Capturing the impact of navigational app usage on road traffic from a gametheory point of view

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by

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Abstract

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The rise of mobile internet has changed routing behavior in traffic networks. With ubiquity of traffic information and the increased use of routing apps, urban and suburban areas in the US and abroad have seen a recent rise in "cut-through" traffic and related congestion patterns. The phenomenon is shown to have negative impact on communities near congested major roads. Chapter 1 presents data that shows the rerouting phenomenon from highway to side-roads on the I-210 corridor in the LA Basin.

The differences in the routing behavior of routing-app users and non routing-app users remains uncertain to this day. Therefore, the impact of these applications on traffic is unclear. Chapter 2 introduces three traffic assignment models that differentiate app users' and non-app users' routing behaviors. Two of them are based on static traffic assignment models – the so called **cognitive cost path choice model** and the **restricted path choice model** – and the third one on dynamic traffic assignment models using the microsimulator *Aimsun*.

Chapter 3 provides a criterion to evaluate the impact of routing apps usage on road traffic at a macroscopic level: **the average marginal regret**. Derived from game theory, the average marginal regret of an observed state of traffic can be seen as a distance between this observed state and a user equilibrium state of traffic (the so called Nash equilibrium for routing in traffic networks). Experiments – using the two previously introduced static models – demonstrate that the average marginal regret decreases with an increase of app usage. Similar results are shown using the dynamic model in *Aimsun*. A sensitivity analysis of the restricted path choice model equilibrium with respect to the app usage ratio even proves that the average marginal regret monotonically converges to zero with an increase of app usage. Therefore, chapter 3 shows that an increase of app usage stirs a state of traffic toward a user equilibrium state. This is plausible as one can expect such property. Recall that a user equilibrium is most likely not socially optimal. Therefore, app usage might leads to an increase of the average travel time in the network at a system level.

To Marine Gasc and Dr. Alexander Keimer

For supporting the work I do from two totally different perspectives.

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Acknowledgments

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Chapter 1

The rise of routing using navigational applications and the impact on communities

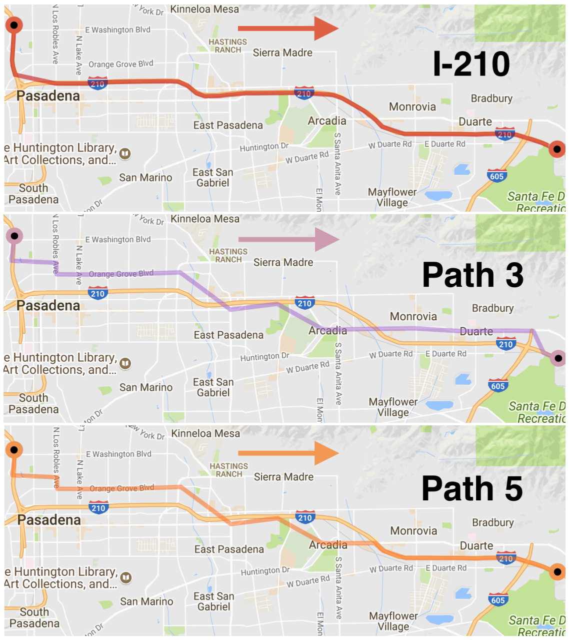
1.1 Context

The importance of reducing traffic. Road traffic costs the U.S. billions in GDP every year [7]. Reducing road congestion is a way to reduce fuel consumption and emissions, to increase economic productivity and to improve the daily life of motorists. During the last decade, new mobility services have grown with the rise of the mobile internet (*mobility as a Service*, *GPS-routing apps*, *carpooling*). These new mobility services have not translated in reduced road congestion however. On the contrary, some recent traffic services have been blamed for contributing to alleged new congestion patterns [15, 9, 17, 16] due to the increase in "through traffic".

New traffic patterns generated by routing apps. The last decade has witnessed the explosion of cell phone use [5], in particular in the context of mobility. Today, INRIX, HERE, Google, Apple, Waze or TomTom are used by a large number of motorists [3]. These routing apps have created new traffic patterns called *cut-through* traffic [17]. As the number of app users increases, arterial roads are subjected to higher flows of vehicles due to cut-through traffic [4]. Solving this problem will require new public policy approaches, such as regulation of congestion, and routing to reduce the negative externalities of cut-through traffic. [12].

Selfish routing

Routing apps typically provide vehicles with shortest paths based on the current state of the network [20]. We assume that people using apps are routed on the fastest possible path based on the current state of the network. This is desirable for individuals looking to minimize their own travel time. Unfortunately, shortest travel time routing (i.e. "selfish" routing) does



Path	I-210	2	3	4	5
Distance (mi)	13.7	15.8	14.8	14.6	14.5
Free flow travel time (min)	13	34	31	26	25

Figure 1.1: I-210 freeway section in Passadena, CA., with four alternative arterial paths and the distance and the free flow travel time of each path.

not lead to socially optimal travel patterns [11, 14]. At best, traffic conditions under selfish routing can achieve a *Nash equilibrium*, sometimes also referred to as Wardrop's equilibrium in traffic theory [11, 19].

1.2 Analysis of reroutes from loop and arterial probe data [4]

In this section, we use field data to quantify reroute phenomena for a highly congested day. We then present trends over a longer period to display increasing travel times on arterial roads due to an increase in arterial flow and a subsequent decrease in speed.

A sub-network of the LA Basin along the eastbound I-210 corridor is studied. Because of the geography of the corridor, paths parallel to the I-210 are good candidates for alternate routes. Five paths (see Figure 1.1) that lead from northwest Pasadena to Azusa (in northeast Los Angeles) are considered, one along I-210 and four alternative routes. These paths were chosen among the routinely suggested routes provided by Google Maps. The four alternative paths thus provide viable rerouting options for app users when the freeway is highly congested and, hence, it is expected that, in these scenarios, drivers tend to reroute over the alternative paths (paths 2,3,4,5 see Figure 1.1), as opposed to staying on the high-capacity route (path 1 see Figure 1.1).

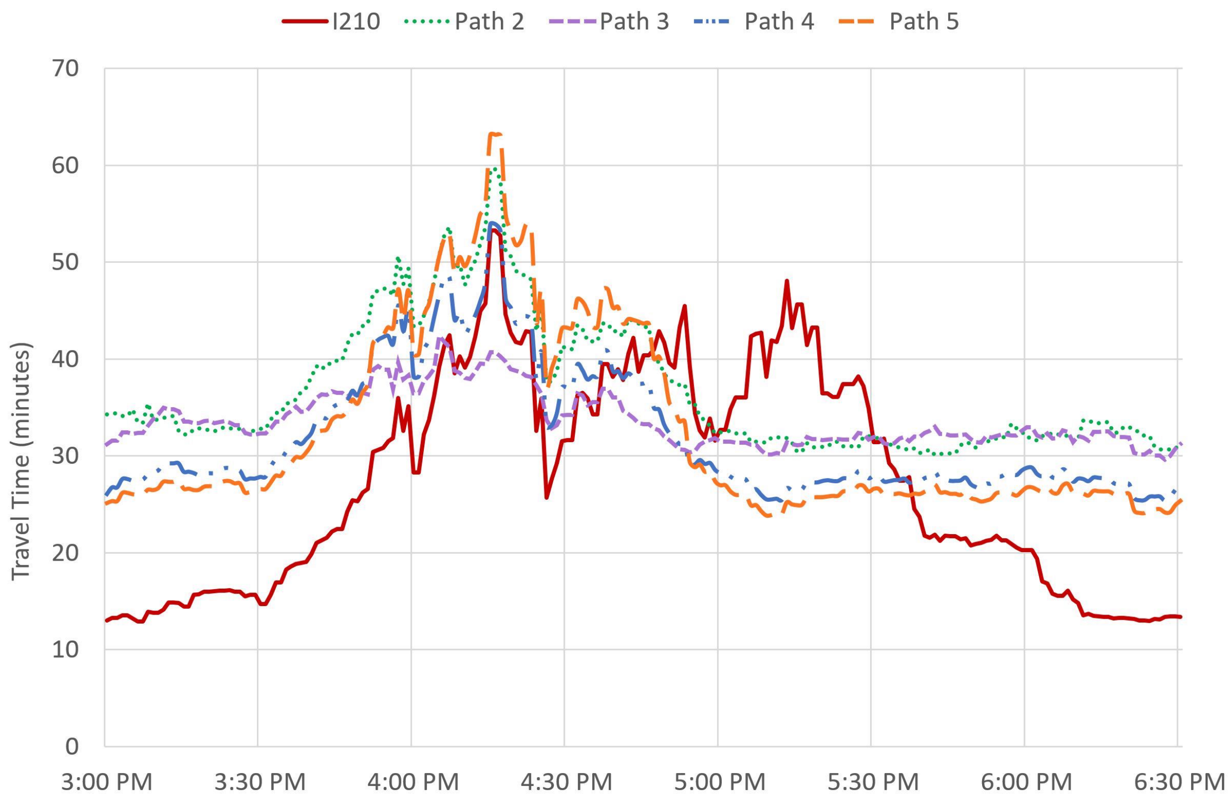


Figure 1.2: Evolution of travel times on the paths parallel to the I-210 considered in Figure 1.1 (March 10, 2014).

Data source

Data used in this analysis is taken from both INRIX and the Performance Measurement System (PeMS) [18]. INRIX data includes instantaneous link speeds (from which link travel times can be calculated) from 2014 through 2015. The travel times of the five considered paths have been computed from this data. We chose instantaneous travel time but could have chosen achieved travel time.

Travel time equalization during peak congestion

This section assesses the rise in travel time on the I-210 and the equalization of travel times between this freeway and the arterial routes parallel to it. Compared to average conditions shown in INRIX, the domain of interest shown in Figure 1.1 was highly congested during the evening peak hours (3:30 PM until 6:30 PM) on March 10, 2014.

As seen in Figure 1.2, at the start of the peak hour on March 10, 2014, around 4 PM, the travel time spikes, along with the travel times of the alternative paths. Corresponding with this spike is a decrease in the average difference in travel time between alternative paths and the I-210 freeway. At the beginning and end of the peak hour, the travel time on the freeway is close to the free flow travel time, and the freeway is faster than the arterial road routes. However, when the freeway travel time increases, the arterial detours become beneficial alternative routes (up to 20% faster at 4:20). This figure shows that in high congestion, drivers can reroute themselves to arterial roads in order to reduce their travel time, leading to travel time equalization among possible parallel routes.

This increase and convergence of travel times is expected, as drivers using navigation applications are rerouted to arterial roads to minimize their own travel times.

Observation of yearly trends

In addition to observing the effect of navigational apps during peak hour of a single day, we examine trends on a larger time scale. As the number of app users increases, it is expected that the travel time on arterial streets will increase as well, due to increased flow rerouting around congestion on the freeway.

This phenomenon is observed in the evolution of the travel times shown in Figure 1.3. The average travel time on the paths was computed during peak hours for each week from January to June in each year. As expected, in a one year time span, the travel times along the alternative paths increased by roughly 20%, i.e around five minutes. However, without direct access to Google Maps/Waze user data, we cannot be certain that rerouted cars actually used navigation apps. Possible alternate reasons for this increase could include demographic growth, urban activities development or other causes.

Figure 1.3 also shows that the I-210 is always faster than the arterial roads. The paths were suggested by Google Maps and chosen because they contained primarily arterial roads (which made computation of travel time easier). It is possible that the reroute paths suggested by apps are highly variable over time. These variable paths could include only portions of the arterial roads that are considered here. This would still explain the increase in the travel times of the alternative paths.

The travel time on the I-210 path oscillates around 15 minutes over the two years, remaining roughly constant over time. This occurrence can be explained by *latent demand* or by the very low marginal cost of extracting vehicles from the freeway (when a typical freeway lane capacity is around 2000 veh./h).

Additionally, PeMS data was analyzed, which consisted of flow data from inductive loop sensors embedded along the I-210 roadway (path 1) and its associated ramps. The median (chosen to reduce the effect of outliers) of the total evening peak flow was found over the weekdays of the month of March. Specifically, flows exiting the freeway were analyzed at 4 different off ramps and the yearly trend was examined. The selected off ramps include exits commonly suggested by applications like Google Maps (Michillinda Ave and Baldwin Ave exits), and the exits directly before and after them. All off ramps along this route were examined and the selected off ramps showed significant change as they were most often suggested by the applications. Figure 1.3 presents the evolution of the average travel time

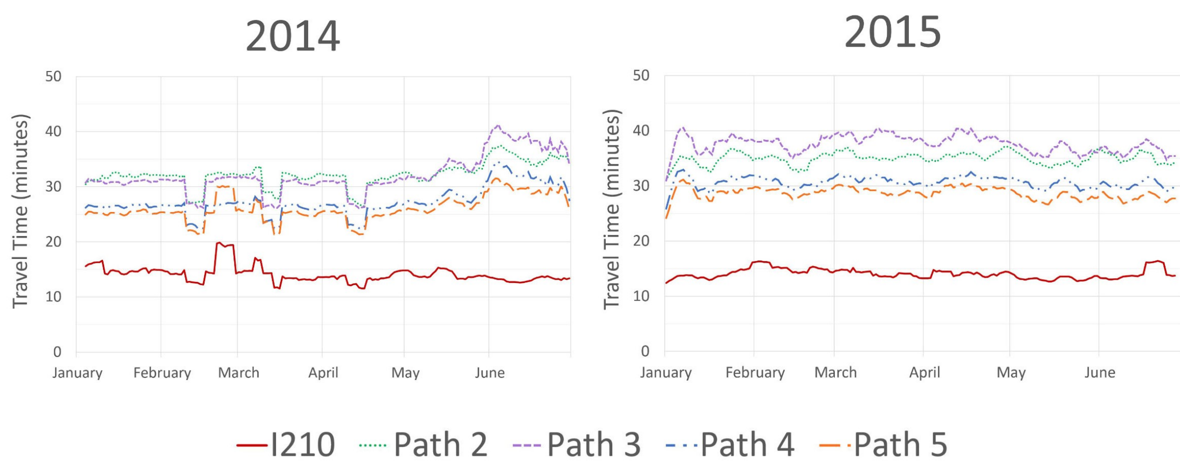


Figure 1.3: Average travel time computed with INRIX data on the five paths during peak hours (4:30 to 5:30 PM) in 2014 and 2015 from January to June.

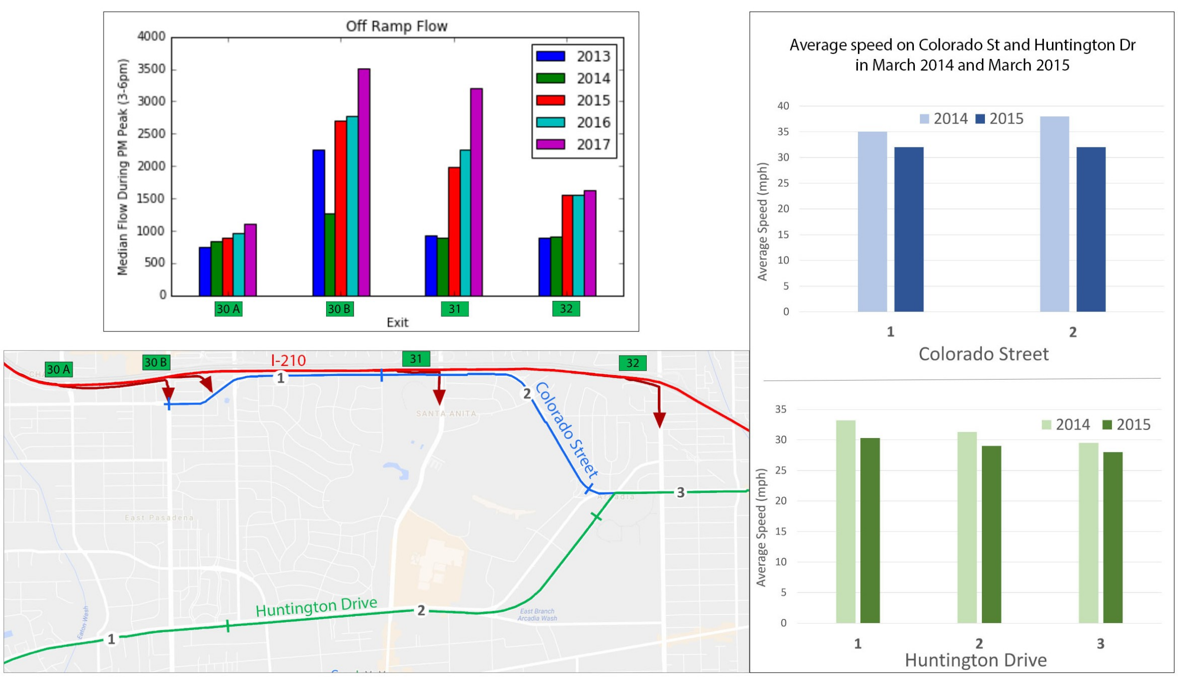


Figure 1.4: Median off ramp flow from I-210 during March weekday peak hours IN 2013, 2014, 2015, 2016 and 2016 and the evolution of speed on parallel paths.

computed with INRIX data on the five paths during peak hours (4:30 to 5:30 PM) considered in 2014 and 2015, for each week from January to June. While the travel time on the I-210 remains roughly constant over two years, alternative paths suffer a 20% increase in travel time. The observed drops in 2014 are irregularities from data flaws.

Figure 1.4 presents the evolution of the median off ramp flow from I-210 during March weekday peak hours and the evolution of speed on parallel paths. A significant increase in flow using these off ramps is observed between 2013 and 2017, as shown in Figure 1.4. Specifically at the exits for Michillinda Avenue and Baldwin Avenue, we see a 1.5- and 3-fold increase respectively over 4 years. While some of this increase can be explained by an increase in demand to Arcadia, it is unlikely that this can be explained solely by demand growth. Additionally, since these exits are often suggested by navigation-apps which have increased in popularity during the same time period, it is likely that the increased flow can be partially explained by app usage. The significant increase in off ramp flow that occurs over the years coupled with the decrease in speed on parallel paths provides evidence in favor of app-induced arterial rerouting patterns.

Chapter 2

Model of information-enabled routing

2.1 Static models [4]

Two models are introduced in this section to capture the rerouting phenomenon due to app usage. In numerous traffic network models, drivers are assumed to possess perfect information over the state of the network. The first model presented is the *cognitive cost model*. It separates drivers into two populations: those who use navigational applications to route themselves (app users) and those who do not (non-app users). In this model, non-app users incur a "cognitive cost" to access arterial roads. However, this cost does not depend on the particular journeys of non-app users, as they are uniformly discouraged from taking arterial roads. This type of modeling does not perfectly account for an actual lack of information on behalf of non-app users.

In order to extend the set of features encompassed by the aforementioned model, we also introduce a new mathematical approach that integrates this lack of information differently is presented - the *restricted path choice* model. This approach considers the limited knowledge of non-app drivers by restricting their path-choice set, which is also dependent on the specific *od* pair of the trip.

We first introduce general network notation, which applies to both models.

Framework

Definition 2.1.1 (Network, paths, and demand)**.** *Given a finite strongly connected directed graph $\mathcal{G}$ with vertex set $\mathcal{V}$ and edges set $\mathcal{E}$, i.e. $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, for each origin $o \in \mathcal{V}$ and destination $d \in \mathcal{V}$:*

- *Let $\mathcal{P}_{od}$ be the set of feasible paths without cycles from o to d.*
- *Let $d_{od} \geq 0$ be the total number of vehicles that make the journey $o \to d$, per unit of time. We denote the demand matrix $\boldsymbol{d} = (d_{od})_{o,d \in \mathcal{V}}$.*

Definition 2.1.2 (Path flows). *For each path* $\mathbf{p} \in \mathcal{P} = \bigcup_{o,d \in \mathcal{V}} \mathcal{P}_{od}$*:*

- *Let* $h_{\mathbf{p}}$ *be the flow of vehicles using path* $\mathbf{p}$*, in vehicles per unit of time (path flows). We denote the path flow vector* $\mathbf{h} = (h_{\mathbf{p}})_{\mathbf{p} \in \mathcal{P}}$*.*
- *We define* $\delta_{\mathbf{p}} := (\delta_{\mathbf{p}}(e))_{e \in \mathcal{E}}$*, where* $\delta_{\mathbf{p}}(e) = \begin{cases} 1 & \text{if } e \in \mathbf{p} \\ 0 & \text{else} \end{cases}$

  This is the indicator vector of the links included in path $\mathbf{p}$*. We denote the incidence matrix* $\Delta = (\delta_{\mathbf{p}})_{\mathbf{p} \in \mathcal{P}}$

Definition 2.1.3 (Link flows). *For each link* $e \in \mathcal{E}$*, let* f_e *be the flow of vehicles using link* e *per unit of time (link flow). We denote the link flow vector* $\mathbf{f} = (f_e)_{e \in \mathcal{E}}$*.*

Remark 2.1.1 (Static model). *Static equilibrium conditions are assumed. This assumption is commonly made in 15 minutes increments in the practitioner's community. Therefore, for any path* $\mathbf{p}$*,* $h_{\mathbf{p}}$ *remains constant over time and* $\mathbf{f} = \Delta \mathbf{h}$*.*

Definition 2.1.4 (Feasible assignment). *Given a demand matrix* $\boldsymbol{d} \in \mathbb{R}_+^{|\mathcal{V}| \times |\mathcal{V}|}$*:*

- *Let* $\mathcal{H}_{\boldsymbol{d}} = \left\{ \mathbf{h},\ \mathbf{p} \in \mathcal{P} : h_{\mathbf{p}} \in \mathbb{R}_+,\ o, d \in \mathcal{V} : \sum_{\mathbf{p} \in \mathcal{P}_{od}} h_{\mathbf{p}} = d_{od} \right\}$ *be the set of feasible path flow allocations.*
- *Let* $\mathcal{F}_{\boldsymbol{d}} = \{\Delta \mathbf{h},\ \mathbf{h} \in \mathcal{H}_{\boldsymbol{d}}\}$ *be the set of feasible link flow allocations.*

Definition 2.1.5 (Travel time). *For each link* $e \in \mathcal{E}$*, let* t_e *be the travel time on link* e*. We denote the travel time vector* $\mathbf{t} = (t_e)_{e \in \mathcal{E}}$*. For each path* $\mathbf{p} \in \mathcal{P}$*, we define* $t_{\mathbf{p}}$ *as the travel time on path* $\mathbf{p}$ *as the sum of the travel times on each link that is included in path* $\mathbf{p}$*, i.e.* $t_{\mathbf{p}} = \delta_{\mathbf{p}}^{\intercal} \cdot \mathbf{t}$*.*

Remark 2.1.2 (Separability of travel time). *For each* $\mathbf{f} \in \mathcal{F}_{\boldsymbol{d}}$*, we denote* $\mathbf{f} = (f_e)_{e \in \mathcal{E}}$ *(link flow allocation).* t_e *is assumed to be only a function of* f_e*:* $t_e(f_e)$*. So* $\mathbf{t}(\mathbf{f}) = (t_e(f_e))_{e \in \mathcal{E}}$*.*

Definition 2.1.6 (Cost function). *For every path allocation* $\mathbf{h} \in \mathcal{H}_{\boldsymbol{d}}$*, the cost function of each path is given by* $\mathbf{p} \in \mathcal{P}$ *by* $t_{\mathbf{p}}(\Delta \mathbf{h})$*.*

Definition 2.1.7 (User equilibrium). *Given a traffic demand* $\boldsymbol{d}$*, a path allocation* $\mathbf{h} \in \mathcal{H}_{\boldsymbol{d}}$ *is a user equilibrium if and only if:*

$$\forall o, d \in \mathcal{V},\ \forall \mathbf{p} \in \mathcal{P}_{od},\ h_{\mathbf{p}} \cdot (t_{\mathbf{p}}(\mathbf{h}) - \min_{\mathbf{q} \in \mathcal{P}_{od}} t_{\mathbf{q}}(\mathbf{h})) = 0 \tag{2.1}$$

Remark 2.1.3 (Wardrop's first condition [19]). *At a* ***user equilibrium****, the travel time on all used routes between an od pair are equal, and less than those which would be experienced by a single vehicle on any unused route in the network.*

Cognitive cost routing model

In the cognitive cost model [17], drivers are separated into two categories: app users and non-app users. App users have perfect knowledge of the traffic network. Their cost function is their travel time, as opposed to non-app users who are made to pay a multiplicative "cognitive cost" C for accessing arterial roads. This cognitive cost C is mathematically accounted for in the cost function of non-app users, $t^{nr}_{\mathbf{p}}$: $t^{nr}_{\mathbf{p}} = \delta^{\intercal}_{\mathbf{p}} \cdot (c^{nr}_e)_{e\in\mathcal{E}}$ where:

$$\forall e \in \mathcal{E},\ c^{nr}_e(f_e) = \begin{cases} C \cdot t_e(f_e) & \text{if } e \text{ arterial road} \\ t_e(f_e) & \text{if } e \text{ freeway} \end{cases} \tag{2.2}$$

We denote $t^r_{\mathbf{p}}$ the cost function for a routed users on the path $\mathbf{p}$.

Definition 2.1.8 (Flow of routed and non-routed users, feasible path flow allocation)**.** *For any $\mathbf{p} \in \mathcal{P}$, $h^r_{\mathbf{p}}$ and $h^{nr}_{\mathbf{p}}$ are the flow of routed users and non-routed users on the edge e, respectively. Thus, we have $h_{\mathbf{p}} = h^{nr}_{\mathbf{p}} + h^r_{\mathbf{p}}$. Likewise, $\mathcal{H}^r_{\boldsymbol{d}}$ and $\mathcal{H}^{nr}_{\boldsymbol{d}}$ are the feasible path flow allocation given the demand of app users, d^r_{od}, and non-app users, d^{nr}_{od}. We denote $\mathbf{h}^r = (h^r_{\mathbf{p}})_{\mathbf{p}\in\mathcal{P}} \in \mathcal{H}^r_{\boldsymbol{d}}$ and $\mathbf{h}^{nr} = (h^{nr}_{\mathbf{p}})_{\mathbf{p}\in\mathcal{P}} \in \mathcal{H}^{nr}_{\boldsymbol{d}}$ and we have $d_{od} = d^r_{od} + d^{nr}_{od}$.*

Property 2.1.1 (Cognitive cost variational inequality)**.** *In this model, computing the user equilibrium (Wardrop's conditions) is equivalent to:*
Finding $(\mathbf{h}^r, \mathbf{h}^{nr}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$, such that:

$$\forall (\mathbf{g}^r, \mathbf{g}^{nr}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}},\ t^r_{\mathbf{p}}(\mathbf{h})^{\intercal} \cdot (g^r_{\mathbf{p}} - h^r_{\mathbf{p}}) + t^{nr}_{\mathbf{p}}(\mathbf{h})^{\intercal} \cdot (g^{nr}_{\mathbf{p}} - h^{nr}_{\mathbf{p}}) \geq 0 \tag{2.3}$$

Remark 2.1.4. *The equation above can be simplified. If one defines $c(\mathbf{h}) = (c^r(\mathbf{h}), c^{nr}(\mathbf{h}))^{\intercal}$, and $\tilde{\mathbf{h}} = (\mathbf{h}^r, \mathbf{h}^{nr})$, then the inequality is equivalent to :*

$$\forall \mathbf{g} \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}},\ c(\mathbf{h})^{\intercal} \cdot (\mathbf{g} - \tilde{\mathbf{h}}) \geq 0 \tag{2.4}$$

Remark 2.1.5. *For any path $\mathbf{p}$, and for any $\mathbf{g} = (\mathbf{g}^r, \mathbf{g}^{nr}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$, denote $g_{\mathbf{p}} = (g^r_{\mathbf{p}}, g^{nr}_{\mathbf{p}})$ and define $c_{\mathbf{p}}(\mathbf{h}) \otimes g_{\mathbf{p}} = c^r_{\mathbf{p}}(\mathbf{h}) \cdot g^r_{\mathbf{p}} + c^{nr}_{\mathbf{p}}(\mathbf{h}) \cdot g^{nr}_{\mathbf{p}}$*

Proof: Suppose Wardrop's user equilibrium is reached at $\tilde{\mathbf{h}} = (\mathbf{h}^r, \mathbf{h}^{nr}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$. Therefore, if $h^r_{\mathbf{p}} > 0$, then $c^r_{\mathbf{p}}(\mathbf{h}) = \pi^r_{o,d} := \min_{p\in\mathcal{P}} c^r_{\mathbf{p}}$, and if $h^{nr}_{\mathbf{p}} > 0$, then also $c^{nr}_{\mathbf{p}}(\mathbf{h}) = \pi^{nr}_{o,d} := \min_{p\in\mathcal{P}} c^{nr}_{\mathbf{p}}$, whenever $\mathbf{p}$ connects o to d. Hence, if $\pi_{o,d}$ denotes the vector $(\pi^r_{o,d}, \pi^{nr}_{o,d})$, then

$$\pi_{o,d} \otimes \tilde{\mathbf{h}} = c_{\mathbf{p}}(\mathbf{h}) \otimes \tilde{\mathbf{h}}$$

Let $\mathbf{g} := (g^r_{\mathbf{p}}, g^{nr}_{\mathbf{p}}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$. Now, compute :

$$\begin{aligned}
c(\mathbf{h})^T\mathbf{g} &= \sum_{(o,d)\in\mathcal{V}} \sum_{\mathbf{p}\in\mathcal{P}_{od}} c_{\mathbf{p}}(\mathbf{h}) \otimes g_{\mathbf{p}} \\
&\geq \sum_{(o,d)\in\mathcal{V}} \pi_{o,d} \otimes \sum_{p\in\mathcal{P}_{od}} g_{\mathbf{p}} \\
&= \sum_{(o,d)\in\mathcal{V}} \sum_{\mathbf{p}\in\mathcal{P}_{od}} \pi_{o,d} \otimes \tilde{h}_{\mathbf{p}} \quad \text{because } \mathbf{h}, \mathbf{g} \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}},\ \sum_{p\in\mathcal{P}_{od}} g^\star_{\mathbf{p}} = \sum_{p\in\mathcal{P}_{od}} h^\star_{\mathbf{p}} = \mathrm{d}^\star_{od} \\
&= \sum_{(o,d)\in\mathcal{V}} \sum_{\mathbf{p}\in\mathcal{P}_{od}} c_{\mathbf{p}}(\mathbf{h}) \otimes \tilde{\mathbf{h}} \quad \text{by the previous remark} \\
&= c(\mathbf{h})^T\tilde{\mathbf{h}}
\end{aligned}$$

Now suppose that we can find $\tilde{\mathbf{h}} = (\mathbf{h}^r, \mathbf{h}^{nr}) \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$ that satisfies the variational inequality defined in Property 2.1.1. Suppose, to the contrary, that Wardrop's conditions are not reached, i.e. that there exists an origin o and a destination d, and a path $\mathbf{p}$ that connects o to d such that $c^*_{\mathbf{p}}(\mathbf{h}) > \pi^*_{o,d}$ and $h^*_{\mathbf{p}} > 0$, where $* = r$ or nr. Then define $\mathbf{g}$ as follows : take on the path $\mathbf{p}$, $g^*_{\mathbf{p}} = h^*_{\mathbf{p}} - \varepsilon$, and on a path $\mathbf{q}$ such that $c^*_{\mathbf{q}}(\mathbf{h}) = \pi_{o,d}$, take $g^*_{\mathbf{q}} = \varepsilon$; and any other component of $\mathbf{g}$ equal to the corresponding component of $\tilde{\mathbf{h}}$. If ε is small enough so that $\mathbf{g} \in \mathcal{H}^r_{\boldsymbol{d}} \times \mathcal{H}^{nr}_{\boldsymbol{d}}$, then it is easy to see that:

$$c(\mathbf{h})^\intercal(\mathbf{g} - \tilde{\mathbf{h}}) = \varepsilon(c^*_{\mathbf{q}}(\mathbf{h}) - c^*_{\mathbf{p}}(\mathbf{h})) = \varepsilon(\pi^*_{o,d} - c^*_{\mathbf{p}}(\mathbf{h})) < 0$$

This contradicts the variational inequality.

Restricted path choice routing model

In the restricted path choice routing model, app users possess perfect knowledge of the path set $\mathcal{P}_{od}$, as apps such as Google Maps or Waze can efficiently compute the set $\mathcal{P}_{od}$ and determine the optimal path (the path minimizing travel time) leading from o to d. On the other hand, non-app users do not have access to such extensive knowledge of the set $\mathcal{P}_{od}$: they tend to select routes empirically or let their navigation be determined by road signs. In this model, this heterogeneity is accounted for by reducing the path choice set of non-app users, $\mathcal{P}^{nr}_{od}$, to a subset of the path choice set of app users, $\mathcal{P}_{od}$, i.e. $\mathcal{P}^{nr}_{od} \subset \mathcal{P}_{od}$. If $\pi^r_{o,d}$ and $\pi^{nr}_{o,d}$ are the minimal travel times for the journey $o \to d$ for app users and non-app users respectively, the user equilibrium of the system is given by Wardrop's conditions:

$$\begin{aligned}
h^r_{\mathbf{p}} \cdot \left(t_{\mathbf{p}}(\Delta \mathbf{h}) - \pi^r_{o,d}\right) &= 0 && \forall \mathbf{p} \in \mathcal{P}_{od} \\
h^{nr}_{\mathbf{p}} \cdot \left(t_{\mathbf{p}}(\Delta \mathbf{h}) - \pi^{nr}_{o,d}\right) &= 0 && \forall \mathbf{p} \in \mathcal{P}^{nr}_{od} \\
h^r_{\mathbf{p}} &\geq 0 && \forall \mathbf{p} \in \mathcal{P}_{od} \\
h^{nr}_{\mathbf{p}} &\geq 0 && \forall \mathbf{p} \in \mathcal{P}^{nr}_{od} \\
\pi^r_{o,d} &\geq 0 && \forall o, d \in \mathcal{V} \\
\pi^{nr}_{od} &\geq 0 && \forall o, d \in \mathcal{V} \\
\sum_{p \in \mathcal{P}^{nr}_{od}} h^{nr}_{\mathbf{p}} &= \mathrm{r}^{nr}_{od} && \forall o, d \in \mathcal{V} \\
\sum_{p \in \mathcal{P}_{od}} h^{r}_{\mathbf{p}} &= \mathrm{r}^{r}_{od} && \forall o, d \in \mathcal{V}
\end{aligned}$$

Property 2.1.2 (Equivalence to the minimization problem)**.** *Finding the user equilibrium of the system is equivalent to solving the minimization problem:*

$$\begin{aligned}
&\min_{\mathbf{f} \in \mathbb{R}^{|\mathcal{E}|}} \sum_{e \in \mathcal{E}} \int_0^{f_e} c_e(x) \cdot dx \\
s.t. \quad \mathbf{f} &= \sum_{\mathbf{p} \in \mathcal{P}} h^r_{\mathbf{p}} \cdot \delta_{\mathbf{p}} + \sum_{\mathbf{p} \in \mathcal{P}^{nr}} h^{nr}_{\mathbf{p}} \cdot \delta_{\mathbf{p}} \\
r^r_{od} &= \sum_{\mathbf{p} \in \mathcal{P}_{od}} h^r_{\mathbf{p}} \\
r^{nr}_{od} &= \sum_{\mathbf{p} \in \mathcal{P}^{nr}_{od}} h^{nr}_{\mathbf{p}}
\end{aligned} \tag{2.5}$$

Proof: A very similar variational inequality to that in Property 2.1.1 can be derived. Interpreting the $c(\mathbf{f})$ as a gradient yields the above conditions.

2.2 Dynamic model

Dynamic routing model using Aimsun

One can argue that a key difference between app users and non-app users is that non-app users will never dynamically change their routes once in the network. Unfortunately because the two previous models are both based on static equilibrium, they cannot model the dynamic rerouting phenomena. Dynamic traffic assignment models need to be used in order to model dynamic rerouting.

The effect of information on routing behavior can be explicitly modeled by considering two different type of vehicles: app users and non-app users. Dynamic traffic simulations can be perform through microsimulators like *Aimsun*. *Aimsun* uses a car following model to describe the movement of individual vehicles through a network. Using *Aimsun*, one can model non-app users as vehicles that follow prescribed paths. For example, one might assume that non-app users mainly follow road signs. One can model app users as vehicles that can dynamically reroute in the network in order to always follow a fastest route recommended by the apps. For example, app users choose the lowest cost paths (i.e. lowest travel time) with high probability and are allowed to change paths throughout the simulation as path costs change due to congestion.

Chapter 3

Assessing the impact of app usage from a game theoritical approch: the average marginal regret

3.1 The average marginal regret

Game theoretical static definition

In this section, we formulate **the average marginal regret**, which quantifies how much time an average driver can expect to save by changing their path to the optimal one. Then, some properties of the average marginal regret are presented.

Definition 3.1.1 (Best path, optimal flow pattern)**.** *Given a flow allocation* $\mathbf{f} \in \mathcal{F}_{\boldsymbol{d}}$ *(c.f. definition 2.1.4), which provides the cost vector* $\mathbf{t}(\mathbf{f})$ *(c.f. definition 2.1.5), we define:*

- *An optimal path between* o *and* d*, as* $\mathbf{p}^*_{od}(\mathbf{f}) \in \underset{\mathbf{p} \in \mathcal{P}_{od}}{\operatorname{argmin}} \delta_{\mathbf{p}}^\top \mathbf{t}(\mathbf{f}) = \mathcal{P}^*_{od}(\mathbf{f})$
- *An all-or-nothing allocation* $\mathbf{y}(\mathbf{f})$ *based on the travel times at the flow* $\mathbf{f}$*, as* $\mathbf{y}(\mathbf{f}) = \sum_{o,d \in \mathcal{V}} d_{od} \cdot \delta_{\mathbf{p}^*_{od}(\mathbf{f})}$ *for* $\delta_{\mathbf{p}^*_{od}(\mathbf{f})} \in \mathcal{P}^*_{od}(\mathbf{f})$*. In this definition, the full od demand* d_{od} *is allocated to an optimal path (between* o *and* d*) computed with the current flow allocation* $\mathbf{f}$*.*

Remark 3.1.1 (Existence and non-uniqueness)**.** *For all* $\mathbf{f} \in \mathcal{F}_{\boldsymbol{d}}$*,* $\mathbf{p}^*_{od}(\mathbf{f})$ *and* $\mathbf{y}(\mathbf{f})$ *exist but might not be unique.*

We define the average marginal regret as the inner product of the travel time vector and the actual flow allocation minus the all-or-nothing flow allocation normalized with the total demand.

Definition 3.1.2 (Average marginal regret). *We define the average marginal regret of the flow pattern* $\mathbf{f} \in \mathcal{F}_{\boldsymbol{d}}$ *as follows:*

$$\bar{\mathcal{R}}\left(\mathbf{f}\right) = \frac{1}{\|\boldsymbol{d}\|_1}\mathbf{t}(\mathbf{f})^\top(\mathbf{f} - \mathbf{y}(\mathbf{f})) \tag{3.1}$$

where $\|\boldsymbol{d}\|_1 = \sum_{o,d \in \mathcal{V}} d_{od}$ *and* $\mathbf{y}(\mathbf{f})$ *is an all-or-nothing solution as in definition 3.1.1.*

Remark 3.1.2 (Measuring the average marginal regret). *Because the average marginal regret is a function of only the link flow, the link travel time and the traffic demand, it can be accessed with loop detectors and demand survey. Knowing the path flows is not required to measure the average marginal regret.*

After defining the average marginal regret, we introduce its properties.

Definition 3.1.3 (Shortest travel time). *Any optimal path* $\mathbf{p}^*_{od}(\mathbf{f})$ *(see definition 3.1.1) is a shortest path (with respect to cost) between* o *and* d *given the cost on each link* $\mathbf{t}(\mathbf{f})$*:* $\mathbf{t}(\mathbf{f})^\top \delta_{\mathbf{p}^*_{od}(\mathbf{f})} = \min_{\mathbf{p} \in \mathcal{P}_{od}} \mathbf{t}(\mathbf{f})$. *For every* $o, d \in \mathcal{V}$*, we define:*

$$\pi_{od}(\mathbf{f}) = \mathbf{t}(\mathbf{f})^\top \delta_{\mathbf{p}^*_{od}(\mathbf{f})}$$

Remark 3.1.3 (Interpretation of the average marginal regret). *Since* $t_{\mathbf{p}}(\mathbf{f}) - \pi_{od}(\mathbf{f})$ *represents the time a driver on path* $\mathbf{p} \in \mathcal{P}_{od}$ *could save by choosing the best path for their trip,* $\bar{\mathcal{R}}$ *can be interpreted as the average time a driver could save by changing unilaterally their path.*

Using $\mathbf{f} = \Delta\mathbf{h}$*, we have:*

$$\begin{aligned}
\mathbf{t}(\mathbf{f})^\top \mathbf{f} &= \sum_{(o,d) \in \mathcal{V}^2} \sum_{\mathbf{p} \in \mathcal{P}_{od}} h_{\mathbf{p}} \cdot t_{\mathbf{p}}(\mathbf{f}) \\
\mathbf{t}(\mathbf{f})^\top \mathbf{y}(\mathbf{f}) &= \sum_{(o,d) \in \mathcal{V}^2} \mathbf{t}(\mathbf{f})^\top \delta_{\mathbf{p}^*_{od}(\mathbf{f})} \cdot d_{od} = \sum_{(o,d) \in \mathcal{V}^2} \pi_{od}(\mathbf{f}) \cdot d_{od} \\
\bar{\mathcal{R}}\left(\mathbf{f}\right) &= \frac{1}{\|\boldsymbol{d}\|_1} \cdot \sum_{(o,d) \in \mathcal{V}^2} \left(\left(\sum_{\mathbf{p} \in \mathcal{P}_{od}} h_{\mathbf{p}} \cdot t_{\mathbf{p}}(\mathbf{f}) \right) - d_{od} \cdot \pi_{od}(\mathbf{f}) \right) \\
\sum_{\mathbf{p} \in \mathcal{P}_{od}} h_{\mathbf{p}} = d_{od} \implies \quad \bar{\mathcal{R}}\left(\mathbf{f}\right) &= \frac{1}{\|\boldsymbol{d}\|_1} \cdot \sum_{(o,d) \in \mathcal{V}^2} \sum_{\mathbf{p} \in \mathcal{P}_{od}} h_{\mathbf{p}} \cdot (t_{\mathbf{p}}(\mathbf{f}) - \pi_{od}(\mathbf{f}))
\end{aligned}$$

Note that this shows that $\bar{\mathcal{R}}$ *is defined even if* $\mathbf{y}(\mathbf{f})$ *is not unique.*

Property 3.1.1 (The average marginal regret is a positive real value and characterizes all user equilibria). *As* $\mathbf{p}^*_{od}(\mathbf{f})$ *is the fastest path between* o *and* d*, we have*

$$\frac{1}{\|\boldsymbol{d}\|_1} \cdot \mathbf{t}(\mathbf{f})^\top(\mathbf{f} - \mathbf{y}(\mathbf{f})) = \max_{\mathbf{x} \in \mathcal{F}_{\boldsymbol{d}}} \frac{1}{\|\boldsymbol{d}\|_1} \cdot \mathbf{t}(\mathbf{f})^\top(\mathbf{f} - \mathbf{x}) \geq 0 \tag{3.2}$$

thus:

$$\bar{\mathcal{R}}\left(\mathbf{f}\right) = 0 \iff \forall \mathbf{f}' \in \mathcal{F}_{\boldsymbol{d}},\ \mathbf{t}(\mathbf{f})^\top(\mathbf{f}' - \mathbf{f}) \geq 0 \tag{3.3}$$

Remark 3.1.4. *The variational inequality tells us that none of the players can have a better outcome by choosing a different path in isolation. The average marginal regret identifies which travel time any player could expect to save by rerouting.*

Property 3.1.2 (The average marginal regret as a measure of driver efficiency)**.** *Given* $\epsilon \in \mathbb{R}_{>0}$*, from remark 3.1.3, it is straightforward that* $\forall \mathbf{f} \in \mathcal{F}_{\boldsymbol{d}},\ \bar{\mathcal{R}}(\mathbf{f}) \leq \epsilon \iff \mathbf{f}$ *is an average-*ϵ*-Nash equilibrium (definition 2.8 of [1]).*

So, the average marginal regret is a good way to characterize how close to a user equilibrium the state of traffic is.

A player is defined as efficient if they take one best route between their origin and destination as their path. Then $\bar{\mathcal{R}}$ *can be interpreted as a measure of the efficiency of the drivers.*

The closer $\bar{\mathcal{R}}$ *is to 0, the less inclined players are to change their paths. If* $\bar{\mathcal{R}} = 0$*, the state of traffic is a user equilibrium.*

Property 3.1.3 (Continuity)**.** *The average marginal regret* $\bar{\mathcal{R}}(\mathbf{f})$ *is continuous with respect to* $\mathbf{f}$*.*

Proof. We have $\bar{\mathcal{R}}(\mathbf{f}) = \frac{1}{\|\boldsymbol{d}\|_1}\mathbf{t}(\mathbf{f})^\top(\mathbf{f} - \mathbf{y}(\mathbf{f})) = \frac{1}{\|\boldsymbol{d}\|_1}\left(\mathbf{t}(\mathbf{f})^\top\mathbf{f} - \min_{\tilde{\mathbf{f}} \in \mathcal{F}_{\boldsymbol{d}}} \mathbf{t}(\mathbf{f})^\top\tilde{\mathbf{f}}\right)$. Because $\mathbf{t}(\mathbf{f})$ is continuous with respect to $\mathbf{f}$, it suffices to show that $\min_{\tilde{\mathbf{f}} \in \mathcal{F}_{\boldsymbol{d}}} \mathbf{t}(\mathbf{f})^\top\tilde{\mathbf{f}}$ is continuous with respect to $\mathbf{f}$. This is a linear program (LP) ($\mathcal{F}_{\boldsymbol{d}}$ defined in definition 2.1.4 is a polytope). The optimal objective value of an LP is continuous with respect to perturbation on the objective function [2]. □

Extension to dynamic traffic models

The average marginal regret notion can be extend to dynamic traffic models using remark 3.1.3. For a given o, d pair, if all vehicles are on the shortest paths for this o, d pair, then the travel time should be identical for all vehicles. However, when there are over a certain percentage of vehicles traveling on different paths, there will be a difference in travel times experienced by different vehicles. Then, the vehicles taking the paths with longer travel time will "regret" their choices.

For a single vehicle – denoted a – travelling in a specific o, d pair, the "regret" it experiences can be intuitively given by

$$\mathcal{R}_{veh}(a) \equiv t_{veh}(a) - t_{min}(A), \tag{3.4}$$

where A is the set of all vehicles, or agents, travelling in the network, $t(\cdot)$ returns the travel time of an agent, and

$$t_{min}(A) \equiv \min_{a \in A} t(a) \tag{3.5}$$

returns the minimum travel time of a set of agents. It is clear from (3.4) that when agent a has a travel time identical to the minimum travel time in the network, its "regret" is zero.

Expanding the form in (3.4), we can obtain the regret for the entire network, denoted $\bar{\mathcal{R}}_{naive}$, by

$$\bar{\mathcal{R}}_{naive} \equiv \frac{1}{|A|} \sum_{a \in A} (t(a) - t_{min}(A)), \tag{3.6}$$

where $|\cdot|$ is the size of a set. Note how the regret is first accumulated then normalized by the number of agents in the network. This is because without the normalization, network with more agents will result in a larger regret.

Assume there is only one o, d pair and the network flow allocation is a user equilibrium flow allocation, where all vehicles have the same travel time. It is clear that the definition in (3.6) yields zero, which corresponds to the definition of "regret." However, if there are multiple o, d pairs in the network and if the network flow allocation is a user equilibrium flow allocation then the form shown in (3.6) might not be equal to zero. This can be easily shown by creating two o, d pairs, denoted od_1 and od_2, with different minimal travel time, denoted t_{min1} and t_{min2}, where $t_{min1} > t_{min2}$. Then, even if all vehicles in od_1 has a travel time identical to t_{min1}, indicating the regret of agents in od_1 should be zero, the form in (3.6) does not yield zero. Such problem can be solved by defining average marginal regret for a specific o, d pair instead of the entire network, given by

$$\bar{\mathcal{R}}_{i,j} \equiv \frac{1}{|A_{i,j}|} \sum_{a \in A_{i,j}} (t(a) - t_{min}(A_{i,j})), \tag{3.7}$$

where $A_{i,j}$ is the set of agents travelling from origin i to destination j (i.e., o, d pair (i, j)). Since the definition in (3.7) is the average regret of all agents in a specific o, d pair, we refer to it as average marginal regret.

The average marginal regret will be equal to zero for each o, d pair when the network flow allocation is a user equilibrium flow allocation. The average marginal regret of the entire network can then be computed trivially with a weighted sum of the average marginal regret of each O/D pair, with the weights being the number of vehicles travelling in each o, d pair. However, we know that the average marginal regret might change over time, which is a property not captured in (3.7). For instance, when there are car accidents in a network, the average marginal regret will increase because the travel time of non-app users will increase while the travel time of app users remain similar. Therefore, we define a time-dependent average marginal regret, given by

$$\bar{\mathcal{R}}_{i,j,T_k} \equiv \frac{1}{|A_{i,j}(T_k)|} \sum_{a \in A_{i,j}(T_k)} (t(a) - t_{min}(A_{i,j}(T_k))), \tag{3.8}$$

where

$$A_{i,j}(T_k) \equiv \{a | a \in A_{i,j} \text{ and } (k \times S) \leq t_{entrance}(a) \leq ((k+1) \times S)\} \tag{3.9}$$

is the set of all agents traveling in o, d pair (i, j) and enters the network during time step k, $t_{entrance}(\cdot)$ returns the time an agent enters the network, and S is the step size, typically set between one to ten minutes. With the definition in (3.8), we are able to analyze how the average marginal regret changes over time.

3.2 Implementation of traffic models at scale

Static models

In this section, simulations of the two models introduced in section 2.1 are performed on the LA network (figure 3.1). Simulations use the cognitive cost static model [17] with app user percentages ranging from 0% to 100%, with a 1% increment. For each of these simulations, traffic demand data is collected from the American Community Survey, composed of 96,077 *od* pairs. The network is built from Open Street Map. Traffic demand is set consistently at rush hour levels to find the effects of app usage when networks are congested.

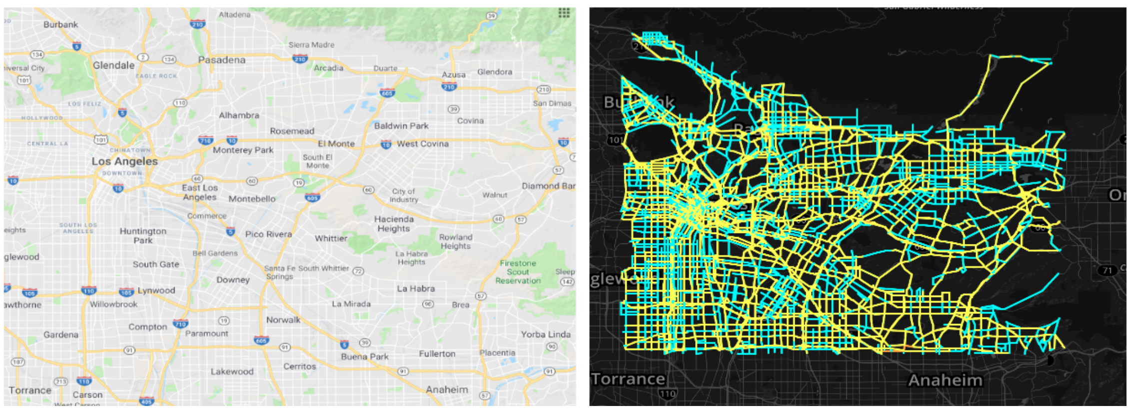


Figure 3.1: LA network considered: a map of the LA basin and the corresponding graph used to model the network.

We see that the average marginal regret decreases monotonically with the increase of navigational app usage (figure 3.2). The fact that the decrease is monotonic is important here. This shows that, whatever the percentage of app users is, the traffic will be closer to a user equilibrium when app usage increases.

Remark 3.2.1. *For every simulation run and on every type of network, the average marginal regret monotonically decreases with the increase of navigational app usage. This is not the case with the price of anarchy, which depends on the network configuration.*

We show the evolution of path flow for a particular o, d pair. This o, d pair has been chosen to be one of the (o, d pairs) with the highest demand. This particular (o, d pair) starts in slightly southeast of Compton and ends just north of Burbank. Figure 3.2 shows the top five paths taken for this o, d pair. Almost all of these paths take the SR 2 through Glendale. One takes the I-210 through Pasadena (the green one).

In the 0% to 35% app usage range, almost all app users take the green path, which is the fastest. But then, with 35% app usage, app users begin to take other paths, particularly the blue (Path 1) and red (Path 3) ones. 35% app usage is exactly when the travel time of path green, blue and red equalize. App users always follow the fastest paths. Then, after

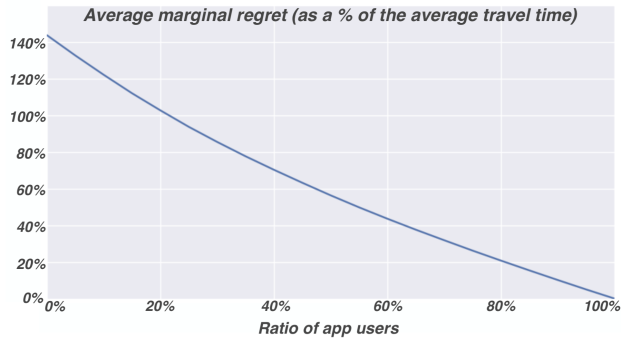


Figure 3.2: Average marginal regret as a function of the percentage of app usage on the LA network.

35% app usage, the travel time of the other paths fall below that of the green path and all app users leave the green path for other paths.

Remark 3.2.2 (Travel time evolution)**.** *It is important to see that here the travel time of these paths depends on other* o, d *pairs. Even after 35% app usage, when no rerouting occurs, the path travel time still varies, Mainly because drivers from other* o, d *pairs still change their path while the ratio of app usage increases.*

Dynamic traffic simulations using Aimsun

To evaluate the impact of routing apps on traffic we perform microsimulations using *Aimsun*. *Aimsun* uses a car following model to describe the movement of individual vehicles through a network. We explicitly model the effect of information on routing behavior by considering app users and non-app users.

We differentiate app users from non-app users in the scenarios by prescribing different routing behavior between the two groups. App users choose the lowest cost paths (i.e. lowest travel time) with high probability and are allowed to change paths throughout the simulation as path costs change due to congestion. We assume that apps give the fastest route and app users follow the recommendation of the apps. Non-app users follow prescribed paths. Their prescribed paths are determined by solving the static user equilibrium problem for the same demand. We assume that non-app users mainly follow road signs. So, they follow a prescribed path. We also assume that these paths induced by the road signs are designed to be the path obtained by solving the static user equilibrium. Therefore, non-app users are required to follow the paths that were found by solving the static user equilibrium problem. Non-app users are unable to change routes during the simulation since they are following predetermined routes.

Since path costs (i.e./ path travel times) are an essential component of app user behavior, these costs have to be updated frequently in order to guarantee that vehicles are routed based on up-to-date travel time information. A high cost cycle (e.g. 20 minutes in a 60 minute

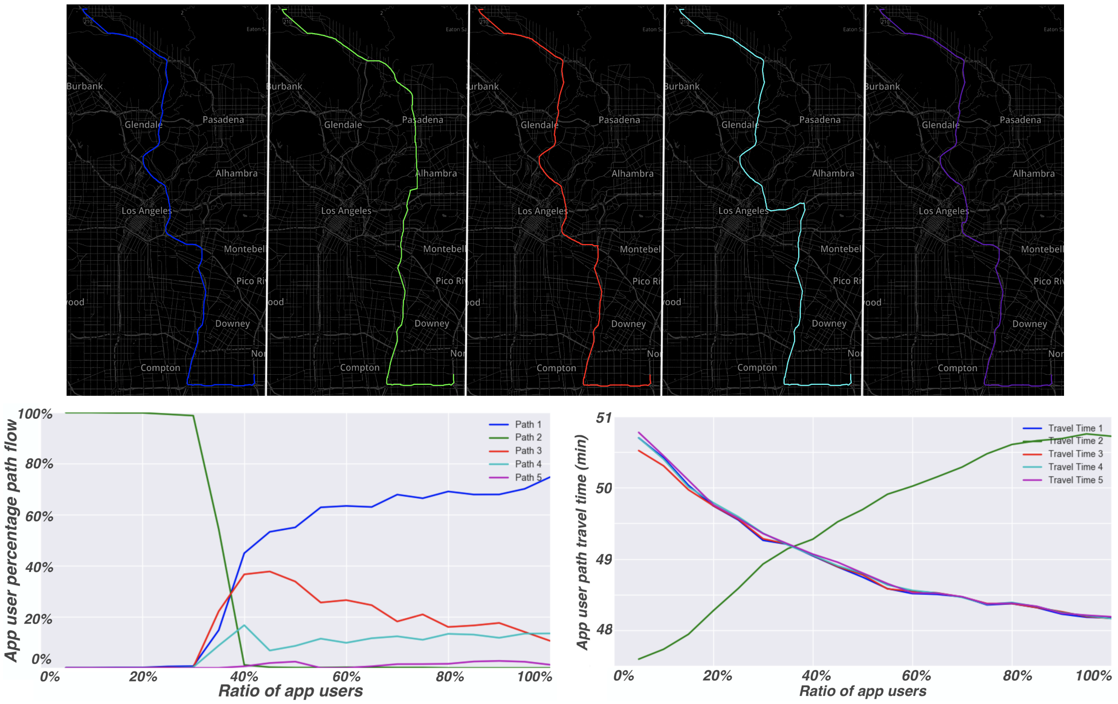


Figure 3.3: Impact of the increase of app usage on path choice and path travel time for a specific o, d pair with the increase of app usage.

Above: the 5 main paths are used by app users. The blue path is the main path used by non app users. **Below on the right:** the travel time of the 5 paths as a function of the app usage. **Below on the left:** the percentage of flow of app users on the 5 paths as a function of the app usage. When there are no app users, every vehicle uses the highway. The side-road green is a shortcut for app users. When there is more than 35% of app users, the green path is not a shortcut anymore. This path gets congested because of other motorists that use this path for their trips. App users always use paths that have the smallest travel time.

simulation) will lead to undesired effects. For instance, assume that a path has low travel time (i.e. a low cost) due to low traffic flow. App users will start routing themselves onto the path, which will lead to the congestion of the path. However, the cost of the path is not updated (since the cost cycle is high) so app users will continue to route onto the path, further worsening the congestion. To prevent such effects, we use a one minute cost cycle time.

Scenario setup

Simulations on a benchmark network – shown in 3.4 – are conducted. We demonstrate the use of average marginal regret on the benchmark network in order to show how it can be

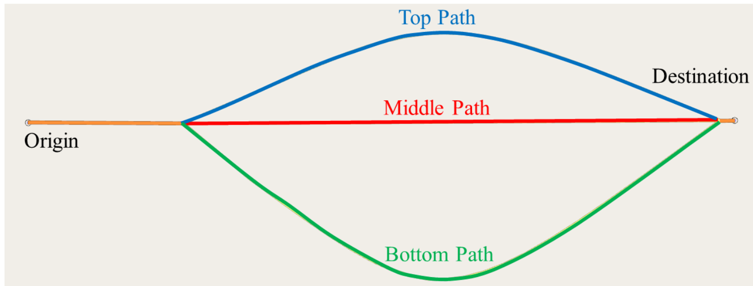


Figure 3.4: Illustration of the benchmark network.

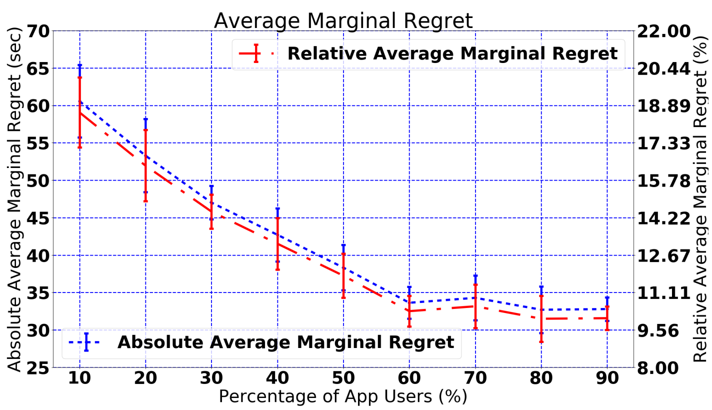


Figure 3.5: The benchmark network scenario without an accident shows the decreases in average marginal regret as the percentage of app users in the network increases.

calculated and interpreted. In the benchmark network we consider the general impact of increased app usage on the state of the network in addition to the impact of a capacity decrease due to an accident.

The second network that we consider is the I-210 corridor in LA. The *Aimsun* model of the I-210 is part of an ongoing project to build a calibrated corridor model [13]. Data from the California DOT freeway loop sensors and city traffic studies are used to establish realistic OD demand. Traffic control plans from the California DOT, Arcadia, and Pasadena are incorporated into the model. The Connected Corridors project is a fundamental component of creating response plans for incident response and congestion mitigation in the I-210 corridor. As a result, the *Aimsun* model of the I-210 realistically simulates the evolution of traffic over the network.

For the both the benchmark network and the I-210 corridor simulation, we consider only impact of increased app usage and use the average marginal regret to quantify the evolution of the traffic state. In both networks, we fix the demand between OD pairs and perturb the percentage of app users between a single OD pair, starting with 10% app users and increase to 90%, using 10% increments. Negative externalities of app usage have previously been shown using static traffic assignment models and field data from the I-210 corridor [4].

3.3 The decrease of the average marginal regret with the increase of app usage in the restricted path choice model

Definition 3.3.1 (User equilibrium of the restricted path choice model)**.** *The Wardrop's first condition for the restricted path choice model can be express as (section 2.1):*

$$h^r_{\mathbf{p}} \cdot (t_{\mathbf{p}}(\Delta \mathbf{h}) - \pi^r_{od}) = 0 \qquad \forall o,d \in \mathcal{V},\ \forall \mathbf{p} \in \mathcal{P}_{od} \tag{3.10}$$

$$h^{nr}_{\mathbf{p}} \cdot (t_{\mathbf{p}}(\Delta \mathbf{h}) - \pi^{nr}_{od}) = 0 \qquad \forall o,d \in \mathcal{V},\ \forall \mathbf{p} \in \mathcal{P}^{nr}_{od} \tag{3.11}$$

$$h^r_{\mathbf{p}} \geq 0 \qquad \forall o,d \in \mathcal{V},\ \forall \mathbf{p} \in \mathcal{P}_{od} \tag{3.12}$$

$$h^{nr}_{\mathbf{p}} \geq 0 \qquad \forall o,d \in \mathcal{V},\ \forall \mathbf{p} \in \mathcal{P}^{nr}_{od} \tag{3.13}$$

$$\pi_{od} \geq 0 \qquad \forall o,d \in \mathcal{V} \tag{3.14}$$

$$\pi^{nr}_{od} \geq 0 \qquad \forall o,d \in \mathcal{V} \tag{3.15}$$

$$\sum_{p \in \mathcal{P}^{nr}_{od}} h^{nr}_{\mathbf{p}} = (1-\alpha) d_{od} \qquad \forall o,d \in \mathcal{V} \tag{3.16}$$

$$\sum_{p \in \mathcal{P}_{od}} h^{r}_{\mathbf{p}} = \alpha d_{od} \qquad \forall o,d \in \mathcal{V} \tag{3.17}$$

With the assumption of strictly increasing travel time functions, only a unique flow allocation satisfies the above Wardrop's condition ([4, Property 4.2] and [2]). We denote it $\mathbf{f}^\star_\alpha = \Delta(\mathbf{h}^{a,\star} + \mathbf{h}^{na,\star})$. For this flow allocation, app users are routed on the shortest path inside $\mathcal{P}_{od}$ and non app users routed themselves on the shortest path inside $\mathcal{P}^{nr}_{od}$. Therefore, app users do not have "regrets" while non-app users "regret" to not know $\mathcal{P}_{od}$. We can express the average marginal regret associated with $\mathbf{f}^\star_\alpha$ (as in remark 3.1.3):

$$\bar{\mathcal{R}}(\mathbf{f}^\star_\alpha) = \sum_{o,d \in \mathcal{V}} \sum_{p \in \mathcal{P}_{od}} h_{\mathbf{p}} \cdot (t_{\mathbf{p}}(\mathbf{f}^\star_\alpha) - \min_{\tilde{p} \in \mathcal{P}_{od}} t_{\tilde{p}}(\mathbf{f}^\star_\alpha))$$

$$\bar{\mathcal{R}}(\mathbf{f}^\star_\alpha) = \sum_{o,d \in \mathcal{V}} \sum_{p \in \mathcal{P}_{od}} h^r_{\mathbf{p}} \cdot (\pi_{od}(\mathbf{f}^\star_\alpha) - \pi_{od}(\mathbf{f}^\star_\alpha)) + h^{nr}_{\mathbf{p}} \cdot (\pi^{nr}_{od}(\mathbf{f}^\star_\alpha) - \pi_{od}(\mathbf{f}^\star_\alpha))$$

$$\bar{\mathcal{R}}(\mathbf{f}^\star_\alpha) = \sum_{o,d \in \mathcal{V}} \sum_{p \in \mathcal{P}_{od}} h^{nr}_{\mathbf{p}} \cdot (\pi^{nr}_{od}(\mathbf{f}^\star_\alpha) - \pi_{od}(\mathbf{f}^\star_\alpha))$$

Then eq. (3.16) gives:

$$\bar{\mathcal{R}}\left(\mathbf{f}_\alpha^\star\right) = (1-\alpha) \sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left(\pi_{od}^{nr}(\mathbf{f}_\alpha^\star) - \pi_{od}(\mathbf{f}_\alpha^\star)\right) \tag{3.18}$$

Theorem 3.3.1 (Continuity of the average marginal regret with the ratio of app users)**.** *the average marginal regret $\bar{\mathcal{R}}\left(\mathbf{f}_\alpha^\star\right)$ is continuous as a function of α.*

Proof. This follows from the continuity of the average marginal regret and of the continuity of $\mathbf{f}_\alpha^\star$ with respect to α. The continuity of $\mathbf{f}_\alpha^\star$ with respect to α is due to the convexity of the restricted path choice model (see [4, Property 4.2] for the convexity, and [8, Theorem 1] for a more detailed proof). □

Theorem 3.3.2 (Monotonicity and convergence to Nash)**.** *For α_1, α_2 such that $0 \le \alpha_1 \le \alpha_2 \le 1$:*

$$\bar{\mathcal{R}}\left(\mathbf{f}_{\alpha_2}^\star\right) \le \bar{\mathcal{R}}\left(\mathbf{f}_{\alpha_1}^\star\right) \text{ and } \lim_{\alpha_2\to 1} \bar{\mathcal{R}}\left(\mathbf{f}_{\alpha_2}^\star\right) = 0$$

Proof. First, given that $\bar{\mathcal{R}}\left(\mathbf{f}_\alpha^\star\right)$ is continuous with α (theorem 3.3.1), $\bar{\mathcal{R}}\left(\mathbf{f}_{\alpha=1}^\star\right) = 0$ (eq. (3.18)) gives that $\lim\limits_{\alpha\to 1} \bar{\mathcal{R}}\left(\mathbf{f}_\alpha^\star\right) = 0$.

Then, we can use the sensitivity analysis of the travel cost $\pi_{od}(\mathbf{f}_\alpha^\star)$ and $\pi_{od}^{nr}(\mathbf{f}_\alpha^\star)$ with respect to α (as in [6, 10]). By using eq. (3.18), we have:

$$\begin{aligned}
\bar{\mathcal{R}}\left(\mathbf{f}_{\alpha_1}^\star\right) - \bar{\mathcal{R}}\left(\mathbf{f}_{\alpha_2}^\star\right) &= (1-\alpha_1) \sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left(\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}(\mathbf{f}_{\alpha_1}^\star)\right) \\
&\quad - (1-\alpha_2) \sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left(\pi_{od}^{nr}(\mathbf{f}_{\alpha_2}^\star) - \pi_{od}(\mathbf{f}_{\alpha_2}^\star)\right) \\
&= (\alpha_2-\alpha_1) \sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left(\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}(\mathbf{f}_{\alpha_1}^\star)\right) \\
&\quad + (1-\alpha_2) \sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left(\left(\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}^{nr}(\mathbf{f}_{\alpha_2}^\star)\right) - \left(\pi_{od}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}(\mathbf{f}_{\alpha_2}^\star)\right)\right)
\end{aligned}$$

Because $\alpha_1 \le \alpha_2$ and $\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) \ge \pi_{od}(\mathbf{f}_{\alpha_1}^\star)$ then $(\alpha_2-\alpha_1)\sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot (\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}(\mathbf{f}_{\alpha_1}^\star)) \ge 0$.

Using Dafermos sensitivity analysis of travel cost with respect to the demand [6, Theorem 4.2], we will show that $\sum_{o,d\in\mathcal{V}} \mathrm{d}_{od} \cdot \left((\pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}^{nr}(\mathbf{f}_{\alpha_2}^\star)) - (\pi_{od}(\mathbf{f}_{\alpha_1}^\star) - \pi_{od}(\mathbf{f}_{\alpha_2}^\star))\right) \ge 0$. Since $(1-\alpha_2) \ge 0$, it will complete the proof.

Changing the problem (in definition 3.3.1) into a stationary traffic assignment problem by vectorizing it, we denote $\tilde{\pi}_{o,d} = (\pi_{od}(\mathbf{f}_{\alpha_1}^\star),\ \pi_{od}^{nr}(\mathbf{f}_{\alpha_1}^\star))$, $\tilde{\mathrm{d}}_{o,d} = (\alpha_1\ \mathrm{d}_{od},\ (1-\alpha_1)\ \mathrm{d}_{od})$, and $\tilde{\pi}_{o,d}^\star = (\pi_{od}(\mathbf{f}_{\alpha_2}^\star),\ \pi_{od}^{nr}(\mathbf{f}_{\alpha_2}^\star))$, $\tilde{\mathrm{d}}_{o,d}^\star = (\alpha_2\ \mathrm{d}_{od},\ (1-\alpha_2)\ \mathrm{d}_{od})$. This notation is inspired by Dafermos [6]. Then, the Dafermos sensitivity analysis of the travel cost with respect to the demand [6, Theorem 4.2] gives:

$$\sum_{o,d \in \mathcal{V}} (\tilde{\pi}^{\star}_{o,d} - \tilde{\pi}_{o,d})^{\top} (\tilde{\mathrm{d}}^{\star}_{o,d} - \tilde{\mathrm{d}}_{o,d}) \geq 0$$

Going back to previous notations:

$$\sum_{o,d \in \mathcal{V}} (\pi_{od}(\mathbf{f}^{\star}_{\alpha_2}) - \pi_{od}(\mathbf{f}^{\star}_{\alpha_1}))^{\top} ((\alpha_2 - \alpha_1)\ \mathrm{d}_{o,d}) - (\pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_2}) - \pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_1}))^{\top} ((\alpha_2 - \alpha_1)\ \mathrm{d}_{o,d}) \geq 0$$

$$(\alpha_2 - \alpha_1) \sum_{o,d \in \mathcal{V}} \mathrm{d}_{o,d} \cdot \big(((\pi_{od}(\mathbf{f}^{\star}_{\alpha_2}) - \pi_{od}(\mathbf{f}^{\star}_{\alpha_1})) - (\pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_2}) - \pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_1})) \big) \geq 0$$

$$\sum_{o,d \in \mathcal{V}} \mathrm{d}_{o,d} \cdot \big(((\pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_1}) - \pi^{nr}_{od}(\mathbf{f}^{\star}_{\alpha_2})) - ((\pi_{od}(\mathbf{f}^{\star}_{\alpha_1}) - \pi_{od}(\mathbf{f}^{\star}_{\alpha_2})) \big) \geq 0$$

This shows the claim $\bar{\mathcal{R}}\,(\mathbf{f}^{\star}_{\alpha_1}) \geq \bar{\mathcal{R}}\,(\mathbf{f}^{\star}_{\alpha_2})$. □

The average marginal regret decreases monotonically to zero when the ratio of app users increases uniformly using the restricted path choice model.

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