Model Predictive Control Approach to Electric Vehicle Charging in Smart Grids

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by

Somil Bansal

A thesis submitted in partial satisfaction of the
requirements for the degree of
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in

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Committee in charge:

Professor Claire Tomlin, Chair
Associate Professor Murat Arcak
Assistant Professor Anil Aswani

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Abstract

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In this work, I present a method to design a predictive controller for handling Plug-and-Play (P&P) requests of Electric Vehicles (EVs) in a power distribution system. The proposed method uses a two-stage hierarchical control scheme based on the ideas of Model Predictive Control (MPC) tracking for periodic references to ensure that bus voltages track the closest possible (reachable) periodic reference to the nominal voltage while minimizing the required generation control and guaranteeing satisfaction of system constraints at all times. Next, the problem of handling real-time P&P requests is considered using a prepration-phase before the actual connection/disconnection of EVs. The only assumption made on the load is that it is time-periodic with a period of 24 hours. Under this assumption, it is proved that the proposed controller is recursively feasible, is exponentially stable and both the EVs' State of Charge (SOC) and bus voltages converge to the desired SOC and to the optimal reference trajectory, respectively. Finally, the proposed scheme is illustrated in a set of examples.

To my dear mother and father . . .

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Chapter 1

Introduction

In a typical distribution system, changes in demand result in fairly mild and predictable voltage fluctuations. Capacitor banks are generally used to regulate the voltages in such systems [3], [2], [1]. However, the increasing penetration of EVs in the distribution system will introduce rapid and random fluctuations in the voltages. This, coupled with proliferation of renewable energy sources such as photovoltaic devices, poses some key challenges in terms of grid management and network operation. Capacitor banks alone are not adequate to regulate voltages in such a scenario (see [4], [14] and references therein). Renewable energy sources are connected to the grid through inverters, and control schemes such as *inverter volt/var control* have been proposed to stabilize the voltages [6], [5]. In such schemes, reactive power is pushed into or pulled from the distribution system at a much faster rate compared to capacitor banks to stabilize voltage fluctuations [7]. (Since these control schemes are based on local generation at a bus, we will refer to them as *generation-based control* schemes in this work.)

Although EVs represent an additional load on the distribution systems, this load is controllable and thereby offers an important opportunity. As per the smart grid initiatives in most countries that envision their integration with renewable energy sources as a dispatchable load [8], EVs not only are a sustainable alternative to the fuel-based automobiles but can also provide increased reliability of the distribution system [13], [14], [10]. Motivated by these ideas, our goal in this work is to develop a control scheme that integrates EVs in the distribution network while taking into account additional loads and network requirements. There are three key challenges in designing a control scheme to achieve this integration. First, the proposed scheme should be able to charge all connected EV batteries to their desired level while minimizing the voltage fluctuations in the system. Second, it should be able to handle variations in the number of connected EVs, i.e. plugging in and out operations. In a real scenario, a user can request to connect or disconnect an EV at any desired time and bus. This will modify the overall distribution system, and the controller designed for the current system might be infeasible and/or unstable for the modified system, requiring a redesign of the controller. Third, network constraints, or more specifically voltage constraints, should be satisfied at all times. Assuming that the remaining load profiles are fixed, this has to be achieved by scheduling EV charging accordingly. According to the smart grid literature [8], this will be enabled by the communication infrastructure allowing to control the charging power of individual EVs. In this work we will address these three issues and provide a constrained optimal control scheme that integrates charging and generation-based control. Under the proposed scheme, constraints on EVs as well as on the network are satisfied at all times. Moreover, a plug and play MPC (P&P MPC) approach is proposed to handle the real time plugging in and out of EVs.

Note that although a generation-based control scheme is capable of stabilizing voltage fluctuations, this control input is limited by the capacity of the generator. Strain on the grid can increase significantly if several EVs are connected to the distribution system during peak hours and charged according to uncoordinated charging schedules (i.e. charging starts immediately after connection at a fixed charging power). Generation-based control alone may not be able to stabilize the voltages in such an environment as the required capacities may be prohibitive. Integrating both control inputs (i.e. charging and generation-based control) in one control scheme however allows to control the charging such that voltage constraints are satisfied with a given capacity of generators and hence is more cost efficient.

In this study, we will use a Model Predictive Control (MPC) approach to design our controller. MPC is an attractive tool, being capable of minimizing a control objective while ensuring constraint satisfaction. Moreover the 'look ahead' characteristic of MPC provides improved performance, particularly in the presence of load forecast. Since the load can be assumed (approximately) periodic, ideas of MPC tracking for periodic references [15] are employed to make sure that the bus voltages track a trajectory that both minimizes deviation to the nominal voltage and generator control inputs. Ideally we want the bus voltages to track the nominal voltage exactly, but, in general, this may be impossible due to limits imposed by network constraints or model inconsistency. In such cases a periodic reference is calculated and tracked, which is the optimal tradeoff between deviation from the actual reference and required control effort, and consistent with system dynamics and constraints (from hereon called *optimal periodic reference*). We will also employ the ideas of the P&P MPC concept introduced in [17] to deal the connection and disconnection of EVs from the grid.

In this work, we will first consider the problem of designing a controller for the static distribution system (i.e. no P&P requests are made). To do this, we first compute the optimal periodic reference and then design a control scheme that regulates the bus voltages to this reference, while minimizing the required generation input and charging the EVs to their desired level, assuming a periodic load profile. We will then explain how we can handle plug-in and unplug requests. To do this, a procedure for updating the controller together with a transition scheme is proposed, which prepares the system for the requested modifications. To summarize, the main contributions of our work are:

- the system voltage is regulated to the optimal periodic reference, trading off deviation from nominal voltage and generation control effort;

- the load and network constraints are explicitly taken into account, and constraint satisfaction is guaranteed at all times;
- the control scheme ensures an exponential rate of convergence of voltages to the optimal periodic reference, and of EVs' batteries to their desired charge level;
- the proposed control scheme can handle the real-time plugging and unplugging of EVs;
- all results are established under a time varying load as opposed to constant load considered so far in most of the studies in the literature.

The document is organized as follows: Chapter 2 introduces the distribution network model and its power flow equations. In Chapter 3, various network constraints are discussed. Chapter 4 explores the different representation of a distribution network that are suitable for the controller design. In chapter 5, control objectives are defined and an MPC controller is proposed to track the optimal periodic reference and to charge the EVs to provide optimal charging and generator control with respect to system constraints and objectives, and to handle plug and play requests. Chapter 6 presents a numerical simulation demonstrating the advantages of the proposed control scheme and chapter 7 provides concluding remarks.

This work was done with the advice of Dr. Melanie N. Zeilinger.

Chapter 2

Power Flow Equations for a Radial Network

A distribution system consists of several buses connected together through power lines. In this work, radial distribution networks are considered, which is a common model adopted in the power systems literature. In a radial distribution network, each bus has exactly one parent bus. In this section, I first setup the network model for a radial distribution network and then characterize the power flow in such a network.

2.1 Network Model

To represent the network we use the notation introduced in [5] (restated here in Table 2.1 for completeness), along with some other parameters.

2.2 Power Flow Equations

To characterize the power flow in a radial distribution system, DistFlow equations are used (first introduced in [3]). These DistFlow equations can be used to find the operating point of the network using Newton-Raphson method and shown to have nice convergence properties [3]. Following the same approach as in [3], we first consider a special case where there is only one main feeder. The general case for any radial distribution system is considered next.

Special Case: Radial Main Feeder

We can now write power flow equations for the line network in a recursive fashion using the approach described in [3].

Table 2.1: Variables for a radial distribution network

$\mathcal{N}$	Set of buses, $\mathcal{N} := \{1, \ldots, n\}$
$\mathcal{L}$	Set of lines between the buses in $\mathcal{N}$
$\mathcal{L}_i$	Set of lines on the path from bus 0 to bus i
p_i^l, p_i^v	Real power consumption by load and EVs at bus i
q_i^l, q_i^g	Reactive power consumption and generation at bus i
z_{ij}	Impedence of line $(i, j) \in \mathcal{L}$
r_{ij}, x_{ij}	Resistance and reactance of line $(i, j) \in \mathcal{L}$
S_{ij}	Complex power flow from bus i to j
P_{ij}, Q_{ij}	Real and reactive power flows from bus i to j
V_i	Voltage (complex) at bus i
v_i	Magnitude of voltage at bus i
I_{ij}	Current (complex) from bus i to j
l_{ij}	Squared magnitude of complex current from bus i to j
M_i	Number of EVs connected at bus i

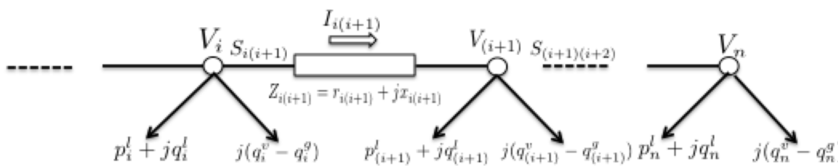

Figure 2.1: Line diagram of a distribution network with one main feeder

Power flow equations at node i gives (following equations hold for all time instants so I am dropping the time index t in all equations):

$$S_{(i-1)i} = S_{i(i+1)} + l_{(i-1)i}z_{(i-1)i} + (p_i^l + p_i^v) + j(q_i^l - q_i^g)$$

By applying KVL across the impedence of the line connecting node $i-1$ and i :

$$I_{(i-1)i} = (V_{i-1} - V_i)/z_{(i-1)i}$$

From above equation:

$$v_i = |V_{i-1} - I_{(i-1)i}.z_{(i-1)i}| \tag{2.1a}$$

$$v_i^2 = v_{i-1}^2 + |I_{(i-1)i}.z_{(i-1)i}|^2 - 2Real(V_{i-1} * \overline{I_{(i-1)i}.z_{(i-1)i}}) \tag{2.1b}$$

By definition of complex power,

$$V_{i-1} * \overline{I_{(i-1)i}} = S_{(i-1)i}$$

Hence we have

$$P_{(i-1)i} = p_i^l + p_i^v + r_{(i-1)i}l_{(i-1)i} + P_{i(i+1)} \tag{2.2a}$$

$$Q_{(i-1)i} = q_i^l - q_i^g + x_{(i-1)i}l_{(i-1)i} + Q_{i(i+1)} \tag{2.2b}$$

$$v_i^2 = v_{i-1}^2 + (r_{(i-1)i}^2 + x_{(i-1)i}^2)l_{(i-1)i} - 2(r_{(i-1)i}P_{(i-1)i} + x_{(i-1)i}Q_{(i-1)i}) \tag{2.2c}$$

$$l_{(i-1)i}v_{i-1}^2 = P_{(i-1)i}^2 + Q_{(i-1)i}^2 \tag{2.2d}$$

As stated in Table 2.1, $P_{(i-1)i}$ and $Q_{(i-1)i}$ are the real and reactive power from bus $i-1$ to bus i respectively. Similarly, v_i and $l_{(i-1)i}$ denote the voltage magnitude at bus i and squared magnitude of the current flowing from bus $i-1$ to bus i respectively. We also have the following terminal conditions:

- at the substation, voltage is known at all times, i.e.

$$V_0 = V^s$$

- at the end of the main feeder;

$$P_{n(n+1)} = Q_{n(n+1)} = 0$$

General radial network

The DistFlow equations for a single feeder can be generalized to include laterals as follows:

$$P_{ij} = p_j^l + p_j^v + r_{ij}l_{ij} + \sum_{k:(j,k)\in\mathcal{L}} P_{jk} \tag{2.3a}$$

$$Q_{ij} = q_j^l - q_j^g + x_{ij}l_{ij} + \sum_{k:(j,k)\in\mathcal{L}} Q_{jk} \tag{2.3b}$$

$$v_j^2 = v_i^2 + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) \tag{2.3c}$$

$$l_{ij}v_i^2 = P_{ij}^2 + Q_{ij}^2 \tag{2.3d}$$

Note also that we have the following terminal conditions:

- at the substation, voltage is known at all times:

$$V_0 = V^s$$

- here we do not need to worry about the second boundary condition unlike single feeder case as our notation in equation (2.3) (i.e. using $\mathcal{L}$) already takes care of that (because the last bus has no neighbors in a lateral and hence the summation term in the equation (2.3) will be zero)
- we will assume that load profile is known at every time instant i.e. p_j^l and q_j^l are known for all times.

2.3 Approximate Power Flow Equations

Constraints imposed by the power flow equations are non-linear and non-convex and hence they are a bit difficult to be handled by MPC. To overcome this problem several relaxations and approximations have been proposed in the literature. In this work, we will discuss two important such techniques namely formulation using second order cone constraint and neglecting losses in the system.

Reformulating Power Flow Equations Using Second Order Cone Constraint

To reformulate the power flow equations as convex constraints, we convert them in equivalent second order cone constraints which roughly speaking are norm-constraints. Consider the following relaxation of the original power-flow equations:

$$P_{ij} = p_j^l + p_j^v + r_{ij} l_{ij} + \sum_{k:(j,k) \in \mathcal{L}} P_{jk} \tag{2.4a}$$

$$Q_{ij} = q_j^l - q_j^g + x_{ij} l_{ij} + \sum_{k:(j,k) \in \mathcal{L}} Q_{jk} \tag{2.4b}$$

$$y_j = y_i + (r_{ij}^2 + x_{ij}^2) l_{ij} - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) \tag{2.4c}$$

$$l_{ij} \geq \frac{(P_{ij}^2 + Q_{ij}^2)}{y_i} \tag{2.4d}$$

We have essentially substituted v_i^2 by y_i in power flow equations (2.3). The last equatlity constraint in power flow equations is replaced with an inequality constraint, which essentially represents an upper bound on the current flowing in the line connecting node i and j. The inequality constraint (2.4d) can be expressed as a second order cone constraint as follows:

$$\left\| \begin{matrix} 2P_{ij} \\ 2Q_{ij} \\ l_{ij} - y_i \end{matrix} \right\| \leq l_{ij} + y_i \tag{2.5}$$

Above relaxation was first introduced and proved to be exact in [6]. For above relaxation to be exact, over-satisfaction of loads should be allowed (i.e. suppose load profile at bus j is given as $p_j^l = p$ then instead of this equality constraint we will consider the constraint $p_j^l \geq p$). Due to the exactness of the above relaxation, (2.4) is expected to reach the same solution as that of (2.3) [6].

Approximating Power Flow Equations by Neglecting Power Losses

We next want to reformulate the power flow equations (2.3) in a fashion which is more suitable for the stability analysis and optimization. Following [5] we assume losses are small as compared to the load power i.e. $l_{ij}r_i = l_{ij}x_i = 0$ for all $(i,j) \in \mathcal{L}$ in (2.3). This approximation neglects the higher order real and reactive power loss terms that are generally much smaller than power flows P_{ij} and Q_{ij}, and only introduces a small relative error, typically on the order of 1% [5]. With above approximation these equations reduce to:

$$P_{ij} = p_j^l + p_j^v + \sum_{k:(j,k)\in\mathcal{L}} P_{jk} \tag{2.6a}$$

$$Q_{ij} = q_j^l - q_j^g + \sum_{k:(j,k)\in\mathcal{L}} Q_{jk} \tag{2.6b}$$

$$v_j^2 = v_i^2 - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) \tag{2.6c}$$

As in [5], we further assume that $v_i \approx 1$ (in per unit). With this approximation, voltage equation in (2.6) further simplifies to:

$$v_j - v_i = (r_{ij}P_{ij} + x_{ij}Q_{ij}) \tag{2.7}$$

as $(v_j + v_i) \approx 2$. We can now recursively write the voltage equation (2.7) to get the following linear equation (see [5] for more details on this derivation):

$$v = \overline{v}_0 - R(p^l + p^v) - X(q^l - q^g) \tag{2.8}$$

where,

$$R_{ij} = \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} r_{hk} \tag{2.9a}$$

$$X_{ij} = \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} x_{hk} \tag{2.9b}$$

and $\overline{v}_0 = (v_0, \ldots, v_0) \in \mathbb{R}^n$. The other variables in equation (2.8) are generation input (column) vector $q^g := (q_1^g, \ldots, q_n^g) \in \mathbb{R}^n$ and EV load vector $p^v := (p_1^v, \ldots, p_n^v) \in \mathbb{R}^n$.

We assume that the substation voltage v_0, is given and is constant. Further, load profiles p^l and q^l are time-varying but their 24 hour forecast is assumed to be given. To establish the desired results we will make the following assumption on the load profile:

Assumption 1: The load profile (i.e p^l and q^l) is time periodic with period length 24h.

This assumption is a tradeoff between a freely varying load over days (real scenario) and a constant load (a stringent assumption). However it is reasonable to expect the load profile to be approximately periodic with a period of 1 day, because the load on the distribution system is expected to be almost the same at a given hour of the day on two consecutive days.

Let v_{nom} denote the nominal value of bus voltage. Also, let $\widetilde{v} = \overline{v}_0 - Rp^l - Xq^l$, i.e. a periodic vector due to Assumption 1. Then the model (2.8) reduces to:

$$v = Xq^g - Rp^v + \widetilde{v} \tag{2.10}$$

In this work we will use (2.10) as our power flow equation.

Chapter 3

Electric Vehicle Charging Model and Network Constraints

In this chapter, I will discuss the battery model for Electric Vehicles that is used in this work. Next, various network constraints have been discussed that need to be satisfied while designing any control scheme for the network.

3.1 Electric Vehicle Charging Model

We adopt a linear state space model governing the battery State Of Charge (SOC) of an Electric Vehicle (EV):

$$e(k+1) = e(k) + Tc(k) \tag{3.1}$$

where $e \in \mathbb{R}$ is the battery SOC (kWh), $c \in \mathbb{R}$ is the charging power supplied for charging the device (kW), and T is the sampling time, which is assumed to be fixed. For EV charging loads, the battery storage capacity, and the maximum charging power are limited by the following constraints:

$$e_{min} \leq e(k) \leq e_{max} \tag{3.2}$$

$$0 \leq c(k) \leq c_{max} \tag{3.3}$$

Remark: The EV charging model introduced in this section has been chosen for the sake of simplicity. Nevertheless, more advanced linear models can be similarly considered (e.g. including some efficiency of charging as in [9]).

3.2 Network Constraints

Depending on the load, bus voltages can fluctuate significantly. For reliable operation of the distribution network it is required to maintain the bus voltages v within a tight range around the nominal value at all times:

$$v_{min} \leq v - v_{nom} \leq v_{max} \tag{3.4}$$

In addition, there are inherent physical limitations on the generator control input, which is limited to:

$$q_{min} \leq q^g \leq q_{max} \tag{3.5}$$

Remark: The generation power output has both active and reactive power components. However due to inverters connected at the generator output, one can always control it to output a constant active power and varying reactive power [5]. In our work this constant active power is merged in the total active load, p^l. Moreover the reactive power has time varying limits [6]. In this work, we will work with constant limits, but the proposed method applies directly for the time-varying limits.

The other important constraint that we can impose is on the charging time required to charge an EV. Since we are interested in the real time plugging of EVs, we need some estimate of how much time we have for charging and require a reasonable assumption on the time period for which EV remains connected to charging station once it is plugged in. There are different types of models that can used for this purpose:

- We guarantee a minimum average charging power rate for the entire period for which EV is connected to the network. Note that in this case we don't need to assume anything about the time for which an EV remains connected to the grid.

- We assume a minimum buffer time. For example suppose if we charge an EV at maximum possible power, it takes 4 hours to fully charge it. But if one really wants to control the charging he should have some extra time in which charging rate can be dropped from c_{max} to some lower level while still ensuring that EV is fully charged after a given time interval i.e. we require some buffer time. So if we assume that once an EV is connected, it will remain connected for next 5 hours then we can control the charging in these 5 hours so as to fully charge the EV at the end of 5 hours.

- The other implicit way to take care of the charging time constraint is introducing a cost in the objective function that increases monotonically with the charging time. This is an easy way to approach this problem without losing important insights. In our formulation this method has been used.

Chapter 4

Representation of Overall System in Standard Linear System Form

Now that we have developed the network and EV model in last two chapters, we will represent our overall system with constraints in standard linear system forms. In particular, three different representations have been discussed. In the first representation, SOCs are states and bus voltages are outputs. In second representation, bus voltages are also represented as states by introducing a delay in the controller output. Finally, the distribution structure of the system is explored.

4.1 System as an input-output model: Representing voltages as outputs

In this section we will represent the overall system in the standard linear input-output system form with bus voltages as outputs and SOCs as states. We start with noting that the EV load, $p_i^v \in \mathbb{R}$, denotes the net load of all vehicles connected for charging at bus i i.e. $p_i^v = \sum_{j=1}^{M_i} c_j$ where M_i is the number of EVs connected to bus i. Note that M_i can vary over time due to plugging and unplugging operations. Thus (2.10) can be rewritten as:

$$v = Xq^g - RKu_2 + \widetilde{v} \tag{4.1}$$

where $u_2 \in \mathbb{R}^M$ and $K \in \mathbb{R}^{n \times M}$, where M is the total number of EVs connected to the grid, i.e. $M = \sum_{j=1}^{n} M_j$. In particular $u_2 := (c_1, \ldots, c_{M_1}, \ldots, c_M)^T$, and $K_{ij} = 1$ if and only if EV j is connected to bus i and 0 otherwise. So the control objective of v tracking v_{nom} is equivalent to $(v - \widetilde{v})$ tracking the periodic reference $r_k := v_{nom} - \widetilde{v}_k$. Note that r is the actual reference that we would ideally like to track and should not be confused with the optimal periodic reference. As stated before, it may be impossible to track r exactly so the optimal periodic reference is tracked instead.

Users connect their EVs to the grid to charge them to the desired SOC, e_{des} (specified by the user at the time of connection). Rewriting the SOC dynamic equation (3.1) in terms of $\overline{e} := e_{des} - e$

$$\overline{e}(k+1) = \overline{e}(k) - Tc(k) \tag{4.2}$$

Writing equation (4.2) for all EVs will give us the following standard representation for our system:

$$x(k+1) = Ax(k) + Bu(k) \tag{4.3a}$$
$$y(k) = Cx(k) + Du(k) \tag{4.3b}$$

$$(x(k), u(k), y(k)) \in \mathcal{Z}_k \tag{4.4}$$

where,

$$x = (\overline{e}_1, \ldots, \overline{e}_{M_1}, \ldots, \overline{e}_M)^T_, \; u = \begin{bmatrix} q^g & u_2 \end{bmatrix}^T_, \; y = (v - \widetilde{v})$$

$$A = I, \quad B = \begin{bmatrix} 0 & -T \end{bmatrix}, \quad C = 0, \quad D = \begin{bmatrix} X & -RK \end{bmatrix}$$

$$\mathcal{Z}_k = \left\{ \begin{matrix} (x(k), u(k), y(k)) : e_{min} \le e_{des} - x(k) \le e_{max} \\ q_{min} \le q^g(k) \le q_{max}, \; 0 \le u_2(k) \le c_{max} \\ v_{min} - \widetilde{v}(k) \le y(k) - v_{nom} \le v_{max} - \widetilde{v}(k) \end{matrix} \right\}$$

Note that due to the periodic time varying $\widetilde{v}(k)$, constraint set $\mathcal{Z}_k$ is also time varying and periodic.

4.2 System as an input-output model: Representing voltages as states

In above system if we choose a control scheme such that both u_2 and q^g at time $(k+1)$ are functions of the system variables till time k then we can write

$$u_2(k+1) = l(k) \tag{4.5a}$$

$$q^g(k+1) = d(k) \tag{4.5b}$$

Now introducing $u_2(k)$ as a state, we can rewrite our system as:

$$x(k+1) = Ax(k) + Bu(k) \tag{4.6a}$$
$$y(k) = Cx(k) + Du(k) \tag{4.6b}$$

$$(x(k), u(k), y(k)) \in \mathcal{Z}_k \tag{4.7}$$

where,

$$x = (v - \widetilde{v} \;\; \overline{e} \;\; u_2)^T_, \quad u = \begin{bmatrix} d \; l \end{bmatrix}^T_, \; y = (v - \widetilde{v})$$

where, $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & -T \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} X & -RK \\ 0 & 0 \\ 0 & I \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $D = 0$

$$\mathcal{Z}_k = \left\{ \begin{matrix} (x(k), u(k), y(k)) : e_{min} \leq e_{des} - \overline{e}(k) \leq e_{max} \\ q_{min} \leq d(k) \leq q_{max},\ 0 \leq l(k) \leq c_{max} \\ v_{min} - \widetilde{v}(k) \leq y(k) - v_{nom} \leq v_{max} - \widetilde{v}(k) \end{matrix} \right\}$$

Again due to the periodic time varying $\widetilde{v}(k)$, constraint set $\mathcal{Z}_k$ is also time varying and periodic.

4.3 Distributed Representation of the System

We next develop a distributed representation of the system where only local communication between buses is required to determine system operating point. We will prove that 3-level communication is sufficient to solve the problem in a distributed fashion under certain assumptions. In particular to update the controller of an EV connected to bus i, we need communication between bus i, its parent bus, its first children, and its second children so a total of 3-level communication. To establish this result we start with defining some mathematical preliminaries. Let η_i denotes the set of buses which are first child of bus i. Also denote its parent bus by $\alpha(i)$. Writing the approximate power flow equations (**??**) for link $(\alpha(i), i)$

$$P_{\alpha(i)i} = p_i^l + p_i^v + \sum_{k \in \beta_i} P_{ik} \tag{4.8a}$$

$$Q_{\alpha(i)i} = q_i^l - q_i^g + \sum_{k \in \beta_i} Q_{ik} \tag{4.8b}$$

$$v_{\alpha(i)} - v_i = r_{\alpha(i)i} P_{\alpha(i)i} + x_{\alpha(i)i} Q_{\alpha(i)i} \tag{4.8c}$$

$$\text{So we have,} \quad v_{\alpha(i)} - v_i - r_{\alpha(i)i}(p_i^l + p_i^v) - x_{\alpha(i)i}(q_i^l - q_i^g) = \tag{4.8d}$$

$$r_{\alpha(i)i} \sum_{k \in \eta_i} P_{ik} + x_{\alpha(i)i} \sum_{k \in \eta_i} Q_{ik} \tag{4.8e}$$

Note that each power flow equation consists of two types of terms: one term deontes load and control power, and second term is power transferred to the next links. So if we similarly write power flow equation for each $k \in \eta_i$ and transfer control(and load) terms on the LHS we will have

$$v_{\alpha(i)} - v_i - r_{\alpha(i)i} \sum_{k \in i \cup \eta_k} (p_k^l + p_k^v) - x_{\alpha(i)i} \sum_{k \in i \cup \eta_k} (q_k^l - q_k^g) = \tag{4.9a}$$

$$r_{\alpha(i)i} \sum_{k \in \eta_i} \sum_{j \in \eta_k} P_{kj} + x_{\alpha(i)i} \sum_{k \in \eta_i} \sum_{j \in \eta_k} Q_{kj} \tag{4.9b}$$

Also, volatge equation for each $k \in \eta_i$ will give,

$$v_i - v_k - r_{ik}(p_k^l + p_k^v) - x_{ik}(q_k^l - q_k^g) = r_{ik} \sum_{j \in \eta_k} P_{kj} + x_{ik} \sum_{j \in \eta_k} Q_{kj} \tag{4.10}$$

Similarly we can write voltage equation for each $j \in \eta_k$ where $k \in \eta_i$,

$$v_k - v_j = r_{kj} P_{kj} + x_{kj} Q_{kj} \tag{4.11}$$

Summing above equation for all $j \in \eta_k$ will give

$$\sum_{j \in \eta_k} v_k - \sum_{j \in \eta_k} v_j = \sum_{j \in \eta_k} r_{kj} P_{kj} + \sum_{j \in \eta_k} x_{kj} Q_{kj} \tag{4.12}$$

Now if we assume that all first child links of a bus have same impedence i.e. $z_{km} = z_k$ for all $m \in \eta_k$ then we can solve equations (4.10) and (4.12) simultaneously to get $\sum_{j \in \eta_k} P_{kj}$ and $\sum_{j \in \eta_k} Q_{kj}$. Same process can be repeated for each $k \in \eta_i$. Substituting these values in equation (4.9a) will give us,

$$v_i = f\Bigg(v_{\alpha(i)}, v_k, v_j, z_{\alpha(i)}, z_i, z_k, p_i^v, p_i^l, q_i^v, q_i^g, p_k^v, p_k^l, q_k^v, q_k^g\Bigg) \quad ; k \in \eta_i, j \in \eta_k$$

Some important remarks:

- function f is a linear function
- as can be seen from the arguments of the above function, voltage at bus i can be determined from voltage of its parent bus, voltage of its first and second child, and control (and load) at first child buses. We don't need load/control information of it's parent bus and second child buses. Moreover we don't need information about each EV or load individually, we only need to know the net load EV active and reactive power at each child bus. This is an advantageous scenario from the security point of view as SOC of an EV can be considered as a personal information of an user.
- above result holds under the assumption that all first child links have same impedence. This assumption is trivially satisfied for line network as there is only one child per bus.
- we can also derive the same result even if all child links don't have the same impedence but if every fisrt child link is either purely resistive or purely reactive or if resistance is same as reactance(immediate from equation (4.10) and (4.11)).

In this work we will use the formulation discussed in section 4.1.

Chapter 5

Controller Design

In this chapter our goal is to design a controller that captures three control objectives: bus voltages track the optimal periodic reference while minimizing the required generation control, charge the EVs to their desired SOC, and enable P&P operations. The designed controller should then try to achieve these goals subject to the system dynamics and the system constraints. The proposed control scheme addresses these three objectives by solving the following three subproblems subject to (4.3) and (4.6):

- compute a periodic reference, which is an optimal tradeoff between deviation from nominal voltage and the required generation control input;
- design a controller, which ensures that bus voltages indeed track the above computed optimal periodic reference and EVs are charged to their desired SOCs;
- if a modification is requested, determine feasible P&P time, prepare the system for the modification and, redesign/update this controller such that it is feasible for the modified system.

In subsection 5.1, we handle subproblems 1 and 2 using the two-stage hierarchical control scheme based on an MPC scheme for tracking periodic references [15]. Subproblem 3 is handled in subsection 5.2 using the P&P MPC concept introduced in [17].

5.1 Periodic Reference Tracking MPC Integrating EV Charging and Generation-based Control

Following the ideas proposed in [15] for tracking periodic reference signals, we design a predictive controller that regulates y exponentially to the optimal periodic reference while satisfying system constraints (4.3) and (4.6) at all times. Moreover the regulation is achieved with the minimum generation control.

For the purpose of this section we assume that the number of EVs connected to the grid is constant i.e. no new EVs are connected or disconnected from the system. We will relax this assumption in the next section. In general, in the reference tracking problem, it may be impossible to track the given reference signal due to the limits imposed by the constraints (see [15], [12] and references therein). In such cases, it is a common practice to first calculate a reachable trajectory, in our case periodic, which is the closest trajectory to the given reference signal (with respect to some objective function) that satisfies system dynamics and constraints, and then this reachable trajectory is tracked instead [15]. If the given reference signal is also a reachable trajectory then in theory one can design a controller that exactly tracks it. In either case, the control problem can be solved using a two-layer structure [15]. At the first level, a reachable trajectory y^s is calculated to be as close as possible to the reference signal r (with period N_r) by solving the following optimization problem (referred as *stage-1* in this work):

$$(x^s(k), u^s(k), y^s(k)) = \underset{\tilde{x},\tilde{u},\tilde{y}}{\operatorname{argmin}}\, V_1(r_k; \tilde{x}, \tilde{u}, \tilde{y}) \tag{5.1a}$$

$$\text{s.t} \qquad \tilde{x}(i+1) = A\tilde{x}(i) + B\tilde{u}(i) \tag{5.1b}$$

$$\tilde{y}(i) = C\tilde{x}(i) + D\tilde{u}(i) \tag{5.1c}$$

$$\tilde{x}(0) = A\tilde{x}(N_r - 1) + B\tilde{u}(N_r - 1) \tag{5.1d}$$

$$(\tilde{x}(i), \tilde{u}(i), \tilde{y}(i)) \in \mathcal{Z}_i; \quad i = 0, \ldots, N-1 \tag{5.1e}$$

where,

$$V_1(r_k; \tilde{x}, \tilde{u}) = \sum_{i=0}^{N_r-1} (||\tilde{y}(i) - r(k+i)||^2_{T_1} + ||\tilde{u}(i)||^2_{T_2} + ||\tilde{x}(i)||^2_{T_3})$$

and T_1, T_2, T_3 are positive semi-definite weight matrices of appropriate sizes. The resulting y^s is what was referred as optimal periodic reference. Throughout this work we use the term optimal periodic reference to denote both y^s and $v^s := y^s + \tilde{v}$ whenever it is clear. v^s in our case represent an optimal voltage profile, trading off deviation from v_{nom} and required generation control.

Remark: From (4.3) it is clear that x is monotonically decreasing in u so due to the periodicity constraint on x in the first stage, the optimal solution is $x^s, u^s \equiv 0$.

At the second level, a predictive controller is designed to track the calculated reachable trajectory $(x^s(k), u^s(k), y^s(k))$. To achieve this, we propose the following tracking MPC scheme (referred as *stage-2* in this work):

$$\min_{\overline{u}} \quad V_2(x, x^s(k), u^s(k), y^s(k); \overline{u}) \tag{5.2a}$$

$$\text{s.t} \qquad \overline{x}(i+1) = A\overline{x}(i) + B\overline{u}(i) \tag{5.2b}$$

$$\overline{y}(i) = C\overline{x}(i) + D\overline{u}(i) \tag{5.2c}$$

$$\overline{x}(0) = x(k) \tag{5.2d}$$

$$\overline{x}(N) = x^s(N|k) \tag{5.2e}$$

$$(\overline{x}(i), \overline{u}(i), \overline{y}(i)) \in \mathcal{Z}_i; \quad i = 0, \ldots, N-1 \tag{5.2f}$$

where,

$$\begin{aligned} V_2(x, x^s, u^s, y^s; \overline{u}) = & \sum_{i=0}^{N-1} (||\overline{y}(i) - y^s(i)||^2_{R_1} \\ & + ||\overline{u}(i) - u^s(i)||^2_{R_2} + ||\overline{x}(i)||^2_{R_3}) \end{aligned}$$

R_1, R_2, R_3 are positive definite weight matrices of appropriate sizes and N is the prediction horizon of the MPC problem. MPC problem (5.2) is solved at every sampling time, returning the optimal control sequence $\boldsymbol{u}^*(x)$ (we suppress the dependence of $\boldsymbol{u}^*$ on the optimal reference trajectory for the ease of notation). The optimal control law is defined in a receding horizon fashion by $\kappa(x) = u_0^*(x)$, where $u_0^*(x)$ denotes the first control input of the sequence.

Remarks: (1) In stage-2 a cost function is chosen that penalizes the deviation of voltage from the nominal voltage, the difference between the current SOC and the desired SOC, the generation control input, and the EV input, but with a very small weight as the total amount of energy required to charge the EV is fixed. (2) Due to the penalty introduced on the input in the cost function, the above hierarchical control structure ensures that the bus voltages track nominal voltage as close as possible with minimum control input while charging EVs.

We will conclude this section with formally establishing the exponential convergence of bus voltages to the optimal reference trajectory (i.e. $v \to v^s$), and of the SOCs to their desired SOCs (i.e. $x \to 0$).

Theorem 1: Assume that the reference trajectory r is periodic with period N_r. Let $\mathcal{X}_N$ denotes the set of intial states for which (5.2) is feasible then for any $x \in \mathcal{X}_N$ the proposed control law $\kappa(x)$ ensures that the system constraints are satisfied at all times, and bus voltages and SOC converge exponentially to v^s and desired SOC respectively.

Proof. We first prove that the stage-2 problem is feasible at all times if the initial state is feasible. This will ensure that system constraints are satisfied at all times.

Noting that the constraint set $\mathcal{Z}_i$ is periodic, recursive feasibility can be easily established by using the control sequence proposed in Theorem 2 in [12]. With this control sequence we have,

$$\begin{aligned} V_2(k+1) - V_2^*(k) = & -||\overline{y}(k) - y^s(0|k)||^2_{R_1} - ||\overline{x}(k)||^2_{R_3} \\ & - ||\overline{u}(k) - u^s(0|k)||^2_{R_2} \end{aligned} \tag{5.3a}$$

Exponential convergence of $y \to y^s$ (equivalent to $v \to v^s$) and $x \to 0$ follows from the Lyapunov exponential stability theorem proving the result. ■

Remarks: We have assumed periodic references in Theorem 1 for simplicity of presentation. However, using the one stage formulation in [12] these results can be extended to slowly varying periodic references.

5.2 Plug-And-Play EV Charging

In real distribution systems users can connect or disconnect their EVs randomly. This changes the overall load on the system and can affect bus voltages significantly. This section extends the MPC scheme to the case where the system dynamics in (4.3) change, due to EVs joining or leaving the network by employing the concept of P&P MPC in [17]. The plug-and-play capabilities in a system pose two key challenges [17], [16]: 1. Feasibility of the network change has to be assessed and the system must be prepared for this change; 2. The control law has to be redesigned for the modified dynamics. In the considered case, the problem is reduced to only the first, since the new controller for the modified system is directly given by applying the MPC scheme in Section 5.1 with the dynamics replaced by the modified dynamics. In this section we address the first challenge by means of a preparation phase ensuring recursive feasibility and stability during P&P operation.

As discussed earlier, sudden changes in the system may lead to constraints violation (4.6). Consider for example the scenario, where a large number of EVs are connected to an already heavily loaded bus. In such an environment bus voltage may fluctuate significantly and it may not be possible for the system to satisfy voltage constraint (3.4) under current state. This problem is addressed by using the concept of the transition phase (first introduced in [17]), where first a steady-state is computed that allows P&P operation and then system is controlled to this steady-state. After reaching this steady-state the PP operation is performed and the new controller is applied to the modified system.

The steady state (x^{ss}, u^{ss}) is chosen such that it is reachable from the current system state under the previous dynamics and starting from the steady state, there exist a control sequence such that the optimization problem (5.2) is feasible for the modified system. In particular, let $\mathcal{S}$ and $\mathcal{S}_{mod}$ be the set of current EVs and the modified set of EVs (after the P&P operation) respectively. For any set $\mathcal{D}$, denote by $x_{\mathcal{D}}$ the state of EVs in that set. Also let x be the current state of the system. This results in the optimization problem:

$$\min_{\overline{x}^{ss},\overline{u}^{ss},} \sum_{i=0}^{d-1} (||x_{\mathcal{S}}(i)||^2) \tag{5.4a}$$

$$\text{s.t} \qquad \overline{x}^{ss} = A\overline{x}^{ss} + B\overline{u}^{ss} \tag{5.4b}$$

$$x_{\mathcal{S}}(i+1) = Ax_{\mathcal{S}}(i) + Bu(i) \tag{5.4c}$$

$$y(i) = Cx_{\mathcal{S}}(i) + Du(i) \tag{5.4d}$$

$$(x_{\mathcal{S}}(i), u(i), y(i)) \in \mathcal{Z}_i \tag{5.4e}$$

$$x_{\mathcal{S}_{mod}}(d+k+1) = Ax_{\mathcal{S}_{mod}}(d+k) + Bu(d+k) \tag{5.4f}$$

$$y(d+k) = Cx_{\mathcal{S}_{mod}}(d+k) + Du(d+k) \tag{5.4g}$$

$$(x_{\mathcal{S}_{mod}}(d+k), u(d+k), y(d+k)) \in \mathcal{Z}_{d+k} \tag{5.4h}$$

$$x_{\mathcal{S}}(0) = x, \quad x_{\mathcal{S}}(d) = \overline{x}^{ss}, \quad x_{\mathcal{S}_{mod}}(d+N) = 0 \tag{5.4i}$$

$$i = 0, \dots, d; \quad k = 0, \dots, N-1$$

where d is determined to be as small as possible while providing feasibility of problem (5.4). If a P&P is ever possible for the distribution system, optimization problem (5.4) is feasible. Note that the minimization of d not only prepares the system for the P&P request but also minimizes the waiting time for the EV before it gets accepted by the system (i.e. minimize the duration of required transition phase). It is also important to note that due to the time varying loads, this plug and play problem really becomes a problem of when to plug-in/-out and it is not enough for safe P&P to be only at a given state, but at a given state at a given time, for example, the same steady-state may not allow a P&P operation when the system is heavily loaded.

Denote the optimal solution of (5.4) by (d^*, x^{ss}, u^{ss}) so the P&P operation is performed after time d^*T. In order to ensure that the system reaches the steady-state within the specified d^* time steps, the control sequence obtained in (5.4) can be applied open loop, or a shrinking horizon MPC scheme can be applied. Once the system reaches steady-state, constraint satisfaction is guaranteed for the modified system by (5.4). Hence we can use the method described in Section 5.1 to design a controller for the modified system that tracks y^s and exponential convergence is again ensured by the design technique.

Remark: In the P&P procedure, the optimal periodic reference does not have to be recomputed for the modified system as it does not depend on EVs (see Remark after stage-1 in Section 5.1).

Chapter 6

Numerical Examples

Example 1: In this example, we will apply the method discussed in Section 5.1 on a 9-branch linear feeder system to compute and track the optimal periodic reference of the system, and to charge the EVs to their desired SOCs. We will also show the effect of not using a penalty on the generation input in (5.1). In general, the optimization problem (5.1) computes an optimal periodic reference, which is a tradeoff between deviation from v_{nom} and generation-based control effort. So if we don't penalize generation input, the computed optimal periodic reference is the closest possible reachable reference to v_{nom} subject to system constraints.

The study is performed for the 9-bus network used in [3] with network data rescaled for the household load. A typical household load profile is used at buses, which peak at evening hours as in [11]. The number of EVs connected to the system is 11 and assumed to be constant for the purpose of this example (i.e. no P&P requests are made). Other parameters used in this example are given in Table 6.1. We consider two different cases. In

Table 6.1: Other System Parameters for Example 1 in per unit (pu)

T	e_{min}	e_{max}	c_{max}	v_0
5 min	0.2 pu	1.0 pu	0.6 pu	1 pu
v_{nom}	v_{min}	v_{max}	q_{min}	q_{max}
1 pu	-0.1 pu	0.1 pu	-1.0 pu	1.0 pu

case1 we choose T_2 to be an identity matrix and in case2 it is chosen to be a zero matrix. All other weight matrices in (5.1) and (5.2) are chosen to be the identity.

Without loss of generality, the simulation starting time is taken as $t = 0$. Results are shown in Figures 6.1-6.4. Figure 6.1 shows the reference trajectory (v_{nom}), the calculated optimal periodic refernece (v^s), and the actual voltage trajectory (v) in two cases. In both cases the optimal reference trajectory is tracked eventually with zero error. As expected, the optimal periodic reference in case2 is much closer to the nominal voltage as compared to case1. This is because more generation input is expended to find a reachable trajectory that is closer to v_{nom} in case2. This is also evident from the required generation input curve in the two cases (shown in Figure 6.2). From the optimization problem (5.1) it is clear that this is also the closest trajectory to v_{nom}, which is reachable subject to system dynamics and constraints.

Figure 6.3 shows how the voltage magnitude at bus 9 changes over time in both cases and compares our control scheme with the uncoordinated charging scheme (i.e. charging starts as soon as EV is plugged in at a constant charging power of C_{max}). In particular, voltage constraints are satisfied at all times by the designed controller whereas uncoordinated charging doesn't and causes a huge deviation of more than 30% from the nominal voltage.

All the EVs are charged to their desired state of charge as evident from Figure 6.4. Corresponding charging control is also shown.

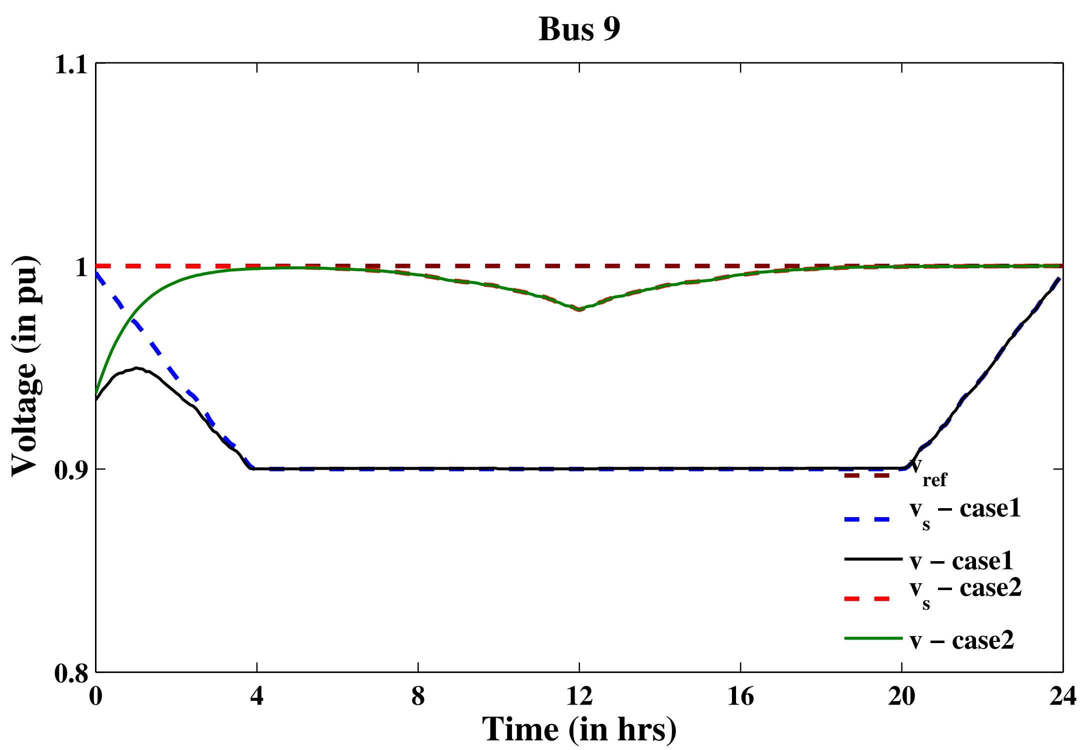

Figure 6.1: Optimal periodic reference and voltage trajectory at bus 9 for two cases, illustrating convergence of voltage trajectory. Optimal reference in case2 is closer to v_{nom} because there is no penalty on generation-effort.

Example 2: In this example our goal is to show how a distribution system can handle P&P requests using the methodology described in Section 5.2. To illustrate this we simulate a 45-bus radial distribution system used in [6]. 85 EVs are already connected to the system. All other parameters are the same as in Table 6.1 except g_{min} and g_{max}, which are now $-0.45pu$ and $0.45pu$ respectively. We will process 4 different P&P requests over 5 hours (starting at $t = 10$). Details of the P&P requests are in Table 6.2. The load profile at bus 10 for these 5 hours is shown in Figure 6.5. Note that the load reaches its maximum value at $t = 12$.

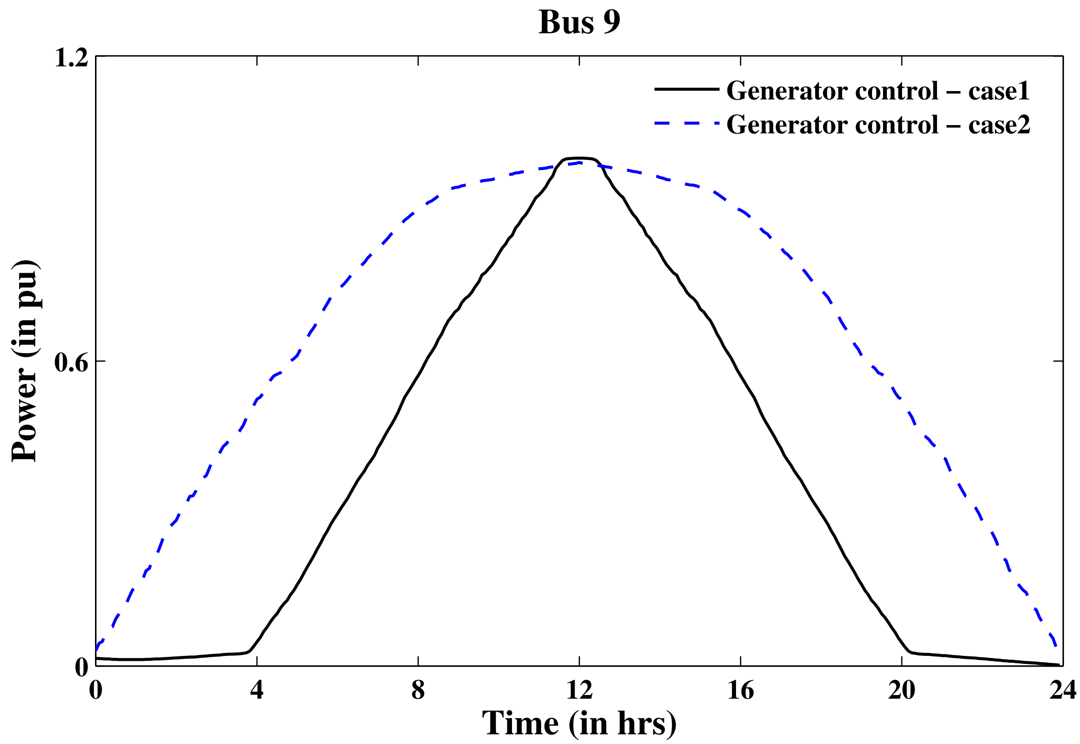

Figure 6.2: Generation control effort required at bus 9 to track the optimal periodic reference. More generation control is used in case2 to find a closer reference (v^s) to v_{nom}.

Table 6.2: P&P Requests for Example 2

Type of request	At bus	Number of EVs	Request time	d^*	Request accepted at
Connection	10	2	10.8	1	Immediately
Connection	10	6	11.2	15	12.5
Disconnection	10	3	12.9	1	Immediately
Connection	22	8	13.3	1	Immediately

Results are shown in Figures 6.6-6.9. The first P&P request is accepted immediately due to the mild load conditions at that time. However, for the second P&P request, due to the high load demand within the prediction horizon of the requested connection time, the steady-state computation determines that it will not be feasible to accommodate the request immediately and starts preparing the system to regulate it to the best steady-state to allow this desired change. The shortest horizon in problem (5.4) is $d^* = 15$ i.e. new EVs have to wait for 75 minutes before they start charging.

The disconnection request is accepted instantly because it is reducing strain on the distribution system and hence no feasibility issues arise. This is expected to be the case in most disconnection scenarios.

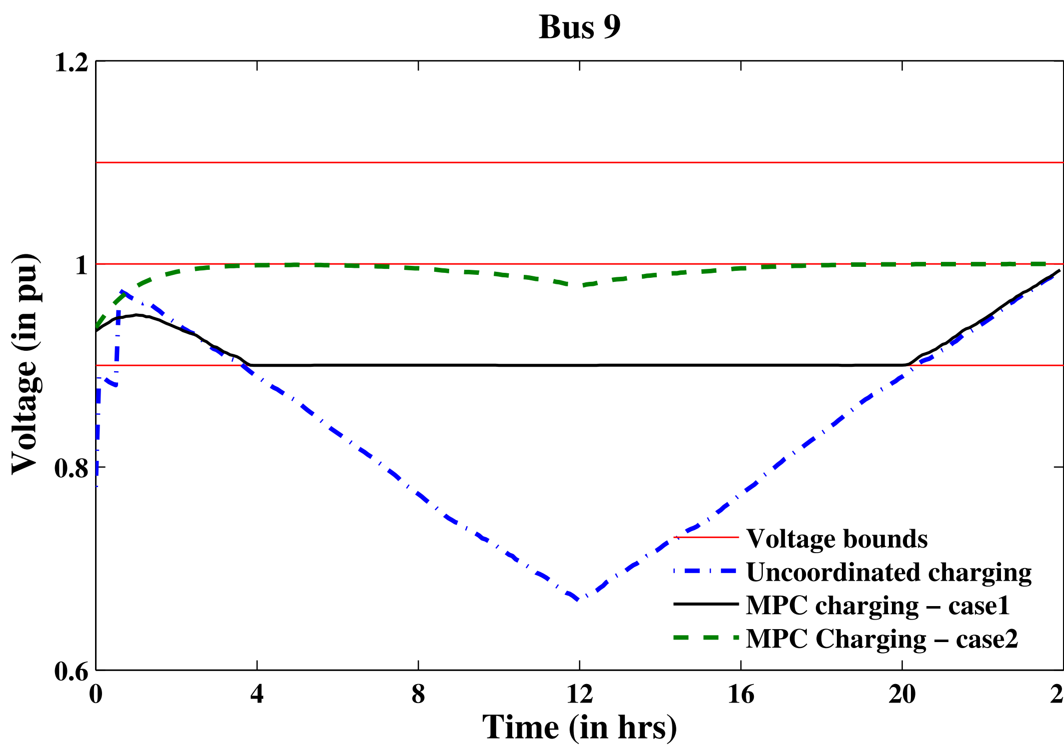

Figure 6.3: Voltage fluctuations at bus 9 under proposed control scheme and uncoordinated charging. Uncoordinated charging is not able to satisfy system constraints and causes large voltage drop.

Interestingly, the third plug-in request is also accepted instantly despite the high load on the system and the large number of new EVs. This is due to the prediction capability of MPC. Since the MPC algorithm can 'look-ahead', it determines that the load is going to reduce soon and the system will have more room to accommodate new EVs and hence they can be connected right away.

P&P requests modify the system and as a result the voltage trajectory deviates from its optimal reference trajectory, however it again tracks the reference trajectory eventually (see Figure 6.7). Our control scheme ensures that voltage constraints are satisfied at all times during and after the processing of P&P requests as shown in Figure 6.8. We next compare the required generation control in this example to the scenario when no P&P requests are made. In particular, when a new plug-in is requested more generation control is used to stabilize the voltage fluctuations caused due to the modification in the system (see Figure 6.6). Finally we show the SOC dynamics and charging control of one of the two EVs connected at $t = 10.8$ in Figure 6.9. To handle the upcoming P&P request of 6 EVs, the system charges this EV at the maximum possible charging power, C_{max} so that new EVs can be accommodated on the same bus with the minimum possible waiting time (note that this is not generally the case without the P&P operation see for example Figure 6.4 where the control input is

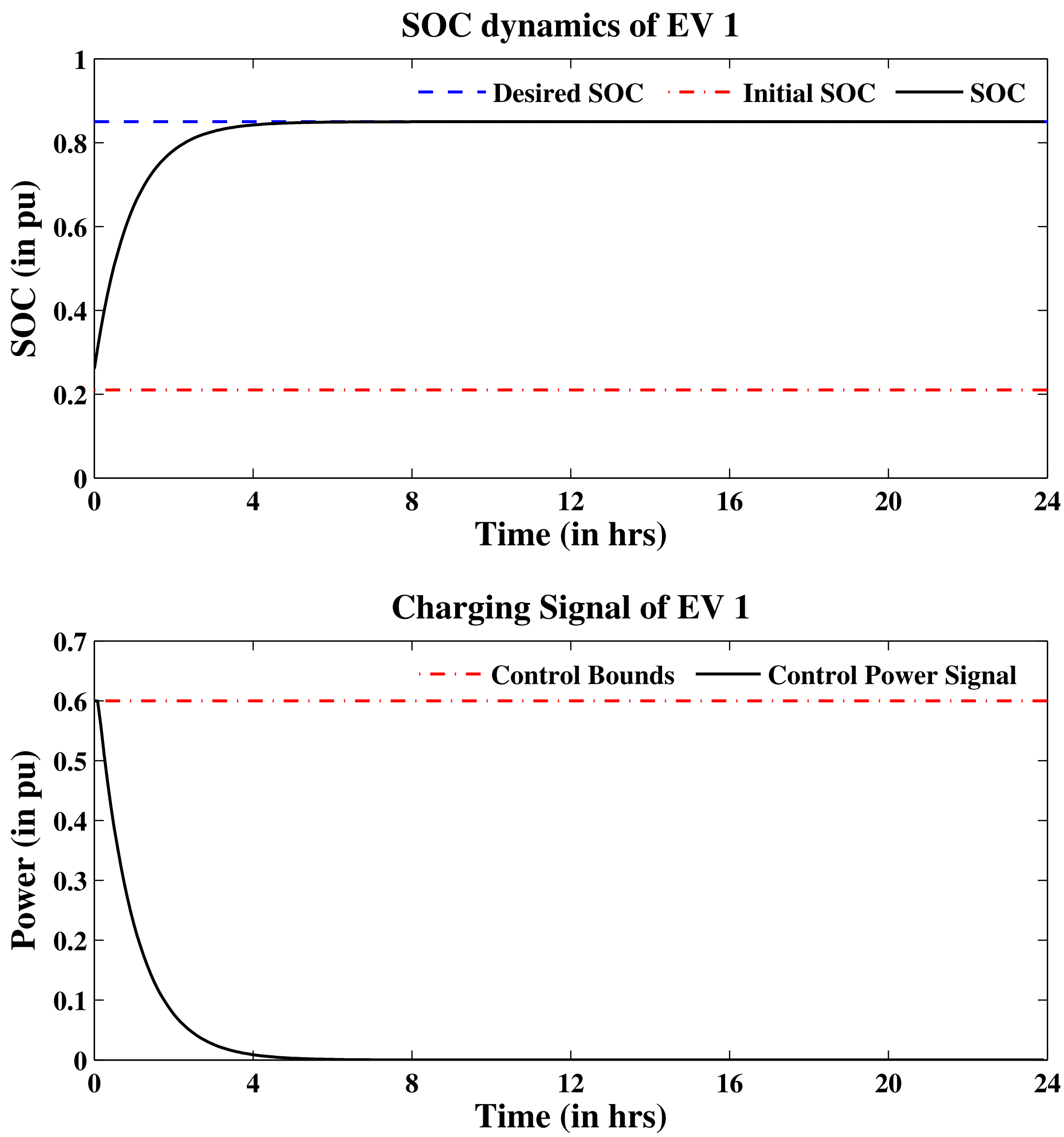

Figure 6.4: SOC dynamics and charging control for EV 1 in case1. EV is charged to its desired SOC in 4 hours.

strictly decreasing). Due to this high charging power, the voltage at bus 10 deviates from its reference trajectory (see Figure 6.7). But due to the maximum load at $t = 12$ (Figure 6.5) charging power is dropped for a while and then it rises again to the optimal level allowed by the load profile of the system.

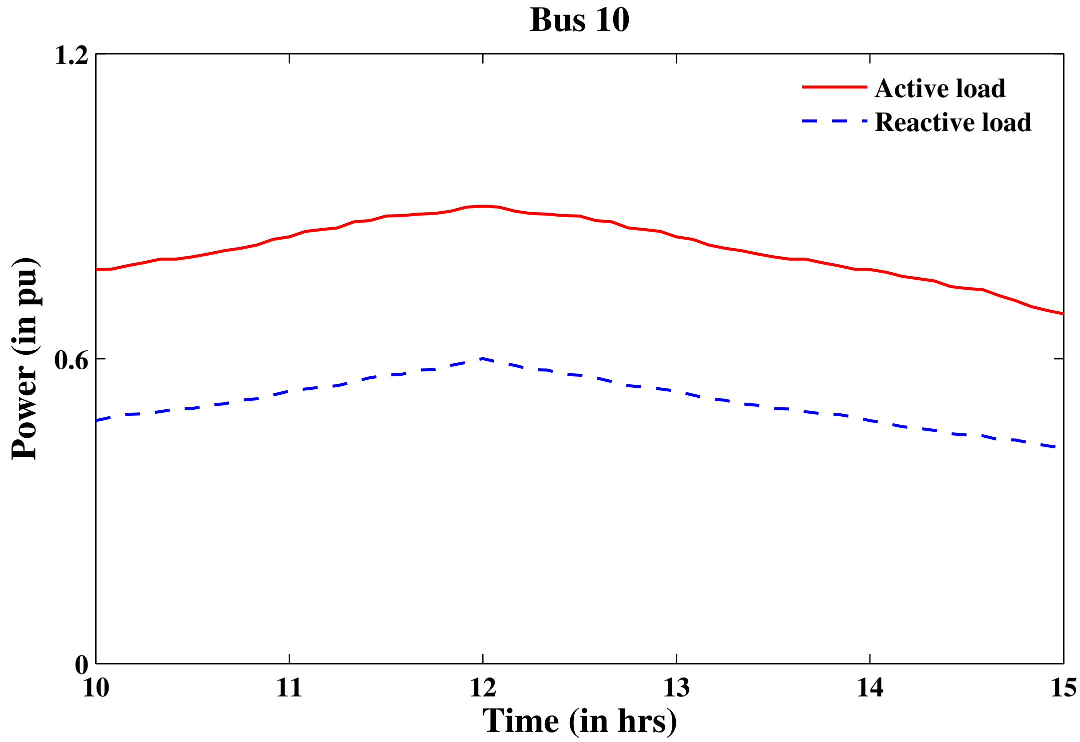

Figure 6.5: Load profile at bus 10. Load is maximum at $t = 12$.

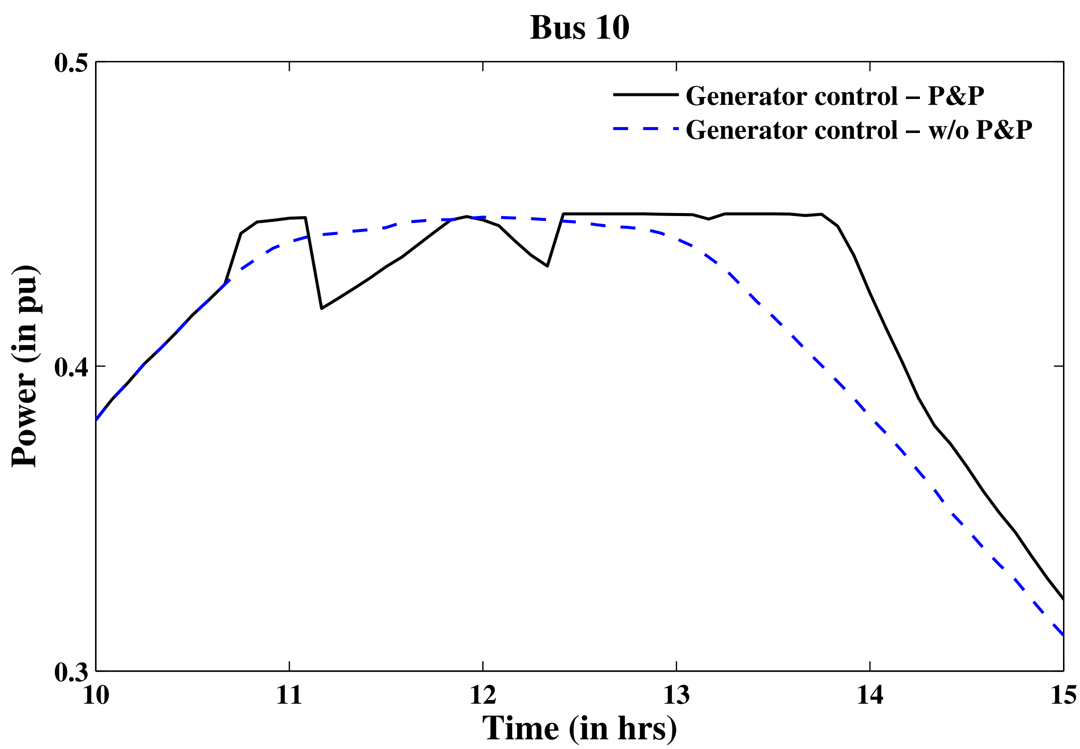

Figure 6.6: Generation control required in example 2 vs when no P&P request is made. More generation control effort is required to stabilize voltage fluctuations caused due to the system modification.

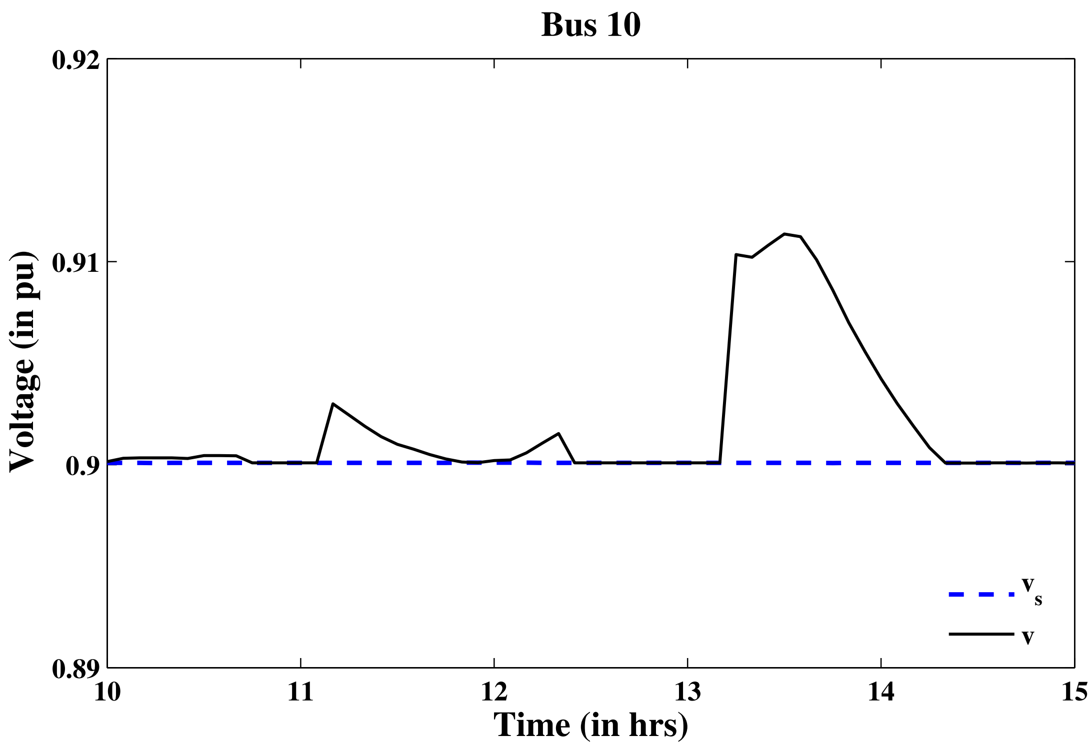

Figure 6.7: Optimal periodic reference and voltage trajectory at bus 10. Voltage deviates from the reference trajectory to accommodate new EVs but again tracks the reference eventually.

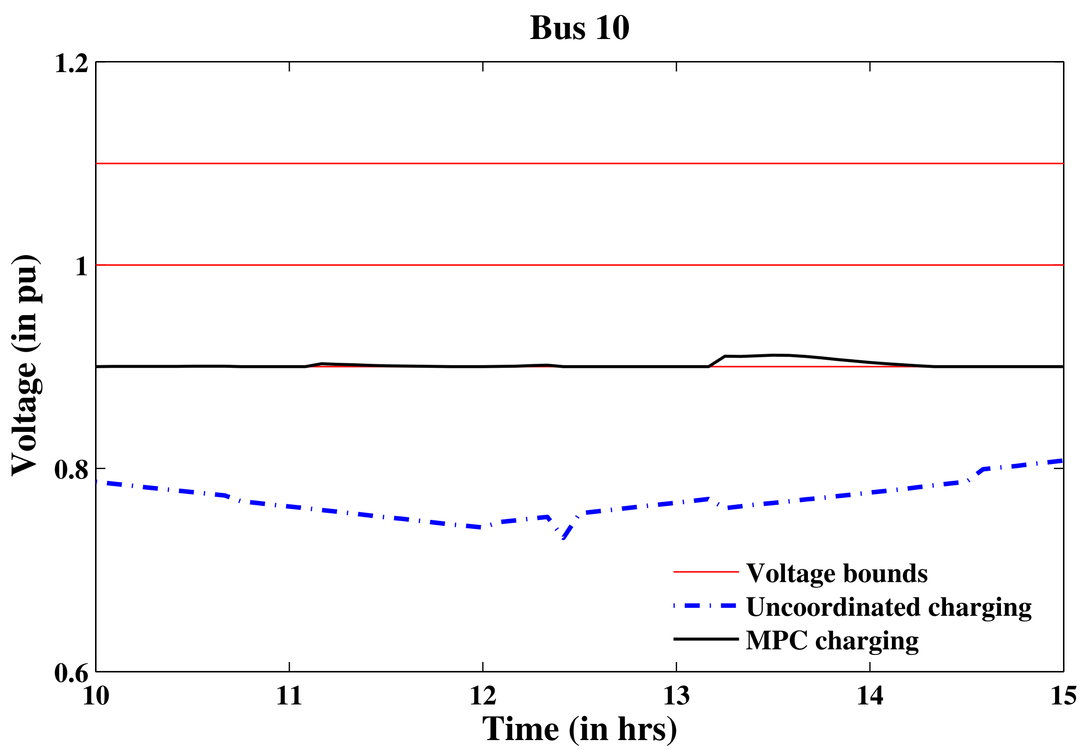

Figure 6.8: Voltage fluctuations in the proposed control scheme and the uncoordinated charging scheme, illustrating the inability of the latter to ensure constraint satisfaction.

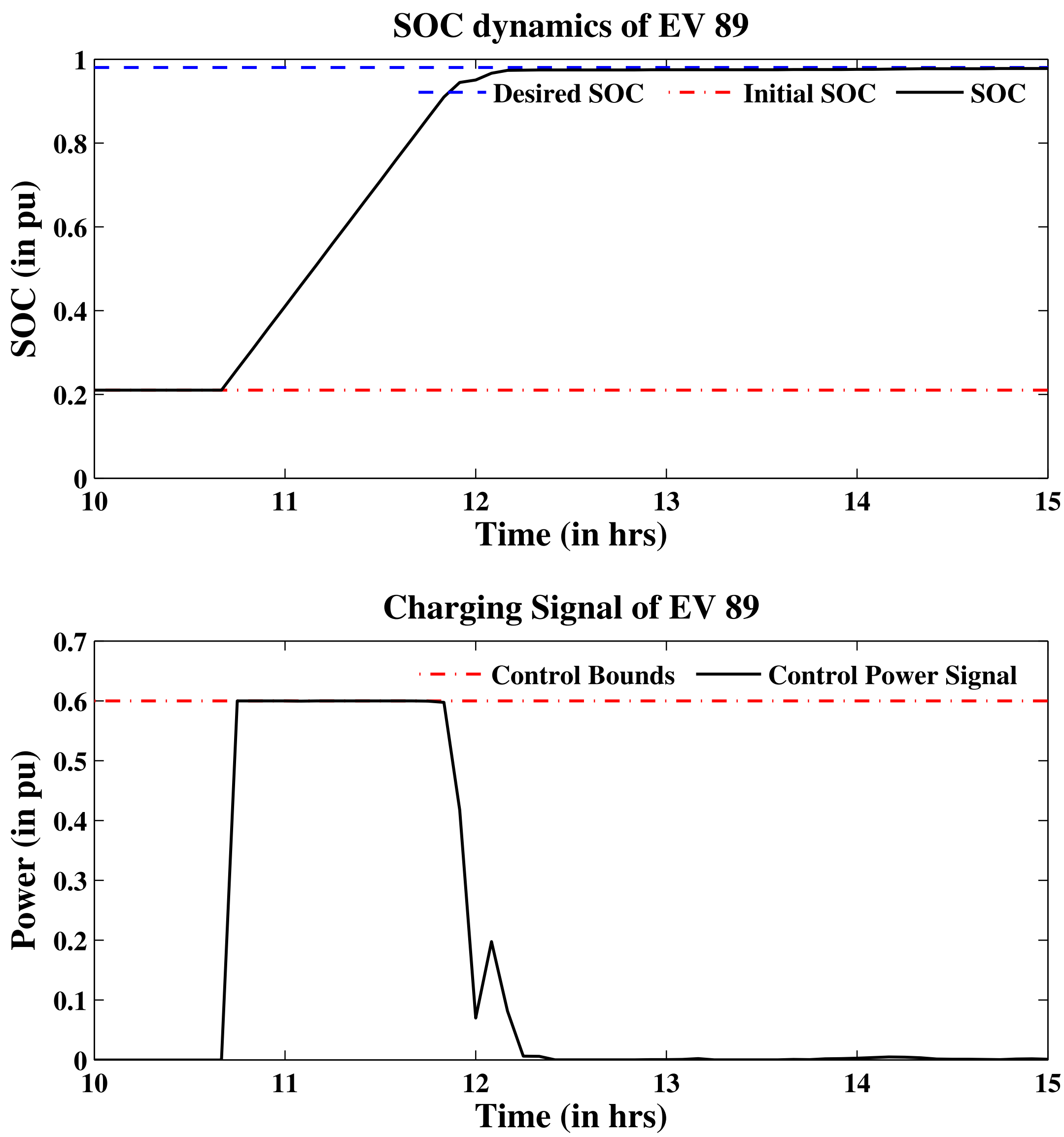

Figure 6.9: SOC dynamics and charging control of the EV connected at $t = 10.8$. EV is charged at the maximum possible charging power so as to accommodate the new EVs - connected at $t = 11.25$ - as soon as possible.

Chapter 7

Conclusion

In this work, we considered the design of a predictive controller that is capable of handling the real-time P&P requests of Electric Vehicles. The proposed controller minimizes the waiting time for connection/disconnection of EVs from the system. We also used the ideas of MPC for tracking periodic reference to achieve an optimal tradeoff between minimizing the voltage fluctuations and minimizing the required generation control in the distribution system. A reachable reference is calculated which is exponentially tracked without error. The exponential convergence of SOCs to their desired values is provided under the assumption of periodic loads, whose future evolution is known. The performance of the proposed method was demonstrated for the control of two radial distribution system, an illustrative example with 9 and a large example with 45 buses was presented.

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