REDUCED ORDER MODEL OF TRANSMISSION LINES BY MULTIPLE POINTS MOMENT MATCHING AND PASSIVITY PRESERVATION

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1 Introduction

With the rapid increase of signal frequency and decrease of feature sizes of IC's, interconnects play more and more important roles. On the MCM and PCB levels, interconnects are modeled as lossy and lossless transmission lines.

In recent years, many papers have been published in dealing with the modeling and simulation of transmission line networks, among them the moment matching models are widely used [4]. Except for few cases [1], the passivity of the moment matching models is not guaranteed, and very often they will meet with instability problem when *Padé* approximation is used to form a rational approximation of the characteristic functions of the lines. Also, if the moment matching models are formed by the computation of the moments first and the formulation of a rational function next, it is also suffered from the numerical ill-conditioning problem.

Since 1994, there have been a number of papers using the Lanczos or Anordi algorithms to do model order reduction of RLC networks [5, 6], which can overcome the numerical ill-conditioning problem. In [2], the *Padé* via *Lanczos* (PVL) process was used to do model order reduction for transmission lines, but neither the passivity nor the stability of the model is concerned there. In [3], a technique called SCT (split congruence transform) is provided to guarantee the passivity of the reduced order model of an RLC network, but its disadvantage is that the order of the model is doubled when splitting is done.

In this paper, we will provide a method to form a reduced order model for general RLGC transmission lines. The passivity of the model is guaranteed and the moment matching at multiple points is also provided.

2 Discrete model of transmission lines

The first step of our algorithm is to form a discrete model for an RLGC line system.
Consider an $n+1$ conductor coupled transmission line system with length d and R_0, L_0, G_0, C_0 as its per-unit-length resistance, inductance, capacitance and conductance matrices. We form a discrete model of the system by m T-typed network for each line, where each floating branch consisting of a series connection of an inductor of $L_d/2$ and a resistor of $R_d/2$, and a grounded branch with a parallel connection of a capacitor of C_d and a conductor of G_d, where $R_d = R_0 d/m$, etc.. Let the input voltage and current vectors be V_s and I_s, the internal node voltage vector be

$$V_x = [V_1^T, V_2^T, \ldots, V_m^T]^T \tag{1}$$

where V_i has n components, and I_l be the inductor current vectors where

$$I_l = [I_1^T, I_2^T, \ldots, I_{m+1}^T]^T \tag{2}$$

Let

$$x = \begin{bmatrix} V_x \\ I_l \end{bmatrix} \tag{3}$$

be the state vector of the system. Then , the state equations of the system is

$$Hx = bu \tag{4}$$

where

$$H = \begin{bmatrix} G+sC & A_{Lx} \\ -A_{Lx}^T & R+sL \end{bmatrix} \tag{5}$$

where

$$G + sC = diag(G_0 + sC_0) \tag{6}$$

$$R + sL = diag(\frac{R_d + sL_d}{2}, R_d + sL_d, \ldots, R_d + sL_d, \frac{R_d + sL_d}{2}) \tag{7}$$

$$A_{lx} = \begin{bmatrix} -I & I & & & \\ & -I & I & & \\ \cdots & \cdots & \cdots & \cdots & \\ & & \cdots & -I & I \end{bmatrix} \tag{8}$$

where I is an $n \times n$ identity matrix,

$$b = \begin{bmatrix} 0 \\ A_{ls}^T \end{bmatrix} \tag{9}$$

and

$$u = V_s \tag{10}$$

where A_{ls} consists of an I at its upper left corner and a $-I$ at its lower right corner and zero elements elsewhere. Note that H is a positive-real matrix. When we form the admittance matrix of the system, the output vector is I_s and the output equations are

$$I_s = A_{ls} I_l = b^T x \tag{11}$$

3 Reduced-order model via congruent transformation

The number of the state variables in the above state equations is $q = n(2m+1)$. If a congruence transform matrix $P \in R^{q \times k}$ with $k \leq q$ is applied to the system, i.e., let $x = P\hat{x}$, then we have

$$\hat{H}\hat{x} = \hat{b}u \tag{12}$$

and

$$I_s = \hat{b}^T \hat{x} \tag{13}$$

where $\hat{H} = P^T H P$ and $\hat{b} = P^T b$, and the order of the system is reduced to k. The input admittance matrix of the reduced order system is

$$\hat{Y}(s) = \hat{b}^T \hat{H}^{-1} \hat{b} \tag{14}$$

and we have the following important result:
Theorem 1
If P is of full rank, then $\hat{H}$ is positive-real and the reduced-order system is passive.

4 Congruent transformation with block Anordi algorithm

Let $H = sM + N$, then the state equations can be transformed into the following form:

$$sAx = x + cu \tag{15}$$

where $A = -N^{-1}M$ and $c = -N^{-1}b$, and the admittance matrix is

$$Y(s) = b^T (sA - I)^{-1} c \tag{16}$$

The j-th order moment of $Y(s)$ at $s = 0$ is

$$m_j(0) = -b^T A^j c \qquad j \geq 0 \tag{17}$$

the j-th order moment at $s = \infty$ is

$$m_{-j} = b^T A^{-j} c \qquad j \geq 1 \tag{18}$$

and the j-th order moment at $s = s_i$ is

$$m_j(s_i) = -b^T\{(I - s_iA)^{-1}A\}^j(I - s_iA)^{-1}c \quad j \geq 0 \tag{19}$$

Let the matching order at $s = 0$, $s = \infty$ and $s = s_i$ be n_0, n_∞ and n_i respectively. Let $K_0 = \{c, Ac, \ldots, A^{n_0}c\}$, $K_\infty = \{A^{-n_\infty}c, A^{-n_\infty+1}c, \ldots, A^{-1}c\}$ and $K_i = \{(I - s_iA)^{-1}c, (I - s_iA)^{-1}A(I - s_iA)^{-1}c, \ldots, \{(I - s_iA)^{-1}A\}^{n_i}(I - s_iA)^{-1}c\}$, and let

$$K = \{K_\infty, K_0, K_1, \ldots, K_r\} \tag{20}$$

The block Anordi algorithm can be used to find an orthonormal matrix

$$P = [P_\infty, P_0, P_1, \ldots, P_r] \tag{21}$$

where there are q rows in P, and the number of columns in each submatrix is the same as the number of elements in the corresponding subset of K. The column vectors of P form an orthonormal basis of the Krylov subspace K, i.e., $span(P) = span(K)$. When P is applied to the original system, the moment matching at the specified frequencies with the required orders is obtained.

5 Summary

In this paper, we provide an algorithm to form a reduced-order model of multiconductor coupled lines. The algorithm is based on the congruence transformation on the state equations of the discrete model of the lines. The passivity and the multi-point moment matching are preserved in the model. Compared with [3], no split is needed for the congruence transform matrix, so the model order for the same moment matching is lowered.

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