

Using Perspective Rays and Symmetry to Model “Duality”

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Technical Report No. UCB/EECS-2016-13

<http://www.eecs.berkeley.edu/Pubs/TechRpts/2016/EECS-2016-13.html>

March 17, 2016

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Acknowledgement

This work was performed as part of the 2015/16 URAP program under the direction of Professor Carlo H Sequin.

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Abstract

This technical report will detail a process for creating a 3D model of Charles Perry’s sculpture “Duality.” The sculpture can be described as four ribbons whose ends are connected by two 4-way junctions. By plotting points on a picture of the sculpture, parametric rays can be created and manually adjusted to give depth to said points and approximate the sculpture’s shape in 3D space. Inserting multiple pictures with different viewpoints into the CAD environment provides more conditions to guide the approximation. “Duality” also has three mutually orthogonal symmetry axes. This is exploited by only modeling one quarter of the sculpture and then fitting it together with three of its copies, each of which are rotated by 180° around one of the three symmetry axes.

Introduction

While working as a research apprentice under Professor Carlo Séquin for UC Berkeley’s URAP program, I was assigned the task of creating a 3D model of Charles Perry’s sculpture “Duality” (Fig. 1). The CAD software I used was Berkeley’s SLIDE language [1]. With this tool I was able to design and render various 3D geometries, and its capacity for embedded Tcl code allowed for dynamic parametrizations. These parametrizations made working in 3D space much easier, since fine manual adjustments could be made in an interactive manner on the fully rendered model.

My main objective was to develop a systematic way of extracting the sculpture geometry from a few photographs taken at various angles, and turning this shape into a printable CAD model. When a photograph of an object is taken, each point on the object reflects a ray of light that travels in a straight path to the camera lens. Working backwards, rays can be drawn from the camera to points on the photograph. The true location of a point in 3D space lies somewhere on the corresponding ray, and through manual adjustment and comparison with views from other eye points, the overall shape of the object can be approximated.

Exploiting the symmetry of the object drastically cuts down the number of perspective rays needed. This technical report will walk through these steps for modelling “Duality,” so that readers may also utilize this method in their own CAD projects.

The Geometry of “Duality”

“Duality” is a 2-sided 2-manifold surface with two borders. One way to describe the sculpture is as four ribbons whose ends are connected by two 4-way junctions. The top and bottom ribbons both have 360° twists, and the middle two ribbons have no twist. The sculpture also has three C_2 rotational symmetry axes at right angles with each other (Fig. 1c) for which a 180° rotation results in the exact same shape. This means that only one quarter of the sculpture needs to be modeled, and then, by rotating three copies of the modeled piece around different axes, the entire sculpture comes together.

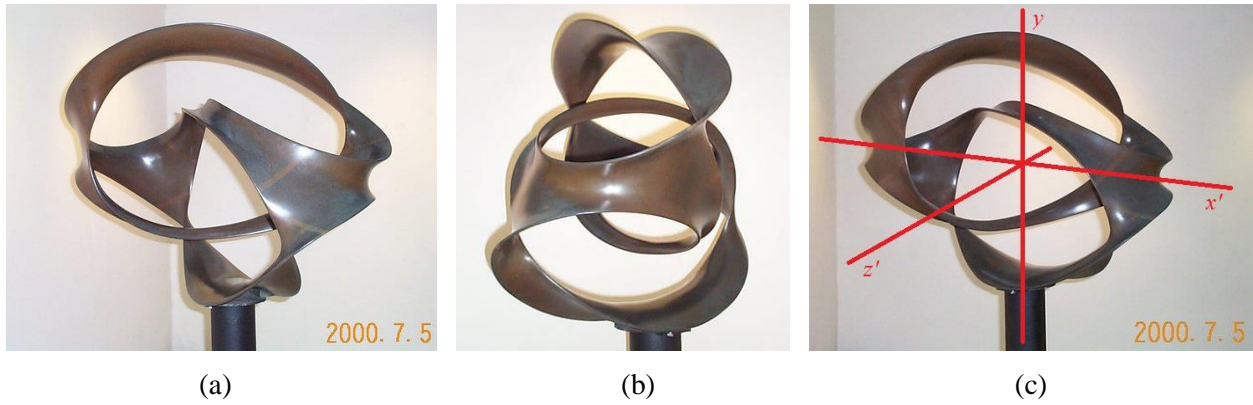


Figure 1: (a) Charles Perry's "Duality"; (b) alternate view, showing one "4-way junction";
 (c) C_2 symmetry axes drawn.

Perspective Rays

After identifying exploitable symmetries, the next step is to create a frame by locating the 3D coordinates of some points on the edges of the sculpture. Take Figure 1a as our primary view. We will treat the photograph as if it exists in the x - y -plane. Points on this photograph are selected and their x and y coordinates are recorded with respect to an arbitrarily chosen origin (Fig. 2a). Then, parametric rays are created by selecting a point on the z -axis, where one believes the camera to have been when the photograph was taken (Fig. 2b). For the pictures of "Duality" that I was provided, the photographer's distance from the sculpture were unknown, so I left this quantity as an adjustable variable.

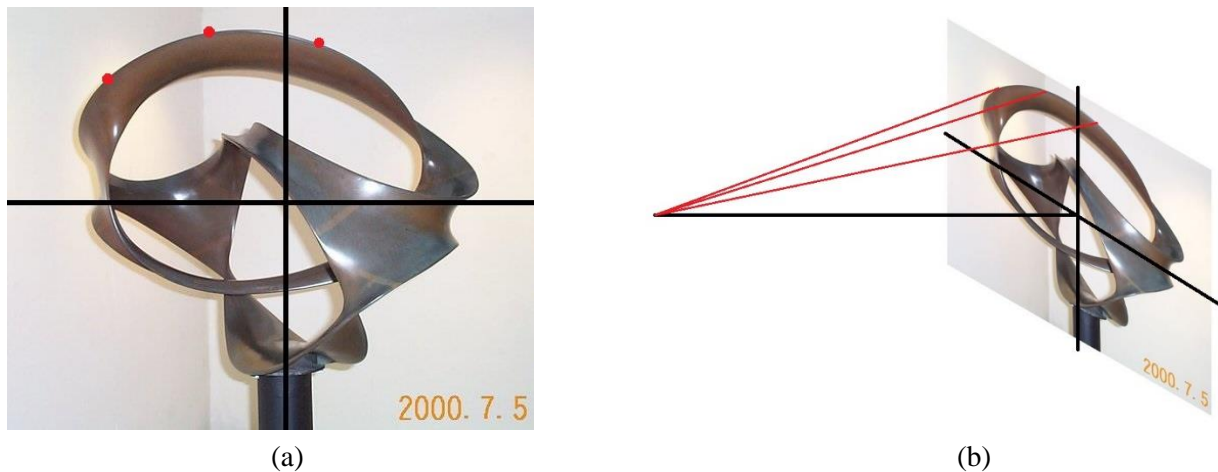


Figure 2: (a) Points plotted for x_0 and y_0 values; (b) corresponding perspective rays from point $(0, 0, z_0)$.

The parametric equation for a perspective ray is as follows, where x_0 and y_0 are the coordinates of the point on the photograph, and z_0 is the location of the camera:

$$(x, y, z) = (x_0 t, y_0 t, z_0(1 - t)) \quad (1)$$

As seen from the equation, when the t -value is 1, the point will lie in the x - y -plane, and when the value is 0, the point is at the camera location. Once enough perspective rays have been programmed, each t -value is set to 1 so that each point's initial position lies directly on its location in the photograph (Fig. 3a). The picture of the primary view is imported into the CAD model to ensure all the points line up properly, and the points are connected with polylines so that visualizing the edges of the sculpture is easier.

Tcl sliders are programmed to interactively control each point's t -value. Reducing a point's t -value brings the point closer to the camera, and increasing it pushes the point away. The information from one photograph is not enough to give the points' depths, so another photograph with a different view of the sculpture is imported (Fig. 3b). The Tcl sliders are then used to line up each point with the second view as best as possible (Fig. 3c).

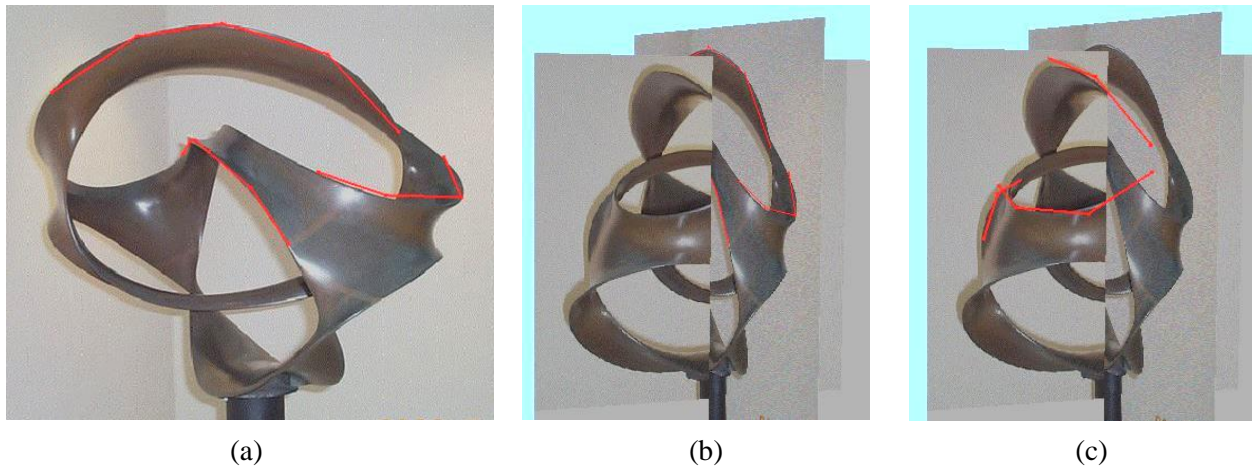


Figure 3: (a) Polylines placed over the primary view in SLIDE; (b, c) t -values adjusted to provide depth.

Adjusting the C_2 Symmetry Axes

Reexamining Figures 1a - 1c, it is clear that one of the C_2 symmetry axes is the vertical y -axis. Creating a rotated copy of a point for this axis is as simple as negating the x and z values of the original point. The other two symmetry axes are orthogonal and lie in the x - z plane, but it is unclear in which directions they point. Given that the symmetry axes x' and z' are at an angle α from the true x and z axes (Fig. 4a), I made α an adjustable variable, which is again controlled by a Tcl slider. In order to create the reflected points, a rotation matrix is used.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 2 \sin^2 \alpha - 1 & 0 & -\sin(2\alpha) \\ 0 & -1 & 0 \\ -\sin(2\alpha) & 0 & 2 \cos^2 \alpha - 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (2)$$

With the unprimed coordinates representing the original points, the primed coordinates are the copies rotated 180° about the z' -axis. For the copies rotated about the x' -axis, one can apply this matrix to the copies that were rotated about the y -axis.

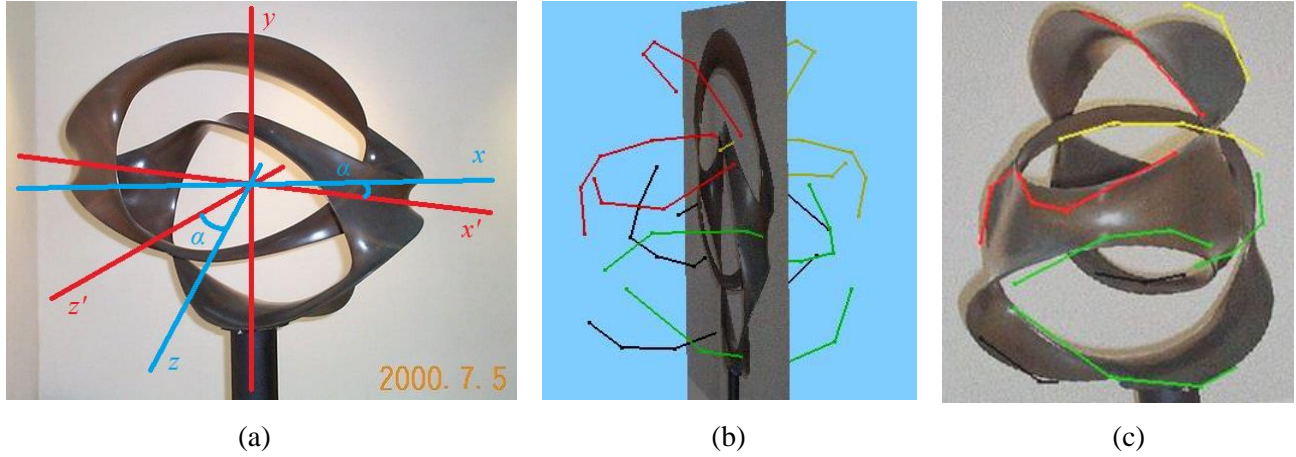


Figure 4: (a) Fig. 1c with true axes in blue; (b) original points in red; rotated copies in yellow, green, and black corresponding to the y , x' , and z' axes respectively; (c) α adjusted to align with 2nd view

Making a Model

The steps detailed so far are enough to model all the edges of “Duality.” However, the geometry entails more than the edges, because the ribbons and junctions also have lateral curvature. To model these “channels,” midpoints between the two edges are created and then offset in the direction of the ribbon’s surface normal. The position of a point p_i in the middle of the curved ribbon is given by the following vector equations:

$$\vec{p}_i = \vec{m}_i + \vec{c}_i \quad (3)$$

$$\vec{m}_i = \frac{\vec{v}_i + \vec{u}_i}{2} \quad (4)$$

$$\vec{c}_i = \frac{\vec{v}_i - \vec{u}_i}{2} \times \frac{\vec{m}_{i-1} - \vec{m}_{i+1}}{|\vec{m}_{i-1} - \vec{m}_{i+1}|} \quad (5)$$

The vector m_i represents the midpoint between v_i and u_i , which are two points on opposite edges of the ribbon. The midpoint m_i is then translated by the vector c_i to give point p_i . This translation vector c_i is the cross product of the lateral direction of the ribbon and the lengthwise direction of the ribbon. The lateral direction is defined by a vector pointing from v_i to u_i . The lengthwise direction is defined by a vector between the two neighboring midpoints of m_i , which are denoted by vectors m_{i-1} and m_{i+1} , respectively. The magnitude of c_i defines the depth of the channel; by default it is set to be half of the ribbon’s width. A lower magnitude can be used for areas where the ribbons flatten out. The sign of c_i is not reflected in these equations because it depends on the specific channel.

Each point on an edge is paired with the nearest point on the opposite edge to make a midpoint, and this works out with no leftover points because opposing edges are rotated copies of each other. At the 4-way junctions, a few midpoints are constructed using other midpoints, but the formulas are still the same.

Finally, with the edges and the translated midpoints all in place, a mesh of faces can be generated (Fig. 5a). By running the faces through a few iterations of Catmull-Clark subdivision [2], a first smooth geometrical model is created (Fig. 5b).

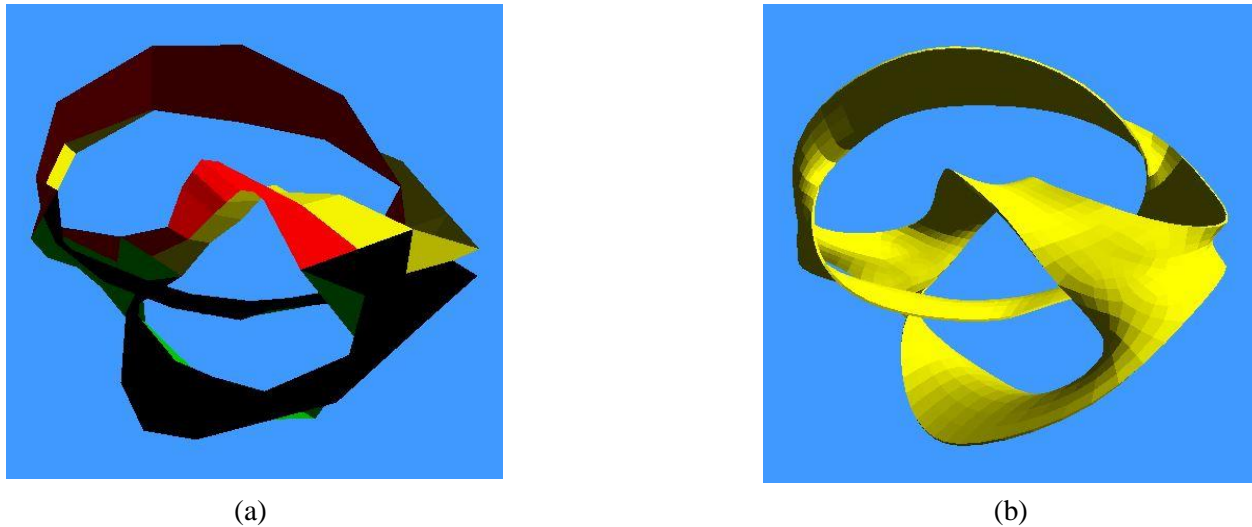


Figure 5: (a) Mesh of faces, with colors denoted in Fig. 4b; (b) smooth CAD model of “Duality”

In order to make the CAD model a physical reality, the SLIDE offset module is used to give the surface some thickness. The module creates two offset surfaces above and below the original surface and connects them along their borders. A 2mm thickness is sufficient for printing a small, robust model 3-4 inches in size on a 3D printer that uses Fused Deposition Modeling (Fig. 6a). Figure 6b shows the final model after all the support structure has been removed.

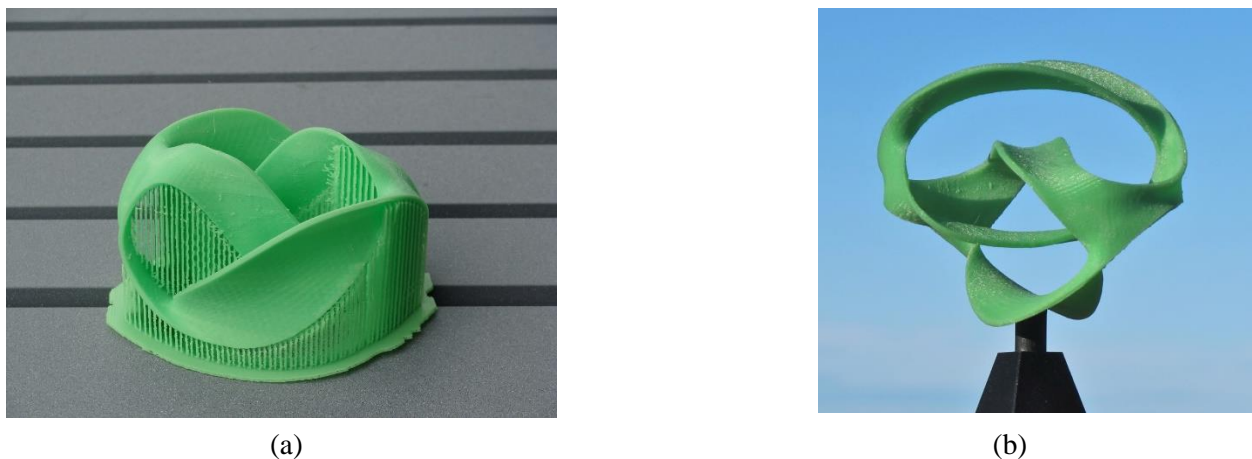


Figure 6: (a) Model as it comes out of the machine; (b) cleaned-up and mounted on a small black stand

Conclusions

The printed model comes out fairly close to the “Duality” sculpture depicted in the photographs. There are a few noticeable differences; some are due to unknown parameter values that made the perspective rays imprecise. Other flaws are simply due to the fact that modeling a 3D object on the 2D computer screen is difficult; some shape details are less visible while the model is still in digital CAD form. Comparing the 3D model to the available photographs shows that the top and bottom ribbons are approximated well, but the middle two ribbons are too wide and should be smoother. Moreover, the 4-way junctions aren’t wide enough, and their “channel” curvatures flatten out too soon. Nevertheless, the overall geometry has been captured adequately, and its symmetry is clearly visible.

The perspective rays are a useful tool for parameterizing a 3D model, but even with only around a dozen t -values that need to be adjusted manually, this model still inconvenient to fine-tune. For example, in order to make a ribbon narrower, each point on the ribbon must be repositioned individually until the entire ribbon has the desired width. One way to ameliorate this would be to define each ribbon primarily by its central sweep path, and define the rim points as symmetrical pairs, offset from this path by a parameterized ribbon-width.

To improve the overall smoothness of the model, some automated post-processing could be applied. In this process the sequence of points that defines the sweep path of a ribbon would be fine-tuned algorithmically so as to minimize the change in each ribbon’s longitudinal curvature.

References

- [1] J. Smith, *SLIDE design environment*. (2003). – <http://www.cs.berkeley.edu/~ug/slide/>
- [2] E. Catmull and J. Clark. *Recursively generated B-spline surfaces on arbitrary topological meshes*. *Computer-Aided Design* 10 (1978), pp 350-355.