MULTIRATE SIMULATION FOR HIGH FIDELITY HAPTIC INTERACTION WITH DEFORMABLE OBJECTS IN VIRTUAL ENVIRONMENTS

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Abstract

Haptic interaction is an increasingly common form of interaction in virtual environment (VE) simulations. This medium introduces some new challenges. In this paper we study the problem arising from the difference between the sampling rate requirements of haptic interfaces and the significantly lower update rates of the physical models being manipulated. We propose a multirate simulation approach which uses a local linear approximation. The treatment includes a detailed analysis and experimental verification of the approach. The proposed method is also shown to improve the stability of the haptic interaction.

Keywords — Balanced model reduction, deformable models, haptic interfacing, virtual reality, virtual environments.

1 Introduction

Haptic interaction is an increasingly common form of interaction in virtual environment (VE) simulations, especially since the commercial availability of high fidelity haptic devices such as the Phantom (Sensable Technologies Inc., Cambridge, MA) and Impulse Engine (Immersion Corp., San Jose,CA). This relatively new medium introduces some new challenges, which are being studied in the literature. Ensuring stability of haptic interaction with the virtual environment is an important problem. Several groups have considered the effects of model sampling time on stability [7, 3]. Colgate *et.al.* point out the non-passive nature of the discrete implementations of virtual environments as a major source of instability [3], and propose a virtual coupling network to improve stability [4]. Adams and Hannaford give a design algorithm to ensure stability of the haptic interface coupled to arbitrary passive virtual environments, therefore separating the design of the virtual environment and the haptic interface [1]. Simulation of stiff walls and hard contact is another interesting research topic. The penalty based approach is the most common way to simulate stiff walls. Zilles and Salisbury propose using "god-objects" to eliminate problems with penetration into the virtual objects in a penalty based approach [13], and Salcudean and Vlaar report that using a braking pulse greatly improves the perception of a stiff wall [8].

The authors' interest on this topic stems from the ongoing project of development of a VE based surgical training simulator [10, 6]. This application involves construction of realistic three dimensional geometric models of the anatomy; deformable object models simulated in real-time capable of manipulation, cutting, and suturing; as well as development of methods for teaching procedures, tasks, and skills transferable to actual surgery.

In a VE simulation of interaction with deformable bodies, for example in a surgical simulator, typically the physical model is updated at the visual update rates of 10 Hz order of magnitude. But haptic interfaces require much higher update rates, typically in the order of 1 kHz. It is not possible to increase the update rate of the physical model to the haptic rate with its full complexity due to computational limitations. The current practice is to apply the same force between the model updates, or to low-pass filter this generated force to the bandwidth of the model update rate. These effectively reduce the haptic update rate to the visual update rate, and therefore impair the fidelity of the haptic interaction. This is especially significant when the high frequency interaction forces are significant, for example in nonlinear phenomena like contact.

Astley and Hayward propose to use a multiscale multirate finite element model to address this problem. In their method, a coarse linear finite element mesh models the behavior of the overall object and a finer finite element mesh running at a higher update rate is used locally where there is an interaction [2]. Their work is based on decoupling the coarse mesh and the fine mesh by using the Norton equivalents as interfaces. This is only applicable to the linear finite element case, and the update rates reported were still significantly below 1 kHz required by the haptics.

In this paper we propose a multirate simulation approach to handle the difference between the update rate requirements for the haptics and the physical model during haptic interaction in VE simulations, complete with theoretical and experimental verifications of the approach. The proposed method is justified by model reduction techniques from system theory, and the approach is applied to nonlinear physical models.

We will start our treatment in this paper with a demonstration of the problem. This will be followed by the description of the proposed method, analysis of the critical parts, implementation, a short discussion of stability implications, and concluding remarks.

The discussion here is limited to lumped element models (also referred to as mass-spring-damper models in the literature), but the arguments can easily be extended to deformable models based on finite element analysis.

2 Demonstration of the problem

We first consider a haptic interface interacting with a simulated nonlinear spring in one dimension, and evaluate the fidelity of the force output of different simulation schemes for a given stimulus. This simple analysis demonstrates the problems that arise from the low model update rate and illustrates the basic motivation of the method proposed in this paper.

Four different simulation models, with 1kHz haptic update rates, are considered. In the first model, force feedback is generated by the nonlinear spring model updated at 1 kHz, which corresponds to the case where the model update rate is the same as haptic update rate. It is the baseline for the analysis as it is the ideal case. In the second model, the force is generated from the nonlinear spring model at an update rate of 10 Hz, and maintained constant in between the model updates. This is the counterpart of the case where there is interaction with a deformable model simulated at a larger step size than the haptic sampling

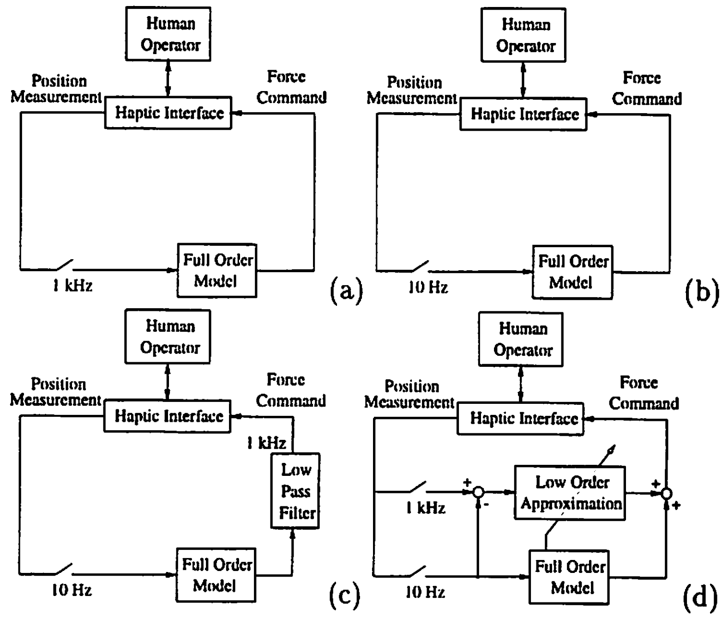

Figure 1: Simulation paradigms.

time. The third simulation model is an improved version of the second one. In this case, the nonlinear spring model is updated at 10 Hz, but the applied force is a low pass filtered version of the piecewise constant force generated from the nonlinear model. The bandwidth of the low pass filter is 10 Hz, and it is running at the haptic update rate of 1 kHz. In the last model, the nonlinear model is again updated at 10 Hz, as in the second and third models. However, the force output in between the nonlinear model updates is calculated from a linear spring model based on the tangent of the nonlinear spring at the last model update. To summarize

$$force_1[n] = f(x[n]) \quad (1)$$
$$force_2[n] = f(x[N]) \quad (2)$$
$$force_3[n] = f(x[N]) * lpf[n] \quad (3)$$
$$force_4[n] = f(x[N]) + f'(x[N])(x[n] - x[N]) \quad (4)$$

where n and N are respectively the haptic and model samples and $lpf[n]$ is the impulse response of the 10 Hz low-pass filter. Note that n runs at 1 kHz, and N runs at 10 Hz.

The nonlinear force-position characteristic of the spring used is based on the experimentally determined force deformation characteristics of the skin of the thigh given in [5]:

$$f(x) = \frac{x}{5.4398 - 0.1418x} \quad (5)$$

In the simulations, quantization noise is added to the position measurements and the force output. The quantization step size used is 0.3 mm for position measurements and 0.07 Newtons for force output. These values are the typical quantization values for the Phantom(TM) version 1.5 haptic interface [9].

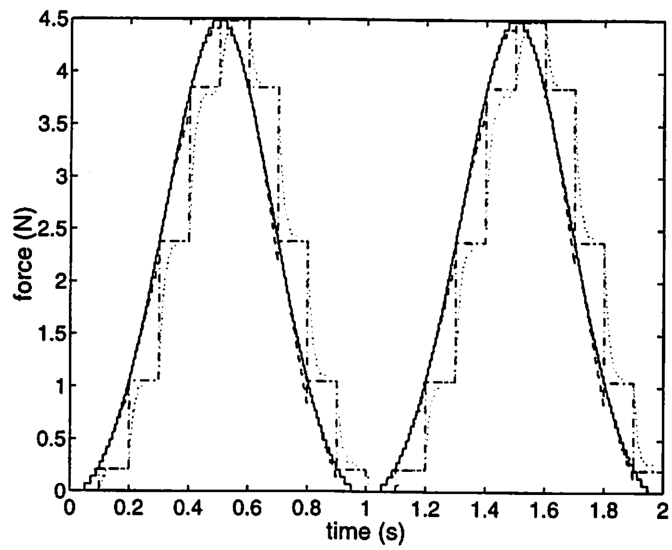

Figure 2: Interaction with a nonlinear spring in one dimension. Solid line is the 1kHz model, dash-dot line is the 10 Hz model, dotted line is the filtered 10 Hz model, and dashed line is the local tangent model.

In the test simulations of Fig. 2, the haptic interface is following a sinusoidal path maintaining contact with the spring, and the interaction force generated by the simulation model versus time is shown in the plot.

If we look at the signal-to-noise ratios (SNR) for the different simulation models, we can see the difference more clearly. The SNR for the ideal case is 89 dB with lag less than 1 ms. The constant force output model has SNR of 43 dB and a lag of 49 ms, and its low pass filtered version has SNR of 53 dB and a lag of 65 ms. The local tangent model has 72 dB SNR and a lag less than 1 ms, which is significantly better performance than the other two approximate models.

The use of a low-pass filter to improve the performance of the constant output method seems to help by reducing contaminating noise at the harmonics of the model update rate. However, this approach has two main limitations. First, low pass filtering may eliminate useful high frequency force information as well, for example in the case of nonlinear stiffness. To avoid this, model update rate has to be higher than the bandwidth of voluntary hand movements, 5–10 Hz, times the harmonics generated by interaction with nonlinear stiffness. Second problem is in the case of contact, where there is significant amount of information in high frequency. Also, the lag introduced by the low pass filter tends to destabilize the haptic interaction, or introduce oscillation.

The performance of the local tangent model gives the motivation of the method proposed in this paper for coping with the problems with the difference between the deformable model and haptic update rates.

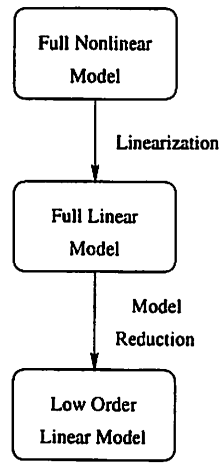

Figure 3: Construction of the low order model.

3 Using a Low Order Linear Approximation to Model Intersample Behavior

When the instrument interacts with the deformable model in a VE simulator, the haptic interface will displace the node(s) it is touching and display the reaction force. Therefore, from the haptic interfaces point of view, it will be interacting with a three-input three-output nonlinear dynamical system, considering the three components of translation and force respectively. However, the underlying dynamical system has a very high order as it includes the deformation of the whole body. For example, when interacting with a $10 \times 10 \times 10$ deformable cube, a mid-sized deformable model, the deformation will have $1000 \times 3 \times 2 = 6000$th order dynamics. This very high order dynamical system, which cannot be simulated in real time, needs to be replaced with a low order approximation for real time haptic performance.

The method we are proposing follows the local tangent approach described in section 2, shown in Fig. 1(d). In this approach, a low order approximation, running at the haptic update rate, is used on top of the full order model to estimate the intersample behavior. The low order approximation is updated by the full model after each step.

To analyze the construction of the low order approximation, we start with the paradigm given in Fig. 3. Linearization is a basic step. The linearized model gives the tangential behavior of the full model. As we want to capture the behavior in between the model updates, the deformation will be small. As illustrated in section 2, use of local tangent instead of constant force output improves the response significantly.

The linearized system will have the same order as the full model, therefore the improvement is limited by just using a linear model, i.e. it will still be difficult if not impossible to simulate in real time. Therefore, model reduction is the critical step of the approach, as it is the means of getting a temporally local haptic model which can be simulated in real time.

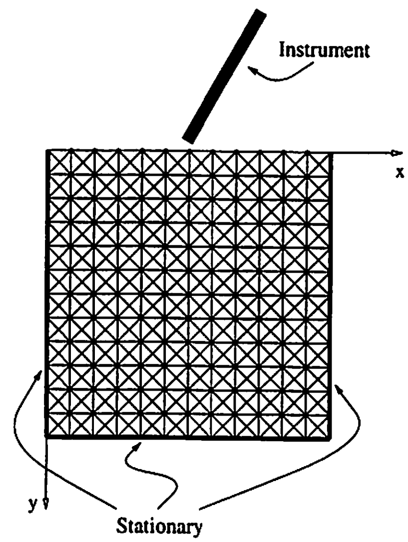

Figure 4: Two dimensional lumped element mesh.

4 Order Reduction

To evaluate the effectiveness of model reduction, consider a two dimensional 12×12 lumped element mesh being indented by an instrument (Fig. 4). Each node of the mesh has a lumped mass, which is connected to the neighboring nodes (diagonal as well as lateral neighbors) with spring and dampers. Three edges of the mesh are constrained to be stationary. Linearization of this system gives a two-input two-output 524th order linear dynamical system.

When we perform a balanced model reduction [12] on this model, we can approximate the system's input-output response with a 10th order system, with the infinity norm of the error resulting from the approximation being less than 1.6×10^{-3}, less than 1% of the full order model. This is a significant reduction in computational complexity while virtually maintaining the accuracy of the model. The frequency responses of the original and reduced order systems are shown in Fig. 5. The responses of the two systems are essentially indistinguishable except in normal-tangential interactions, where the response magnitudes in both conditions are very small (less than -200 dB).

The original states of the system before order reduction are the positions and velocities of the lumped masses at the vertices of the mesh. To visualize the spatial properties of the reduced model, the states of the new model are shown in Fig. 6. The figure shows the magnitude of the components of the new states with respect to the location on the mesh. The input node is at coordinate $(50, 0)$, which is the top middle node.

The states of the new low order model show that it is a local approximation. This result is actually expected, because stress decays inversely proportional to the square of the distance from the load in a semi-infinite linear elastic body under a point load [11].

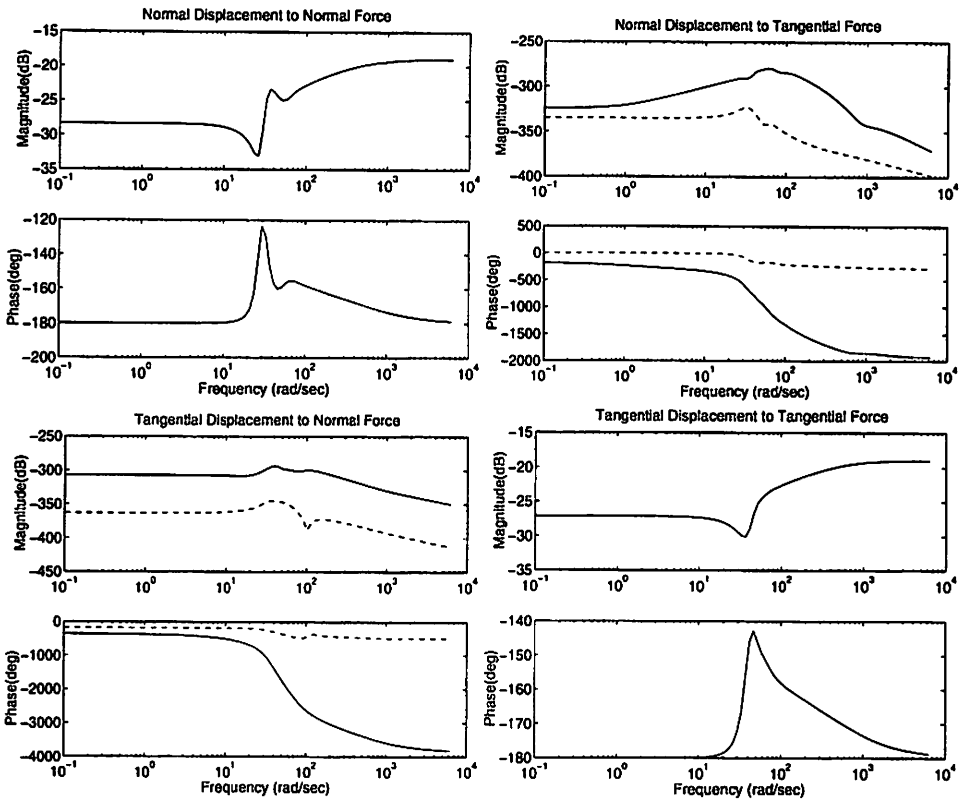

Figure 5: Frequency responses of the original and reduced order systems. Solid line is the reduced order model, dashed line is the full order model.

5 Towards a Real-Time Algorithm

It is important to note that balanced model reduction requires costly calculations as well, which prevents the use of this algorithm as a part of the on-line computation. However, the analysis in the previous section reveals that the approximation given by the balanced model reduction algorithm in a homogeneous medium is a local model, i.e. the force response depends mostly on the states spatially close to the interaction location. So, a natural way to construct a low order approximation with significantly less computation is to construct a local linear model directly from the full order model (Fig. 7). The local linear approximation we will demonstrate in this paper is shown in Fig. 8. It models the local behavior of the mesh with the nodes, springs and dampers near the instrument.

The frequency response of the local linear approximation, along with the frequency responses of the full linear model and a reduced order system with the same number of states as the local linear approximation calculated by balanced order reduction, are shown in Fig. 9. The local model approximates the behavior in the high frequency range, whereas its DC gain is significantly off. However, it is important to note that the local model is used only to estimate the intersample behavior of the full model, and therefore only needs to be close to the full model in the frequency range of around 10–1000 Hz, which is the case here. If necessary, it is possible to improve the low frequency accuracy by increasing the number

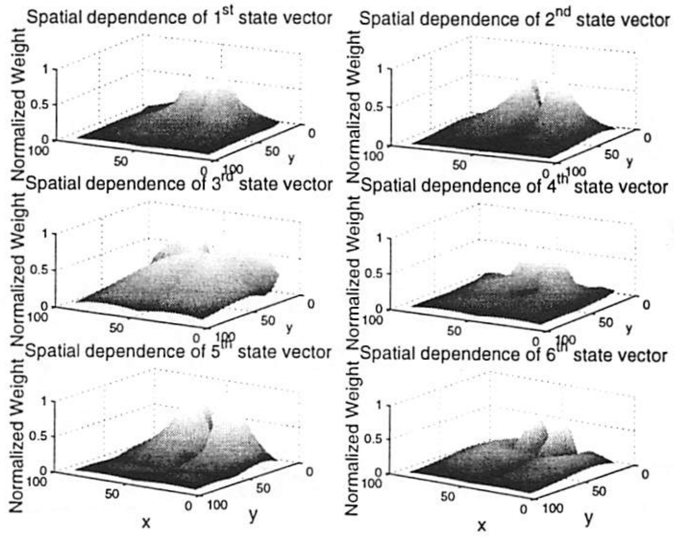

Figure 6: Spatial dependence of the states of the reduced order model.

of layers of nodes included around the instrument.

Another local linear approximation is shown in Fig. 10. This model approximates the behavior of the full order model over a wider frequency range. It includes the local behavior of the mesh from the spring and dampers right around the instrument, and the steady state behavior from the outermost connection of spring and dampers. The interior springs and dampers have the same coefficients as in the original mesh. Coefficients of the outer elements are scaled to reflect the fact that the equivalent stiffness of a fixed sized block changes as the mesh density is changed. The mass parameters of the nodes are also scaled according to their distance to the end of the specimen. The scaling rules are generalized form the one dimensional case, as described in section 5.2. The square root of the scaling estimated from Fig. 16 is used for the surface elements. The frequency response of this local approximation is shown in Fig. 11. Qualitatively, this local approximation captures the first cut-off and the overall shape of the Bode plot of the full order model.

These results show that the local linear approximation is a suboptimal approximation, as expected. But it can be constructed on the fly with minimal computation and give sufficiently accurate behavior in the frequency range of interest.

5.1 One Dimensional Case — Motivation for the Construction of Local Approximations

Consider the lumped element chain shown in Fig. 12. The transfer function from displacement x to interaction force f of this model is calculated as

$$\frac{F}{X} = (k+bs)\left[1 - \frac{(k+bs)\left((ms^2+2bs+2k)^2-(k+bs)^2\right)}{(ms^2+2bs+2k)\left((ms^2+2bs+2k)^2-2(k+bs)^2\right)}\right] \tag{6}$$

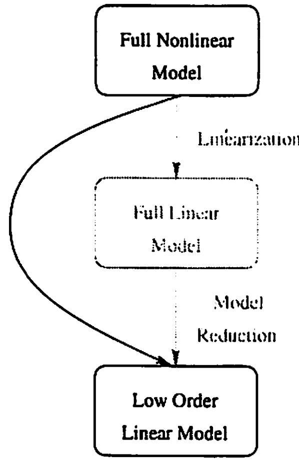

Figure 7: Construction of the low order model.

The poles of this transfer function are at

$$\frac{-(2-\sqrt{2})b \pm \sqrt{((2-\sqrt{2})b)^2 - 4m(2-\sqrt{2})k}}{2m}, \tag{7}$$

$$\frac{-2b \pm \sqrt{4b^2 - 8mk}}{2m}, \tag{8}$$

$$\frac{-(2+\sqrt{2})b \pm \sqrt{((2+\sqrt{2})b)^2 - 4m(2+\sqrt{2})k}}{2m}. \tag{9}$$

It has high frequency asymptote $(k + bs)$ and the DC gain $k/4$.

The first local model proposed considers only the pair of states closest to the interaction, i.e the position and velocity of the first mass, and assumes all the other masses stay stationary (Fig. 13). The transfer function of this model is given by

$$\frac{F}{X} = (k + bs)\left[1 - \frac{(k + bs)}{(ms^2 + 2bs + 2k)}\right] \tag{10}$$

which has poles at

$$\frac{-2b \pm \sqrt{4b^2 - 8mk}}{2m} \tag{11}$$

high frequency asymptote $(k + bs)$ and DC gain $k/2$.

The second local model shown in Fig. 14 tries to approximate the behavior of the full model over the low frequency range as well as the high frequency region by including the interior springs connected to the edge of the object. The coefficients of the interior spring and damper are chosen to be the equivalents[1] of the three interior layers of the full model. The masses of the nodes removed form the full model to get to the two level model is equally distributed to the neighboring nodes in the two level model. The transfer function of this

[1]Note that this spring damper configuration replacing the interior three layers is not exactly equal to the Norton equivalent of these as proposed in [2]. Rather the values used are the stiffness (damping) of the interconnection when only springs (dampers) are used. This results in a lower order approximation, whereas the Norton equivalent of the interconnection would have the same order as the network replaced.

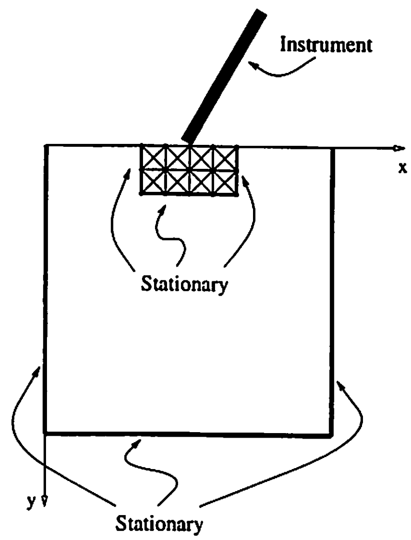

Figure 8: Local low order approximation.

model is given by

$$\frac{F}{X} = (k + bs)\left[1 - \frac{(k + bs)}{(2ms^2 + \frac{4}{3}bs + \frac{4}{3}k)}\right] \tag{12}$$

which has poles at

$$\frac{-\frac{2}{3}b \pm \sqrt{(\frac{2}{3}b)^2 - \frac{8}{3}mk}}{2m} \tag{13}$$

high frequency asymptote $(k + bs)$ and DC gain $k/4$.

It can be observed from the frequency responses of these models shown in Fig. 15 that the first local model can only match the high frequency response, whereas the second local model can approximate the high and low frequency asymptotes as well as the two low frequency poles of the full model.

5.2 Element Coefficient Scaling in the Two and Three Dimensional Cases

In order to be able to generalize the local approximation shown in Fig. 14 to higher dimensions, we need to establish how the equivalent stiffness changes when the mesh density is changed. Fig. 16 shows the change in the largest singular value of the stiffness matrix of two and three dimensional lumped element mesh blocks as the interior mesh density is changed.

6 Implementation

We have implemented the paradigm explained above in a real time VE simulation of manipulation of a deformable object. The object used is a $6 \times 6 \times 6$ lumped element model. The local low order model used is the three dimensional extension of the model shown in Fig. 8. In the simulation, the full nonlinear model is updated at 20 Hz, whereas the haptics

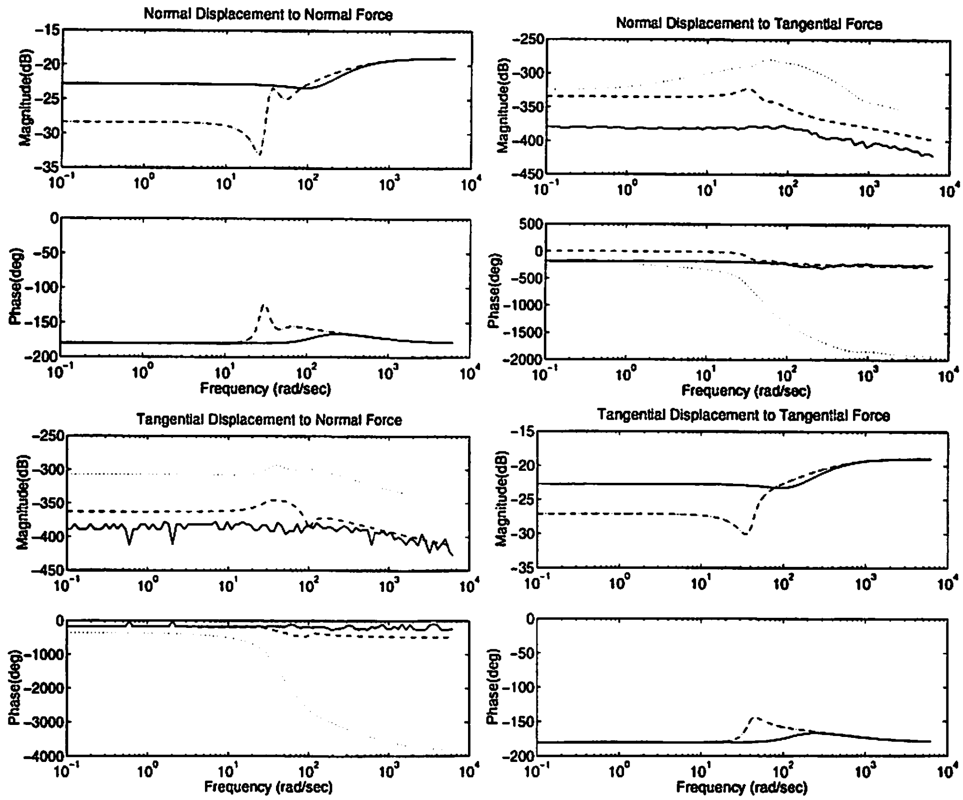

Figure 9: Frequency responses of the local linear approximation (solid line), full linear model (dashed line) and reduced order model (dotted line).

and the local linear approximation is run at 1 kHz. It is important to note that the size of the full order nonlinear object model can be scaled without affecting the performance of the haptic interaction. The computational requirements of the construction of the local model and the haptic loop is fixed and independent of the size of the full order model.

Note that, it is also important to have a contact surface in the model. This is to insure that the linear model will be only pushing the instrument during contact. This is achieved simply by applying the interaction force feedback only if the component of the force in the surface normal direction is smaller than zero, and giving zero force otherwise.

The simulation is implemented in C++, using OpenGL as the graphics library. It is run on a dual processor SGI Octane computer. A Phantom(TM) version 1.5 manipulator is used as the haptic interface.

The force during interaction is shown in Fig. 17. The dashed line shows the force calculated by the low update rate model, and the solid line shows the force displayed by the local linear model at haptic update rate.

Stability Implications

Stability of haptic interaction with VEs is an important consideration for design of haptic interfaces and virtual objects. The update rate of the simulation is one of the critical

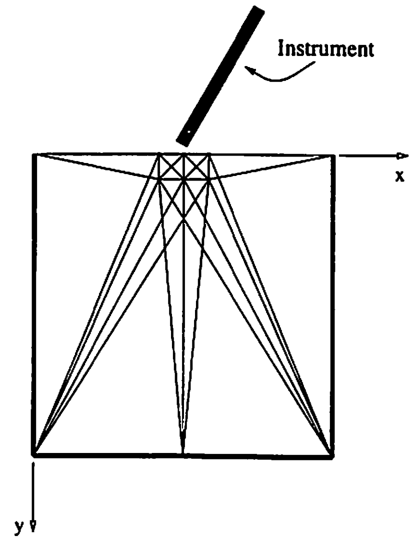

Figure 10: Local low order approximation with better low frequency response.

determinants of the stability of interaction, where increasing the update rate of the model improves stability [7, 3]. In the method we are proposing, having the low order linear model running at a faster update improves the stability of the haptic interaction as the VE model runs at 1 kHz instead of 10 Hz.

This effect can also be observed in the implementation of our method described above. However, stability of our method is difficult to determine because the resulting system is a multirate nonlinear sampled-data system. In the simulation, if the local linear approximation is not used, the haptic interface tends to have oscillatory behavior when the operator loosens his/her grip (Fig. 18). This oscillatory behavior is not present with the local linear approximation even when the operator completely releases the instrument.

7 Discussion

In this paper, a multirate simulation approach to handle the difference between the sampling rate requirements of the haptics and the possible update rates of the physical models during haptic interaction with deformable objects in VE simulations is presented. The proposed method uses a linear approximation to model the intersample behavior of the nonlinear full order model. The natural choice of the linear approximation is to use the linearization of the nonlinear dynamics, which gives the tangent behavior of the dynamical system. However, this linearization does not completely solve the computational complexity problem since the order of the linearized model is still very high. We performed balanced model reduction on the linearized model and showed that it is possible to use a low order local approximation and still get an accurate input output response. Based on this analysis, we proposed a simple local linear approximation which can be computed in real-time and implemented it in a simulation to verify the method.

It is important to note that the local linear approximation used here is not necessarily the best choice. There is room for improvement.

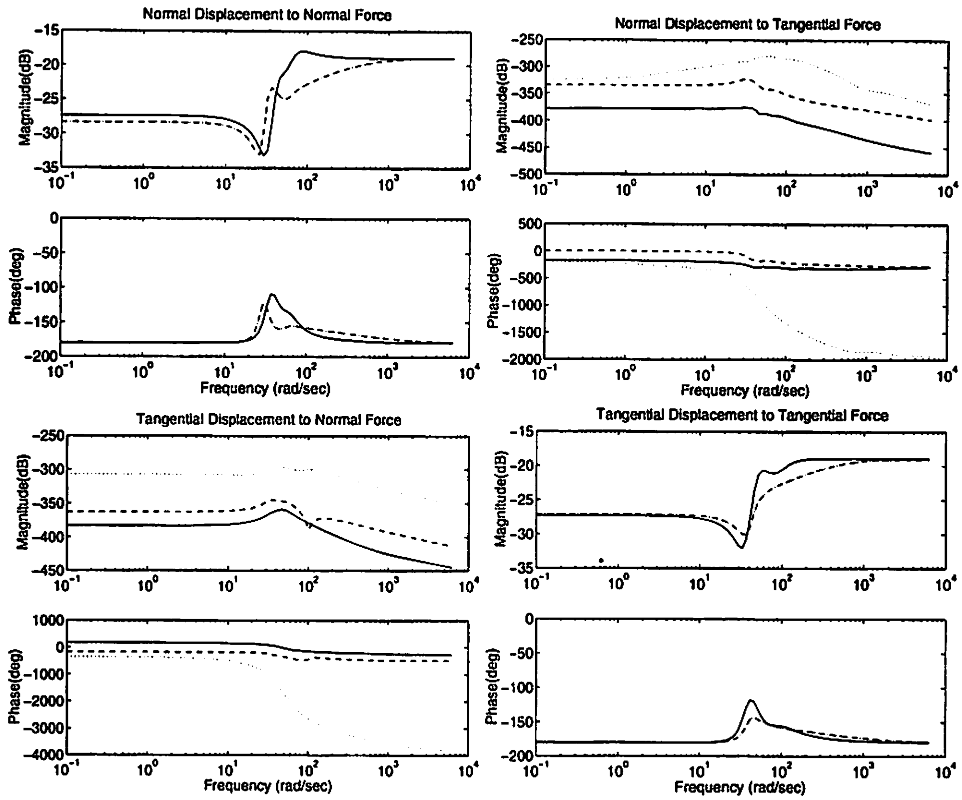

Figure 11: Frequency responses of the second local linear approximation (solid line), full linear model (dashed line) and reduced order model (dotted line).

The major limit of this method is that the local states must be the dominant ones. This could be violated if the material was inhomogeneous, for example if the deep tissue were significantly more compliant than at the surface so that most of the deformation occurred in states far from the interaction. In this case, Astley and Hayward's method [2] would be useful, if the model was linear. Other effects that could violate the dominance of local modes include significant geometric nonlinearities or discontinuities in the tissue that produced large local stresses away from the instrument contact. However, the locality of the dominant modes can always be checked by performing off-line model reduction, as it is done here in section 4.

Acknowledgments

This work is supported in part by ONR under MURI grant N14-96-1-1200 and National Science Foundation under grant CISE CDA 9726362.

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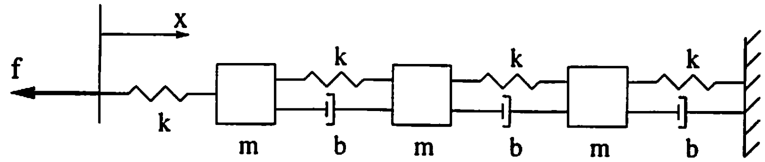

Figure 12: Four layer lumped element chain.

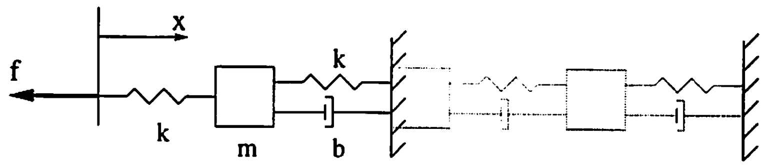

Figure 13: Approximation of the four layer lumped element chain. This local model approximates only the high frequency behavior of the full order model.

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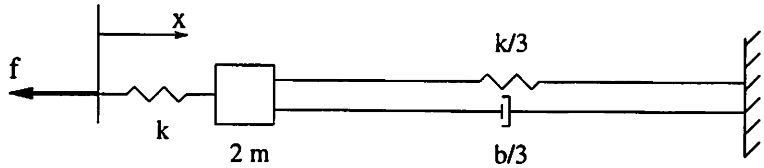

Figure 14: Second approximation of the four layer lumped element chain. This local model is constructed to approximate high and low frequency behavior of the full order model.

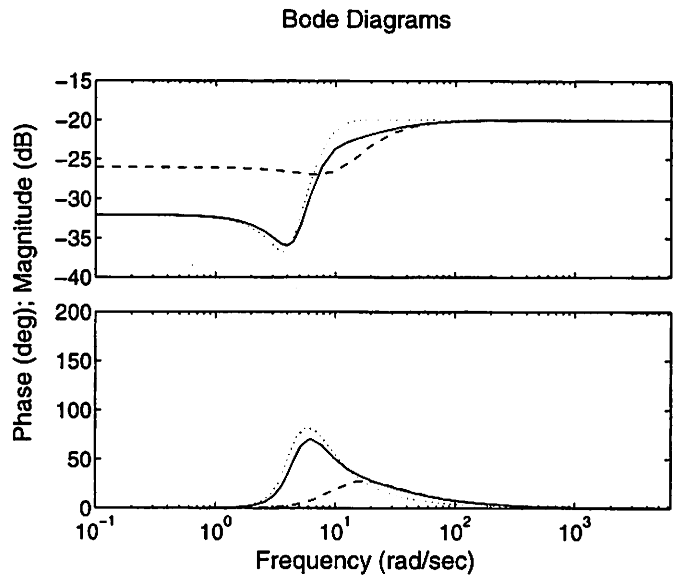

Figure 15: Frequency responses of the full order model (solid) and the first (dashed) and second (dotted) local models.

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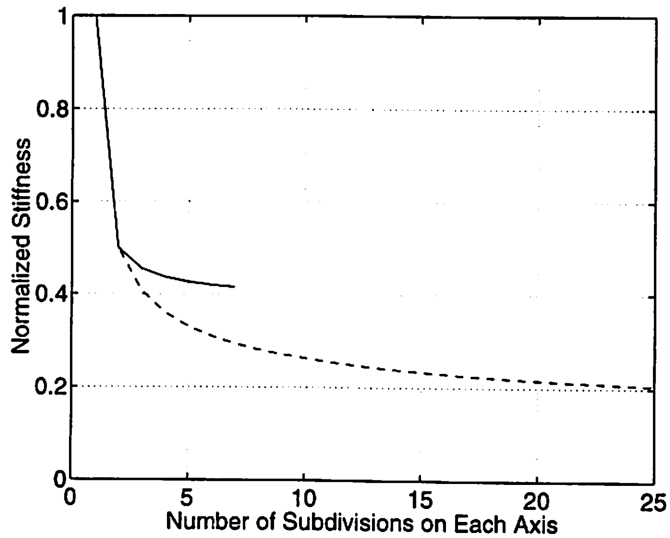

Figure 16: Change in stiffness of a block with change in mesh density for 3-D (solid) and 2-D (dashed) meshes.

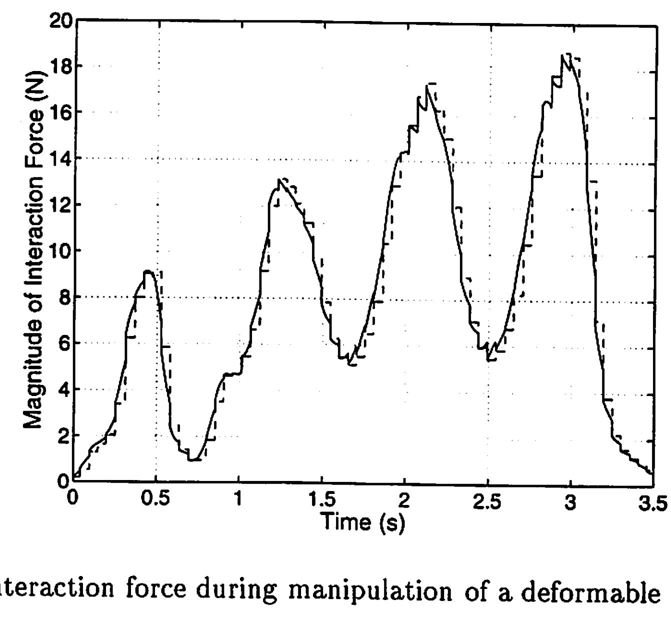

Figure 17: Interaction force during manipulation of a deformable virtual object.

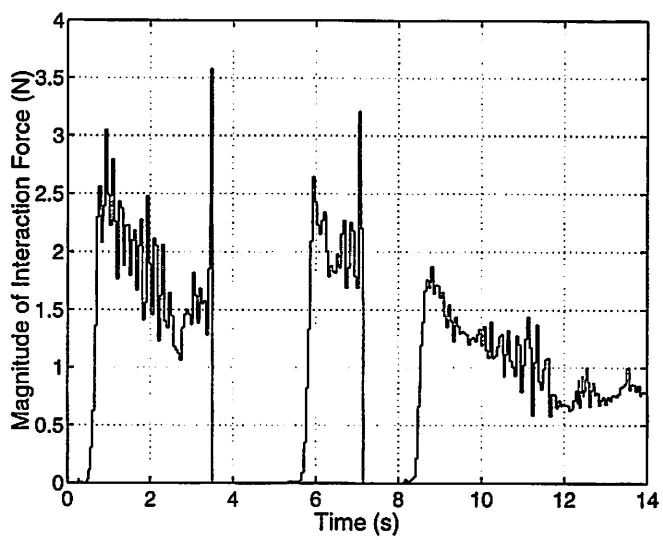

Figure 18: Oscillations observed when the local linear approximation is not used.