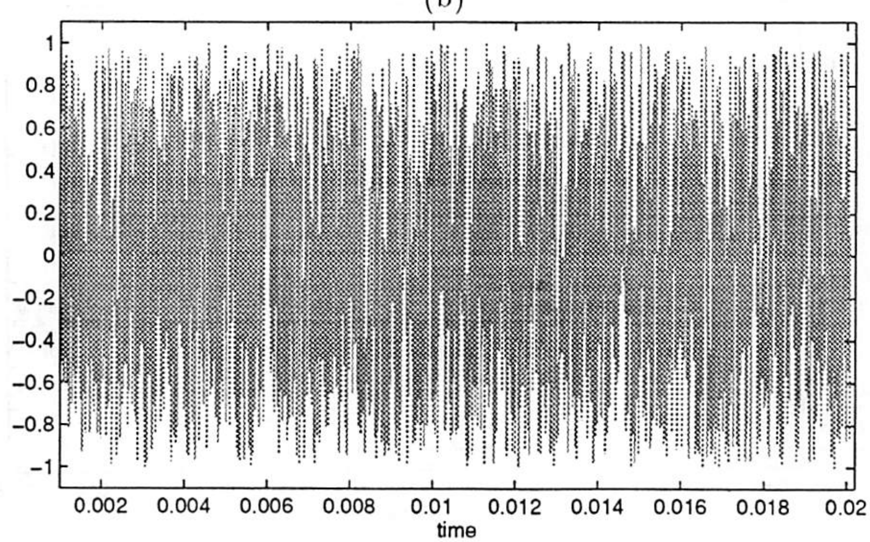
[17] Tao Yang, Chai Wah Wu, and Leon O. Chua. Cryptography based on chaotic systems. *IEEE Transaction on Circuits and Systems—I: fundamental theory and applications*, 44(5):469–472, May 1997.


15
10
5
0
−5
−10
−15
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
time

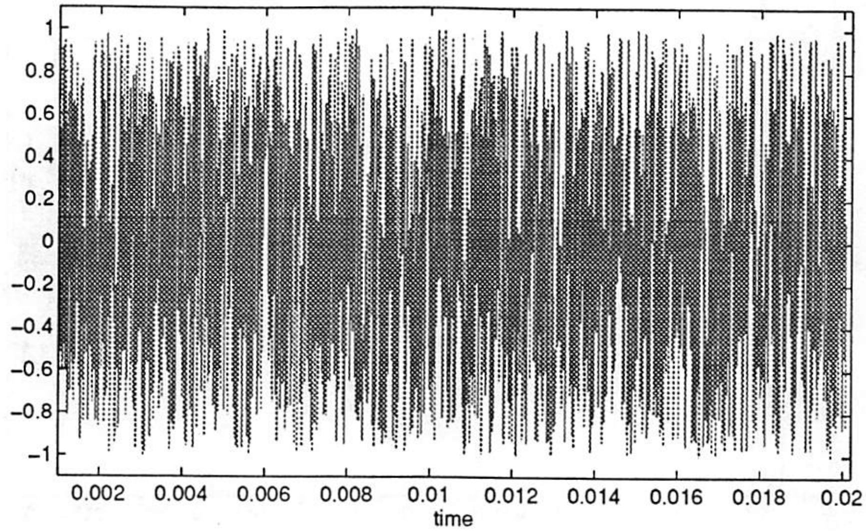
(a)


1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
time

(b)

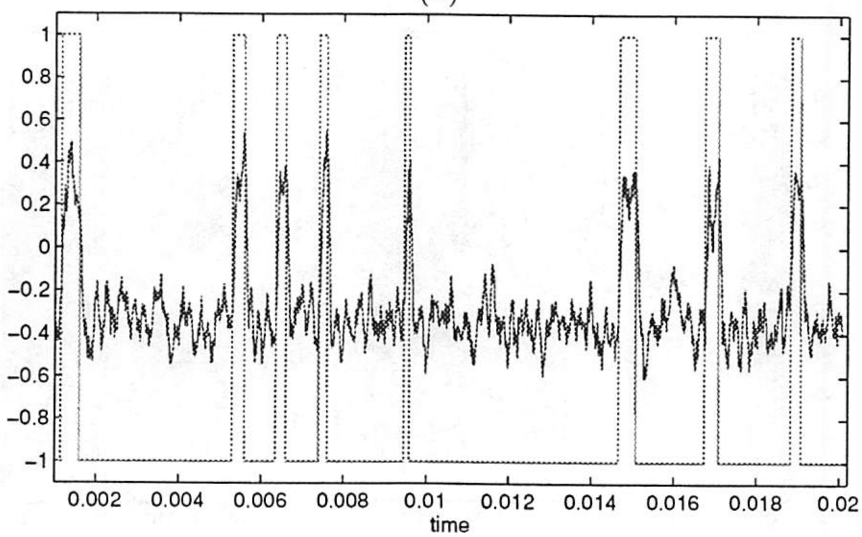

1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
time

(c)



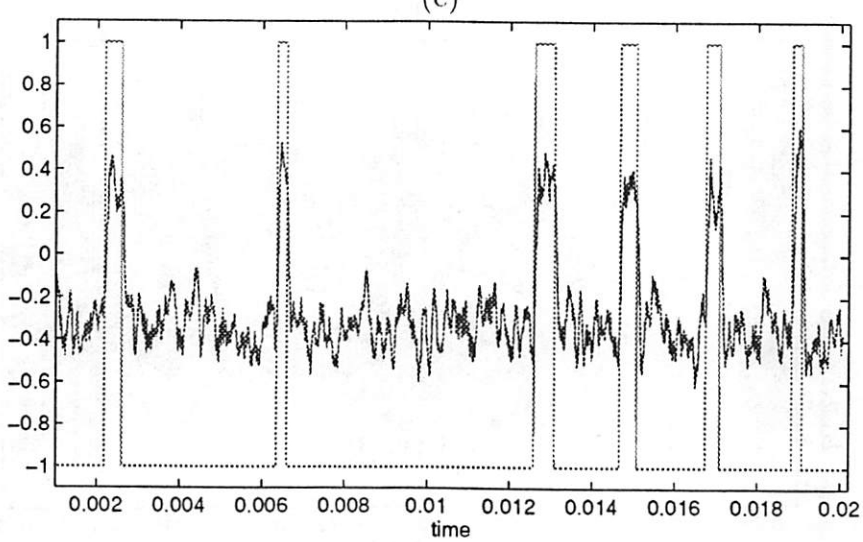
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02
time

(d)



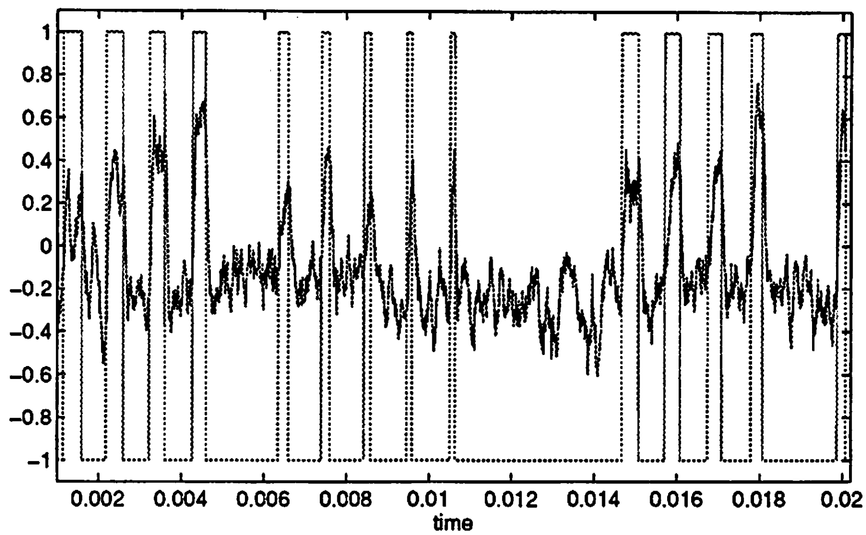
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02
time

(e)



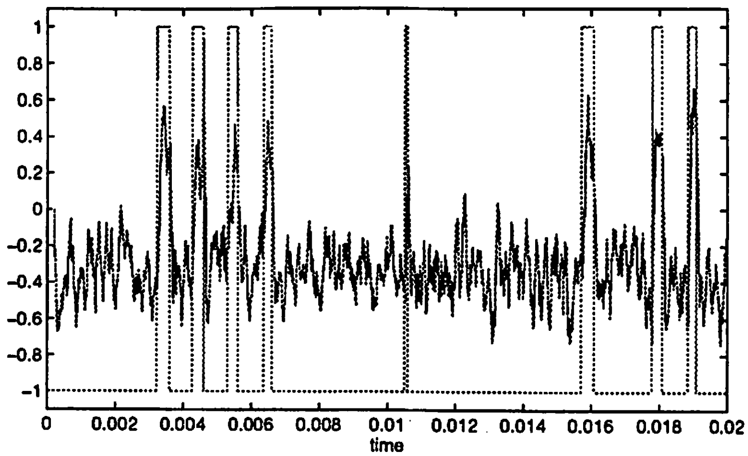
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
0.002 0.004 0.006 0.008 0.01 0.012 0.014 0.016 0.018 0.02
time

(f)

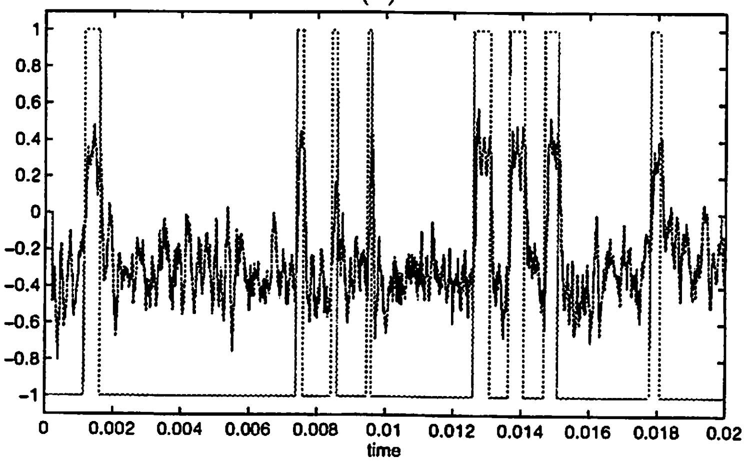


(g)

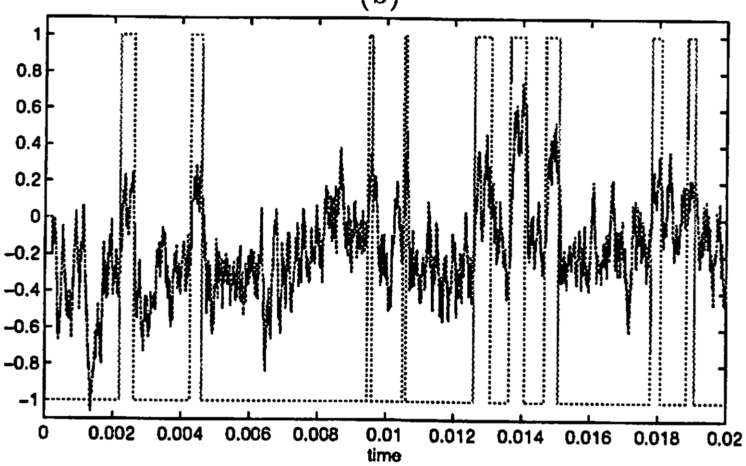
Figure 5: Simulation results of the $(CD)^2$MA system with 60 users/cell and $E_b/N_0 = 4dB$. (a) The mixture of the spreading carriers in the channel. (b) The spread carrier for the user A. (c) The spread carrier for the user B. (d) The spread carrier for the user C. (e) The message signal(green) and the recovered signal(red) for the user A. (f) The message signal(green) and the recovered signal(red) for the user B. (g) The message signal(green) and the recovered signal(red) for the user C.



(a)

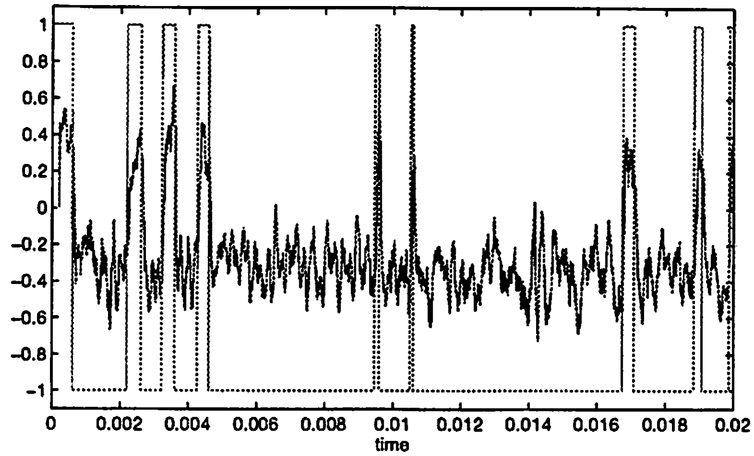


(b)

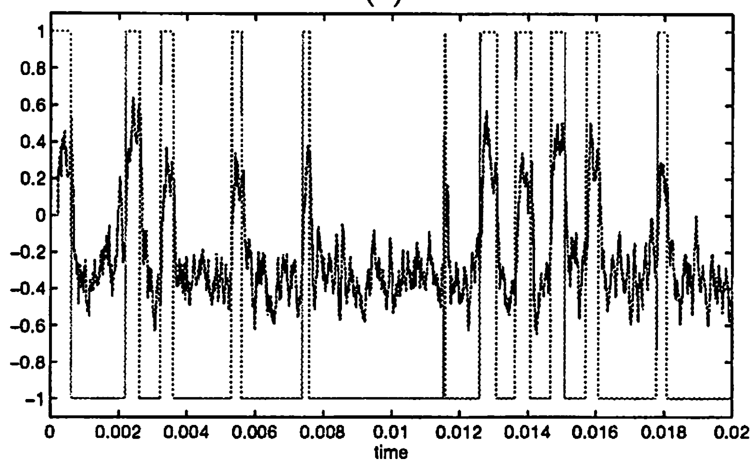


(c)

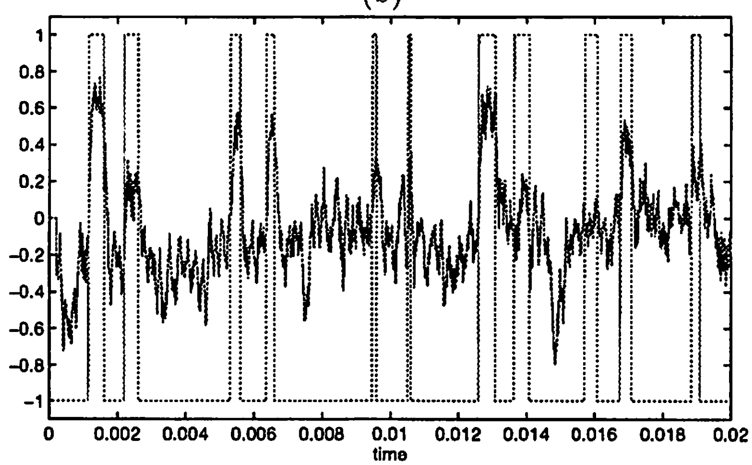
Figure 6: Simulation results of the $(CD)^2MA$ system with 110 users/cell and $E_b/N_0 = 4dB$. (a) The message signal(green) and the recovered signal(red) for the user A. (b) The message signal(green) and the recovered signal(red) for the user B. (c) The message signal(green) and the recovered signal(red) for the user C.



(a)



(b)



(c)

Figure 7: Simulation results of the $(CD)^2$MA system with 90 users/cell and $E_b/N_0 = 4dB$. (a) The message signal(green) and the recovered signal(red) for the user A. (b) The message signal(green) and the recovered signal(red) for the user B. (c) The message signal(green) and the recovered signal(red) for the user C.