# Sensitivity Analysis for AM Detectors

*Simone Gambini*
*Nathan Pletcher*
*Jan M. Rabaey*

Electrical Engineering and Computer Sciences
University of California at Berkeley

Technical Report No. UCB/EECS-2008-31
http://www.eecs.berkeley.edu/Pubs/TechRpts/2008/EECS-2008-31.html

April 6, 2008

Acknowledgement

The authors acknowledge the BWRC Member companies, GSRC and California Energy Commission for support of this research, and STMicroelectronics for foundry service donation.

# Sensitivity analysis for AM detectors

## I. Introduction

The recent interest in low-power radio transceivers for sensor networks and ambient intelligence has led to a resurgence in the popularity of receivers that perform signal downconversion by envelope detection, a concept already employed in early AM radios. The simplest AM-receiver architecture, shown in Figure 1, is composed of several stages of RF gain, followed by a squaring device. Although suboptimal from a communication system perspective, this receiver architecture enables a drastic simplification of the system architecture, as quadrature RF carrier generation can be omitted, and has been recently employed in several ultra-low power receivers using both narrowband ([3],[2]) and ultra-wideband signaling ([1],[9]). It is generally understood that the main limitation of this architecture comes from the decrease in envelope detector gain with decreasing input signal amplitude, and that the most efficient means to improve the sensitivity of such receivers is to increase the amount of RF gain preceding the envelope detector. However, a quantitative analysis aimed at predicting the sensitivity of an AM-receiver given the noise characteristics of its components is unknown to the authors and constitutes the subject of this paper. The paper is organized as follows: in Section 2, we review the noise and conversion gain analysis of a MOSFET peak-detector, with special attention on the cyclostationary noise spectrum generated by the device during downconversion. The results of this section are extended to include noise-self squaring effects. In Section 3, these results are used to perform sensitivity analysis of a complete receiver. Measured results from two experimental AM receivers are presented in Section 4, verifying the presented theory. Section 5 draws conclusions.

## II. Large signal noise analysis of envelope detector circuit

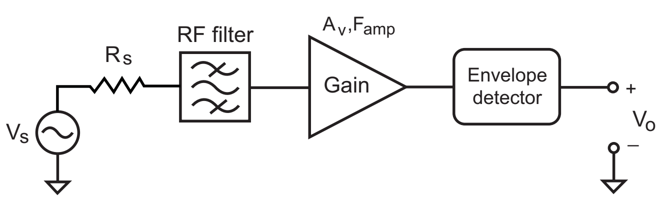


Fig. 1. Envelope detector schematic

Since an envelope detector is by definition a large signal circuit, the validity of standard small-signal analysis in describing its noise behavior should be questioned. In particular, the periodically time-varying DC bias point of the circuit modulates the device noise sources as well as their transfer function to the output, impacting noise generation by this same circuit block, and contributes to downconvert RF noise in the signal baseband. Both these effects are analyzed below.

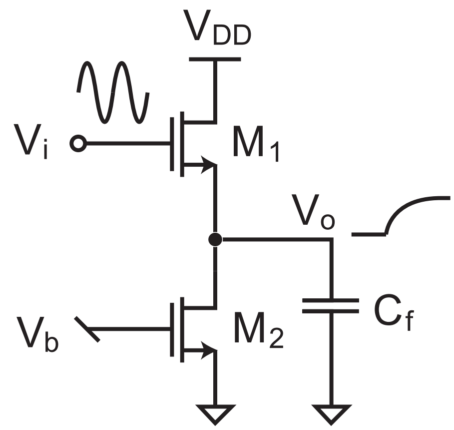


Fig. 2. Envelope detector schematic

### A. Noise generation in the presence of a large signal excitation

The most popular form of RF energy detector is shown in Figure 2 and is simply an MOS implementation of the bipolar circuit described in [4]. It consists of a source follower with output bandwidth much smaller than the incoming carrier frequency. As a result, the full swing of the incoming RF carrier appears across the gate-source of device M1, which is operated in weak inversion to maximize its nonlinear behavior. While the nonlinear V-I characteristic of the device would lead to an increase in the DC drain current, current source M2 counteracts this action generating an increase in the source voltage of the device. Such a source voltage shift can be detected by a following stage and represents the downconverted signal content. Neglecting finite output conductance of M2, the circuit satistifies ([4]).

$$\int_0^{T_{rf}} I_0 \exp\left(\frac{V_{rf}\sin(\omega_{RF}t) - V_s}{nV_t}\right) = I_2 \tag{1}$$

which can be solved for $V_s$ to give

$$V_s = nV_t \log J_0\left(\frac{V_{rf}}{nV_t}\right)$$

where $J_0$ is the zero-th order Bessel function of the first kind. Once the DC value of $V_s$ is known, it can be used in the equation 1 to calculate the exact drain current waveform. From here, we can proceed to calculate the device transconductance $gm$ as a function of time using $gm(t) = \frac{I(t)}{nV_t}$, and the instantaneous noise-current power spectral density $2qI(t)$. Considering noise sources as small signals superimposed on the large signal RF excitation, linear periodically time-varying analysis can be applied to calculate their contribution to output noise. Such noise sources satisfy the following (stochastic) differential equation:

$$I_n(t) = gm(t)V_o(t) + C_L\frac{\partial V_o(t)}{\partial t} \tag{2}$$

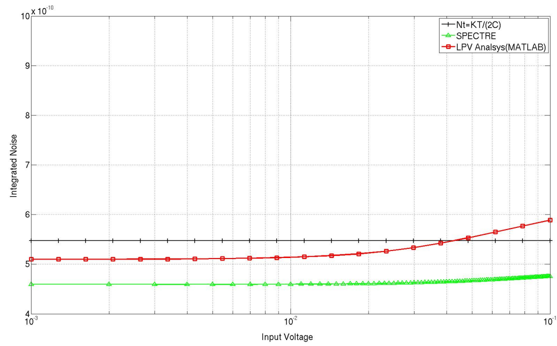


Fig. 3. Noise generated by the envelope detector: linear Vs Cyclostationary analysis

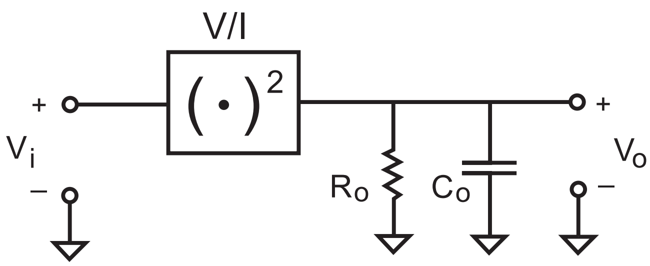


Fig. 4. Simplified envelope detector model

The output voltage generated at time t by a unit charge impulse applied at time $t_0$ reads

$$H(t, t_0) = \exp\left(-\int_{t_0}^{t} \frac{gm(s)dS}{C}\right)$$

Expanding in Fourier series the two quantities and applying the techniques described in [7], the total integrated noise at the output of the envelope detector is found. This calculated value is plotted in Figure 3 versus input RF signal amplitude for the ideal bipolar envelope detector of [4]. In the same figure, SPECTRE-PNOISE simulations of such circuit are also reported. Finally, the total integrated noise as predicted from linear analysis $V_n^2 = \frac{KT}{2C}$ is also reported. The values are within $10\%$ one of the other, and show that for the circuit under examination the total integrated noise at the output of the envelope detector shows insignifant deviation from the linear prediction. Therefore, for the purpose of noise generation, large signal operation of the detector is irrelevant and can be ignored.

### *B. RF Noise downconversion*

Although the nonlinear behavior of the envelope detector does not impact noise generation in this circuit block in a significant way, it downconverts noise present in the input RF band to DC. In this section, we'll approximate the input-output characteristic of the peak detector with a simple squaring function and derive the autocorrelation function of the output noise induced by downconversion of RF noise. In particular, we assume the envelope detector output voltage to be related to the input by the following cascaded operations: first, the input voltage is nonlinearly converted to a current by a squaring device according to $I_o(t) = K \cdot V_{in}(t)^2$, where $K = \frac{1}{2}(\frac{q}{nKT})^2$. Subsequently the current is converted to voltage by a low-pass impedance built of the shunt resistor $R_o = \frac{1}{gm} = \frac{nKT}{q}$ and the output filtering capacitor $C_o$. To simplify derivations, we describe both the signal and the noise in the sampled data domain. Therefore, we'll consider the input signal composed of a sinusoidal signal term of amplitude $V_{sig}$ plus a zero-mean gaussian noise process n(k), we have $V_{in}(k) = V_{sig}\sin(2\pi f_{rf} kT_s) + n(K)$, so that

$$y(k) = I_o(k) = K(V_{in}(k)^2) = K(V_{sig}^2(k) + n^2(k) + 2V_{sig}(k)n(k))$$

Dropping the deterministic component $V_{sig}(k)V_{sig}(k+m)$, calculating the autocorrelation function and dropping odd-moments of n(t)

$$\begin{aligned} \frac{R_{yy}(k, k+m)}{K^2} &= E\{V_o(k)V_o(k+m)\} = \\ &= E\{n^2(k)n^2(k+m)\} + \\ +4V_{sig}(k)&V_{sig}(k+m)E\{n(k)n(k+m)\} \end{aligned}$$

For white noise n(k) with power $\sigma^2$, we finally have after stationarizing the term in $V_s$:

$$\frac{R_{yy}(k, k+m)}{K^2} = (3\sigma^4 + 2V_{sig}^2\sigma^2)\delta(m) \tag{3}$$

The output process also has a finite mean $\mu_o = \sigma^2 + V_{sig}^2\sigma^2$. Therefore, we have

$$\frac{\sigma_o^2}{K^2} = (2\sigma^4 + V_{sig}^2\sigma^2)$$

We recognize a term due to self-mixing of the noise, and a term due signal-noise intermodulation. Defining further $\alpha = \frac{BW_{noise}}{BW_{channel}}$ and as the ratio of the noise bandwidth to the baseband modulation bandiwdth, writing the input SNR as $SNR_i = \frac{V_{sig}^2 BW_{noise}}{2\sigma^2 BW_{channel}}$ we can express the output noise variance as in equation 4

$$\frac{\sigma_o^2}{K^2} = \sigma^2 V_{sig}^2\left(\frac{4\alpha}{SNR_i} + 1\right) \tag{4}$$

The two terms in equation 3 are readily interpreted as the noise variance at the output of the envelope detector in the absence of RF signal, and the noise variance resulting from the downconversion of noise in the RF band to DC in the presence of the input signal. The term in $\sigma^4$ is generated by noise self mixing; it cannot be predicted by conventional LPTV noise analysis, as this neglects terms due to interaction of the noise with itself [8]. Since at sensitivity $SNR_i > 1$, the detector RF noise bandwidth must be much larger than the communication bandwidth ($\alpha \gg 1$) in order for this self-mixing term (3) to be significant. Widening the noise bandwidth can be advantageous in a power constrained regime as it relaxes frequency synthesis requirements [6]. The term $(\frac{2\alpha}{SNR_i} + 1)$ represents then the SNR loss induced by the use of a wide noise bandwidth.

In communication systems analysis results are usually interpreted in terms of noise density and not in noise variance. Since in the envelope detector in [4] the filtering process happens after the nonlinear downconversion, this noise power is still spread uniformly across the whole noise bandwidth so that the noise density in band is just $N_o = \frac{\sigma^2}{BW_{noise}}$. In particular, for the first term in (3) we find

$$\frac{N_{ss}}{K^2} = \frac{2\sigma^4}{BW_{noise}} \tag{5}$$

The output of the envelope detector when RF signal is present is given by a square wave superimposed to noise. The amplitude of the square wave is given by $V_o^{peak} = \frac{KV_{sig}^2}{2}$, while the noise power spectral density is the $N_o$ calculated above. If the envelope detector output filter is brickwall with bandwidth $BW_{channel}$, and neglecting the term in $\sigma^4$, when the output of the envelope detector is sampled at rate $2BW_{channel}$ and sliced, the bit-error rate performance is given by $BER = \frac{1}{2}erfc(\frac{V_o^{peak}}{2\sigma_o}) = \frac{1}{2}erfc(\frac{\frac{K \cdot V_{sig}^2}{2}\alpha}{K \cdot V_{sig}\sigma}) = \frac{1}{2}erfc(\frac{V_{sig}\alpha}{4\sigma})$. This can be re-expressed as

$$BER = \frac{1}{2}erfc(\frac{\sqrt{SNR_o}}{2}) = \frac{1}{2}erfc(\frac{\sqrt{SNR_i}}{2\sqrt{2}}) \tag{6}$$

and represents an absolute limit on the performance of a receiver employing envelope-detection. In fact, (6) shows that the slicer after the envelope detection operates with an SNR reduced by 3dB with respect to the input SNR. Intuitively, this 3dB degradation arises from the fact that while the signal component at the output is downconverted through harmonic distortion, the noise component is downconverted by second order intermodulation, which is known to have a gain twice larger.

## III. Sensitivity analysis for an AM receiver

Defined

$$G_i = \frac{V_{out,i}}{V_{in,i}}$$

$$F_i = \frac{SNR_{in,i}}{SNR_{out,i}}$$

The noise performance of the cascade connection of an arbitrary number of linear blocks is described by Friis Formula (7).

$$F = F_1 + \sum_j \frac{F_j - 1}{\Pi_{i \le j} G_i^2} \tag{7}$$

This analysis cannot be directly applied to the case of an ED-based receiver for two reasons:

1) The signal gain through the envelope detector is nonlinear
2) Typical envelope detector implementations have substantial transmission for DC inputs, thus passing baseband noise present at the input to the output

As a result of these two phenomena, the computation of the envelope detector output noise must take into account the noise downconversion by intermodulation with the incoming signal and by self mixing, and the DC noise leakage to the output. Since classical cascaded stages analysis can be still applied to all the linear stages preceding the ED, the RF noise at its input port can be characterized with no loss of generality by a noise factor F and a linear voltage gain G. However, these quantities do not allow one to calculate noise at the DC input of the envelope detector, so that this contribution must be taken into account separately by introducing the noise power spectral density $N_{bb}$ at such input.

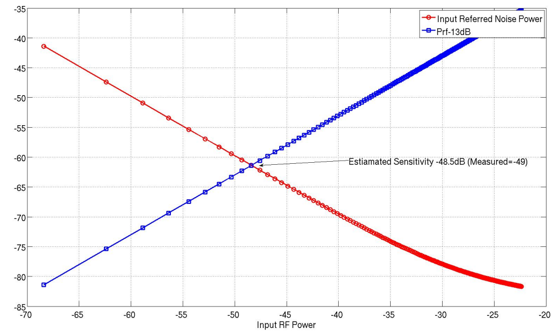


Fig. 5. Calculate Sensitivity for receiver in [5]($(A_v = 18dB, NF_{amp} = 9dB)$

Noise calculations are performed as follows: for a given value of RF input power, the input voltage at the input of the detector port $V_1$ is calculated, and the conversion gain $k = \frac{V_o}{V_1}$ is calculated. Knowing $k$, the input referred noise of the entire chain is calculated as

$$N_{irn} = 4KTR_a(2F) + \frac{(N_{bb} + N_{ss})}{G^2k^2} \tag{8}$$

Note the factor of 2 in front of the linear front end noise factor F, which arises for the same reasons underlying the 3dB SNR penalty mentioned in the preceding section. The term $N_{ss}$ stands instead for the noise downconverted to baseband by self-mixing, which can be related to the variance of the noise impinging the envelope detector as shown in the preceding section, while $N_{bb}$ is the noise predicted by classical linear analysis at the output of the envelope detector, resulting from the combined effect of DC feedthrough of noise at its input as well as slicer input-referred noise. Since the receiver input noise level is now dependent on input level, the receiver sensitivity is found solving graphically the equation

$$N_{irn}(P_{sens}) \mid_{dBm} + 10\log_{10}(BW) = P_{sens} \mid_{dBm} + SNR$$

## IV. Comparison to experimental results

We tested the validity of the analysis by comparing it to measured data performed on the two experimental data receivers [5] and [6] (see Figures 5,6). In [5], the ED is preceded by a single stage of combined gain and filtering which provides G=20dB and NF=9dB. An FBAR bulk-acoustic-wave resonator limits the RF bandwidth to 7MHz, and the total integrated RF noise power to $\sigma^2 = 3.67e - 10V^2$. The ED schematic is shown in Figure 2, and has a DC gain close to unity.

The sensitivity plot is shown in Figure 4. The calculated sensititivty is -48.5dBm, which is within $1dB$ of the measured -49.2dBm ([5]). For the receiver in [6], a gain of 53dB is achieved in front of the envelope detector employing a mixer followed by broadband IF gain stages. The noise figure of this amplification chain is 22dB. Substituting the values in table I results in a predicted sensitivity of -71.5dBm, which is again within 1dB of the measured -72dBm.

| | $G_{amp}[dB]$ | $F_{amp}$ | $N_{bb}[V/\sqrt{Hz}]$ | $\sigma^2(V^2)$ | $N_{ss}[V/\sqrt{Hz}]$ | $BW_{IF}$ | $BW_{channel}$ |
|---|---|---|---|---|---|---|---|
| [5] | 20 | 8 | 2.2e-7 | 3.67e-10 | 3.24e-9 | $\approx$ 7 MHz | 200KHz |
| [6] | 51 | 130 | 7e-8 | 7.9e-4 | 7e-7 | 100MHz | 200KHz |

TABLE I

NOISE ANALSYSIS PARAMETERS FOR RECEIVERS IN [5] AND IN [6]

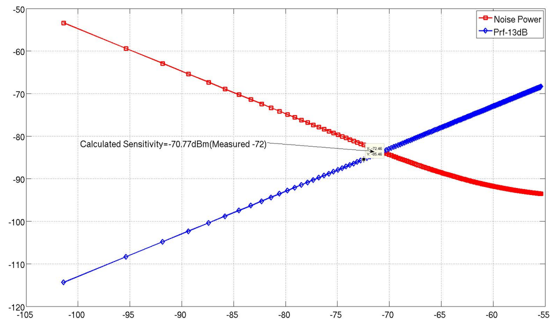


Fig. 6. Calculate Sensitivity for receiver in [6] ($A_v = 52dB, NF_{amp} = 23dB$)

## V. CONCLUSIONS

We present an analysis of the noise performance of radio receivers based on energy-detection downconversion. Experimental results on two different energy detection receiver chips show an agreement better than 1 dB with the calculated values.

## REFERENCES

[1] F.S.Lee and A.Chandrakasan *A 0.65V sub-banded UWB receiver in 90nm CMOS* IEEE Journal of Solid-State circuits, December 2007.

[2] D.C.Daly and A.Chandrakasan *An energy efficient OOK transceiver for wireless sensor networks* IEEE Journal of Solid-state Circuits, May 2007.

[3] B.Otis,R.Lu,Y.H.Chee,N.Pletcher and Jan Rabaey *An ultra-low power MEMS-based two channel transceiver for wireless sensor networks* Proceedings of the 2004 Symposium on VLSI Circuits.

[4] R.G. Meyer *Low-power monolithic RF peak detector analysis* IEEE Journal of Solid-State Circuits, January 1995.

[5] N.Pletcher,S.Gambini,J.Rabaey *A* $65\mu W$*, 0.5 V, 1.9 GHz RF to digital baseband wakeup receiver for wireless sensor nodes* Proceedings of the 2007 IEEE Custom Integrated Circuits Conference.

[6] N.Pletcher,S.Gambini,J.Rabaey *A 2GHz* $52\mu W$ *wake-up-receiver with -72dBm sensitivity using uncertain-IF architecture* Proceedings of the 2008 International Solid State Circuits Conference

[7] J.Roychowdhury,P.Feldman *Cyclostationary noise analysis of large RF circuits with multi-tone excitations* Proceedings of the 1997 Custom Integrated Circuits Conference.

[8] K.Kundert *Introduction to RF simulation and its applications* IEEE Journal of Solid-State Circuits, Vol.34, Issue 9, September 1999.

[9] T.Phan,V.Krizhanovskii,S.Lee *Low Power CMOS energy detection transceiver for impulse-radio system* Proceedings of the 2007 Custom Integrated Circuits Conference.