High-Frequency Trading

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Technical Report No. UCB/EECS-2012-130
http://www.eecs.berkeley.edu/Pubs/TechRpts/2012/EECS-2012-130.html

May 30, 2012

Center for Entrepreneurship & Technology

University of California, Berkeley

Coleman Fung Institute for Engineering Leadership

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Industry Strategy Project

Engineering Leadership Program

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Abstract

One of the most significant market structure developments in recent years is high frequency trading (‘‘HFT’’). By utilizing modern high-speed computers and computation technologies, HFT firms are able to execute electronic transactions within milliseconds even microseconds and therefore taking advantage of small discrepancies in prices, and regulator are weighing new rules for high-speed trading.

Although the equity market is drastically competitive, the potential opportunities are to develop advanced technologies including trading models and algorithms. The technical part mainly discussed mathematical models to process the Limit Order Book (LOB) Information, especially for the discrete model, which is quite popular in research on high frequency trading.

This article mainly focus on the market making strategies, we developed two major strategies and tested them using real market data. Details of implementations of these strategies and their pros and cons are also provided.

1. Introduction

For years, high-frequency trading firms have operated in the shadows, often far from Wall Street, trading stocks at warp speed and reaping billions while criticism rose that they were damaging markets and hurting ordinary investors. Now they are stepping into the light to buff their image with regulators, the public and other investors.

High frequency trading (‘‘HFT’’), typically is used to refer to professional traders acting in a proprietary capacity that engage in strategies that generate a large number of trades on a daily basis. Characteristics often attributed to proprietary firms engaged in HFT are: (1) The use of extraordinarily high-speed and sophisticated computer programs for generating, routing, and executing orders; (2) use of co-location services and individual data feeds offered by exchanges and others to minimize network and other types of latencies; (3) very short time-frames for establishing and liquidating positions; (4) the submission of numerous orders that are cancelled shortly after submission; and (5) ending the trading day in as close to a flat position as possible .

Furthermore, the SEC has significantly reduced the trading cost recent years. As trading costs have diminished, smaller and smaller opportunities have become profitable to trade. This leads to the growth of higher trading volumes. These higher trading volumes then exert further downward pressure on trading costs, creating a virtuous cycle.

High frequency firms use strategies to make market fluctuate and earn tenths of pennies millions of times from the price imbalances. HFT firms weren’t holding on to their stock for a period of time. All the trading was creating massive price volatility. One of its benefits is adding liquidity to the market, however, high frequency trading has not become without controversy. It has been criticized for destabilizing markets. However, as algorithm trading accounts for 70 percent of average daily trading volume in trading market, high frequency trading became a key issue in financial market.

This article is organized as follows. In section 2 we discuss the work that has been done on market making strategy, including the basic framework and simulation results of basic strategies that are discussed in these papers. In section 3 we provide the outline of three strategies we developed and how we test them using the market data. In section 4 we show the simulation results and compare performance between these strategies. In last section we have conclusion and potential improvements we can make in the future.

Key words: High frequency trading, Limit order book, market making, smoke strategy

2. Literature Review

In the paper "Two stock-Trading Agents: Market Making and Technical Analysis [1]," it implements two market maker strategies using Penn-Lehman Automated Trading simulator (PLAT). The simulations described in this paper have no commissions or tax charges. Therefore it focuses on the behaviors of order book dynamics and methods to optimize profits using proposed strategies. For the basic approach, it focuses on only one stock at a time and places a pair of sell and buy orders of same volume on order book simultaneously. The key feature of this strategy is that it executes without predicting the direction of price movements and believes there are a lot of price fluctuations. The way it works is that when price is beyond market maker's ask price the sell order matches, once the price goes below the price the market maker sold, it gains profit and vice versa. In figure 1 we demonstrate the basic idea of how a market maker places his orders using this strategy.

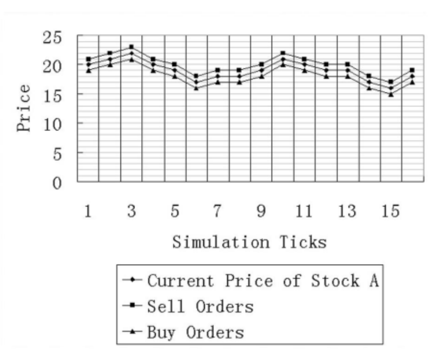


Fig.1 Basic Idea of Market Making

The main concern of applying this basic approach is at what price the market maker should place his orders. In this paper it suggests using a parameter *n* which can be any integer from one to number of existing orders. The order will be placed in front of nth orders in both queues and have same price difference *x*. In figure 2 it shows the order book after the orders are being placed with *n* equals 1 and *x* equals $0.0001.

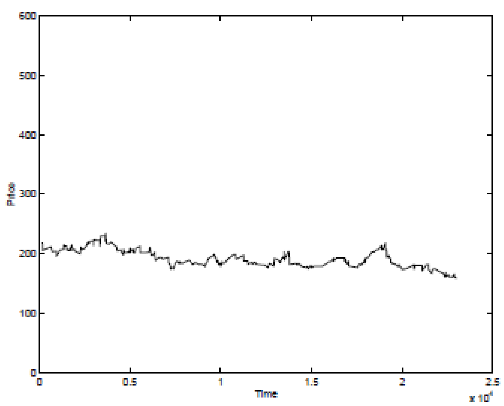
Buy Order Book		Sell Order Book	
Price	Volume	Price	Volume
24.0360	500	24.0700	350
24.0061	**1000**	**24.0889**	**1000**
24.0060	1500	24.0890	600
24.0010	800	24.0950	2000
23.9700	1000	24.0950	1200

Fig.2 Limit Order Book

The performance of this strategy turns out to be very good when end price of a day is very close to the start price. Although this ideal condition makes this strategy become very limited but it is valuable to our project in a sense that we can explore the choice of parameter *n* and *x* using data set we are provided and hence improve it furthermore.

In another paper "Electronic Market Makin: Initial Investigation [2]," it models the stock market using Penn Exchange Simulator (PXS) and explores the optimal price spread for electronic market makers to place their orders. There are two major components of electronic market makers. The first one is establishing bid-ask spread and the second one is updating spread information. In this paper the author further decompose the first part into predictive and non-predictive. Similar to the strategy discussed in the previous paper, non-predictive strategy assumes market goes up and down and does not try to predictive the direction of price movements. There are several conclusions which are very important and we should take them into accounts to our project. The first one is that faster update allow to follow market more closely and increase profitability. However, by putting the quotes deeper into the order book, we can compensate the effects of time delays and narrow price spread. The main drawback of this strategy is that when price fluctuates too much in a short period of time it does not work well.

In the paper "Modeling Stock Order Flows and Learning Market-Making from Data [3]," I found the result is very useful to our project. The authors build a more complex market model based on real market data and employ reinforcement learning algorithm to derive a market making strategy. In figure 3 and 4 it shows the price changes over time when market maker quote with bid/ask spread of $1 and $10.



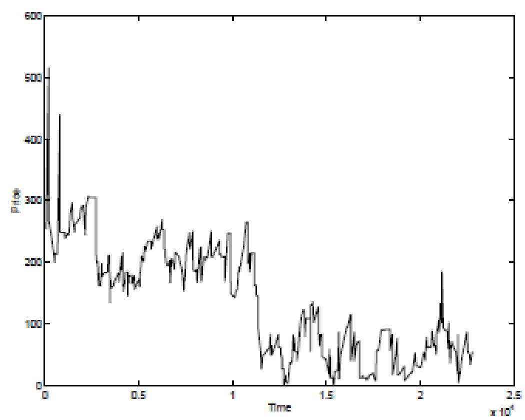


Fig.3 Bid/Ask Spread $1

Fig.4 Bid/Ask Spread $1

It is clear that when market maker has larger price spread of his quotes, it has more influence on the price of the stock. This paper also suggests that when the system is being trained with more data the profitability also increases, although this is not within the scope of our project.

3. Methodology

3.1 Markovian Queuing Model

Motivated by the fact that it is sufficient to focus on the dynamics of the best bid and ask queue if one is primarily interested in the level I order book dynamics, we then decided to follow the Markovian queuing model [4] to test its validity on our data where the limit order book is driven by orders at the bid and ask side, represented as a system of two interacting Markovian queues. We will first introduce the setup of R. Cont's queuing model [4], and then elaborate our modifications according to the empirical analysis.

3.1.1 Model Setup

To simplify the initial model, we use the following terms to represent the limit order book:

- The bid price s_t^b and the ask price $s_t^a = s_t^b + \delta$, which captures the majority of the market situation.
- The size of the bid queue q_t^b which represents the outstanding limit buy orders at the bid.
- The size of the bid queue q_t^a representing the outstanding limit buy orders at the ask.

The state of the limit order book is thus described by the triplet $X_t = (s_t{}^b, q_t{}^b, q_t{}^a)$ which takes values in the discrete state space $\delta.\mathbb{Z} \times \mathbb{N}^2$.

The state X_t of the order book is modified by order book events: limit orders (at the bid or ask), market orders and cancelations. According to the Rama's works, we first assume that these events occur according to independent Poisson processes:

- Market buy (resp. sell) orders arrive at independent, exponential times with rate μ.
- Limit buy (resp. sell) orders at the (best) bid (resp. ask) arrive at independent, exponential times with rate λ.
- Cancellations occur at independent, exponential times with rate θ.
- These events are mutually independent.
- All orders sizes are equal (assumed to be 1 without loss of generality).
- All the previous sequences are independent.

Under these assumptions $q_t = (q_t^{\ b}, q_t^{\ a})$ is thus a Markov process, taking values in $\mathbb{N}^2$, whose transitions correspond to the order book events $\{T_i^a, i \geq 1\} \bigcup \{T_i^b, i \geq 1\}$.

When the bid or ask queue is depleted, the price moves up or down to the next level of the order book. Analogous to the heavy traffic model, the new queue sizes are sampled from the empirical pdf $f^{b/a}(x, y)$. We assume that the order book contains no 'gaps' (empty levels) so that these price increments are equal to one tick (in our case, 0.05 HKD):

- When the bid queue is depleted, the price decreases by one tick.
- When the ask queue is depleted, the price increases by one tick.

In summary, the process $X_t = (s_t^{\ b}, q_t^{\ b}, q_t^{\ a})$ is a continuous-time process with right-continuous, piecewise constant sample paths whose transitions correspond to the order book events $\{T_i^a, i \geq 1\} \bigcup \{T_i^b, i \geq 1\}$. At each event:

- If an order or cancelation arrives on the ask side i.e. $T \in \{T_i^a, i \geq 1\}$:

$$(s_T^{\ b}, q_T^{\ b}, q_T^{\ a}) = (s_{T-}^{\ b}, q_{T-}^{\ b}, q_{T-}^{\ a} + V_i^a) 1_{q_T^{\ a} > -V_i^a} + (s_{T-}^{\ b} + \delta, R_i^b, R_i^a) 1_{q_{T-}^{\ a} \leq V_i^a}$$

- If an order or cancelation arrives on the bid side i.e. $T \in \{T_i^b, i \geq 1\}$:

$$(s_T{}^b, q_T{}^b, q_T{}^a) = (s_{T-}{}^b, q_{T-}{}^b + V_i^b, q_{T-}{}^a)1_{q_T{}^b > -V_i^b} + (s_{T-}{}^b - \delta, R'{}_i^b, R'{}_i^a)1_{q_{T-}{}^b \leq V_i^b}$$

Where $(V_i^a)_{i>1}$ and $(V_i^b)_{i>1}$ are sequences of IID variables, $(R_i)_{i>1} = (R_i^b, R_i^a)_{i\geq 1}$ is a sequence of IID variables with (joint) distribution $f^b(x,y)$, and $(R'_i)_{i>1} = (R'{}_i^b, R'{}_i^a)_{i\geq 1}$ is a sequence of IID variables with (joint) distribution $f^a(x,y)$.

3.2 Model Applications

After completing Markovian model, we start designing trading strategy for market makers based on it. We also analyze our strategy with real market data.

3.2.1 Market Making Strategy

A market maker is an individual or a company that place quotes on both buy and sell sides in a financial market and attempts to make profit from bid/ask spread. In our model we have several assumptions. Firstly, market maker has to post N quotes in time period T and it has freedom to post limit orders at any levels of order book. The market maker has to close his positions in time T by selling or buying all his quotes. The key issue we are interested in is that at what level of order book and should he post his orders. From literature review we know that the deeper market maker goes in he is more likely to make profits. However it is more risky at the same time. In our simulations we test with different values of completion time T and shares N and different levels at limit order book by running independent Monte Carlo simulation. In the next section we provide details implantation of strategies we developed.

3.2.2 Smoking Strategy

The basic idea of smoking strategy [9] is that the market maker places quotes that below the current best ask or higher than current best bid in order to lure market buys or market sells. Then right before market buy or sell reaches the market, our market maker cancels the lure quotes and other market makers hits the large ask or bid previously posted by our market maker. In the figures below we show graphically the process of smoking strategy.

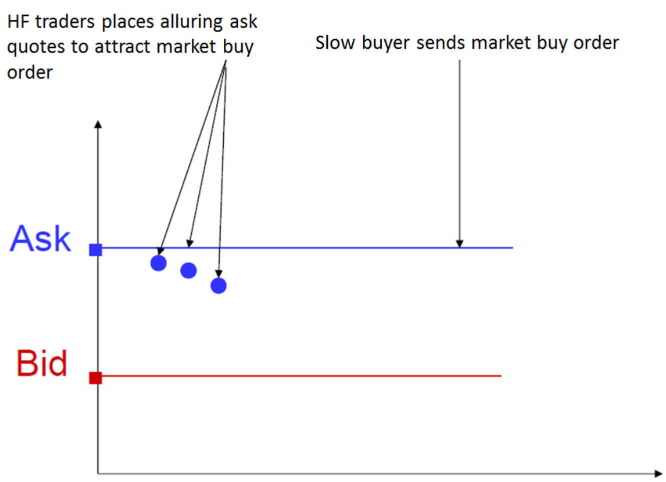


Fig.5 Luring quotes are posted by market maker

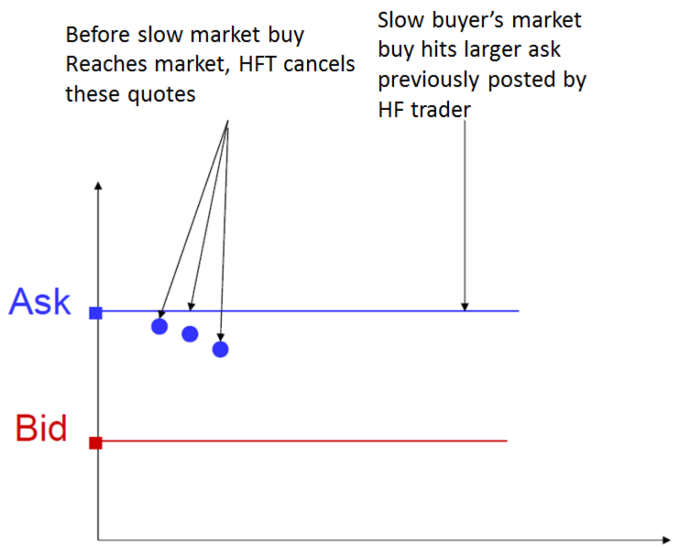


Fig.6 Luring quotes are canceled

There are several hypotheses we make when designing smoking strategy. First of all, the response time of our market maker need to be faster than their competitors. This means higher hardware and software requirements. Otherwise the luring quotes will be actually taken and our market maker will lose money eventually. Moreover, when luring quotes is no longer the best bid or ask price on order book, the market maker need to cancel the previous luring quotes and place new ones. In figure 7 we show an example of how market maker places his orders.

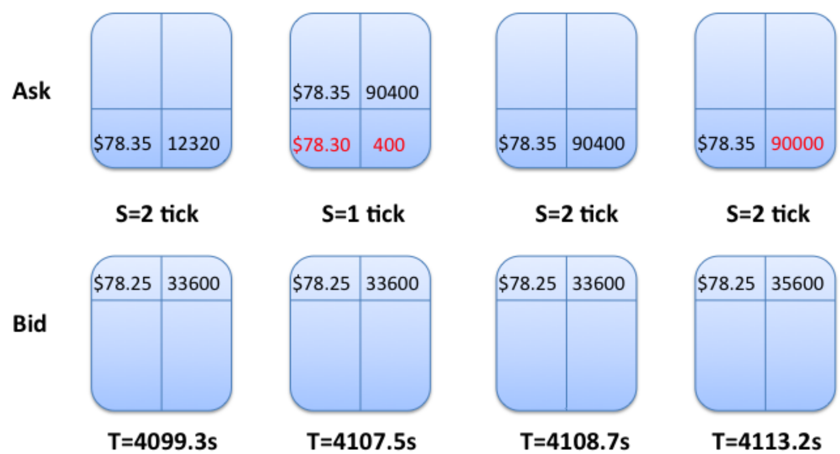


Fig. 7 Example of smoking strategy

It is obvious at this point that speed is the crucial factor that determines whether market maker is going to succeed with smoking strategy. This is also the major characteristic of High-Frequency Trading. We examined the 5 days Hong Kong traded stock data and found out that there are 20 occurrences happened in 5 days (7 bid side, 13 ask side). The average alluring quote size is 7,280 shares, the average cancellation duration is 1.98s, and the average trading volume is 2,160 shares. The histogram of cancellation duration is shown in figure 8.

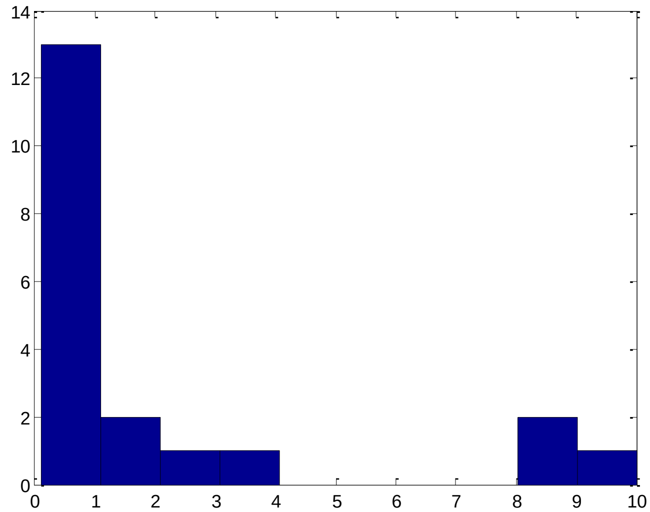


Fig. 8 Cancellation duration histogram

It is clear that the key parameter for this strategy is cancellation duration, we test it based on our model by running independent Monte Carlo simulation.

3.2.3 Balancing Strategy

After posting the initial shares at time t (which is the assumption in the market making strategy), most market makers simply follow the trend of the market and adjust the portfolio according to the market performance in real time. One way is to rebalance the position around the current price, since it assures the equal probability for both sides to be executed. Here, besides the model problems in the previous session, I also integrated the balancing strategy, which shows as follows:

At each end of time interval Δt

- If currently no position is on the best level and both bid and ask size hasn't been depleted yet, rearrange the remaining shares to be balance around the current market price.

- If both sides haven't been depleted and the current price is already balanced, decrease the margin of both sides by one tick.

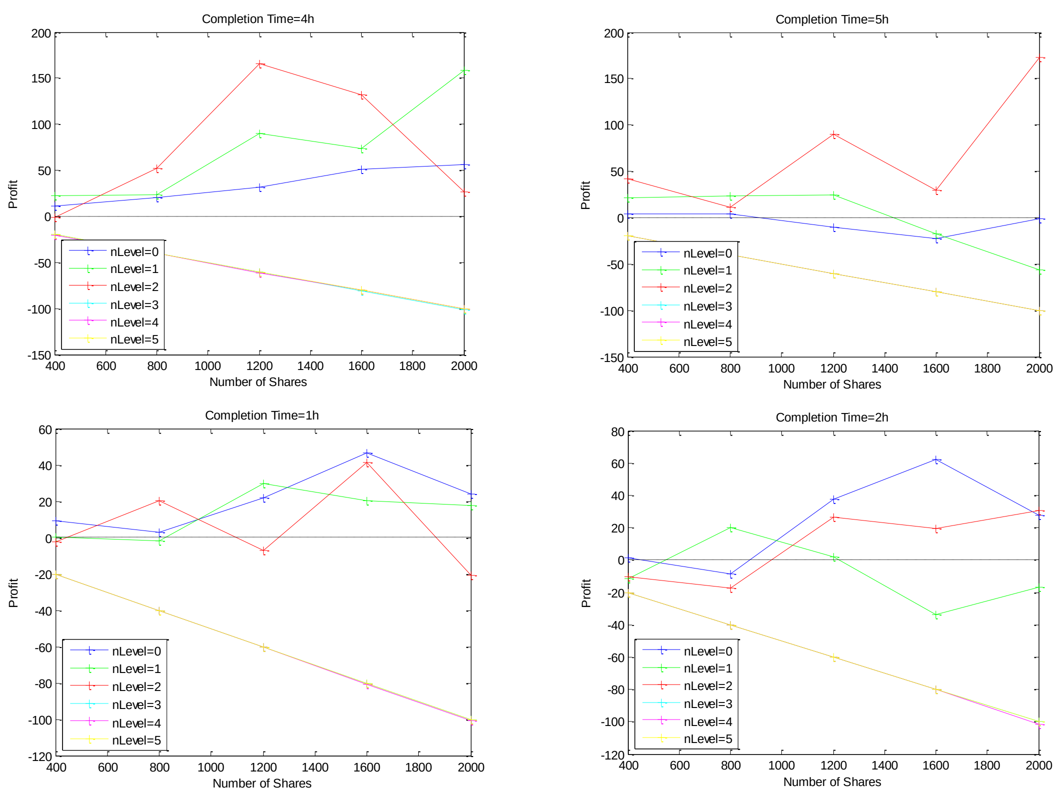
- If one size already depleted, try to increase the other side by one tick.

In the model setup, we assume there is no cancellation fees for cancellation the original orders. The reason is that in most exchanges, there will be a "liquidity award" for posting limit order, which is approximately the same amount of the cancellation fee. We tested balancing strategies with different completion time T, number of shares N and level of order placement by running independent Monte Carlo simulation. The results will be analyzed in the next section.

4. Discussion/Result

4.1 Market Making Strategy Simulation Result

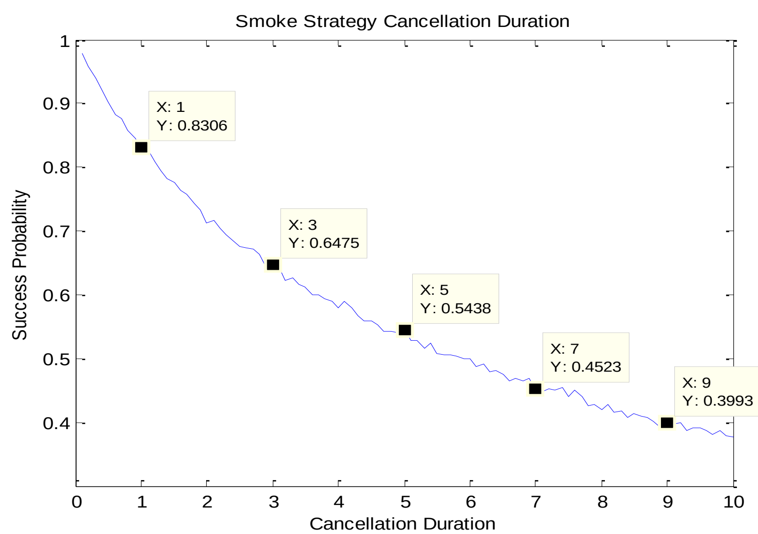
We tested the market making strategy based on the Markovian Queuing System. Parameters are *nLevel*, which range from 0 to 5, representing the level market maker place the orders; Completion Time T, which range from 3600s to 18000s (1 to 5 hours), representing the total amount of time for closing the position; Number of shares N, which range from 400 to 2,000, representing the total shares used by the market making strategy. The results are as follows:



We got conclusion that in general placing order deeper (higher level on order book) and with larger number of shares results in more profit. This makes sense in reality because placing orders at higher levels make them easier to be executed. Also when time T is shorter this allows market maker to close positions faster.

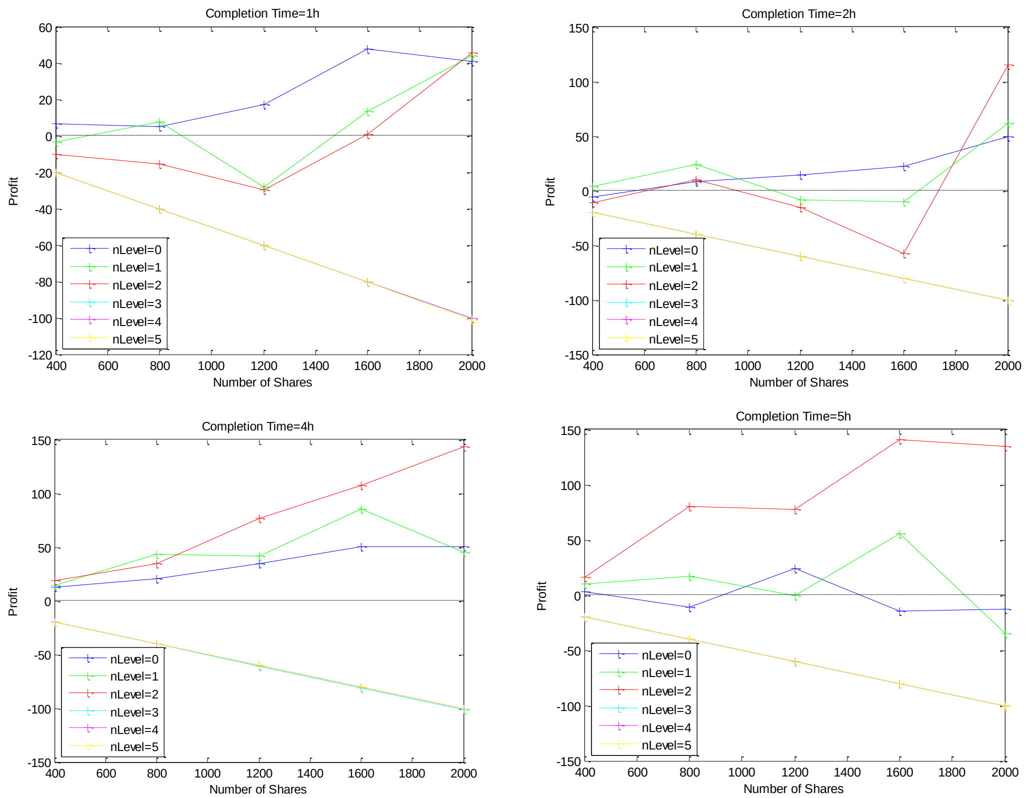
4.2 Smoking Strategy Simulation Result

Due to the limitations of Markovian Queuing model that it only focuses best level of limit order book, we cannot completely simulate smoking strategy with it. However, we can simulate the most important factor of smoking strategy which is the cancellation duration. Institutively, shorter cancellation time indicates lower probability of alluring orders being hits by other parties. When the cancellation duration goes to zero, there is nearly 100% probability that the alluring orders will not be hit. Therefore we define the probability of the alluring orders successfully cancelled as the *Success Probability*, and ran 10000 independent Monte Carlo simulations against the model. The simulation results are shown as follows:



4.3 Balancing Strategy Simulation Result

In this simulation, we fix $\Delta t = 1800s$, which is half hour. Simulation results are as follows:



Compare to market making strategy without balancing, we can see a huge improvement on performance. We can see that by applying this strategy, market maker can utilize larger share numbers and making more profits by balancing his orders on both sides. This result shows that balancing is a good way for market makers to optimize their profits.

5. Conclusion

In this article we starts from reviewing work that helps us developing market making strategies. Then we have Markovian model that helps us to describe limit order book dynamics. The potential improvement we can make is to future improve our models so that we can include dynamic of level II order book to allow more complicated simulations like smoking strategy. Moreover, the current model we have does not include the impact of large orders which would be good if we can also incorporate that into our model in the future.

6. Summary

In order to perform a more detailed implementation of Markovian queuing model [4] we analyze the stock price dynamics and order arrival/cancelation rate. The heavy traffic model [5] was also being investigated although Markovian model is more suitable for simulation purposes. Moreover, we also study the price fluctuation model [6] to understand the impact of each trade.

We also use L1/2 Regularization Algorithm [7] to develop GARCH model [8] on HFT. The use of GARCH model to estimate volatility has been done before and our result is consistent with previous studies. On top of that we add trading volume and additional 34 variables in GARCH model to investigate which can capture the GARCH effect. After performing real market experiment it turns out that the L1/2 algorithm is relatively effective.

Besides implementing Markovian queuing model and simulating two different trading strategies base upon it, we also focus on business side of HFT. We perform marketing analysis to search for potential customers of our HFT trading software. However, we cannot ignore the impact of regulators since May Flash Crash has already caused intense public attentions on HFT. Factors like improvement of technologies either on hardware or software can also have huge impact on the world of HFT.

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