A Synthetic Inductor Implementation of Chua's Circuit

Bharathwaj Muthuswamy
Tamara Blain
Kyle Sundqvist

Electrical Engineering and Computer Sciences
University of California at Berkeley

Technical Report No. UCB/EECS-2009-20
http://www.eecs.berkeley.edu/Pubs/TechRpts/2009/EECS-2009-20.html

January 30, 2009

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***Abstract*—We show how to build an inductorless version of the classic Chua's circuit. The goal is to build Chua's circuit quickly and easily. To this end, only off-the-shelf components are used. A suitable inductor for Chua's circuit is often hard to procure. Here, the inductor is replaced by a single op-amp synthetic inductor circuit. The synthetic inductor is novel in the sense that it is not a gyrator, it is a one port device and thus a simple black-box analogy to an inductor is possible. We illustrate the robustness of the synthetic inductor by synthesizing two different inductor values.**

I. Introduction

IN this paper, we propose an off-the-shelf implementation of Chua's circuit. This circuit is the paradigm for generating chaotic attractors [1]. A schematic of Chua's circuit is shown in Fig. 1 [4]. The i_R-v_R graph of the nonlinear resistor N_R (also called as the *Chua diode*) is shown in Fig. 2 [4].

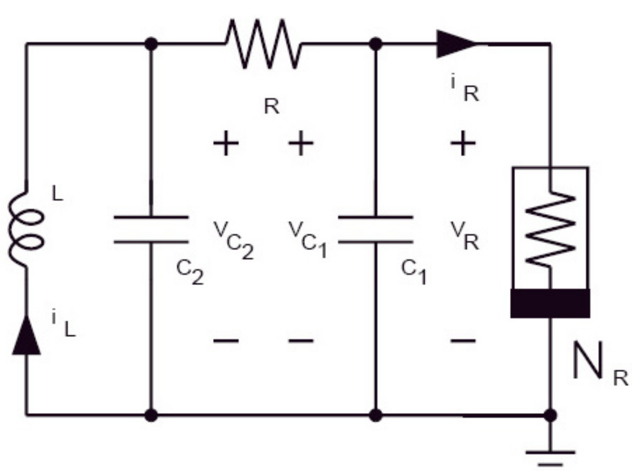

Fig. 1. Chua's Circuit Schematic. The circuit consists of a linear inductor L, a linear resistor R, two linear capacitors C_1 and C_2 and a nonlinear resistor N_R.

Bharathwaj Muthuswamy, Kyle Sundqvist and Tamara Blain are with the University of California, Berkeley, CA, 94720 USA. Contact: mbharat@cory.eecs.berkeley.edu,kylesun@cosmology.berkeley.edu,eb-9d9@eecs.Berkeley.edu

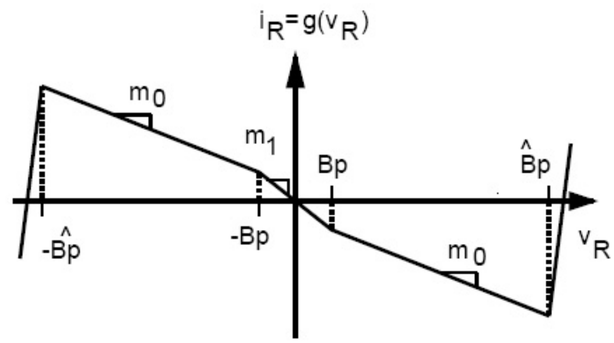

Fig. 2. The i-v characteristic of the nonlinear resistor N_R. Every physically realizable nonlinear resistor is eventually passive - the outermost segments must lie within the first and third quadrants of the v-i plane for sufficiently large $|v|$ and $|i|$.

The state equations for Chua's circuit are shown below.

$$\begin{aligned} C_1\frac{dv_{C_1}}{dt} &= \frac{v_{C_2}-v_{C_1}}{R} - i_R \\ C_2\frac{dv_{C_2}}{dt} &= \frac{v_{C_1}-v_{C_2}}{R} + i_L \\ L\frac{di_L}{dt} &= -v_{C_2} \end{aligned} \tag{1}$$

In (1), $i_R = g(v_R) = g(v_{C_1})$ is a *piecewise-linear* function defined by [4]:

$$g(v_R) = m_0 v_R + \frac{1}{2}(m_1 - m_0)[|v_R + B_p| - |v_R - B_p|] \tag{2}$$

From Fig. 2, it can be inferred that the slopes in the inner and outer regions are m_1 and m_0 respectively; $\pm B_p$ denote the breakpoints. Good references for understanding Chua's circuit are [5] and [4].

It is difficult to obtain precise values of the inductor needed to build the circuit in Fig. 1. Also, the inductor is quite bulky and it cannot be integrated on a chip. Moreover, many commercially available inductors have a core that is added to increase the inductance (and hence reduce the number of windings needed). But, this is known to have the effects of adding distortion to the signal via hysteresis [8]. Given the sensitive dependence on initial conditions for the chaotic circuit, this would be largely undesirable. One solution is to simulate the inductor using a gyrator, as demonstrated by [7], which uses a two op-amp implementation of the gyrator. In this paper, we use a simpler synthetic inductor implementation. Specifically:

1) We use only one op-amp for the synthetic inductor.
2) All components are off-the-shelf (a parts list is included at the end of this paper).

The organization of this paper is as follows: in Section II, we give a brief overview of the component values used in the $18mH$ inductor version of Chua's circuit. In Section III, we give the expression for the impedance of the synthetic inductor (the derivation is given in the Appendix). In Section IV, we show simulation results using a 30-day fully functional trial version of National Instruments' MultiSim suite [3]. This circuit simulator was chosen over PSPICE because MultiSim is easier to use. In Section V, we show experimental results using oscilloscope waveforms from the physical implementation of the circuit. In Section VI, we implement a different inductor value using the synthetic inductor (the $8.2\ mH$Chua's circuit from [5]). We conclude the paper with a parts list, suggestions for future work and acknowledgments.

II. Chua's Circuit Component Values

Fig. 3 shows the realization of Chua's circuit that will be used in this paper. In comparison to Fig. 4, the inductor in Fig. 3 has been replaced by our synthetic inductor and the Chua diode has been implemented using Kennedy's two op-amp implementation [4]. The component values for everything but the synthetic inductor were obtained from [4]. The component values for the synthetic inductor are given in the next section.

From Fig. 3 and [4], the parameters m_0, m_1 and $\pm B_p$ for the Chua diode i-v in Fig. 2 can be computed as:

$$m_0 = -0.409mS, m_1 = -0.756mS, B_p = 1.08V \quad (3)$$

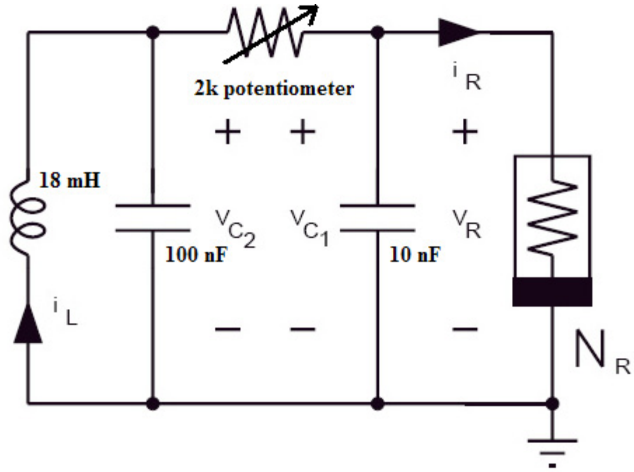

Fig. 4. Chua's Circuit with component values for investigating chaos, the component values are from [4]. Compare with Fig. 3.

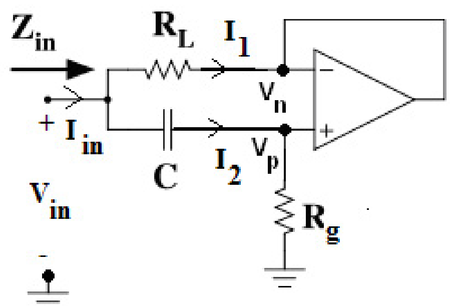

Fig. 5. The synthetic inductor circuit. The goal is to derive an expression for Z_{in}. We assume the op-amp is operating in the linear region. The derivation is in the appendix

III. The Synthetic Inductor Impedance

Fig. 5 shows our modified version of the synthetic inductor from [2].

If $R_L = 10^{-4}R_g$ (so that $R_L << R_g$) in Fig. 5, we have:

$$\boxed{Z_{in} \approx R_L + j\omega R_L R_g C} \quad (4)$$

The derivation of (4) is given in the Appendix. In our case, $R_L = 10\Omega$, $R_g = 100k\Omega$ and $C = 18nF$, so we have the equivalent circuit of Fig. 6 for an $18mH$ inductor.

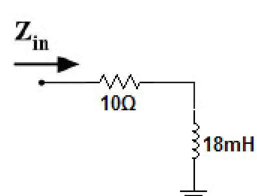

Fig. 6. $18mH$ synthetic inductor circuit.

IV. MultiSim Simulation Results

MultiSim 10 has been used to simulate Chua's circuit. A free 30-day evaluation version of MultiSim 10 can be downloaded from [3]. Please download the *professional edition* of MultiSim 10. This edition has the simulation models for the LMC6482 op-amp. A MultiSim 10 simulation file for the Chua's circuit discussed in this paper can be downloaded from [6].

Fig. 7 shows simulated attractors obtained from MultiSim 10. We show a period-doubling route to chaos by varying C_1 [4], the other parameters are fixed. Attractor periods shown are period-1, period-2, period-4 and a Double-Scroll Chua attractor.

TABLE I
Parts list for 18 mH and 8.2 mH synthetic inductor versions of Chua's circuit

L	R_L	C	R_g	C_2	R	C_1
18 mH	10 Ω	18 nF	100 $k\Omega$	100 nF	2 $k\Omega$ pot.	10 nF
8.2 mH	10 Ω	8.2 nF	100 $k\Omega$	47 nF	2 $k\Omega$ pot.	4.7 nF

TABLE II
Chua Diode parts list for 18 mH and 8.2 mH synthetic inductor

L	R_1	R_2	R_3	R_4	R_5	R_6
18 mH	220 Ω	220 Ω	2.2 $k\Omega$	22 $k\Omega$	22 $k\Omega$	3.3 $k\Omega$
8.2 mH	220 Ω	220 Ω	2.2 $k\Omega$	22 $k\Omega$	22 $k\Omega$	3.3 $k\Omega$

V. Experimental Results

Fig. 8 shows a series of measured attractors from the physical circuit. Note that the experimental C_1 values used for illustrating the period doubling route to chaos closely match the C_1 values from the simulated version (refer to Fig. 7).

VI. 8.2 mH inductor version of Chua's circuit

To examine the robustness of this circuit, let us implement Chua's circuit with component values in [5]: $L = 8.2\ mH$, $C_2 = 55\ nF$, $R = 1.33\ k\Omega$, $C_1 = 5.5\ nF$. However, the capacitor values are not off-the-shelf, therefore we implemented $L = 8.2\ mH$, $C_2 = 47\ nF$, $C_1 = 4.7\ nF$. The 2k potentiometer has been set to $1.5\ k\Omega$. The synthetic inductor capacitor has been changed to $C = 8.2\ nF$ to implement the $8.2\ mH$ inductor. The schematic of this circuit is shown in Fig. 9. A simulated and experimental Double-Scroll is also shown.

VII. Conclusions and Future Work

A. Parts List

The parts list for both the $18\ mH$ and the $8.2\ mH$ circuit are given in Tables I and II. All resistors are 5% tolerance. Capacitors are mylar and have 10% tolerance.

The op-amp that was used in this paper is the LMC6482 from National Semiconductor. However, we replaced the LMC6482 in the circuit with the TL082 and the AD822AN (pin-for-pin compatible op-amps with the LMC6482). Both the TL082 and the AD822AN circuits displayed bifurcation and chaos phenomenon. Therefore, possible op-amps that can be used are the LMC6482, TL082 and the AD822AN. Moreover, the TL082 and the AD822AN can be powered using $\pm 9\ V$ supplies. This makes them attractive for use with $\pm 9\ V$ batteries.

B. Future Work

In this paper, we discussed a single op-amp synthetic inductor version of Chua's circuit. We implemented Chua's circuit for two synthetic values of inductance: $18\ mH$ and $8.2\ mH$. An interesting problem would be to explore the maximum possible bandwidth of this circuit.

Appendix
The Synthetic Inductor Impedance Derivation

We will now derive the equivalent impedance of the synthetic inductor, as seen from its input terminals. Refer to Fig. 5.

Using Thevenin's theorem, we can write an expression for Z_{in}:

$$\begin{aligned} Z_{in} &= \frac{V_{in}(j\omega)}{I_{in}(j\omega)} \\ &= \frac{V_{in}(j\omega)}{I_1(j\omega) + I_2(j\omega)} \\ &= \frac{V_{in}(j\omega)}{\frac{V_{in}(j\omega) - V_n(j\omega)}{R_L} + \frac{V_{in}(j\omega)}{R_g + \frac{1}{j\omega C}}} \end{aligned} \tag{5}$$

Assuming the op-amp is operating in the linear region:

$$\begin{aligned} V_n(j\omega) &= V_p(j\omega) \\ V_n(j\omega) &= \frac{R_g}{R_g + \frac{1}{j\omega C}} \cdot V_{in}(j\omega) \end{aligned} \tag{6}$$

Substituting for $V_n(j\omega)$ in (5) from (6) and simplifying:

$$\begin{aligned} Z_{in} &= \frac{V_{in}(j\omega)}{\frac{V_{in}(j\omega) - \frac{R_g}{R_g + \frac{1}{j\omega C}} \cdot V_{in}(j\omega)}{R_L} + \frac{V_{in}(j\omega)}{R_g + \frac{1}{j\omega C}}} \\ &= \frac{1}{\frac{1 - \frac{R_g}{R_g + \frac{1}{j\omega C}}}{R_L} + \frac{1}{R_g + \frac{1}{j\omega C}}} \\ &= \frac{R_L}{1 - \frac{R_g}{R_g + \frac{1}{j\omega C}} + \frac{R_L}{R_g + \frac{1}{j\omega C}}} \end{aligned} \tag{7}$$

Let $R_L = 10^{-4} R_g$ (so that $R_L << R_g$). Then substituting for R_L in the denominator of (7):

$$\begin{aligned} Z_{in} &= \frac{R_L}{1 - \frac{R_g}{R_g + \frac{1}{j\omega C}} + \frac{10^{-4} R_g}{R_g + \frac{1}{j\omega C}}} \\ &\approx \frac{R_L}{1 - \frac{R_g}{R_g + \frac{1}{j\omega C}}} \\ &= \frac{R_L(R_g + \frac{1}{j\omega C})}{R_g + \frac{1}{j\omega C} - R_g} \\ &= \frac{R_L(R_g + \frac{1}{j\omega C})}{\frac{1}{j\omega C}} \\ &= j\omega R_L C(\frac{1}{j\omega C} + R_g) \\ &= R_L + j\omega R_L R_g C \end{aligned} \tag{8}$$

Thus, if $R_L = 10^{-4} R_g$ (so that $R_L << R_g$), we have:

$$\boxed{Z_{in} \approx R_L + j\omega R_L R_g C} \tag{9}$$

Fig. 10 shows the circuit equivalent of (9).

If $R_L = 10\Omega$, $R_g = 100k\Omega$ and $C = 18nF$, we have the equivalent circuit of Fig. 6 for an $18mH$ inductor.

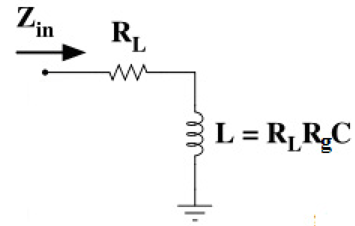

Fig. 10. The synthetic inductor circuit from Fig. 5 can be modelled as a parasitic resistance R_L in series with an inductance $R_L R_g C$.

Acknowledgment

Many thanks to Prof. Pravin Varaiya for insightful discussions on chaos. Prof. Joos Vandewalle was very helpful in providing valuable comments. Many thanks to the students in EE100 Summer 2007 at the University of California, Berkeley. They provided valuable feedback on how to build a simple off-the-shelf version of Chua's circuit.

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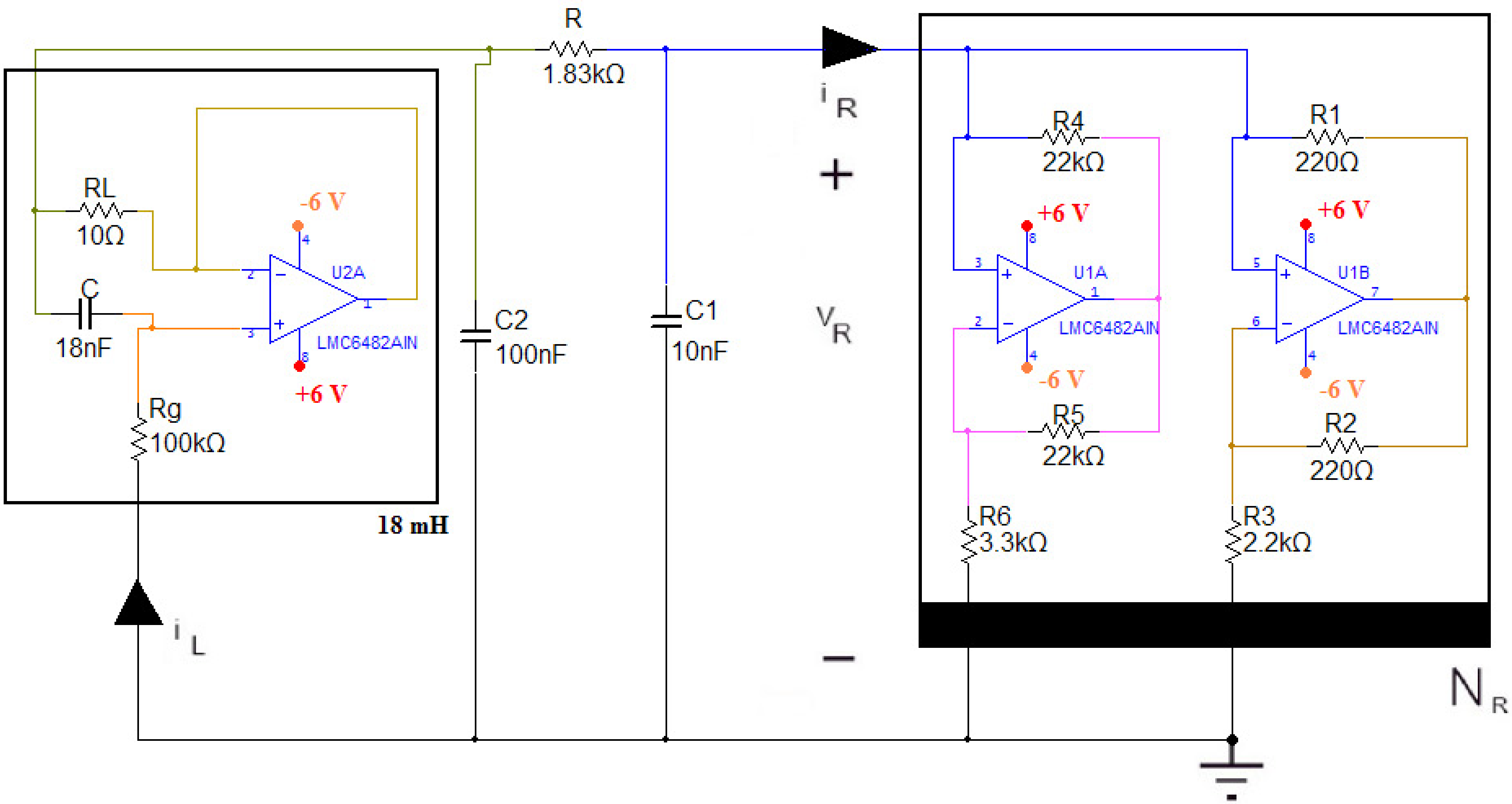
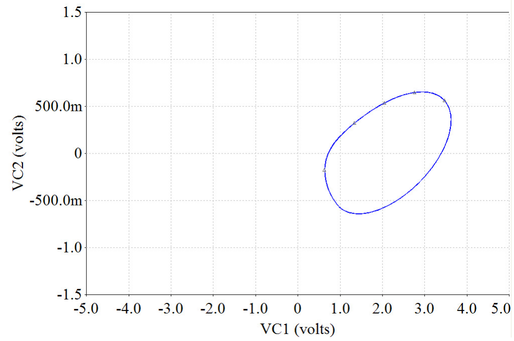
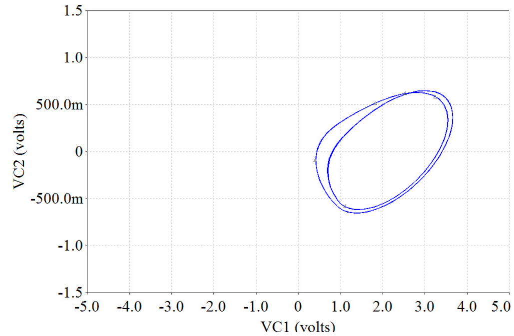


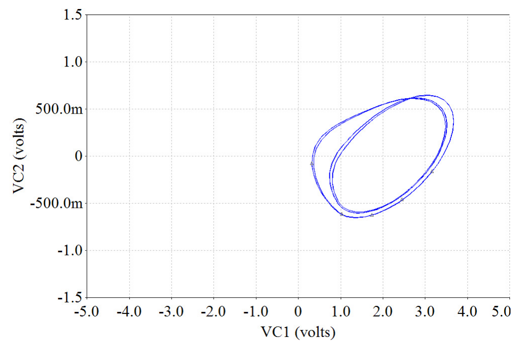
Fig. 3. Chua's Circuit screen capture from MultiSim's schematic editor. The 18 mH inductor has been replaced with the single op-amp synthetic inductor and the Chua diode has been implemented using Kennedy's robust two op-amp implementation [4]. The batteries are not shown for clarity purposes, rather we have indicated the power supply voltages at the respective nodes.



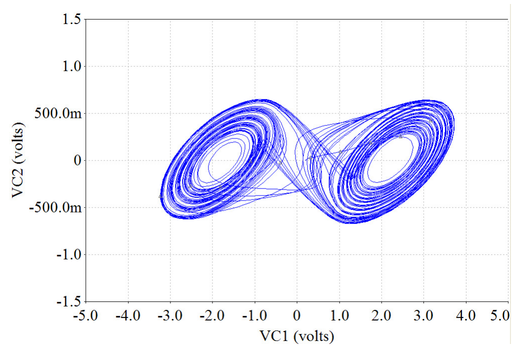
(a) $C_1 = 10.7\ nF$



(b) $C_1 = 10.4\ nF$

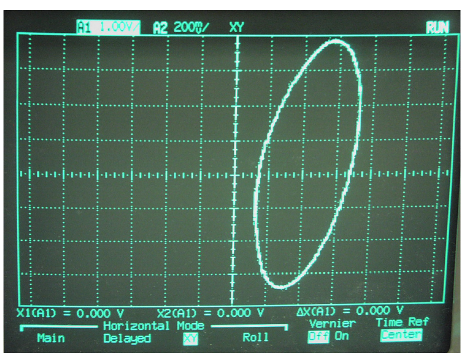


(c) $C_1 = 10.3\ nF$

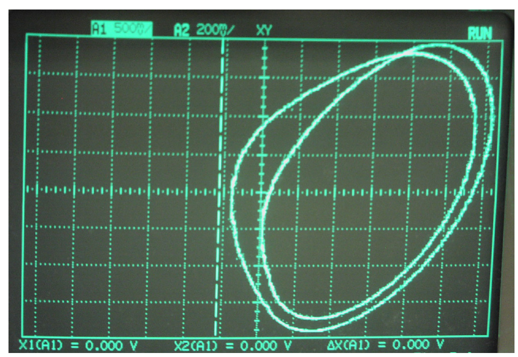


(d) $C_1 = 9.8\ nF$

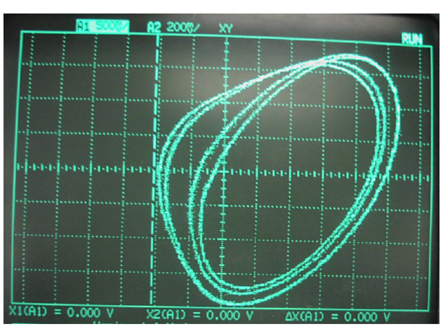
Fig. 7. Simulated attractors from MultiSim 10 with component values $R_L = 10\ \Omega$, $R_g = 100\ k\Omega$, $C = 18\ nF$, $C_2 = 100\ nF$, $R = 1.83\ k\Omega$. C_1 is varied to show the period-doubling route to chaos. Horizontal axis is v_{C_1}; Vertical axis is v_{C_2}.



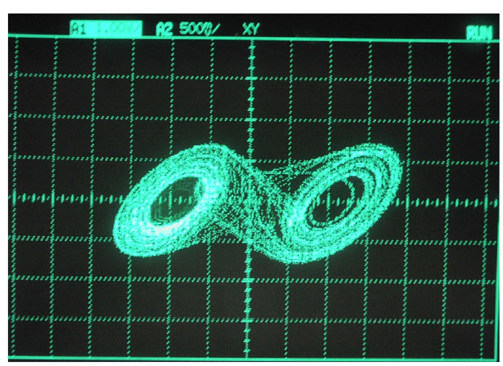
(a) $C_1 = 13.2\ nF$



(b) $C_1 = 12.7\ nF$

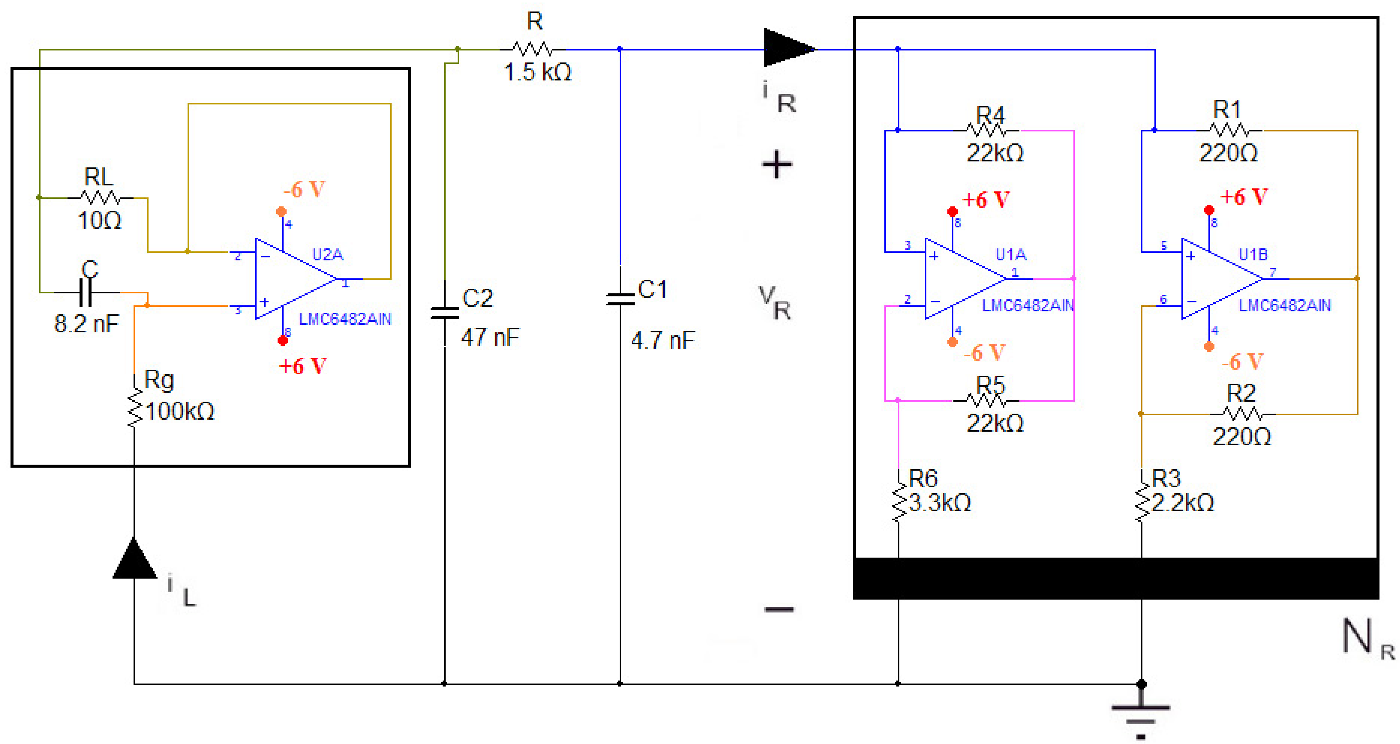


(c) $C_1 = 12.2\ nF$

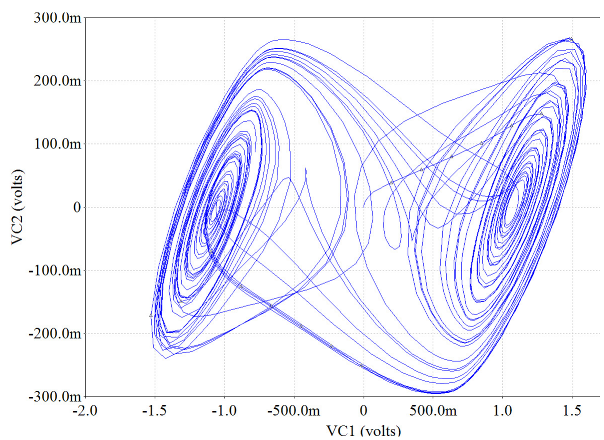


(d) $C_1 = 10\ nF$

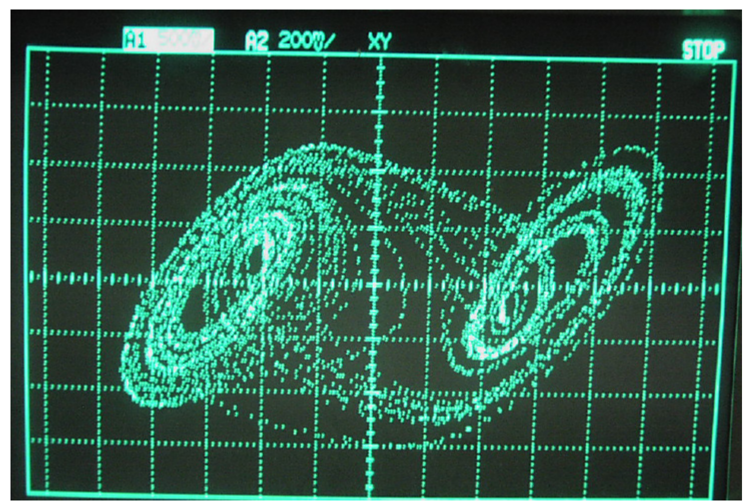
Fig. 8. Measured attractors (using an HP54645D oscilloscope) with component values $R_L = 10\ \Omega$, $R_g = 100\ k\Omega$, $C = 18\ nF$, $C_2 = 100\ nF$, $R = 1.83\ k\Omega$. C_1 is varied to show the period-doubling route to chaos. Horizontal axis is v_{C_1}; Vertical axis is v_{C_2}. Scales are Vertical axis: 1.00 V/div for (a), 0.5 V/div for (b) and (c), 1.00 V/div for (d); Horizontal axis: 0.2 V/div for (a), (b) and (c), 1.00 V/div for (d).



(a) Screen capture of MultiSim schematic for 8.2 mH version of Chua's circuit



(b) Simulated Double-Scroll



(c) Experimental Double-Scroll

Fig. 9. MultiSim schematic, simulated Double-Scroll and experimental Double-Scroll for the 8.2 mH version of Chua's circuit. Component values are $C = 8.2\ nF$, $C_2 = 47\ nF$, $R = 1.5\ k\Omega$, $C_1 = 4.7\ nF$. The Chua diode is unchanged from Fig. 3.