A DETERMINISTIC NONLINEAR CAPACITOR MODEL FOR SINGLE-ELECTRON TUNNELING JUNCTIONS

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Abstract

Single-electron tunneling junctions (SETJs) have intriguing properties which make them a primary nanoelectronic device for highly compact, fast, and low-power circuits. However, standard models for SETJs are based on a quantum mechanical approach which makes them very impractical for the analysis and design of SETJ-based circuitry, where a simple, preferably deterministic model is a prerequisite. We verify by physics-based Monte Carlo simulations that the tunneling junction can in fact be modeled by a piecewise linear voltage-charge relation, which, from the circuit-theoretic perspective, is a nonlinear capacitor.

1 Single-Electron Tunneling Junctions

To explain single-electron effects, an "orthodox theory" based on a phenomenological Hamiltonian approach with a tunneling term and the electrostatic energy has proved successful. To analyze circuits with single-electron junctions (SETJs), however, simplified models of the junction characteristics are required. One example is the Monte Carlo model in which classical electrons tunnel through the junctions stochastically with a probability that is a function of the temperature and the change in electrostatic energy. In the limiting case of zero temperature and small average current, it further reduces to a deterministic model where electron tunneling occurs as soon as it decreases the overall electrostatic energy of the system.

Based on these considerations, a deterministic model for the junction characteristics has been proposed which avoids any unnecessary complexities due to the stochastic nature of quantum mechanics and thermal fluctuation [1].

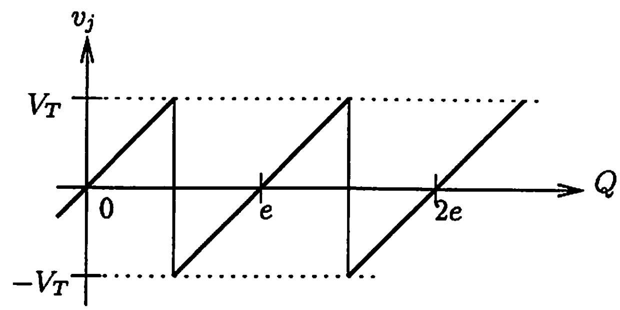


Figure 1: Linear-periodic voltage-charge relation for a SETJ.

In this model, it is assumed that an electron tunnels when the junction voltage v_j reaches the tunneling voltage

$$V_T = \frac{e}{2C_j}. \tag{1}$$

The behavior of the junction can therefore be modeled by a single-valued piecewise linear voltage-charge relation (Fig. 1). This model has been applied for the investigation of phase-locked single-electron tunneling elements [1, 2], but has not been verified so far. In this note, we confirm the model by simulations and put it in a new perspective: we demonstrate that it can be interpreted as a *nonlinear capacitor* – a viewpoint which greatly simplifies the analysis and design of SETJ-based circuits.

2 Simulation of a Current-Biased Single-Electron Tunneling Junction

To verify the validity of this piecewise linear model, we use a very simple test circuit (Fig. 2) consisting of a constant current source and a SETJ characterized by its tunneling resistance R_T and its junction capacitance C_j. For our calculcations, we use SIMON2.0, which is the latest version of a single-electron circuit simulator based on a Monte Carlo simulation of tunneling events. Note that R_T does not have an influence for these simulations provided that its value is larger than $h/e^2 \approx 25.8\text{k}\Omega$. The current is assumed to

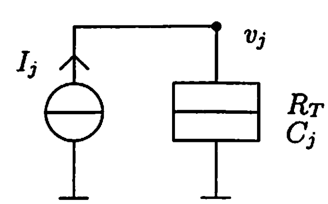
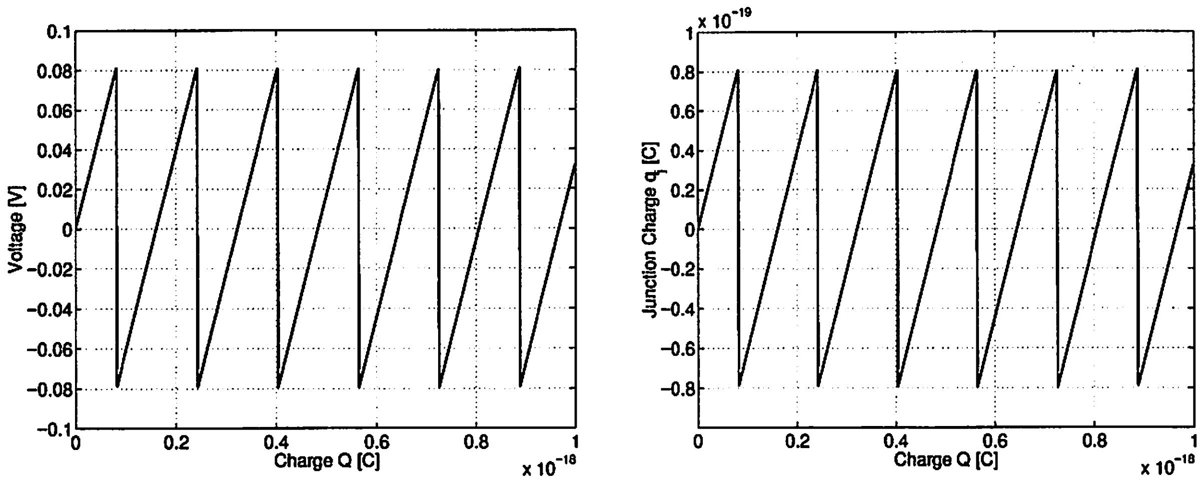


Figure 2: Current-biased SET junction.

be zero for $t < 0$ and constant for $t \geqslant 0$, and the charge is therefore proportional to the



(a) Charge-voltage characteristics.

(b) Charge-junction charge relationship.

Figure 3: Simulation of a single tunneling junction (SIMON2.0).

time, i.e., $Q(t) = I_j\, t$. This allows us to establish a charge-voltage relationship for the tunneling junction.

For the graphs in Fig. 3, we used a junction capacitance of C_j =1aF, which leads to a tunneling voltage of $V_T \approx 0.08V$. Note that the horizontal axis does *not* represent the actual junction charge but the *integrated current* through the junction, i.e.,

$$Q(t) = \int_{-\infty}^{t} i_j(\tau) d\tau \,. \tag{2}$$

Based on the physics of the SETJ, the device is a linear capacitor until the voltage reaches the threshold V_T. Hence, the actual junction charge is given by $q_j(t) = C_j\, v_j(t)$, where v_j is the junction voltage, $|v_j| < V_T$. Observe that q_j cannot grow larger than $e/2$ since this is the threshold junction charge for tunneling. The simulation in Fig. 3 (b) shows the relationship between Q and q_j and confirms the proportionality between junction charge and junction voltage. Mathematically, the junction charge can be described by

$$q_j = f(Q) = Q - n\,e \quad \text{for} \quad \left(n - \frac{1}{2}\right)e \leqslant Q < \left(n + \frac{1}{2}\right)e\,, \quad \forall n \in \mathbb{Z}\,. \tag{3}$$

As expected, $\frac{dq_j}{dQ} \equiv 1$ except at the discontinuities $(n + 1/2)e$, and $f(Q)$ does not depend on the junction capacitance.

Since the junction charge cannot be measured directly, it is desirable to express the junction voltage as a function of the integrated current Q:

$$v_j(Q) = \int_{Q_0}^{Q} \frac{1}{C(q)} dq \tag{4}$$

We use the letter C to emphasize that the integrand's denominator is, in fact, a capacitance, i.e., the ratio between charge and voltage.

Taking the derivative of (4) yields

$$\frac{dv_j}{dQ} = \frac{1}{C(Q)}, \tag{5}$$

and, together with (3) and Fig. 3, we get

$$\frac{1}{C(Q)} = \frac{1}{C_j} - 2\,\delta\Big(\big(n+\frac{1}{2}\big)e - Q\Big)V_T\,, \quad n = \{0, \pm 1, \pm 2, ...\} \tag{6}$$

and

$$C(Q) = \frac{C_j}{1 - 2\,C_j\,\delta\Big(\big(n+\frac{1}{2}\big)e - Q\Big)V_T}\,, \quad n = \{0, \pm 1, \pm 2, ...\}\,, \tag{7}$$

where $\delta(.)$ denotes the Dirac delta function.
Eq. (7) demonstrates that the tunneling junction is, in fact, nothing else than a *nonlinear capacitor* whose capacitance $C(Q)$ depends on the integrated current flowing through it. Its value is equal to the tunneling capacitance C_j except at points $Q = (n{+}1/2)e$, $\forall n \in \mathbb{Z}$.

3 Simulation of a Voltage-Biased Single-Electron Tunneling Junction

For practical circuits such as in [1], the SETJ is often voltage-biased. In this section, we investigate the circuit shown in Fig. 4. The bias voltage V_b is 0.1V, the value of the resistor is R = 1MΩ, and the junction capacitance is again C_j =1aF. We expect the

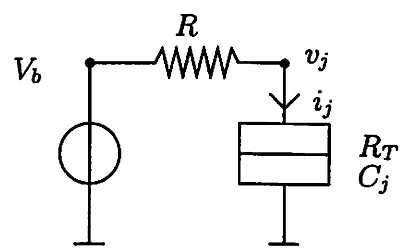
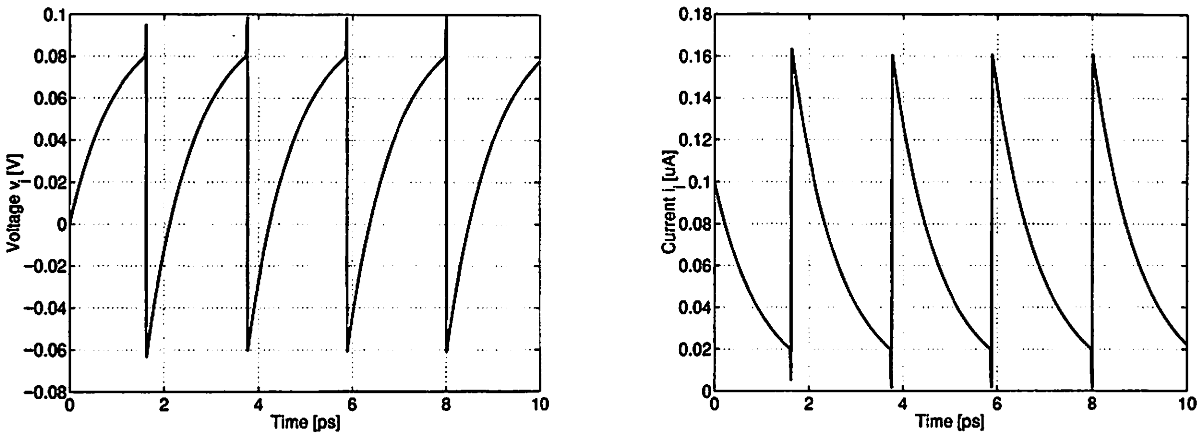


Figure 4: Voltage-biased tunneling junction.

"time constant" of the system to be in the ps range, since RC_j =1ps, which is confirmed by the simulations in Fig. 5. At $t = 0$, when the voltage source is assumed to be switched on, the current is $i_j(0) = V_b/R$, and it decays exponentially to zero, proportional to $\exp(-t/RC_j)$. However, when the junction voltage reaches V_T, the capacitance (7) becomes discontinuous, and the current jumps to a value of about $2i_j(0)$.



(a) Junction voltage.

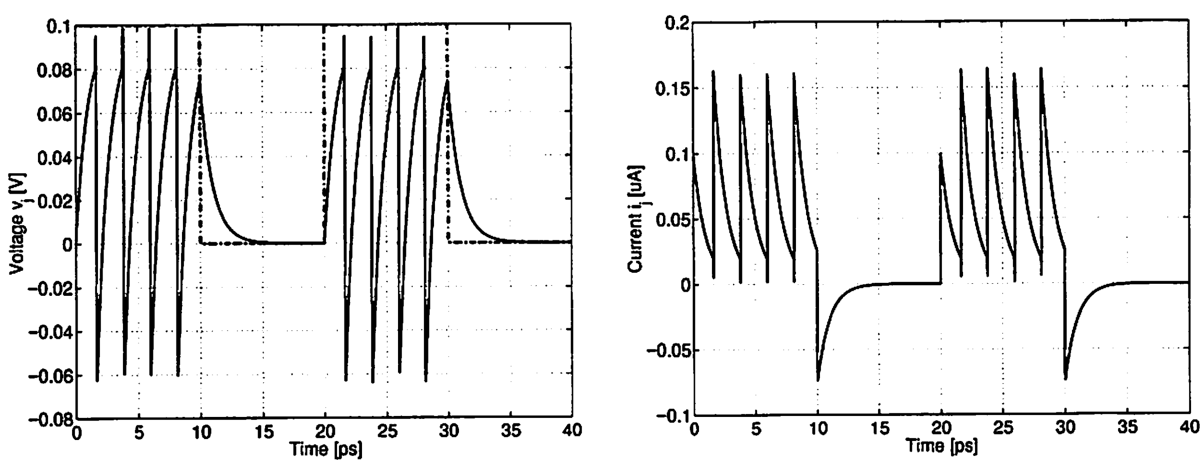
(b) Current.

Figure 5: Simulation of a voltage-biased single tunneling junction (SIMON2.0).

For our last experiment, we replace the constant bias voltage by a pulsed source (cf. dashed curve in Fig. 6 (a))

$$\forall n \in \mathbb{N}_0 : \quad V_b(t) = \begin{cases} 0.1\text{V} & \text{for } n\,20\text{ps} \leqslant t < \left(n + \frac{1}{2}\right) 20\text{ps} \\ 0\text{V} & \text{for } \left(n + \frac{1}{2}\right) 20\text{ps} \leqslant t < (n+1)\,20\text{ps} \end{cases} \tag{8}$$

The graphs in Fig. 6 prove, once again, the validity of the nonlinear capacitor model.



(a) Junction voltage and pulsed bias voltage (dashed).

(b) Current.

Figure 6: Simulation of a pulsed voltage-biased single tunneling junction (SIMON2.0).

For periods with $V_b(t) = 0$, the junction capacitance is discharged like a conventional capacitor, causing a negative current i_j.

4 Concluding Remarks

The foundation of nonlinear circuit theory is based on the axiomatic definition of 4 *basic* (two-terminal and multi-terminal) *circuit elements* [3, 4] from which all electronic devices are modeled. A 2-terminal circuit element defined by a relationship between the voltage $v(t)$ and the charge $q(t)$ is called a two-terminal *capacitor*. Hence the model presented in this note for a SETJ is precisely a 2-terminal nonlinear capacitor with a "sawtooth" $v-q$ characteristic.

It is important to bear in mind that in the *axiomatic* definition of circuit elements, the charge $q(t)$ is defined by the time integral of the current flowing into the terminal of the element, namely, $q(t) = \int_{-\infty}^{t} i(\tau)d\tau$. Observe that if we apply a constant dc current I_0 across an initially uncharged capacitor, we will get

$$q(t) = I_0 t, \quad t \geqslant 0. \tag{9}$$

Hence, the unbounded charge

$$\lim_{t \to \infty} q(t) \longrightarrow \infty \tag{10}$$

should be interpreted merely as a dc current flowing over the infinite time interval $[0, \infty)$, and not as the amount of physical charge stored in the junction, as would have been in the case of a linear capacitor.

Acknowledgment

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