VERBAL PARADIGMS—PART I: MODELING WITH VERBS

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ELECTRONICS RESEARCH LABORATORY

College of Engineering
University of California, Berkeley
94720

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Tao Yang

Department of Electrical Engineering and Computer Sciences,

University of California at Berkeley,

Berkeley, CA 94720, U.S.A.

Tel: (510)-642-5311 Fax: (510)-643-8869

Email: taoyang@fred.eecs.berkeley.edu

http://trixie.eecs.berkeley.edu/~taoyang/cv-taoyang.html

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Abstract

Verbal paradigms are a collective of methods for developing a new systematic framework of artificial intelligence by embedding the knowledge and experiences that human experts expressed or implied with verbs into machine intelligence. Computation with words is a systematic consideration of coping with complexities. While fuzzy logic provides a computing framework of adjectives and adverbs, it does not cope with verbs. It turns out that verbs are very important to describe the dynamics of complexities for the survival of human beings. Verbs should also be a very important building block for transfer and represent human experiences, thus a very important tool for the next generation of expert system. In this paper, the applications of verbs to modeling complexity are discussed. First, the dynamics of linguistic expressions are identified as the usages of verbs. Then nonlinear and linear dynamic systems are used to model and classify verbs. Based on these model of verbs, the dynamic aspects of linguistic expressions, which associate with qualitative descriptions of human experiences, can be described quantitatively. In particular, the verbal models of both simple and complex dynamic systems are provided to show how to use verbal models. Computer simulation results are provided.

1 Introductions

Unlike the foundation of the relativity theory and quantum mechanics at the beginning of this century which were introduced a totally new point of view of the reality as a dramatic revolution, there exists a silent revolution all over the whole scale of our reality and all scientific principles as a whole, due to the challenge of complexity[1]. Complexity is not a new phenomenon though it only falls into the scope of modern science in this century. Human being had survived for such a long time without knowing them (or precisely, without knowing how to model them).

The challenge that complexity proposes to science is the challenge to methods based on precise models. In the history of science, the precise modeling were so excellent such that when fuzzy set theory[8] embedded imprecise and vague into scientific framework, it encountered so many defenses at the beginning. Today, fuzzy set theory has been established and widely used because it meets the natural tendency of human being, which is characterized by using linguistic models[9, 3] to cope with complexity.

From a physical point of view, an electronic digital computer is a simulation running in an analogue computer based on electro-magnetic dynamics of silicon. Similarly, our linguistic modeling process is a symbolic simulation running in an analogue structure called *human brain*. Here I dare not add "computing" before "structure" because "computation" is not well-defined. On the other hand, I wonder if we can find the evidence that our wet-ware based brains are doing "computation". Since this discussion is very sophisticated and in some sense it do not contribute to the substantial process of solving problems, I do not want to discuss it from an engineer's point of view, which is focused on solving problems by using either simulations or real devices.

Encounter with complexities of all scales of the universe, how real our linguistic simulation process should be for coping with the most critical problem—the survival of human species and individuals? Maybe we can also ask this question in the other way: why our human species can survive complexities painlessly while our digital computers (no matter how powerful it may be, from PCs to national supercomputers) can not? The answer may be that our brains simulate the complexities in correct ways and digital computers simulate it in wrong ways.

If we view our languages as dynamic systems of symbols, then how our human beings use these symbolic process to simulate the complexities? It seems to me that all human species use a same structure to simulate complexities because even in China we use analogue method to encode the simulation while in Western world we use symbolic method and in Japan we mix both of them, we can understand each other so well. The only reason should be that we confront the same complexities in our circumstances which are independent to the coding methods.

We can find two kinds of complexities existed in our circumstance. The first one is static such as fractal structures and randomness. The second one is dynamic such as chaotic processes and random processes. The relationship between static complexity and dynamic complexity can not be simply stated that the formal is only a "snap shot" of the latter. Also, the boundaries between this two kinds of complexities are "fuzzy". Form my point of view, the existed debate between fuzzy theory and stochastic theory is unnecessary because they inspect the same complexity from two different aspects. I do not think that anyone can unify complexities into a single framework. In our linguistic system, we have developed different kinds of words to cope with these two kinds of complexities. We use adjectives to describe static complexities while use verbs to describe dynamic complexity.

It is then very clear that fuzzy logic is a quantitative tool of modeling the linguistic process for coping with static complexity represented by adjectives. We still need another quantitative tool of modeling linguistic process for coping with dynamic complexity represented by verbs. In the next section, I will show how verbs are used to model dynamics.

2 What are verbs ?

Can we write down all what we think, feel and want by words? Can we train our offsprings by only letting them reading books? The answer is NO because as we have noticed that there exist some interconnections between human individuals (definitely between animal individuals because they do not have advanced languages and words) far beyond what we can express (speak and write). Two typical examples are cited from [2] as follows.

Example 1

A: 'These two wines taste different.'

B: 'But *how* do they taste different ?'

A: 'It's hard to say: try them yourself.' "

Example 2

A: 'What does it feel like when you do a double back flip on the trampoline?'

B: 'It's hard to say, but when you can do one you'll know when it feels right and when it doesn't.'

It is clear that between the teacher and the learner something behind the "meaningless dialogue" is communicated implicitly. There are may factors attribute to this kind of communication between human individuals such as the same knowledge and experience, the same feelings to some staffs and even the same evolution process encoded in human genes. What the teacher try to do is to show the learner what kind of his own experience should be formed after following these instructions. This kind of experience is full of dynamic complexities implied by verbs such as "taste", "try" and "feel". Since the teacher can not tell this experience explicitly, the learner has to learn by "taste", "try" and "feel" by himself. If this kind of experience is not a dynamic process (in memory), it should be easy to write down into explicit expressions such that the classical expert system can easily recode it and retrieve it very fast and accurate. However, the classical expert system can not recode this kind of experience. If one tries to model all these implicit factors in a single model or paradigm, it is almost impossible because any complex system should have many different aspects and each aspect may need a framework to model.

Computing with word is a characteristic and banner of fuzzy logic[9]. Although "be" is too busy in fuzzy inference, the other verbs in the big collective of words seem to be too leisure to enjoy fuzzy computation. In this sense, it seems that fuzzy set is only the computing with adjectives such as "tall", "good", adverbs such as "very", "extremely"and a verb "be". For example, a typical fuzzy rule set is as follows.

If A is high, then B is low;
If A is low, then B is high;

It is verbs to introduce dynamics into our language and make the transfer of dynamics of brain states between human individuals. Since brain dynamics are closely connected with our intuitions and emotions, there may exist lots of information transfer via verbs using implicit (intuitive) ways among human individuals. Unfortunately, whenever a verb is recorded as a representation(its symbol), the details of dynamics defined by the human individual who spoke it is generally lost and could be only recovered by those human individuals who read it later using their own dynamics associated with this verb (sometimes we call the dynamics associated with a set of verbs as *experiences*).

Since a verb contains so many different dynamics from person to person and even for the same person, with different contexts, it may express different dynamics. The implications behind verbs are very complex. This is the reason why we can efficiently express our feelings if we talk to someone in person instead of communicating only via letters. This is also the reason why we want to develop video telephone and virtual reality.

To keep the dynamics of a verb, we first need to set up a model for describing this verb. Although dynamics of verbs should be very complex, we can qualitatively lumped them into different classes.

1. static verbs

This kind of verb does not has any dynamics, they just fixed at a static point such as "BE". Since this class has been studied very carefully in fuzzy logic, I do not discuss it here.

2. smooth dynamic verbs

This kind of verbs can be described by smoothly dynamic systems whose ω sets may be fixed points(asymptotic processes), limit cycles(oscillated processed), toruses(quasi-periodic processes) and strange attractors(chaotic processes).

Some verbs converge to fixed points like "become" in the following sentence:

$\boxed{\text{Gina becomes a pretty girl.}}$

While "pretty girl" is a fuzzy item which can be modeled by a membership function $x(t) = \mu_{pretty\ girl}(Gina, Time)$, "becomes" is a dynamic process which can only be modeled in different contexts. For example, if I mean that Gina was an ugly girl before, and I only meet her once a year, then "become" is a converge process which approaches its fixed point asymptotically with the initial value $x(0) = \mu_{pretty\ girl}(Gina, 0)$, which is give by

$$\begin{aligned} \dot{x} &= -x + \mu_{pretty\ girl}(Gina, \infty), \\ x(0) &= \mu_{pretty\ girl}(Gina, 0) \end{aligned} \tag{1}$$

If I have a chance to meet Gina more often, say once a month, then in some months Gina may seem to be a little worse than some other months, in this case, the dynamics of "become" can be modeled by an asymptotic oscillating dynamics as

$$\begin{aligned} \frac{d^2x}{dt^2} &= -x + \mu_{pretty\ girl}(Gina, \infty), \\ x(0) &= \mu_{pretty\ girl}(Gina, 0) \end{aligned} \tag{2}$$

If a person can observe Gina in a very fine time resolution, say Gina's mother, then the "become" can be even described by a chaotic process whose average converges to a high value of "pretty", such a system could be

$$\begin{aligned} \dot{x} &= -x + ky + \mu_{pretty\ girl}(Gina, \infty), \\ x(0) &= \mu_{pretty\ girl}(Gina, 0) \end{aligned} \tag{3}$$

where k is a constant and y is a chaotic process given by a chaotic system, say, the Lorenz system[5]

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases} \tag{4}$$

where σ, r, and b are three real positive parameters.

3. impulsive dynamic verbs

In this case, the verbs are modeled by impulsive differential equations whose states are subject to sudden changes (jumps) whenever some conditions are satisfied. This kind of verb can also have different types of ω sets such as: fixed points, limit cycles, toruses and strange attractors. One example is given by the description of a rule from psychotherapy.

Whenever you feel at the edge of crazy, just tell yourself, "calm down, please !", "calm down, please !" ..., you can regain the control of yourself.

To trivialize this statement, let us suppose that "feel crazy" is chaotic status of emotional trajectory in a phase space. When one can control oneself the emotional trajectory is supposed to be a kind of limit cycle. "calm down, please !" can be modeled by a kind of impulsive control effect which only effects whenever some conditions are satisfied. We then use the following Rössler chaotic system[6] to represent "feel crazy" as

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = zx + b - cz \end{cases} \tag{5}$$

where a, b, and c are three parameters chosen as $a = 0.398$, $b = 2$, and $c = 4.0$. The fourth-order Runge-Kutta algorithm with step size of 0.005 is used. The initial condition is given by $(x(0), y(0), z(0)) = (-2.277838, -2.696438, 0.304911)$. Fig.1(a) shows the corresponding chaotic attractor. The "calm down, please !" is modeled by a set of impulsive control law

$$\begin{cases} x(\tau_k^+) = x(\tau_k) \\ y(\tau_k^+) = \psi_2(\mathbf{x}(\tau_k)) = [1 - \lambda sgn(y(\tau_k))]y(\tau_k), \quad |\lambda| < 1 \\ z(\tau_k^+) = z(\tau_k) \end{cases} \tag{6}$$

, which effects only when the condition $x = 0$ is satisfied. $\mathbf{x}^T = (x, y, z)$. The impulsively controlled Rössler system is then given by

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = zx + b - cz \end{cases} , \quad x(t) \neq 0 \tag{7}$$

and

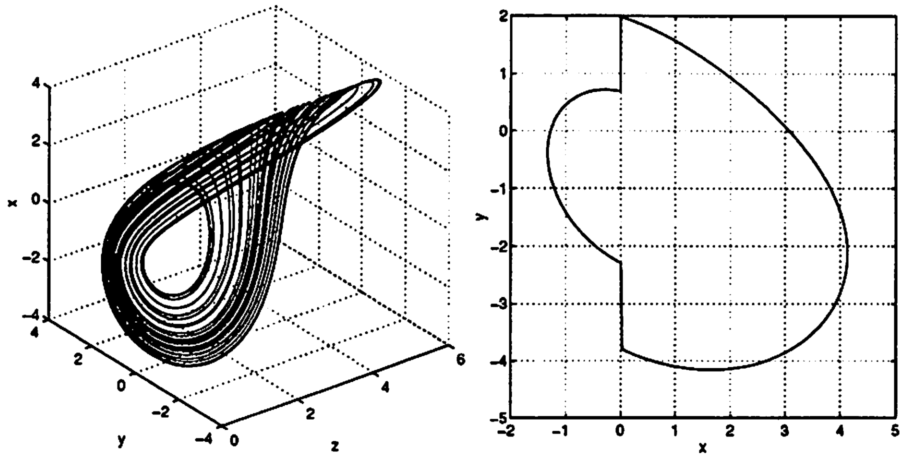
$$\mathbf{x}(\tau_k^+) = \Psi_k(\mathbf{x}(\tau_k)), \quad x(\tau_k) = 0 \tag{8}$$

where $\Psi_k(\mathbf{x})$ is given by Eq.(6). We choose $\lambda = 0.65$, the simulation result is shown in Fig.1(b), which represents a controlled emotional state. The detail of impulsive control of chaotic systems can be find in [7].

3. terminal dynamic verbs

This kind of verb can only effect in a finite time period such as

Gina *arrives* campus.



(a) (b)

Figure 1: Demonstration of impulsive dynamic verbs. (a) The representation of "feel crazy". (b) The result of repeating "calm down, please !".

Although Gina could use different dynamics of "arrive" such as walk, take bus or whatever, "arrive" is terminated whenever she is in campus. This kind of verb can be modeled by dynamic systems with non-Lipschitz singular solutions. One example is given by

$$\dot{x} = -(x - x^*)^{1/3} \tag{9}$$

Since at $x = x^*$ the Lipschitz condition is violated, $x(t)$ will arrive x^* within finite time. The simulation results are given in Fig.2(a). The initial condition is $x(0) = 2$. In Fig.2(a) the four curves correspond to four different x^*'s of $0, 0.5, 1$, and 1.5, respectively. We can see that in each case, the system arrive x^* within finite time. In this simulation, the fourth order Runge-Kutta method with fixed step size 0.02 is used.

Terminal verbs are not always so simple, given some conditions, they may even given some stochastic attractors[10]. One example is as follows.

Gina is keeping changing her idea widely.

If we define the universe of Gina's idea into a two-dimensional space, then If we want to show "keep changing widely" is an irrational process, we can not use a chaotic system to model it. The reversible property of chaotic process seems to be too "rational" because whenever we know the result we can reverse it and find the "cause". A promising candidate for modeling this kind of irrational process is the following terminal dynamic system:

$$\begin{aligned}\dot{x} &= \frac{\lambda x y}{x^2 + y^2} - \epsilon x \\ \dot{y} &= \frac{\lambda y^2}{x^2 + y^2} - \gamma y - (\lambda - \gamma)\end{aligned} \tag{10}$$

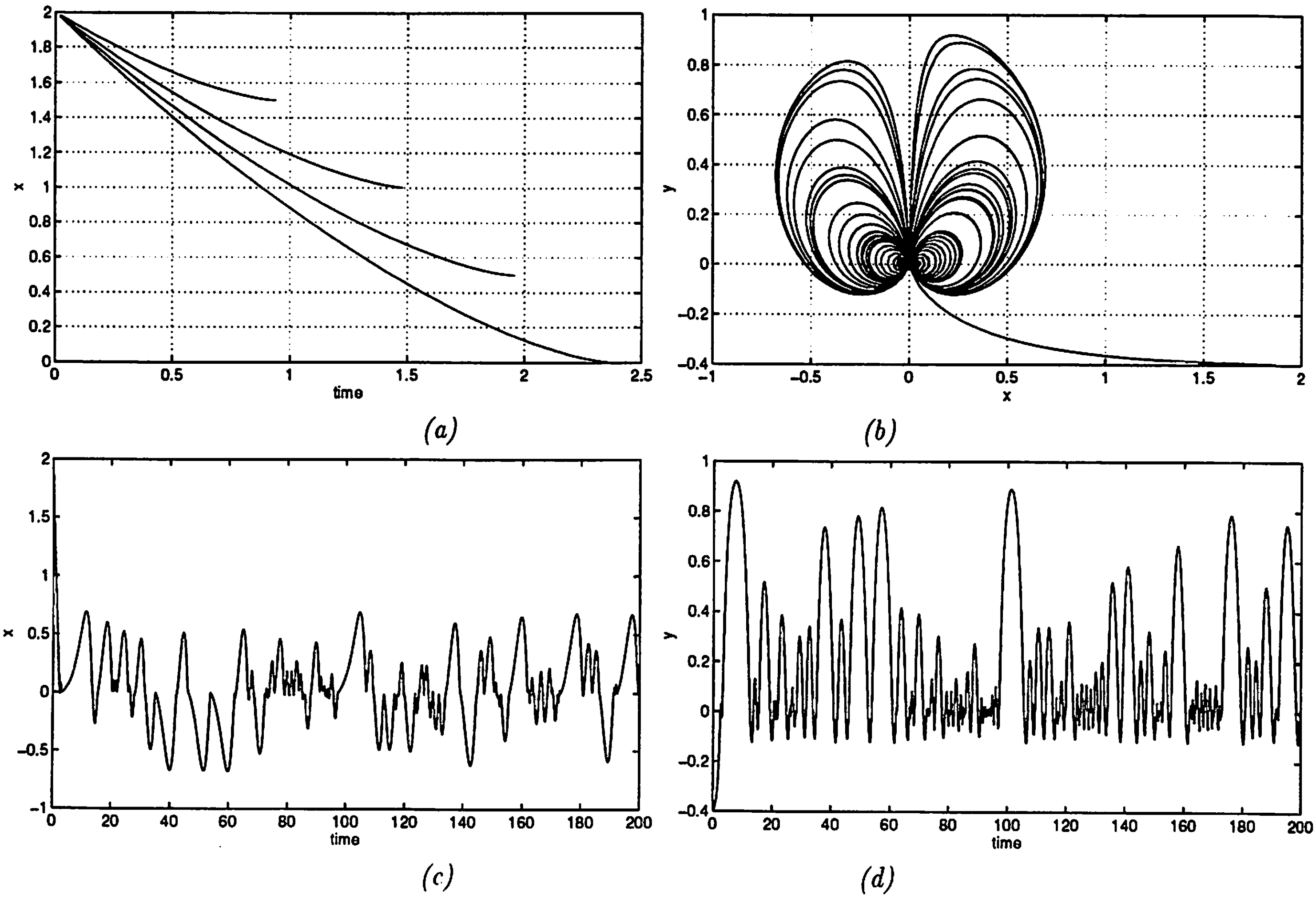


Figure 2: Terminal dynamic verbs and their complexities. (a) The trajectories of "arrive" with different initial conditions. (b) Stochastic attractors for modeling "keep changing widely". (c) The waveform of $x(t)$ of the attractor in (a). (d) The waveform of $y(t)$ of the attractor in (b).

The simulation results are given in Fig.2(b) with $\lambda = 1.0$, $\epsilon = 0.6$, $\gamma = 0.7$. The Lipschitz condition is violated at the origin. The initial condition is $(x(0), y(0)) = (2, -0.4)$. In this simulation, the fourth order Runge-Kutta method with fixed step size 0.1 is used. The waveforms of $x(t)$ and $y(t)$ are also shown in Figs.2(c) and 2(d), respectively. We can see that the waveform of this system is somehow more complex than those of some low-dimensional chaotic systems.

Although the final application of the dynamic models of verbs may be in areas where complexities can not be easily decomposed by using bottom-up methods, for the purpose of demonstrating the usage of verbal models, two examples are presented in the next two sections.

3 The verbal model of a simple system

In this section we build the verbal model for a simple nonlinear dynamic system. We investigate the following 2nd-order piecewise-linear oscillator system. In the outer region, this system is governed by

$$\begin{cases} \dot{x} = 2y - 0.1x \\ \dot{y} = -2x \end{cases}, |x| > 1 \tag{11}$$

while in the inner region, it is governed by

$$\begin{cases} \dot{x} = y + x \\ \dot{y} = -x \end{cases}, |x| \leq 1 \tag{12}$$

The attractor of this system is shown in Fig.3. The initial condition for this trajectory is $(x(0), y(0)) = (0.1, 0.1)$. We can see that the trajectory converges to a limit cycle.

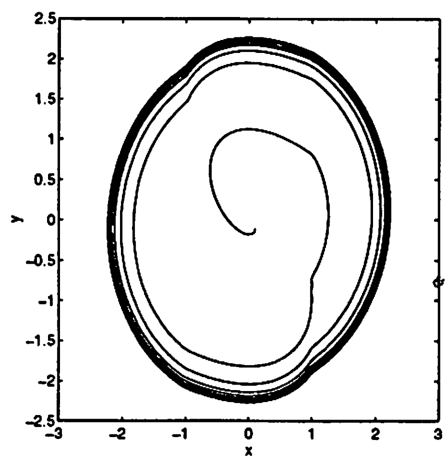


Figure 3: The attractor of the 2nd-order piece-wise linear dynamic system.

By inspecting the trajectory of the limit cycle we can qualitatively describe the evolution of this system using a linguistic statement as

The trajectory *leaves* the inner region and *stay* in the outer region for a while then it *enters* the inner region again, such that it forms a limit cycle.

The above qualitative statement describes a kind of mechanics for forming a limit cycle. This general description is a powerful tool for human being to express, remember and classify different experiences. Human individuals can also change and share their experiences by using this kind of qualitative model because it can be easily understood by the other individuals (including their off-springs). However, if we want to share this kind of experience with an expert system based on machines, we have to introduce some quantitative descriptions of the verbs in the above linguistic statement. For the system in Eqs. (11) and (12) a verbal model is given by

If $\|x-1\| \leq \xi$ and $\dot{x} < 0$ OR $\|x+1\| \leq \xi$ and $\dot{x} > 0$, then the system *enters* the inner region, **else** If $\|x\| > 1$, then the system *stays* in the outer region; If $\|x\| < 1$, then the system *leaves* the inner region.

The verb *stay* and *leave* are respectively given by Eqs.(11) and (12). The verb "enter" is given by the following system:

$$\begin{cases} \dot{x} = w_1 y + w_2 x \\ \dot{y} = -2x \end{cases}, |x-1| \leq \xi \text{ } and \text{ } \dot{x} < 0 \text{ } OR \text{ } |x+1| \leq \xi \text{ } and \text{ } \dot{x} > 0 \tag{13}$$

We choose $\xi = 0.01$. We use the following learning law to train w_1 and w_2

$$\dot{w}_1 = \delta(\dot{x}_o - \dot{x})y \tag{14}$$

$$\dot{w}_2 = \delta(\dot{x}_o - \dot{x})x \tag{15}$$

where $\dot{x}_o$ and $\dot{x}$ are respectively obtained from the trajectories of the original system as shown in Fig.3 and the verbal model system at "enter" status. We choose the learning rate $\delta = 0.1$. The learning processes of parameters for "enter" is shown in Fig.4. The solid line shows $w_1(t)$ and the dashed line shows $w_2(t)$. The initial conditions are $w_1(0) = 0$ and $w_2(0) = 0$. The trained parameters are $w_1 = 0.977687$ and $w_2 = -0.533483$.

Figure 5 shows the performance of the verbal model. The dotted line are show the original trajectory, and the solid line shows the output of the verbal model. One can see that the verbal model gives almost an error-free result for this problem.

Here I do not want to claim that the verbal model has advantage over the other identification schemes to this simple problem. One should always bear in mind that verbal models most possibly used as high-level machine languages for the next generation of expert systems. Verbal models can be viewed as experiences of machines which are supposed to share a kind of *qualitatively related* experience with human experts to the same complex processes.

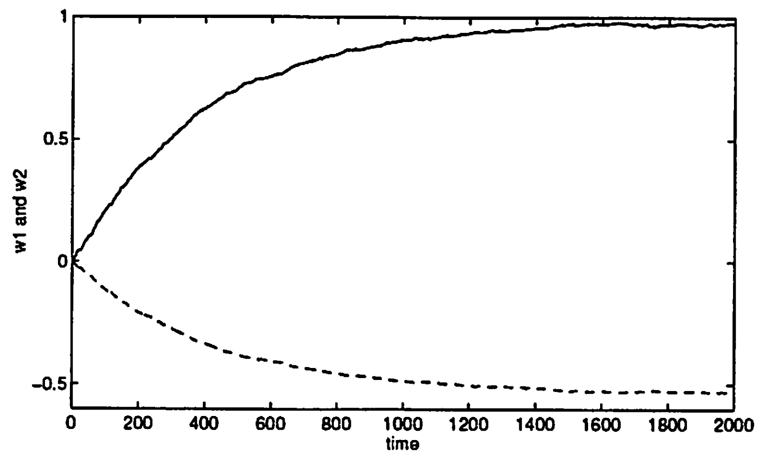


Figure 4: Learning processes of parameters for *enter* for verbal model of the 2nd-order system.

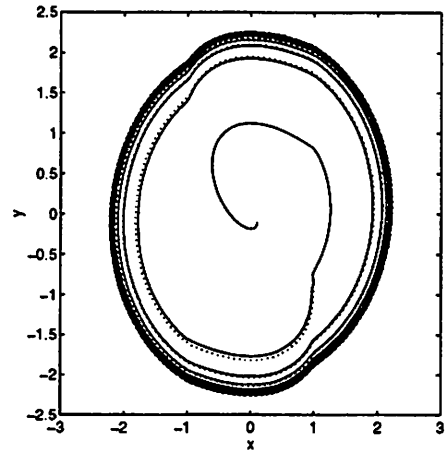


Figure 5: The performance of the verbal model.

4 The verbal model of a complex system

In this section a verbal model is used to describe a complex dynamic system; namely, a chaotic system. We consider the following piecewise-linear chaotic system [4]:

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y - \gamma z \end{cases} \tag{16}$$

where

$$f(x) = bx + \frac{1}{2}(a-b)(|x+1| - |x-1|) \tag{17}$$

One should note that this system is divided into an inner region which is governed by the following linear ODE:

$$\begin{cases} \dot{x} = \alpha(y - x - ax) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y - \gamma z \end{cases} \quad , \quad |x| < 1 \tag{18}$$

and two outer regions which are governed by the following linear ODE:

$$\begin{cases} \dot{x} = \alpha(y - x - bx + (a-b)sgn(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y - \gamma z \end{cases} \quad , \quad |x| > 1 \tag{19}$$

where $sgn(\cdot)$ is the signum function. One of the chaotic attractor of this system is shown in Fig.6(a).

We also use verbs "stay", "leave" and "enter" to model this chaotic system. The verbal model of this system is given by

If $|x-1| < \xi$ and $\dot{x} < 0$ OR $|x+1| < \xi$ and $\dot{x} > 0$, the system *enters* the inner region, **else**
If $|x| > 1$, then the system *stays* in the outer region;
If $|x| < 1$, then the system *leaves* the inner region.

The verb *stay* and *leave* are respectively given by Eqs.(18) and (19).

The verb *enter* represent a kind of transient processes from outer region to inner region. We need to learn parameters for this process. Suppose that the verb *enter* is given by the following system:

$$\begin{cases} \dot{x} = \alpha y + w_1 x \\ \dot{y} = y + x + z \\ \dot{z} = -\beta y - w_2 z \end{cases} \tag{20}$$

We learn parameter w_1 by using an adaptive learning law which minimizes the following cost function

$$E(t) = \frac{1}{2}(\dot{x}_o - \dot{x})^2, |x_o - 1| < \xi \text{ or } |x_o + 1| < \xi \tag{21}$$

where x_o is obtained from the objective trajectory of the original chaotic system within the transient regions. We have

$$\dot{w}_1 = -\delta \frac{\partial E}{\partial w_1}$$

$$
\begin{aligned}
&= \delta(\dot{x}_o - \dot{x})\frac{\partial \dot{x}}{\partial w_1} \\
&= \delta(\dot{x}_o - \dot{x})x \qquad (22)
\end{aligned}
$$

Similarly, we learn parameter w_2 by using an adaptive learning law which minimizes the following cost function

$$E(t) = \frac{1}{2}(\dot{z}_o - \dot{z})^2, |x_o - 1| < \xi \text{ or } |x_o + 1| < \xi \qquad (23)$$

where z_o is the third variable of the objective trajectory of the original chaotic system within the transient region. We have

$$\dot{w}_2 = -\delta(\dot{z}_o - \dot{z})z \qquad (24)$$

The simulation results are shown in Fig.6. In this simulation, we choose the parameters of the chaotic system as $\alpha = 15, \beta = 20, \gamma = 0.56, a = -8/7$ and $b = -5/7$. With these parameters, the strange attractor is shown in Fig.6(a) with initial condition $(x(0), y(0), z(0)) = (-1.087613, 0.012802, 0.312794)$. The learning processes of w_1 and w_2 are shown in Figs.6(b) and (d), respectively. We choose the learning parameter $\delta = 0.1$. The parameter for *enter* is chosen as $\xi = 0.2$. The initial conditions are $w_1(0) = w_2(0) = 0$. We can see that both w_1 and w_2 fluctuate around average values with small biases if time is big enough. The average values of w_1 and w_2 are 1.15 and 1.13, respectively. The output of the verbal model is shown in Fig.6(d). Although there exist some modeling error, we find that the verbal model is a good approximation of the original chaotic system. Since the verbal system is also a chaotic system, in Fig.6(d) we only show the stable strange attractor with the same initial condition as that used in Fig.6(a). Since chaotic system is very sensitive to initial conditions, mismatch of parameters and differences in structure, the verbal model can only repeat the qualitative property of the original system.

Comparing the results in Sections 3 and 4 we can see that the same verbal model can be used to model qualitatively different processes given different definitions of verbs. This makes our language vivid and rich in the sense that human individuals can transfer different experiences by using a small collective of verbs in relatively huge collective of contexts.

5 Concluding remarks

Linguistic description of complexity is a well-developed strategy for human being to survive complexities of circumstance. Besides fuzzy theory, which can quantize static qualitative descriptions in our linguistic statements, we still need a theoretic framework for quantizing

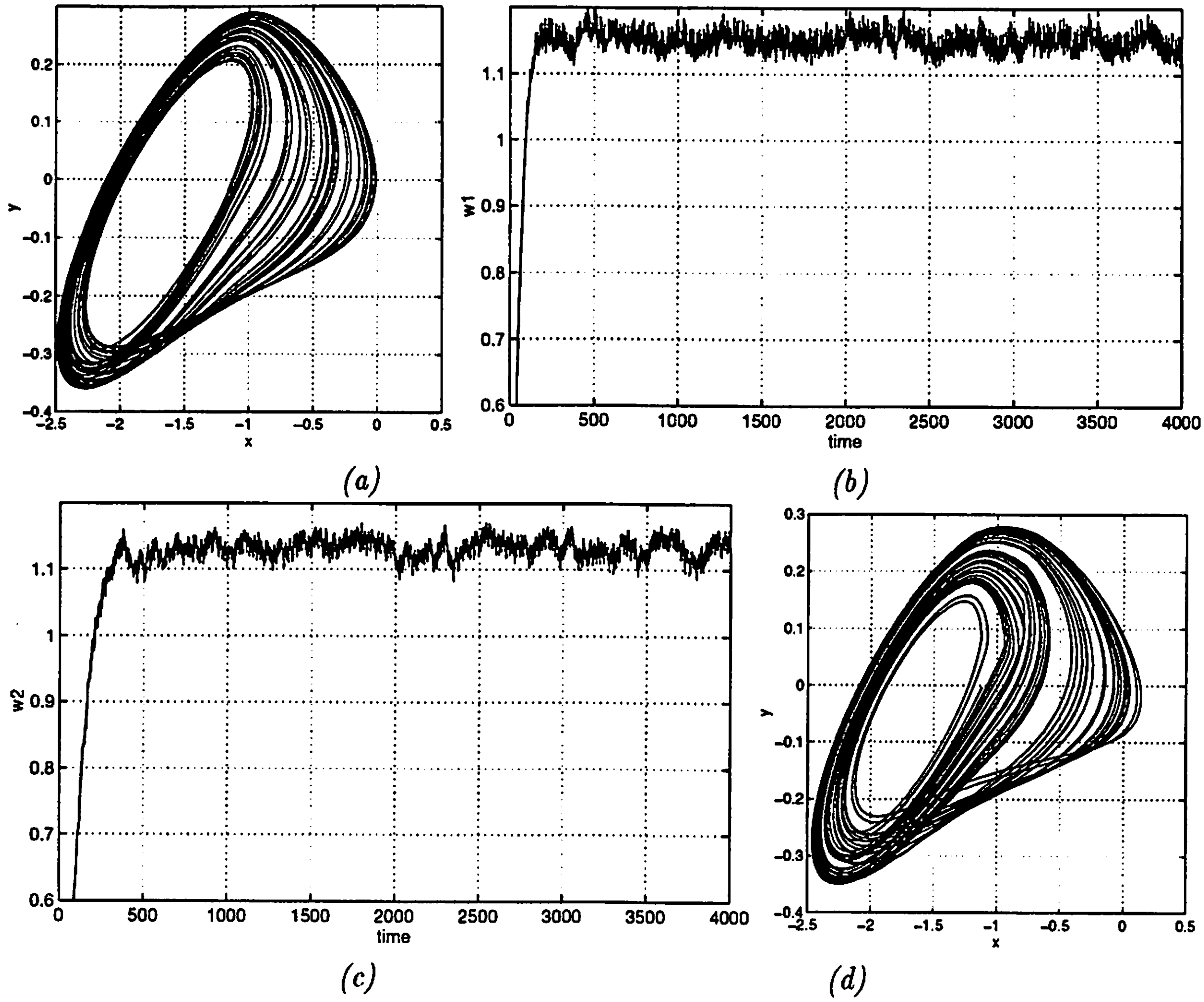


Figure 6: Learning processes of parameters for the verb *enter* for modeling the chaotic system. (a) The original strange attractor. (b) Learning process of w_1 for the verb *enter*. (c) Learning process of w_2 for the verb *enter*. (d) Output of the trained verbal model of the chaotic system.

the dynamic aspects of qualitative processes. This is the systematic consideration of invention of verbal systems.

We can find that the history of science is a process of quantizing the unknown world. However, our human species have survived different kinds of complexities of the nature for such a long time and only developed the linguistic system for handling these complexities, which are far beyond the ability of our modern sciences. We then have two choices: 1. waiting for a long time for the "full-development" of our sciences[1], or 2. to quantize our linguistic system and use an *artificial linguistic system* to help us to cope with the complexity. I choose the second one and want to combine fuzzy theory and verbal systems into an artificial linguistic system for coping with complexity and easing our survival.

[1] Is this possible ?

This paper is the first one of a series of my papers on verbal paradigms, which I believe to be the building blocks of the next generation of artificial intelligence, in particular, expert systems. By using verbal paradigm, we can embed our "dynamic" knowledge and experiences into machine intelligence. This strategy benefits sciences in two folds. First, we can build a kind of machine, which can have some artificial experiences of complexities. Second, it will ease the communication of human experts and machines via sharing experiences. Although we can not hope machine has the same "feeling" as that of a human expert to a certain experience, the qualitatively related nature of these two experiences should also help us a lot for developing a new generation of expert systems.

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