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INTERCONNECT MODELING VIA PASSIVITY

by

Janet M. Wang

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COVER SHEET

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Chapter 1

Introduction

1.0 Background

Wave propagation in multiconductor system has been a research topic since the turn of the century. Originally, signal transmission through telephone and telegraph lines as well as the transient phenomena on power transmission lines were the main subjects of the research. Later, striplines have found their applications as directional couplers and interdigital filters. With the advent of integrated circuit technology, the coupled microstrip transmission lines consisting of multiple conductors embedded in a multilayer dielectric medium have evolved as a new class of microwave networks. Then the need to consider interconnect in digital circuits as lossy transmission lines arose in the early 90's, particularly for multi-chip modules. Increasing lengths of interconnect combined with the faster switching speeds of modern logic have created a situation where interconnect time-of-flight(delay) has become comparable to signal transition times, giving rise to transmission-line effects such as reflection and overshoot. Such phenomena are capable of causing undesired switching. In addition, with the development of the deep submicron techniques, the small cross-sectional areas of typical high-performance interconnect give rise to series loss and the additional phenomena of attenuation and dispersion[1][2].

1.1 Interconnect Modeling

Traditionally, different interconnect models are developed for on-chip and off-chip cases respectively. For CMOS technology, RC lumped models are adequate for on-chip interconnect modeling. Previously, moment-based methods, like Asymptotic Waveform Evaluation (AWE)[3] were

employed to reduce the order of on-chip interconnect models. But, these methods usually suffer from the ill-conditioning and numerical instability problems. In recent years, Lanczos algorithm [4][5] is used in place of AWE. For off-chip interconnect modeling, transmission line models based on moment matching methods are often used. Though Padé Via Lanczos (PVL) is efficient in RC or RLC lumped model order reduction, it has some problems when applied to transmission line models. One advantage of moment based approaches over PVL is that they can construct approximations to systems with irrational system response, i.e. a system with delay elements such as transmission lines, as easily as for a rational response, whereas Krylov-subspace model reduction algorithms for systems with irrational response have yet to be demonstrated[6]. A way to deal with the problem is to treat transmission lines as the interconnection of a large number of RLGC segments, but the efficiency of such a discretization approach may be unsatisfactory.

In this thesis, we consider off-chip interconnect modeling by means of transmission line models.

1.2 The Circuit Simulation Problem

The fundamental difficulty encountered in integrating transmission line simulation into a transient circuit simulator is that circuits containing nonlinear devices or time-dependent characteristics must be characterized in the time domain while transmission lines with loss, dispersion, or discontinuity are best characterized in the frequency domain. To cope with this difficulty, four types of approaches have been proposed in the literature. One uses a network of lumped elements and segments of ideal transmission line to approximate the frequency response of each lossy transmission line or each lossy multiconductor system[8]. These

approximated circuit models are suitable for existing general-purpose circuit simulators. However, a drawback of this approach is that the amount of computation increases due to the large number of extra nodes and elements introduced[9]. The second type of approach adopts the convolution technique. The outputs of linear lossy multiconductor lines are the convolutions of the inputs with the impulse responses (Green's function) of the multiconductor lines. The multiconductor lines are treated as a linear N-Port system. The difficulty of this type of approach lies in how to determine the impulse responses of an arbitrary multiconductor line system. Existing methods rely on the use of the inverse Fast Fourier transformation technique[10], the numerical inverse Laplace transformation technique[11], inverse Fourier transformation of frequency-domain scattering parameters[12][13], and even the explicit analytical approach[9] to determine the impulse response. However, the convolution simulations suffer from a common drawback: the convolution operation needs to extend over the entire past history. The computation time required at any time point t is then proportional to the time point t ; therefore, the convolutions at large time points will be very time-consuming. Furthermore, the inverse Fourier transformations of [10], [12], and [13] will suffer from either the aliasing effects or from too many frequency points needed for the transformations to avoid aliasing.

In order to improve the efficiency of the simulation involving fast-varying signals, the recursive convolution technique is used instead of direct convolution[16], but explicit analytical expressions for the impulse responses are still needed. In order to deal with this difficulty, the Pade technique is used to obtain approximate explicit analytical expression of the transfer function. Impulse response functions are approximated with a sum of exponential functions in the time domain. However, the Pade technique suffers from the instability problem: unstable poles may be generated for known stable networks.

1.2.1 Interconnect Simulation Via Passivity

Most of the research in this area, such as AWE (Asymptotic Waveform Evaluation), depends on Padé approximations of irrational functions and convolution. Although Padé approximation provides a feasible and accurate model for low-order reduced models, it tends to yield unstable poles when the number of poles (order of approximation) exceeds eight. In [17], mixed-exponential functions are introduced to overcome the problem of instability. However, extremely ill-conditioned matrices because of severe numerical problems may still exist when computing high order moments. Further, this method does generate unstable poles and subsequently deletes them to avoid instability. The Complex Frequency Hopping(CFH) method [18] computes high order moments correctly without generating unstable poles. However, difficulties of using special high precision programming techniques hinder its use. Since transmission lines are passive elements, a rational approximation, properly done, should lead to poles in the left half s-plane. In classical network theory, the impedance of a single transmission line is a positive real function, while for multiple transmission lines the impedance is a positive real matrix. Based on the positive real property, the method due to Pillai, Shim and Youla [19] can be used to obtain stable poles during the approximation of the characteristic admittance and transfer function of the transmission line.

1.3 Contribution

A new approach for transient simulation of lossy interconnects terminated in arbitrary non-linear elements is proposed. The approach is based on Pillai method to avoid the instability problem caused by the Padé approximation. By applying the recursive convolution technique, we greatly reduce the computation used to perform convolutions. We have incorpo-

rated the proposed technique in SWEC. The comparisons with other interconnect models indicate that the model we derive is a good model[20] in terms of accuracy, efficiency and stability.

1.4 Organization

Chapter 2 reviews some basic concepts such as power spectrum, positive function, bounded function and maximum entropy. It also gives some related important theorems.

Chapter 3 presents the formulation of the proposed Pillai-like method. We will compare it with the original Pillai method. The reason why the proposed Pillai-like method insures stability will also be described.

In chapter 4, the recursive convolution method will be reviewed together with experimental results.

Chapter 5 suggests possible future work and makes some conclusions of this thesis.

Chapter 2

Background and Review of Basic Concepts

2.0 Power Spectrum

Suppose $H(\cdot)$ is a transfer function of a linear system, power spectrum of $H(\cdot)$ is defined as:

$$S(\omega) = |H(j\omega)|^2 = H(j\omega)H^*(j\omega) \quad (2.1)$$

If the time domain format of $h(t)$ is real, then $H^*(j\omega) = H(-j\omega)$, so it follows

$$S(\omega) = H(j\omega)H^*(j\omega) = H(j\omega)H(-j\omega) = H(s)H(-s)|_{s=j\omega} = S(-\omega) \quad (2.2)$$

$S(\omega)$ is an even function of ω^2 . If $H(\cdot)$ is also rational, $S(\omega)$ can be presented as,

$$S(\omega) = \frac{B(\omega^2)}{A(\omega^2)} = \frac{B(-s^2)}{A(-s^2)} \Big|_{s=j\omega} \geq 0 \quad (2.3)$$

where $A(\cdot)$ and $B(\cdot)$ are real polynomials in s^2 satisfying (2.3). If s_k is a zero of either A or B in (2.3) with nonzero real part, then $-s_k$, s_k^* and $-s_k^*$ will also have nonzero real parts. This can be easily proved as follows: assume s_1 is a root of A; since A is a real polynomial in s^2 , s_1^* is also a root, so that $(s - s_1)(s - s_1^*) = s^2 - (s_1^* + s_1)s + s_1s_1^*$ has real coefficients. By replacing s with $-s$, it can be easily seen that s_k^* and $-s_k^*$ are also

roots. Moreover, from the nonnegativity constraint in (2.3), every zero of A and B on the $j\omega$ -axis must occur with even multiplicity. This allows the representation

$$A(-s^2) = M(s)M(-s) \quad (2.4)$$

$$B(-s^2) = N(s)N(-s) \quad (2.5)$$

where $M(s)$ and $N(s)$ are real Hurwitz polynomials which have no zeros in the open right half plane in the s -domain (or interior of the unit circle in the z -plane). Substituting (2.4) and (2.5) into (2.3) gives

$$S(\omega) = \frac{N(s)N(-s)}{M(s)M(-s)} \Big|_{s=j\omega} = H(s)H(-s) \Big|_{s=j\omega} \quad (2.6)$$

where $H(s) = \frac{N(s)}{M(s)}$. In the z -domain, the power spectrum of $H(z)$ can be expressed as

$$S(z) = |H(z)|^2 = H(z)H^*(z)$$

where $z = e^{-sT}$ and T is the sampling period. Let $H_*(z) = H^*\left(\frac{1}{z^*}\right)$. Since $z = \frac{1}{z^*}$,

$H_*(z) = H^*(z)$. Thus

$$S(z) = |H(z)|^2 = H(z)H_*(z) \quad (2.7)$$

2.1 Positive function

Positive real function is one of the most important concepts required to understand the passivity property of transmission models.

Definition 2.1.1. A function $f(z)$ is said to be *positive function* if

1. $f(z)$ is analytic in $|z| < 1$ (the interior of the unit circle)
2. The real part of $f(z)$, $Re(f(z)) \geq 0$ in $|z| < 1$

Definition 2.1.2. If $f(z)$ is also real for real z then it is said to *positive real* (p.r.).

In the beginning of this century, Schur discovered the interesting correspondence between positive functions and the power spectrum. This is summarized in the following two theorems.

Theorem 2.1. Associated with every power spectrum $S(z)$

$$S(z) = |H(z)|^2 = \sum_{-\infty}^{\infty} r_k z^k$$

there exists a unique positive function[21]

$$Z(z) = r_0 + 2 \sum_1^{\infty} r_k z^k$$

Theorem 2.2. (Schur's Theorem) The power series

$$Z(z) = r_0 + 2 \sum_1^{\infty} r_k z^k$$

defines a positive function iff every Hermitian Toeplitz matrix is positive for every

$$\begin{bmatrix} r_0 & r_1 & r_2 & \dots & r_{k-1} & r_k \\ r_1^* & r_0 & r_1 & \dots & r_{k-2} & r_{k-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{k-1}^* & r_{k-2}^* & \dots & \dots & r_0 & r_1 \\ r_k^* & r_{k-1}^* & \dots & \dots & r_1^* & r_0 \end{bmatrix} \geq 0$$

$k = 0 \rightarrow \infty$, where $r_k^* = r_{-k}$ [21].

Lemma 1. If $S(z)$ is a power spectrum of $H(z)$, then there exists a positive function

$$Z(z) \text{ such that } S(z) = |H(z)|^2 = \frac{Z(z) + Z_*(z)}{2} \text{ where } Z(z) = r_0 + 2 \sum_1^{\infty} r_k z^k \text{ and}$$

$$Z_*(z) = r_0 + 2 \sum_1^{\infty} r_k^* z^{-k} \text{ with } r_k^* = r_{-k}.$$

This lemma can be inferred from Theorem 2.1.

Lemma 2. If $Z(z)$ is a positive function and has rational expression $Z(z) = \frac{b(z)}{a(z)}$ then

$$Z_*(z) = \frac{b_*(z)}{a_*(z)} \text{ and } S(z) = \frac{Z(z) + Z_*(z)}{2} = \frac{a(z)b_*(z) + b(z)a_*(z)}{2a(z)a_*(z)}$$

Let

$$a(z)b_*(z) + b(z)a_*(z) = 2\beta(z)\beta_*(z)$$

thus

$$\frac{a(z)b_*(z) + b(z)a_*(z)}{2a(z)a_*(z)} = \frac{\beta(z)\beta_*(z)}{a(z)a_*(z)}$$

We can find $H(z)$ by using $\frac{\beta(z)\beta_*(z)}{a(z)a_*(z)} = H(z)H_*(z)$. Thus $H(z) = \frac{\beta(z)}{a(z)}$

2.2 Bounded functions

Definition 2.2.1. A function $d(z)$ is said to be *bounded* if

1. $d(z)$ is analytic in $|z| < 1$
2. $|d(z)| \leq 1$ in $|z| < 1$

Definition 2.2.2. If $d(z)$ is also real for real z , then it is said to be *bounded real* (b.r.).

Let $d(z)$ be the bounded function which represents the reflection coefficient associated with a positive function $Z(z)$ normalized to some R_0 with positive real part. Thus

$$d(z) = \frac{Z(z) - R_0}{Z(z) + R_0^*} \quad (2.8)$$

with $R_0 = Z(0)$. $d_1(z)$ has the form

$$d_1(z) = \frac{1}{z} d(z) = \frac{1}{z} \cdot \frac{Z(z) - Z(0)}{Z(z) + Z^*(0)} \quad (2.9)$$

which is analytic in $|z| < 1$. If $d(z)$ is also analytic on $z = e^{j\theta}$, then on the boundary, we have

$$|d_1(e^{j\theta})| = |d(e^{j\theta})| \leq 1$$

and hence from the maximum modulus theorem, $|d_1(z)| \leq 1$ in $|z| < 1$ and is also analytic in $|z| < 1$. More generally, from (2.9), $|d_1(z)| \leq \frac{1}{r}$ for $|z| = r < 1$ and using Schwarz's lemma[27], $|d_1(z)| \leq 1$ in $|z| < 1$. Hence, $|d_1(z)|$ is always a bounded function. Let $Z_1(z)$ represent the positive function associated with $|d_1(z)|$ normalized to $Z(0)$,

$$d_1(z) = \frac{Z_1(z) - Z(0)}{Z_1(z) + Z^*(0)} \quad (2.10)$$

Combining (2.9) and (2.10), we have

$$\frac{1}{z} \cdot \frac{Z(z) - Z(0)}{Z(z) + Z^*(0)} = \frac{Z_1(z) - Z(0)}{Z_1(z) + Z^*(0)}$$

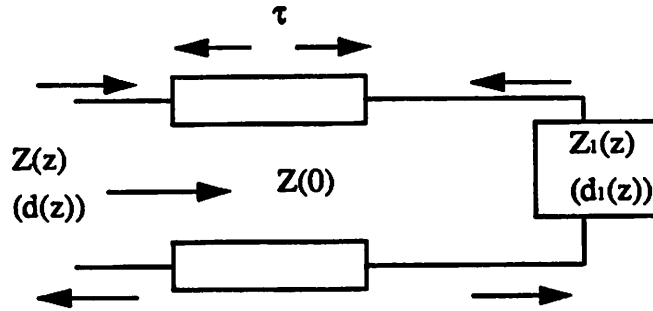


Fig. 2.1 Ideal Transmission Line

The reflection coefficients associated with the two positive functions $Z(z)$ and $Z_1(z)$ so generated are related through the delay operator z when they are both normalized to $Z(0)$. Thus, starting with any positive function $Z(z)$, the above procedure will always give rise to a new positive function $Z_1(z)$ [21]. Equations (2.8) can be given an interesting physical interpretation by making use of an ideal delay line and the incident and reflected waves.

Consider an ideal delay line as Fig 2.1 with one-way delay τ that has been terminated on a positive function $Z_1(z)$ and let $Z(z)$ represent the input positive function. Then, the reflection coefficient $d(z)$ of the input $Z(z)$ normalized to $R_0 = Z(0)$ is given by Eq(2.8) and that of the termination $Z_1(z)$ is given by Eq(2.9). Let $\alpha_1(z)$, $\beta_1(z)$ represent the transforms of the input incident and reflected waves and $\alpha_2(z)$, $\beta_2(z)$ be those at the output terminals of the line. Under the assumption that all these waves are normalized to $Z(0)$, we have

$$\frac{\beta_1(z)}{\alpha_1(z)} = d(z) \quad (2.11)$$

and

$$\frac{\alpha_2(z)}{\beta_2(z)} = d_1(z) \quad (2.12)$$

Since the incident waves $\alpha_1(t)$ and $\alpha_2(t)$ only undergo a one-way delay of τ in the line,

we have

$$\beta_2(t) = \alpha_1(t - \tau)$$

and

$$\beta_1(t) = \alpha_2(t - \tau)$$

Under zero initial conditions, the Laplace transform of the above equations give

$$\beta_2(s) = e^{-s\tau} \alpha_1(s)$$

and

$$\beta_1(s) = e^{-s\tau} \alpha_2(s)$$

Thus, in the z-domain

$$\frac{\alpha_2(z)}{\beta_2(z)} = e^{2s\tau} \frac{\beta_1(z)}{\alpha_1(z)} \quad (2.13)$$

By identifying $e^{-2s\tau}$ to be the delay operator z such that $z = e^{-2s\tau}$, (2.13) changes to

$$\frac{\alpha_2(z)}{\beta_2(z)} = \frac{1}{z} \cdot \frac{\beta_1(z)}{\alpha_1(z)}$$

and it can be further reduced to

$$d_1(z) = \frac{1}{z}d(z)$$

2.3 Generalized Line Extraction Process

$Z_r(z)$ is denoted as the positive function at the r th stage resulting from r successive line extractions. Let

$$R_{r-1} = Z_{r-1}(0)$$

where $r \geq 1$ represent the characteristic impedance of the r th line. The reflection coefficient of $Z_r(z)$ normalized to R_{r-1} , the characteristic impedance of the previous line can be formed as

$$d_r(z) = \frac{Z_r(z) - R_{r-1}}{Z_r(z) + R_{r-1}^*}$$

Hence

$$Z_r(z) = \frac{R_{r-1}(z) + R_{r-1}^* d_r(z)}{1 - d_r(z)} \quad (2.14)$$

To extract the next line, since $\frac{1}{z} \cdot \frac{Z_r(z) - R_r}{Z_r(z) + R_r^*}$ is bounded, $Z_{r+1}(z)$ denotes the positive function

associated with this bounded function. By using Eq(2.14), $Z_{r+1}(z)$ changes to

$$Z_{r+1}(z) = \frac{R_r + R_r^* d_{r+1}(z)}{1 - d_{r+1}(z)}$$

Thus

$$\frac{1}{z} \cdot \frac{Z_r(z) - R_r}{Z_r(z) + R_r^*} = \frac{Z_{r+1}(z) - R_r}{Z_{r+1}(z) + R_r^*} = d_{r+1}(z) \quad (2.15)$$

Physically, $Z_{r+1}(z)$ represents the new positive function after $(r+1)$ line extractions and $d_{r+1}(z)$ the associated bounded function normalized to the characteristic impedance of the last line. Using Eq(2.14) and Eq(2.15), we have

$$Z_r(z) = \frac{R_{r-1}(z) + R_{r-1}^* d_r(z)}{1 - d_r(z)} = \frac{R_r + R_r^* z d_r(z)}{1 - z d_{r+1}(z)}$$

Let

$$d_r(0) = \frac{R_r - R_{r-1}}{R_r + R_{r-1}^*} = S_r$$

represent the r th junction mismatch reflection coefficient. Clearly, $|S_r| = |d_r(0)| \leq 1$ and it gives

$$d_r(z) = \frac{(R_r - R_{r-1}) + z(R_r^* + R_{r-1})d_{r+1}(z)}{(R_r + R_{r-1}^*) + z(R_r^* - R_{r-1}^*)d_{r+1}(z)} \quad (2.16)$$

Referring to Fig 2.2, a line extraction from the input positive function $Z(z)$ has resulted in a set of new positive functions $Z_i(z)$. Notice that the above line extraction is possible only if the normalization is carried out with respect to $R_i = Z_i(0)$. Following the transmission line terminology, we will denote $Z_i(0)$ to be the characteristic impedance of the i th line. This extraction process can be further described by Richards' theorem[23].

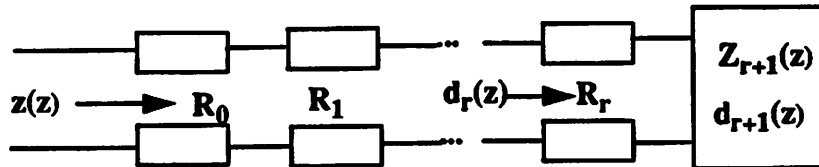


Fig 2.2 Generalized Line Extraction

In the next chapter, we will explain how to apply Pillai or Pillai-like approximation based on line extraction. Before that, we first present an important method, namely, the Maximum Entropy Method, used to estimate power spectrum.

2.4 Maximum Entropy Method[24]

The FFT is not the only way to estimate the power spectrum of a process, nor is it necessarily the best way for all purposes. To see how one might devise another method, let us enlarge our view for a moment, so that it includes not only real frequencies in the Nyquist interval $-f_c < f < f_c$, but also the entire complex frequency plane. Let us transform the complex frequency plane to the z-plane, by introducing the relation $z = e^{2\pi ir\Delta}$, where Δ is, as usual, the sampling interval in the time domain. Notice that the Nyquist interval on the real axis of the s-plane maps one-to-one onto the unit circle in the complex z-plane. We see that the FFT power spectrum estimate can be written as

$$S(z) = \left| \sum_{k=-N/2}^{N/2-1} r_k z^k \right|^2 \quad (2.17)$$

Of course, it is not the true power spectrum, but only an estimate. The estimate is not likely to be exact because in the z-plane, equation (2.17), the finite Laurent series, offers only an approximation to a general analytic function of z. In fact, a formal expression for representing “true” power spectrum is

$$S(z) = \left| \sum_{-\infty}^{\infty} r_k z^k \right|^2 \quad (2.18)$$

This is an infinite Laurent series which depends on an infinite number of values r_k . Equation (2.17) is just one kind of analytic approximation to the analytic function of z rep-

resented by Eq(2.18). It goes under several names, including direct method, all-zero model, and moving average (MA) model. The term “all-zero” in particular refers to the fact that the model spectrum can have zeros in the z-plane, but not poles.

If we look at the problem of approximating Eq(2.18) more generally, it seems clear that we could do a better job with a rational function; one with a series of type Eq(2.17) in both the numerator and the denominator. Less obviously, it turns out that there are some advantages in an approximation whose free parameters all lie in the denominator,

$$S(z) \approx \frac{1}{\sum_{k=-M/2}^{M/2} a_k z^k} \quad (2.19)$$

The a_k 's can be thought of as being determined by the condition that the power series expansion of Eq(2.19) agree with the first $M+1$ terms of Eq(2.18). The differences between the approximations Eq(2.17) and Eq(2.19) are approximations with very different character. Most notable is the fact that Eq(2.19) can have poles, corresponding to infinite power spectral density, on the unit z-circle, i.e. at real frequencies in the Nyquist interval. Such poles can provide an accurate representation for underlying power spectral which have sharp, discrete “lines” or delta-functions. By contrast, Eq(2.17) can have only zeros, no poles, at real frequencies in the Nyquist interval, and must thus attempt to fit sharp spectral features with essentially a polynomial. The approximation Eq(2.19) goes under several names: all-poles model, maximum entropy methods (MEM), autoregressive model (AR).

Chapter 3

Interconnect Modeling Based on Passivity

3.0 The Power Spectra of simple RLGC Lines

It is found in [8] that with significant delays, the macromodel method[25] is no longer accurate. Transmission lines should be modeled separately by propagation functions and characteristic impedances or admittances.

Suppose R, L, G, C are unit length parameters, the characteristic admittance of a single transmission line is defined as

$$Y_c = \sqrt{\frac{sC + G}{sL + R}} = Y_c(\infty) \sqrt{\frac{s + a}{s + b}} \quad (3.1)$$

where $Y_c(\infty) = \sqrt{\frac{C}{L}}, a = \frac{G}{C}, b = \frac{R}{L}$. Its power spectrum is

$$S(\omega) = |Y_c(j\omega)|^2 = Y_c(\infty)^2 \sqrt{\frac{\omega^2 + a^2}{\omega^2 + b^2}} \quad (3.2)$$

The propagation function of a single transmission line is defined as:

$$\Gamma(j\omega) = \exp(-\theta(j\omega)) \quad (3.3)$$

where $\theta(j\omega) = \sqrt{(j\omega L + R)(j\omega C + G)}$. After factoring out the ideal delay, $\Gamma(j\omega)$ can be written as

$$\Gamma(j\omega) = \exp(-j\omega\tau) \exp((- \sqrt{ab})\tau) F(j\omega) \quad (3.4)$$

where $\exp(-j\omega\tau)$ is the ideal delay, $\tau = \sqrt{LC}d$, and d is the length of line. Let

$F(j\omega) = \exp(-\sqrt{(j\omega+a)(j\omega+b)} + j\omega + \sqrt{ab})\tau$. Its power spectrum is $S(\omega) = |F(j\omega)|^2$. Then

$F(j\omega)$ can be approximated via its power spectrum so that $\Gamma(s)$ can be obtained.

To apply Schur's theorem, power spectra of $Y_c(j\omega)$ and $F(j\omega)$ are transformed into the z-domain by a process similar to inverse Fourier transform. Assume $S(\omega) = |H(j\omega)|^2$, we denote its inverse Fourier transform as

$$R(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega \quad (3.5)$$

Let T be the sampling period. Using discrete inverse Fourier transform gives

$$S(\omega) = \sum_{-\infty}^{\infty} r_k e^{-jk\omega T} \quad (3.6)$$

Let $z = e^{-j\omega T}$. Eq(3.6) can then be written as

$$S(z) = \sum_{-\infty}^{\infty} r_k z^k \quad (3.7)$$

Since $S(\omega)$ is an even function, in the z-domain, $r_k^* = r_{-k} = r_k$ with $k = 0 \rightarrow \infty$. Therefore,

$$S(z) = r_0 + \sum_1^{\infty} r_k z^k + \sum_1^{\infty} r_k z^{-k} \quad (3.8)$$

3.1 Proposed Method

After deriving the power spectra of characteristic admittance and propagation function as in section 3.0, we intend to get the rational approximation of the power spectra and further obtain the rational approximation of the original functions.

3.1.1 Levinson Algorithm

As in (3.8), $S(z)$ represents the series expansion of the power spectrum for characteristic admittance $Y_c(z)$ or propagation function factor $F(z)$. Using Maximum Entropy Method gives

$$S(z) = \frac{1}{A(z)A_*(z)} \quad (3.9)$$

where $A(z) = \sum_0^{\infty} a_i z^i$ and $A_*(z) = \sum_0^{\infty} a_i^* z^{-i}$. As $S(z) = H(z)H_*(z)$, $H(z)$ represents characteristic admittance or propagation function factor which are positive real functions.

Hence, $H(z) = \frac{1}{A(z)}$ and $A(z)$ is also positive real. Suppose $A_n(z)$ is an approximation of $A(z)$. Let

$$A_n(z) = a_{n,0} + a_{n,1}z + \dots + a_{n,n}z^n \approx A(z) \quad (3.10)$$

Then

$$S(z) \approx \frac{1}{A_n(z)A_{n*}(z)} \quad (3.11)$$

Moving $A_{n*}(z)$ to the right hand side of Eq(3.11) and multiplying z^n on the both side give

$$\left(r_0 + \sum_{i=1}^{\infty} r_i z^{-i} + \sum_{i=1}^{\infty} r_i z^i \right) \cdot (a_{n,n} + a_{n,n-1}z + \dots + a_{n,0}z^n) = \frac{z^n}{A_n(z)}$$

Because $A_n(z) \neq 0$ in $|z| \leq 1$, let

$$\frac{1}{A_n(z)} = b_{n,0} + b_{n,1}z + \dots + b_{n,n}z^n$$

with $b_{n,0} = \frac{1}{a_{n,0}}$. Comparing the coefficients of $1, z, z^2, \dots, z^n$ on both side, we get

$$\begin{bmatrix} r_0 & r_1 & r_2 & \dots & r_{n-1} & r_n \\ r_1 & r_0 & r_1 & \dots & r_{n-2} & r_{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{n-1} & r_{n-2} & \dots & \dots & r_0 & r_1 \\ r_n & r_{n-1} & \dots & \dots & r_1 & r_0 \end{bmatrix} \begin{bmatrix} a_{n,n} \\ a_{n,n-1} \\ \dots \\ a_{n,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ b_{n,0} \end{bmatrix} \quad (3.12)$$

The coefficient matrix is a symmetric and Toeplitz matrix. A recursive algorithm named after Levinson who was one of the first people using this method can reduce the computation complexity to $O(k)$, where k is the number of coefficients in the series expansion of the original power spectrum.

3.1.2 Rational Approximation of $Z(z)$

Let $A_k(z)$ be the power series approximation of $A(z)$ up to the k th order, T_k be the determinant of coefficient matrix in Eq(3.12). We can write $A_k(z)$ as

$$\begin{aligned} A_k(z) &= \begin{bmatrix} z^k & z^{k-1} & \dots & z & 1 \end{bmatrix} \begin{bmatrix} a_{k,k} \\ a_{k,k-1} \\ \dots \\ a_{k,0} \end{bmatrix} = b_{k,0} \begin{bmatrix} z^k & \dots & z & 1 \end{bmatrix} T_k^{-1} \begin{bmatrix} 0 \\ 0 \\ \dots \\ 1 \end{bmatrix} \\ &= \frac{1}{a_{k,0} \Delta_k} \begin{vmatrix} r_0 & r_1 & r_2 & \dots & r_{k-1} & r_k \\ r_1 & r_0 & r_1 & \dots & r_{k-2} & r_{k-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ r_{k-1} & r_{k-2} & \dots & \dots & r_0 & r_1 \\ z^k & z^{k-1} & \dots & \dots & z & 1 \end{vmatrix} \end{aligned} \quad (3.13)$$

with $k = 0, \dots, n$ and $\Delta_k = T_k$.

By applying Cramer's rule on Eq(3.12) to solve for $a_{k,0}$, we get $a_{k,0} = \sqrt{\frac{\Delta_{k-1}}{\Delta_k}}$ which

finally gives the compact expression of (3.13)

$$A_k(z) = \frac{1}{\sqrt{\Delta_k \Delta_{k-1}}} \begin{vmatrix} r_0 & r_1 & r_2 & \cdots & r_{k-1} & r_k \\ r_1 & r_0 & r_1 & \cdots & r_{k-2} & r_{k-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{k-1} & r_{k-2} & \cdots & \cdots & r_0 & r_1 \\ z^k & z^{k-1} & \cdots & \cdots & z & 1 \end{vmatrix} \quad (3.14)$$

Theorem 3.1 Let A be an $n \times n$ matrix and $\Delta_{nw}, \Delta_{ne}, \Delta_{sw}, \Delta_{se}$ denote the $(n-1) \times (n-1)$ minors formed from consecutive rows and consecutive columns in the northwest, northeast, southwest and southeast corners. Further let Δ_c denote the central $(n-2) \times (n-2)$ minor of A . Then from a special case of an identity due to Jacobi[26],

$$\Delta_c |A| = \Delta_{nw} \Delta_{se} - \Delta_{ne} \Delta_{sw}$$

According to the definition of Δ_k

$$\Delta_k = \begin{vmatrix} r_0 & r_1 & r_2 & \cdots & r_{k-1} & r_k \\ r_1 & r_0 & r_1 & \cdots & r_{k-2} & r_{k-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{k-1} & r_{k-2} & \cdots & \cdots & r_0 & r_1 \\ r_k & r_{k-1} & \cdots & \cdots & r_1 & r_0 \end{vmatrix}$$

by Theorem 3.1,

$$\Delta_k \Delta_{k-2} = \Delta_{k-1}^2 \left((-1)^{k-1} \begin{vmatrix} r_1 & r_2 & r_3 & \cdots & r_{k-1} & r_k \\ r_0 & r_1 & r_0 & \cdots & r_{k-2} & r_{k-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{k-2} & r_{k-3} & \cdots & \cdots & r_0 & r_1 \end{vmatrix} \right)^2$$

From Schur's Theorem (Theorem 2.2), Δ_k and Δ_{k-2} are nonnegative, thus $\Delta_k \Delta_{k-2} \geq 0$.

Denote d_k as

$$d_k = \frac{(-1)^{k-1}}{\Delta_{k-1}} \begin{vmatrix} r_1 & r_2 & r_3 & \cdots & r_{k-1} & r_k \\ r_0 & r_1 & r_0 & \cdots & r_{k-2} & r_{k-1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{k-2} & r_{k-3} & \cdots & \cdots & r_0 & r_1 \end{vmatrix} \quad (3.15)$$

it is easy to verify that $|d_k| \leq 1$. Combining with (3.14), we can get a recursive rule

$$\sqrt{1 - |d_k|^2} A_k(z) = A_{k-1}(z) - z d_k \tilde{A}_{k-1}(z) \quad (3.16)$$

with $\tilde{A}_k(z) = z^k A_{k^*}(z) = z^k A_k^*\left(\frac{1}{z^*}\right)$. We can further rewrite Eq(3.15) as

$$d_k = \left\{ A_{k-1}(s) \sum_{i=1}^k r_i z^i \right\}_k A_{k-1}(0) \quad (3.17)$$

where $\{ \}_k$ denotes the coefficients of z^k in $\{ \}_k$.

Since $S(z) = \frac{1}{A(z)A_*(z)} = \frac{Z(z) + Z_*(z)}{2}$ and $Z(z) = \frac{2B_k(z)}{A_k(z)} + O(z^{k+1})$, denote $Z_k(z)$ as

$$Z_k(z) = \frac{2B_k(z)}{A_k(z)}$$

As $A_k(z)$ and $Z_k(z)$ are approximation of $A(z)$ and $Z(z)$ respectively, we have

$$S(z) \approx \frac{1}{A_k(z)A_{k^*}(z)} = \frac{Z_k(z) + Z_{k^*}(z)}{2} \quad (3.18)$$

hence

$$A_{k^*}(z)B_k(z) + A_k(z)B_{k^*}(z) = 1 \quad (3.19)$$

From Eq(3.19), we can find $B_k(z)$ has the same recursive formula as $A_k(z)$,

$$\sqrt{1-|d_k|^2}B_k(z) = B_{k-1}(z) + zd_k\bar{B}_{k-1}(z) \quad (3.20)$$

where $\bar{B}_k(z) = z^k\bar{B}_{k^*}(z) = z^k B_k^*\left(\frac{1}{z^*}\right)$. Thus

$$A_k(z) = \frac{(A_{k-1}(z) - zd_k\bar{A}_{k-1}(z))}{\sqrt{1-|d_k|^2}}$$

$$B_k(z) = \frac{(B_{k-1}(z) + zd_k\bar{B}_{k-1}(z))}{\sqrt{1-|d_k|^2}}$$

Then, $Z_{k+1}(z)$ can be formed recursively as

$$Z_{k+1}(z) = \frac{2(B_k(z) + zd_{k+1}\bar{B}_k(z))}{A_k(z) - zd_{k+1}\bar{A}_k(z)} \quad (3.21)$$

which has similar form as (2.16). By letting $z = 0$, we could easily find out $A_0(0)$ and $B_0(0)$

whose positive real property is guaranteed by power spectrum. Normalizing $A_k(z)$ and denoting it

as $a_k(z)$ give

$$a_k(z) = \sqrt{\frac{\Delta_k}{\Delta_{k-1}}} A_k(z)$$

Similarly we have

$$b_k(z) = \sqrt{\frac{\Delta_k}{\Delta_{k-1}}} B_k(z)$$

Let $a_0(0) = a_0(z) = 1$ and $b_0(0) = b_0(z) = r_0/2$, at $k+1$ th step,

$$Z_{k+1}(z) = \frac{2(b_k(z) + zd_{k+1}\bar{b}_k(z))}{a_k(z) + zd_{k+1}\bar{a}_k(z)} = \frac{2b_{k+1}(z)}{a_{k+1}(z)} \quad (3.22)$$

with kth term

$$a_k(z) = a_{k-1}(z) + zd_k \bar{b}_{k-1}(z)$$

$$b_k(z) = b_{k-1}(z) + zd_k \bar{a}_{k-1}(z)$$

$$\bar{a}_k(z) = z^k \bar{a}_{k^*}(z) = z^k a_k^* \left(\frac{1}{z^*} \right)$$

$$\bar{b}_k(z) = z^k \bar{b}_{k^*}(z) = z^k b_k^* \left(\frac{1}{z^*} \right)$$

After n steps, by letting $d_{n+1} \leftarrow 0$, we have

$$Z(z) = \frac{2b_n(z)}{a_n(z)} + o(z^{n+1}) \approx \frac{2b_n(z)}{a_n(z)}$$

Since

$$|H(z)|^2 = H(z)H_*(z) = \frac{Z(z) + Z_*(z)}{2} = \frac{a(z)b_*(z) + b(z)a_*(z)}{2a(z)a_*(z)}$$

Using factorization[28] $a_n(z)b_{n^*}(z) + b_n(z)a_{n^*}(z) = 2\beta_n(z)\beta_{n^*}(z)$, we can get

$$H(z) = \frac{\beta_n(z)}{a_n(z)}.$$

3.2 Pillai's method ---- Pade-like Approximation

In contrast to the proposed method, after n steps, if $d_{n+1} \neq 0$ and we assume that it is a

rational function, $d_{n+1} = \frac{f(z)}{g(z)}$, then $Z_n(z) = \frac{b_n(z)g(z) + zf(z)\bar{a}_n(z)}{a_n(z)g(z) + zf(z)\bar{b}_n(z)}$, where $g(z)$ and $f(z)$ are coefficients that are solved by

$$Ax = b$$

where

$$x = [g_k, g_{k-1}, \dots, g_2, g_1 | f_{k-1}, f_{k-2}, \dots, f_1, f_0]^T$$

and

$$b = [0, 0, \dots, a_{2k}, \dots, a_{k+1}]$$

$$A = \begin{bmatrix} a_{2k} & 0 & \dots & 0 & 0 & b_0^* & 0 & \dots & 0 & 0 \\ a_{2k-1} & a_{2k} & \dots & 0 & 0 & b_1^* & b_0^* & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{k+2} & a_{k+3} & \dots & a_{2k} & 0 & b_{k-2}^* & b_{k-3}^* & \dots & b_0^* & 0 \\ a_{k+1} & a_{k+2} & \dots & a_{2k-1} & a_{2k} & b_{k-1}^* & b_{k-2}^* & \dots & b_1^* & b_0^* \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_1 & a_2 & \dots & a_{k-1} & a_k & b_{2k-1}^* & b_{2k-2}^* & \dots & b_{k+1}^* & b_k^* \end{bmatrix}_{2k \times 2k}$$

By equating the denominator coefficients of $z^{k+1}, z^{k+2}, \dots, z^{2k}$, we can derive $g(z)$ and $f(z)$.

It is proved in [19] that $H(z) = \frac{\beta_n(z)}{a_n(z)}$ is stable if $d_{n+1} = \frac{f(z)}{g(z)}$ is a bounded function, that

is $|d_{n+1}| = \left| \frac{f(z)}{g(z)} \right| < 1$. It is also shown in [19] that this method is equivalent to a Padé approxi-

mation when stable poles are obtained. Like Padé approximation, Pillai's method can match twice as many terms of $S(z)$ as our proposed method. We call this property its optimal property. In addition, stable poles can always be obtained by increasing the order of approximation. Moreover the equivalent system is not ill-conditioned because of the positive definite coefficient matrix A of the power spectrum.

3.3 Transformation to Time Domain

From the z -domain, we can directly transform $H(z)$ to time domain by partial fraction expansion.

Suppose $H(z) = \frac{\beta_n(z)}{a_n(z)}$. Let $\frac{H(z)}{z} = \frac{Q(z)}{zP(z)} = \frac{N(z)}{D(z)}$. By partial fraction expansion we

have $H(z) = c_0 + c_1 \frac{z}{z-z_0} + \dots + c_i \frac{z}{z-z_k}$, where $c_i = \frac{N(z)}{D'(z)}$ and $D'(z) = \frac{d}{dz}D(z)$. If in the

time domain, $H(t) = \sum_{i=0}^k c_i \exp(-p_i t)$ then $p_i = x_i + jy_i$ is a pole in the s-domain.

Assume $z_i = a_i + jb_i$ represents a pole of $H(z)$ and T is the sampling period, then we

have $x_i = -\frac{\log|z_i|}{T}$, $y_i = 2k\pi - \arctg\left|\frac{b_i}{a_i}\right|$. Hence, one pole in the z domain maps to infi-

nite number of poles in the s-domain.

Levinson Algorithm

$$R_k \hat{a} = \hat{b}$$

$$f_{00} = 1, \beta_0 = r_1, a_{00} = \frac{1}{r_0}, \gamma_0 = a_{00} r_1,$$

for $i=1$ to k

$$\left(\eta_i = -\frac{\beta_i}{\alpha_i}, \alpha_{i+1} = \alpha_i + \eta_i \beta_i, \right.$$

$$\begin{bmatrix} f_{i+1,0} \\ f_{i+1,1} \\ \dots \\ f_{i+1,i} \\ f_{i+1,i+1} \end{bmatrix} = \begin{bmatrix} f_{i,0} \\ f_{i,1} \\ \dots \\ f_{i,i} \\ 0 \end{bmatrix} + \eta_i \begin{bmatrix} 0 \\ f_{i,i} \\ \dots \\ f_{i,1} \\ f_{i,0} \end{bmatrix}$$

$$\beta_{i+1} = f_{i+1,0} r_{i+2} + f_{i+1,1} r_{i+1} + \dots + f_{i+1,i+1} r_1$$

$$q_i = \frac{b_{i+1} - \gamma_i}{\alpha_{i+1}}$$

$$\begin{bmatrix} a_{i+1,0} \\ a_{i+1,1} \\ \dots \\ a_{i+1,i} \\ a_{i+1,i+1} \end{bmatrix} = \begin{bmatrix} a_{i,0} \\ a_{i,1} \\ \dots \\ a_{i,i} \\ 0 \end{bmatrix} + q_i \begin{bmatrix} 0 \\ a_{i,i} \\ \dots \\ a_{i,1} \\ a_{i,0} \end{bmatrix}$$

$$\gamma_{i+1} = a_{i+1,0} \gamma_{i+2} + a_{i+1,1} \gamma_{i+1} + \dots + a_{i+1,i+1} \gamma_1$$

}/* for loop end */

Chapter 4

Transient Simulation of Lossy Interconnects Based on the Recursive Convolution Technique

4.0 Recursive Convolution Technique

The Telegraph Equation for a RLGC-line in the s-domain are:

$$\begin{aligned}\frac{\partial}{\partial x}V(x, s) &= -(sL + R)I(x, s) \\ \frac{\partial}{\partial x}I(x, s) &= -(sC + G)V(x, s)\end{aligned}$$

Rearranging the solution to the Telegraph Equation at the near end and far end of the transmission line, we get the following formula:

$$Y_0V(0, s) - I(0, s) = \exp(-\lambda l)(Y_0V(l, s) + I(l, s))$$

$$Y_0V(l, s) - I(l, s) = \exp(-\lambda l)(Y_0V(0, s) + I(0, s))$$

with $Y_0 = \sqrt{\frac{sC + G}{sL + R}}$, $\lambda = \sqrt{(sL + R)(sC + G)}$, R,L,G,C are unit length parameters, l is the length of the line. The inverse Laplace transform leads to the following equations:

$$v(0, t) * h_1(t) - i(0, t) = v(l, t) * h_3(t) + i(l, t) * h_2(t)$$

$$v(l, t) * h_1(t) - i(l, t) = v(0, t) * h_3(t) + i(0, t) * h_2(t)$$

where * stands for the convolution operation and:

$$h_1(t) = L^{-1}\{Y_0(s)\}$$

$$h_2(t) = L^{-1}\{\exp(-\lambda(s)l)\}$$

$$h_3(t) = L^{-1}\{Y_0(s)\exp(-\lambda(s)l)\}$$

Since the convolution integration is time-consuming, a special recursive convolution formulation was developed[16]. At time t_{n+1} , for example,

$$\begin{aligned} v(0, t) * h_1(t) |_{t=t_{n+1}} &= \int_0^{t_{n+1}} v(0, \tau) h_1(t_{n+1} - \tau) d\tau \\ &= \sqrt{\frac{C}{L}} v(0, t_{n+1}) + \sqrt{\frac{C}{L}} \sum_{i=1}^n \int_0^{t_{n+1}} q_i v(0, \tau) \exp(p_i(t_{n+1} - \tau)) d\tau \end{aligned}$$

where q_i and p_i are zeros and poles after Pade approximation. By using the Trapezoidal rule, we have:

$$\begin{aligned} \int_0^{t_{n+1}} q_i v(0, \tau) \exp(p_i(t_{n+1} - \tau)) d\tau &= \exp(p_i h_n) \int_0^{t_n} q_i v(0, \tau) \exp(p_i(t_n - \tau)) d\tau \\ &+ \frac{h_n}{2} q_i [v(0, t_n) \exp(p_i h_n) + v(0, t_{n+1})] \end{aligned}$$

which means that we can recursively get $\int_0^{t_{n+1}} q_i v(0, \tau) \exp(p_i(t_{n+1} - \tau)) d\tau$ based on the value of previous time where $h_n = t_{n+1} - t_n$.

4.1 Experiment Results

Fig 4.1 and Fig 4.2 are the magnitude of characteristic admittance and propagation function generated by the proposed method. The maximum frequency is 1.5GHz. In Table 1, we list the poles of both the propagation function and the characteristic admittance of the same example obtained by the proposed method. Example 1 includes only one transmission line, with 5 poles approximation, we compare the results of both proposed method with Pillai method in Fig.4.4. Example 2 is a 5 single transmission line case with nonlinear load. The third example is a clock subnetwork with 73 single lossy lines. From the

results we see that the results of Pillai's method are close to the those generated by Padé approximation. The results produced by the proposed method are within 5%-8% error range. Since by using the proposed method, we can not get an optimal rational approximation as Padé and Pillai methods, we may lose some accuracy while we ensure stability. And because the inverse Fourier transform technique is used in the process, the speed compared with Padé approximation is about 1.5-2 times slower, and we usually have to use higher order than Padé to ensure the accuracy.

$Y \times 10^3$

Magnitude of Characteristic Admittance

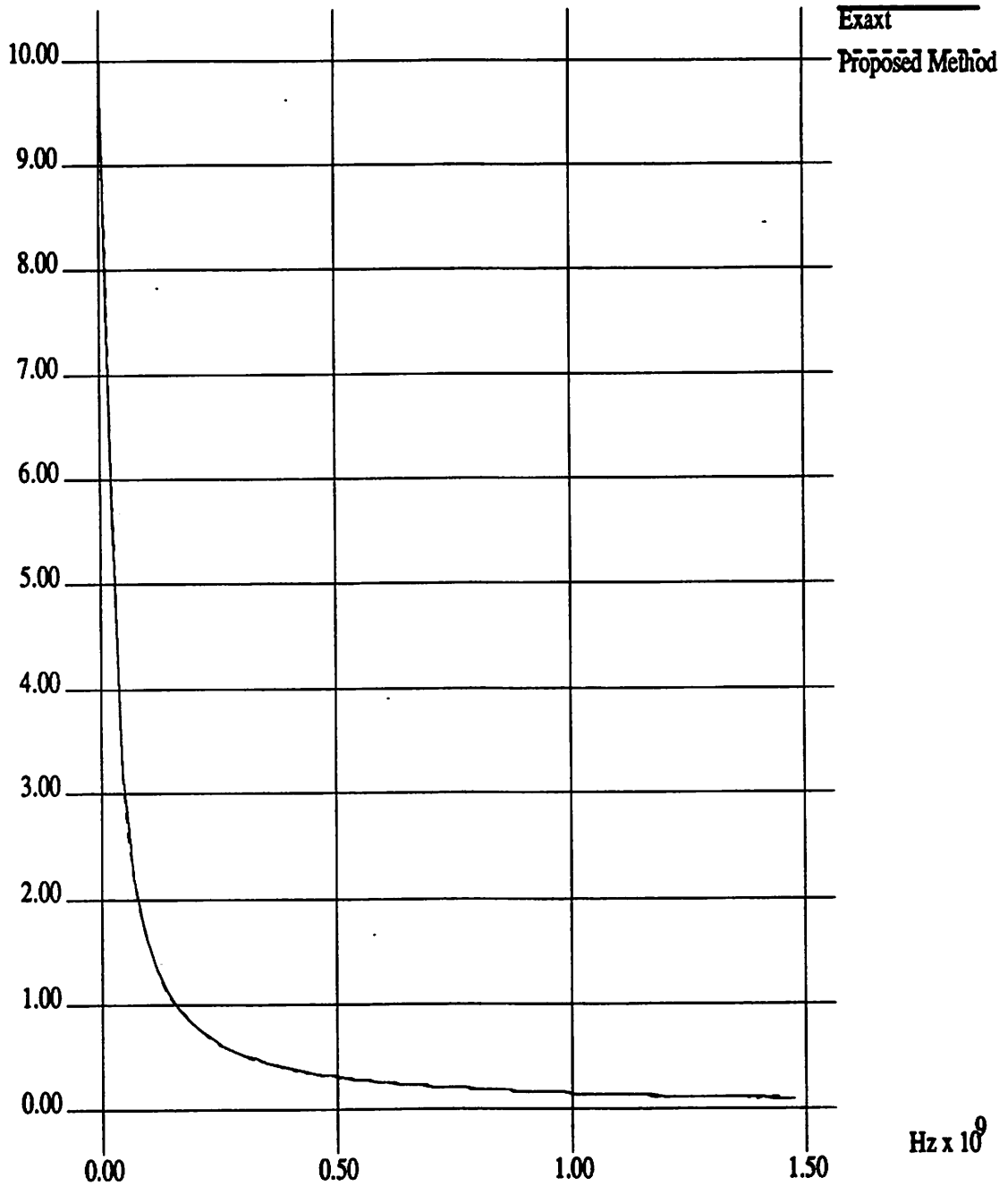


Fig.4.1

f_{max}: 1.5 GHz
a/b: 1.0e4
8 poles
Exmp: clock

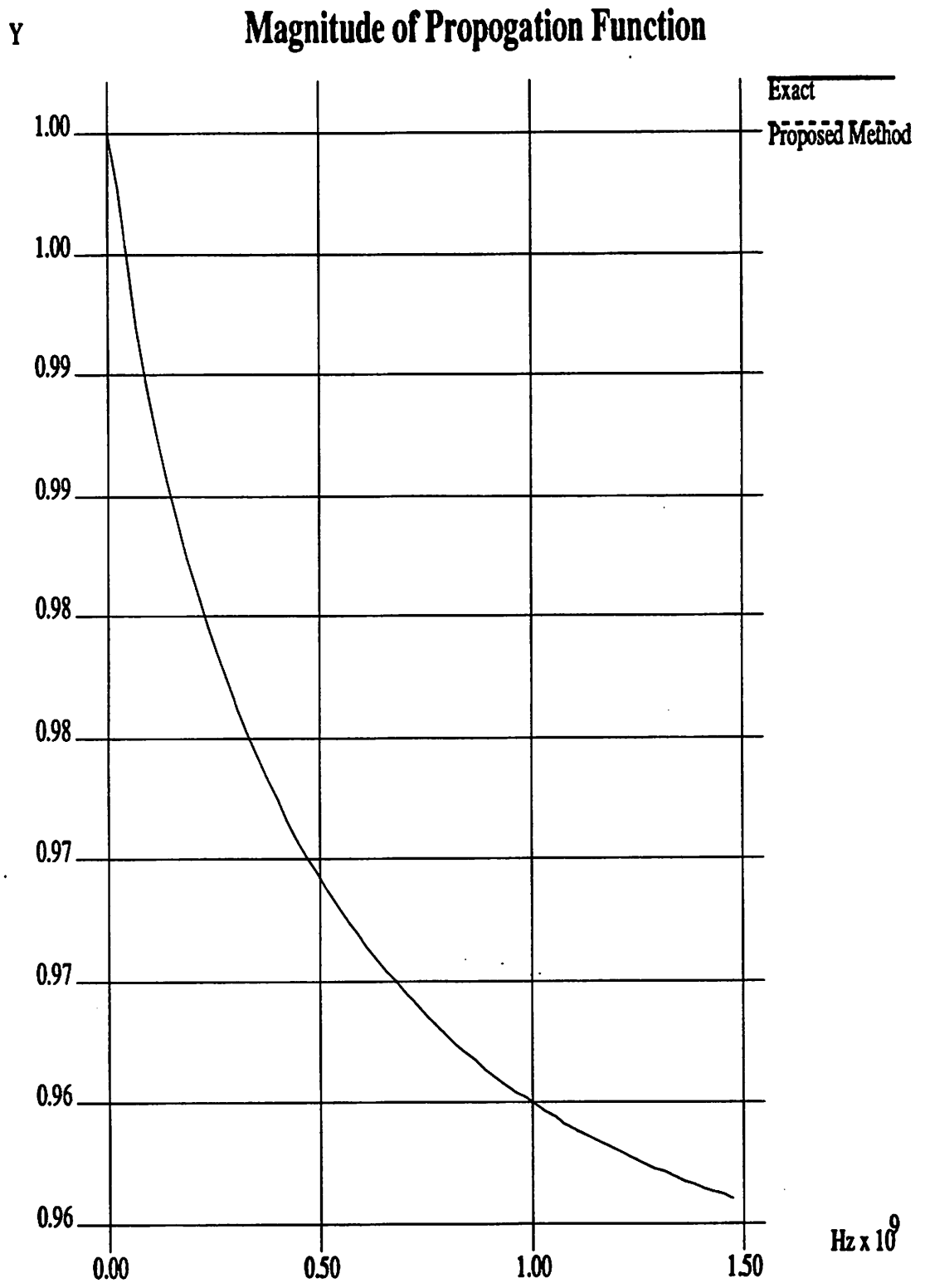


Fig. 4.2

Table 1: Poles of Proposed Method (s-domain)

Pole's number	Characteristic Admittance	Propagation function
1	-911256597.251263	-1703948051.280646 + i * 1939746373.091841
2	-1921458982.374783 + i * 1637922227.260988	-1703948051.280646 - i * 1939746373.091841
3	-1921458982.374783 - i * 1637922227.260988	-1805699460.357218 + i * 4453075067.121104
4	-2088633664.839134 + i * 4253469414.173211	-1805699460.357218 - i * 4453075067.121104
5	-2088633664.839134 - i * 4253469414.173211	-1767330674.945538 - i * 2633458285.983047
6	-2051394397.171135 - i * 2568765801.521321	-1767330674.945538 + i * 2633458285.983047
7	-2051394397.171135 + i * 2568765801.521321	-1655097962.385469
8	-5577962508.113366	-1427442032.579833

Exmp 1. One Single Transmission Line

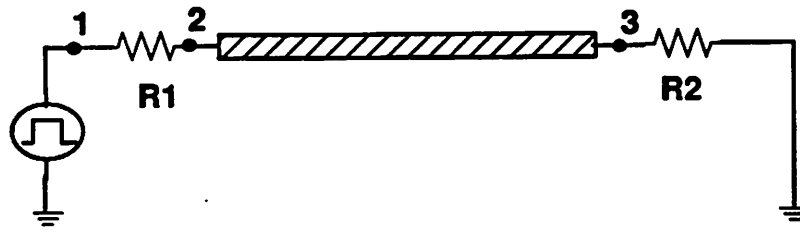


Fig.4.3

$$R = 0.25\Omega/cm \quad L = 2.5e-9H/cm \quad G = 5.0e-9s/cm \quad C = 1.0e-12F/cm$$

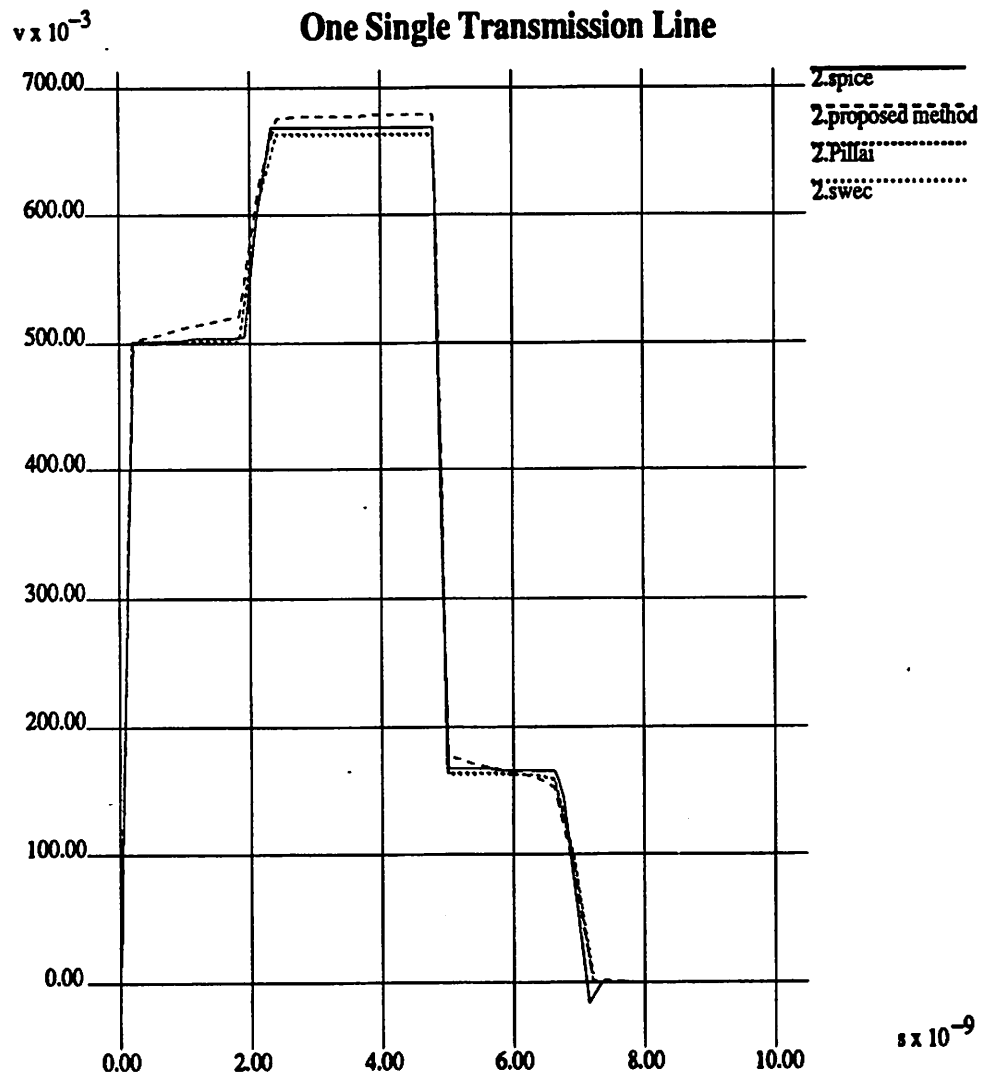


Fig. 4.4

$d = 20cm$ $R1 = 50\Omega$ $R2 = 100\Omega$ Proposed method and Pillai method use 5-pole model

Exmp2. A Small Circuit with Nonlinear Devices

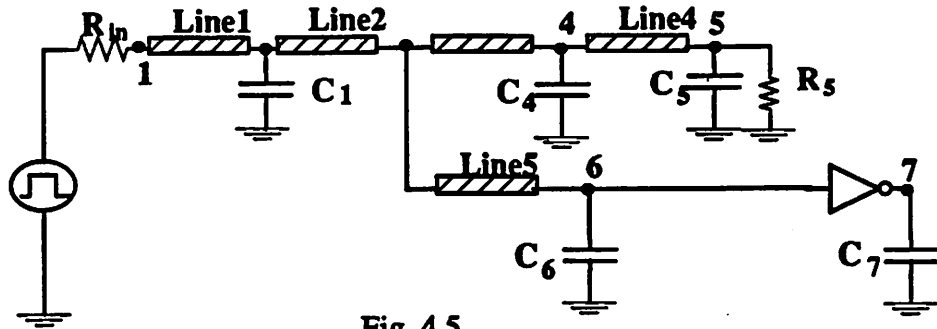


Fig. 4.5

Proposed method and Pillai method use 5-pole model:

v * LOSSY Transmission Line Network With Nonlinear Receiver *

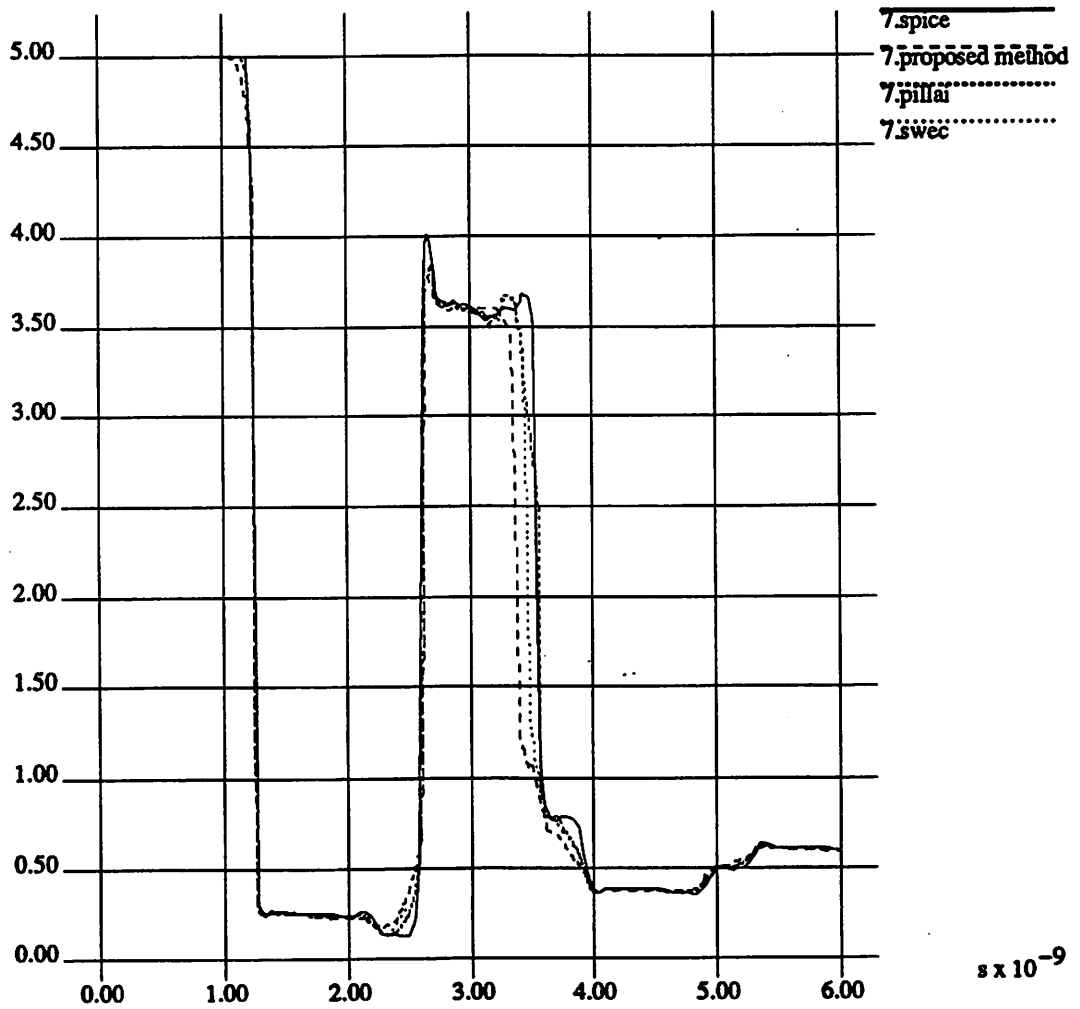


Fig. 4.6

Exmp3. Clock

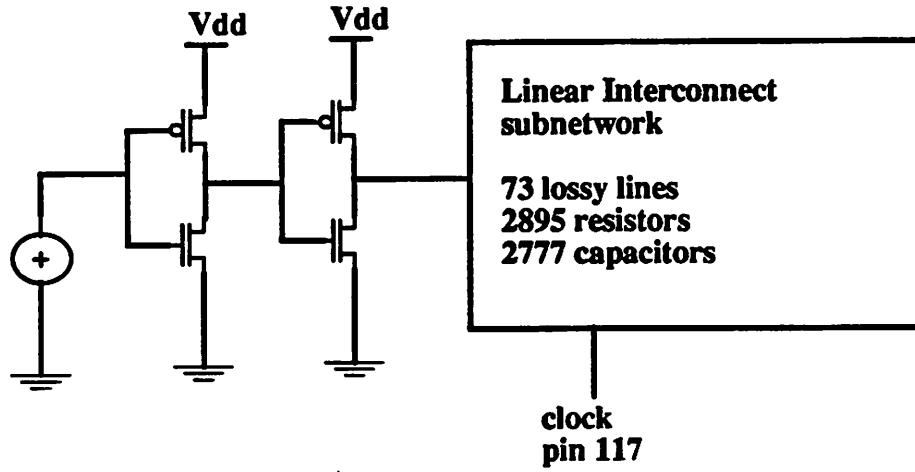


Fig. 4.7

Proposed method and Pillai method use 8-pole model

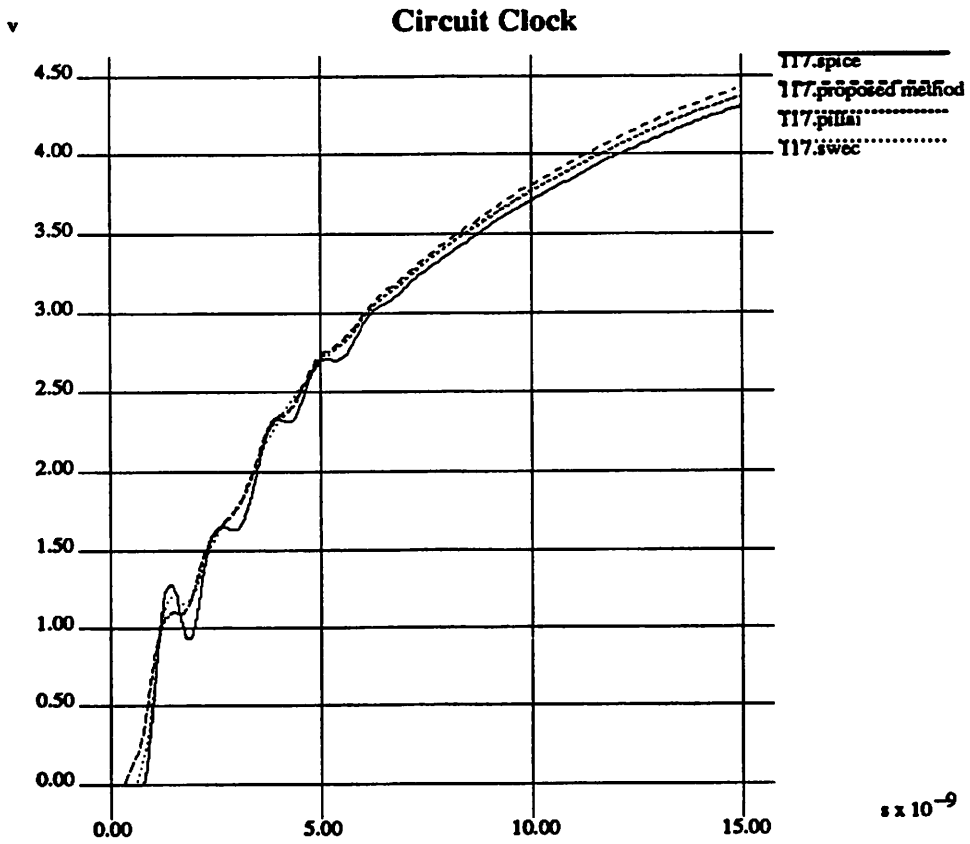


Fig. 4.8

Chapter 5

Conclusion and Future Work

A new approach for transient simulation of lossy interconnects is proposed. This approach is based on Pillai method to avoid the instability problem caused by the Padé approximation. By applying the recursive convolution technique, we greatly reduce the computation used to perform convolutions. We have incorporated the proposed technique in SWEC. The comparisons with other interconnect models indicate that the model we derived can ensure stability while we have to increase the number of order in order to keep the accuracy at the same time.

By comparing the simulation results based on Pillai's method, the proposed Pillai's method and Padé approximation, respectively, we found several interesting facts. 1) Pillai's method belongs to Padé approximation method category. It also achieves optimal rational approximation as Padé approximation does. The advantage of Pillai's method is that it can always find a stable rational approximation by increasing the order of the denominator and nominator. Unfortunately, there is no easy way to find out up to which order the rational approximation would be stable. 2) It is the very last step of Pillai's method which helps find out an optimal solution that leads to unstable rational approximation. Hence, our proposed Pillai method ensures stability by omitting the last step of Pillai's method. 3) There is a trade off between the stability and accuracy. From the

experimental results, we see if proposed Pillai's method is used to force stability, we need higher order rational approximation than Pade approximation to obtain the same accuracy.

When we deal with coupled transmission lines, all the theorems and concepts used in simple line cases can be extended [19]. By considering every entry as a matrix, we have the concept of positive matrix function instead of positive function. And Schur's Theorem has the same form as the original one. Richards' theorem gives rise to two forms of reflection coefficient matrix called right inverse form and left inverse form. This is due to the fact that when $Z(z)$ is a positive matrix function, even if a R has positive Hermitian part, $(Z(z) - R)(Z(z) + R^*)^{-1}$ is not a bounded matrix function unless R is a identity matrix.

So we have to construct the reflection coefficients based on the fact that

$I - d(z)d(z)^* \geq 0$ in $|z| < 1$. And because the impermutability of matrix multiplication,

we have $I - d(z)^*d(z) \geq 0$ in $|z| < 1$. Thus we have two forms when deriving $d(z)$. We also have Levinson Algorithm for matrices polynomial.

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