MOMENT MATCHING IN CONGRUENCE TRANSFORM

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1 Introduction

This report is to prove Proposition 2 in "Multipoint multiport algorithm for passive reduced-order model of interconnect networks".

Consider a system with state equations

$$H(s)x = bu \tag{1}$$

where $x \in R^n$, $u \in R_1$, $H(s) = Ms + N$, and $b = b_0 + b_1 s$; and output functions

$$y = c^T x \tag{2}$$

where $y \in R^m$ and $c = c_0 + c_1 s$. Note that in Proposition 2 only one input is concerned, so we use b instead of b_j for simplicity.
The input admittance function is

$$Y(s) = c^T H(s)^{-1} b \tag{3}$$

Let an orthonormal congruence transform $V \in R^{n \times q}$ with $q \leq n$ be applied to the system. Then we have

$$\hat{H}(s)\hat{x} = \hat{b}u \tag{4}$$

where $\hat{H} = V^T H V = s\hat{M} + \hat{N}$, $\hat{M} = V^T M V$, $\hat{N} = V^T N V$, and $\hat{b} = V^T b$; and

$$y = \hat{c}^T \hat{x} \tag{5}$$

with $\hat{c}^T = c^T V$. For the transformed system, the transfer function becomes

$$\hat{Y}(s) = \hat{c}^T \hat{H}(s)^{-1} \hat{b} \tag{6}$$

For a finite matching point $s = s_q$, let $H(s)^{-1}b$ be expanded as $H(s)^{-1}b = \sum_{k=0}^{\infty} r_k(s_q)(s - s_q)^k$. Let $N(s_q) = N + s_q M$ and $b_0(s_q) = b_0 + s_q b_1$. Then,

$$r_0(s_q) = N(s_q)^{-1} b_0(s_q) \tag{7}$$

$$r_1(s_q) = A(s_q) r_0(s_q) + N(s_q)^{-1} b_1 \tag{8}$$

where $A(s_q) = -N(s_q)^{-1}M$, and

$$r_k(s_q) = A(s_q)r_{k-1}(s_q) \quad k > 1 \tag{9}$$

For $s_q = \infty$, let $H(s)^{-1}b$ be expanded as $H(s)^{-1}b = \sum_{k=0}^{\infty} r_k(\infty)s^{-k}$. Let $B = -M^{-1}N$. Then,

$$r_0(\infty) = M^{-1}b_1 \tag{10}$$

$$r_1(\infty) = Br_0(\infty) + M^{-1}b_0 \tag{11}$$

and

$$r_k(\infty) = Br_{k-1}(\infty) \quad k > 1 \tag{12}$$

For the reduced order model, we have similar equations as Eqs(7) - (12) with M, N, A, B, b_0, b_1 and r replaced by $\hat{M}$, $\hat{N}$, $\hat{A}$, $\hat{B}$, $\hat{b}_0$, $\hat{b}_1$ and $\hat{r}$, respectively. Now let us consdider the moments of $Y(s)$. Let $c = c_0 + sc_1$ where $c_0^T = [G_{px}\ A_{Lp}]$ and $c_1^T = C_{px}$. For finite s_q, let $Y(s) = \sum_{k=0}^{\infty} m^k(s - s_q)^k$ where $m^k = [m_0^k, \ldots, m_m^k]^T$ is a vector of m k-th order moments. Then,

$$m^0 = c_0^T r_0 \tag{13}$$

and

$$m^k = c_0^T r_k + c_1^T r_{k-1} \qquad k > 0 \tag{14}$$

For $s_q = \infty$, when $c_1 = 0$, let $Y(s) = \sum_{k=0}^{\infty} m^k(\infty)s^{-k}$. Then

$$m^k = c_0^T r_k \qquad k \geq 0 \tag{15}$$

When $c_1 \neq 0$, $Y(s) = m^{-1}s + \sum_{k=0}^{\infty} m^k(\infty)s^{-k}$. Then,

$$m^{-1} = c_1^T r_0 \tag{16}$$

and

$$m^k(\infty) = c_0^T r_k + c_1^T r_{k+1} \qquad k \geq 0 \tag{17}$$

2 Lemmas

We first prove some lemmas, where matrix V is supposed to be orthonormal.
Lemma 1
If vector $u \in span(V)$, then $VV^Tu = u$.
Proof.
Vector u can be expressed as $u = Va$. Then

$$VV^Tu = VV^TVa = V(V^TV)a$$

As V is orthonormal, $V^TV = I$, and $V(V^TV)a = Va = u$. □
Lemma 2
Let $K(s_q, n) = \{r_0(s_q), r_1(s_q), \ldots, r_n(s_q)\}$. If $K(s_q, n) \in span(V)$, then

$$\hat{N}(s_q)V^Tr_k = V^TN(s_q)r_k, \qquad 0 \leq k \leq n \tag{18}$$

Proof.
By Lemma 1,

$$\hat{N}(s_q)V^T r_k = V^T N(s_q) V V^T r_k = V^T N(s_q) r_k \quad \square$$

Lemma 3
Under the same assumption of Lemma 2, we have

$$\hat{r}_k(s_q) = V^T r_k(s_q) \qquad 0 \le k \le n \tag{19}$$

Proof.
We first consider the case that s_q is finite. (s_q) will be omitted from the symbols in the following for simplicity.
The proof is done by induction. Note that if the lemma is true for some k, then by Lemma 1,

$$V\hat{r}_k = VV^T r_k = r_k$$

and

$$\hat{M}\hat{r}_k = V^T M V \hat{r}_k = V^T M r_k$$

for the same k.
For k=0,

$$\hat{N} V^T r_0 = V^T N r_0 = V^T b_0 = \hat{b}_0$$

So,

$$V^T r_0 = \hat{N}^{-1}\hat{b}_0 = \hat{r}_0$$

and Lemma 2 is true for $k = 0$.
For $k = 1$.

$$\hat{N} V^T r_1 = V^T N r_1 = V^T(-M r_0 + b_1) = -\hat{M}\hat{r}_0 + \hat{b}_1$$

So,

$$V^T r_1 = -\hat{N}^{-1}\hat{M}\hat{r}_0 + \hat{N}^{-1}\hat{b}_1 = \hat{r}_1$$

and Lemma is true for $k = 1$.
Now suppose that the lemma is true for some $k < n$, we prove it is true for $k+1$.

$$\hat{N} V^T r_{k+1} = V^T N r_{k+1} = -V^T M r_k = -\hat{M}\hat{r}_k$$

So,

$$V^T r_{k+1} = -\hat{N}^{-1}\hat{M}\hat{r}_k = \hat{r}_{k+1}$$

Thus, the Lemma is true for finite s_q.
For the case that $s_q = \infty$, Compare Eqs(10)-(12) with Eqs(7)-(9), it can be seen that if we interchange M and N and b_0 and b_1, one set of equations becomes the other. So the proof is similar to the above and is omitted.

3 Proof of Proposition 2

Now, we are ready to prove the Proposition. For either finite or infinite matching point, from Lemma 3 and Lemma 1, for $0 \le k \le n$,

$$\hat{c}_0^T \hat{r}_k = c_0^T V V^T r_k = c_0^T r_k$$

Similarly,

$$\hat{c}_1^T \hat{r}_k = c_1^T r_k$$

From Eqs(13), (14), (15) - (17), it is clear that the Proposition is true.