CITIES AS PUBLIC GOODS

by

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INTRODUCTION

The importance of collective phenomena in the formation and development of cities has long been recognized. A. C. Pigou wrote -- more than 50 years ago — that "it is as idle to expect a well-planned town to result from the independent activities of isolated speculators as it would be to expect a satisfactory picture to result if each separate square inch were painted by an independent artist. No 'invisible hand' can be relied on to produce a good arrangement of the whole from a combination of separate treatments of the parts. It is, therefore, necessary that an authority of wider reach should intervene and should tackle the collective problems of beauty, of air and of light, as those other collective problems of gas and water have been tackled." Alfred Marshall devoted a whole chapter of his Principles to the collective benefits which resulted from the concentration of many firms to one local area. Furthermore, his distinction between the "public value" of land and the "private value" of land was meant to capture some of the effects of collective phenomena. Alfred Weber likewise gave attention to such phenomena, and so did J. H. von Thünen in "Der Isolierte Staat," almost a hundred years earlier.

To describe the collective phenomena and their effects, the early writers used such words as "economies of agglomeration," "external economies," "advantages of localization," and "increasing division of labor and specialization." These and similar expressions have been parroted by authors ever since -- and with much verbosity, imprecision, and ambiguity. As a consequence, there has been very little cumulative learning in the literature covering such fields as Location Theory and Urban Economics, with respect to the comprehension of collective phenomena.

On the other hand, knowledge in the realm of pure theory has accumulated steadily, especially during the past two decades. The concept of a collective good was defined in a seminal paper by Paul Samuelson on "The Pure Theory of Public Expenditure." That paper gave the impetus for more analytical and rigorous work, aimed at integrating so-called "public goods" into the general theory of value. Although those who worked on the pure theory of public goods, often chose examples from the city, such as street-lighting, to clarify the central concept of a collective good, the fact is that their work has not played any significant role -- as analytical tool and as generator of hypotheses -- in the study of cities and space.

The present paper attempts to use the economic theory of public goods in an analysis of collective phenomena in cities. In the process, we stress that the existing theory needs to be reformulated, in order to be applicable. We suggest that a deeper understanding of important urban phenomena and processes requires the use of a dynamic concept of public goods. It is furthermore suggested that the property of access/no access is common in urban collective goods, and that this property

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cannot be comprehended satisfactorily by means of the concepts of the existing theory of public goods.

The paper has three main sections. In the first section, a broad overview of collective phenomena in cities is given. Since, in the author's view, collective goods played a more significant role in the ancient cities than they generally do in modern cities, an examination of the early cities in history will serve as a convenient starting-point. The second section will lend greater precision to the presentation. There, we shall define three subsets of "goods", namely 1) individual goods, 2) collective goods with the (standard) property of "sharing" and 3) collective goods with the property of "sharing-and-interaction." In the last part, the concepts are put to work in an analysis of urban growth and decay.

COLLECTIVE PHENOMENA IN THE ANCIENT CITY

The central property of a collective good is that of "sharing" or collective concern. This property of sharing eminently characterized the temples and shrines of the ancient world. The early cities were sacred to the gods. The cities functioned as sanctuaries for worship, and their temples served as the visual assurance of divine protection shared by all. The main temple was usually located on a high terrace, as typified by the "acropolis" in Greece and by the "ziggurat" in Sumer.

Whereas the temple, as the tallest and most important building, influenced the spatial layout of the ancient city, much more profound influences -- of a collective nature -- were exerted by religion itself. The social discipline acquiesced in by the city-dwellers, the rituals, the laws, the governmental system, and, more broadly, the social order -- all had their root in religion. As Fustel de Coulanges put it: "The state was closely allied with religion; it came from religion, and was confounded with it. For this reason, in the primitive city all political institutions had been religious institutions, the festivals had been ceremonies of the worship, the laws had been sacred formulas, and the kings and magistrates had been priests. For this reason, too, individual liberty had been unknown, and man had not been able to withdraw even his conscience from the omnipotence of the city. For this reason, also, the state remained bounded by the limits of a city, and had never been able to pass the boundaries which its national gods had originally traced for it. Every city had not only its political independence, but also its worship and its code. Religion, law, government, all were municipal. The city was the single living force; there was nothing above and nothing below it; neither national unity nor individual liberty." One might add that only the Romans were able to change this order, but theirs was also an urban order, an empire of cities governed by one city.

Such then was the nature of the city-states. They had the power and the means to shape the ancient cities into (bundles of) collective assets. But access to many of the collective services was available only to those who were citizens, and to their immediate families. The stranger in the city, or someone deprived of his citizenship, did not have access to religious services, nor did he have the protection of the laws. (In Rome and Athens, the stranger could gain the benefit of some of these collective services by having a citizen as his "patron.")

Excavations at Mohenjodaro and Harappa, in the Indus Valley, as well as discoveries at the sites of the Sumerian cities, indicate that public granaries provided an important form of collective service. But again there is a question of access, at least in some of the early cities. Lewis Mumford, for one, goes so far as to assert that the granaries were used for extortive purposes -- such that the recurrent threat of starvation among the city inhabitants led to forced labor, a "regimentation of the urban labor force." As support for this thesis, he uses evidence that the communal storehouse for grain in Mohenjodaro was situated "within the heavy walls of the citadel, protected against the inhabitants of the city." Whatever purposes such an inner bulwark might have served, it is beyond dispute that the circuit wall that enclosed the ancient city aimed at protection from without. The wall gave security in which all the city-dwellers shared. Indeed, the city wall was a very fundamental collective asset, for thousands of years a necessary condition for continued existence of the city.

Mohenjodaro contained some quite remarkable collective assets, among them a large public bath and an elaborate sewer system. Its sewer system preceded by nearly two thousand years the oldest sewer of Rome, the Cloaca Maxima. (On the other hand, the Roman sewer is still in use, two thousand five hundred years after it was built.)

Rome was the first city ever to reach a population size of one million. It took more than eighteen hundred years before another city, London, exceeded that size. Rome is still unique in the world's history in the sense of having been able to sustain a population of such large size for a continuous period of more than two hundred years. This is truly extraordinary, in view of the relatively very primitive state of the arts at the time -- in transportation and other communications, in industrial technology, and in sanitation and hygiene. How was it possible? Our contention is that Rome's multifarious, and skilfully designed and utilized, collective assets contributed much to her sustenance and viability.

Access to water, a most vital human need, was a serious problem in many of the ancient cities. In Rome, the water supply was so abundant in relation to the needs that access to water could best be described as a public good. Water was conveyed for the collective benefit of the Roman population by means of eight main aqueducts. Just like the city wall, these aqueducts were genuinely collective assets. Aqueducts were indeed built and used before Roman times, but never before had the world seen the use of the arch on such a grandiose scale in the construction of aqueducts. The first aqueduct was constructed as early as the fourth century B.C., and the last main aqueduct was completed by Emperor Trajan at the beginning of the second century A.D. The epoch known as the Early Roman Empire -- a period of approximately 260 years ending with the death of Severus Alexander in 235 A.D. -- saw enormous activity by architects and builders. It was the period described by Hammond as "the finest flowering of the city in the ancient world". Collective assets constructed during the period included roads, bridges, canals, harbors, lighthouses, circuses, baths, market places, theaters, and libraries. As Lewis Mumford expressed it, "the Roman architecc found a mass form for all the collective occasions of life, in the market, the amphitheater, the bath, the racecourse".

Two descriptions of the city of Rome from the fourth century A.D. -- the so-called "Regionaries" -- provide what Mumford called the "first comprehensive inventory of Rome's contents". The descriptions include: 6 obelisks, 8 bridges, 11 public baths, 19 water channels, 2 circuses, 2 amphitheaters, 3 theaters, 28 libraries, 5 nautical spectacles for sea fights, 36 marble arches, 37 gates, 290 storehouses and warehouses, and 254 public bakehouses. Add to this 18 fora or public squares, 8 campi or commons covered with grass, about 30 public parks and gardens, 700 public pools or basins, 500 fountains, about 10,000 memorial monuments, a vast system of lanes and two main avenues, the sewer system, the aqueducts, the temples and the city walls -- and we have a rather substantial list of collective assets in ancient Rome. Some of these assets -- such as the lanes and streets -- had obviously very narrow capacity limits. But the Colosseum had a seating capacity of 45,000, with standing room for another 5,000, the Circus Maximus is reputed to have had a capacity of some 250,000 seats.

However, the important collective services provided by ancient Rome had not always a counterpart in "the stones of the city", as the long list above might suggest. The urban rule system -- the laws, the codes, the rituals, and the social rules -- was a collective good of sorts. So was access to knowledge and information. Access to food was in a sense a collective good. Food in itself is obviously an individual good, and not a collective good. But access to food was assured the Roman populace, in part by the extent of the supply system, with grain being shipped from Sicily, Egypt, North Africa, even Spain, and in part -- as regards the poor inhabitants -- by the practice of regular handouts. The assurance of access was shared by all, and herein lies an aspect of collective good.

In their city-building endeavor, the Romans learnt much from the experience of the Greeks. They assimilated elements of Greek culture, and elements of the political structure of the Greek city-state. But in terms of their sheer physical design and structure, some collective assets in the Greek cities remained distinctly and uniquely Greek. Among these were the Gymnasium, the Stadium, and, above all, the Agora, which Wycherley called "the living heart" of the Greek city. The Agora consisted of an open space and a collection of buildings which formed a civic centre as well as a market-place. In Wycherley's words: "One may doubt whether the public places of any other cities have ever seen such an intense and sustained concentration of varied activities. The agora was in fact no mere public place but the central zone of the city, its living heart. In spite of an inevitable diffusion and specialization of functions, it retained a real share of all its old miscellaneous functions. It remained essentially a single whole, or at least strongly resisted division. It was the constant resort of all citizens, and it did not spring to life on special occasions but was the daily scene of social life, business and politics."

Indeed, the Agora possessed the property of sharing which, as we have suggested, is an essential property of a collective good, but in addition it was a device for interaction. In a later section of this paper, we shall suggest that collective assets with the joint property of sharing-and-interaction form an important ingredient in the very mechanisms of change in the city — that such assets are a key to our understanding of growth and decay processes in the city.

One of the roles of the Agora was to serve as a market-place, and this brings us to one of the most catalytic of all the collective assets that were created in the ancient cities, namely the monetary system. As some form of money became more and more widely accepted as a standard of measurement, or unit of account, all economic activity was greatly facilitated. However, even more significant was the emergence of money as a means of exchange. In that emergence lay — again — processes of sharing and interaction, which have had implications throughout all subsequent economic history.

COLLECTIVE ASSETS IN THE MODERN CITY

By its very nature, its institutions and physical structure, the modern city has retained many of the collective assets of the ancient city. Streets, bridges, and subways, public places and parks, churches and cemetaries, transportation terminals and waste disposal systems are examples of collective assets, which form part of what is nowadays often called "urban infrastructure". The ancient city wall is no more, but the modern city has public health facilities, fire protection, and police services. The city of today also has a much richer and vastly enlarged set of collective assets covering the transmission, selection, assimilation, and accumulation of information and knowledge -- as exemplified by schools, colleges and museums, and by telephone systems, radio and TV. The city has become a storehouse of knowledge. Lewis Mumford wrote that "The great city is the best organ of memory man has yet created."

Yet, an observer of today's cities may be impressed more with the "public bads" in the cities than with their "public goods" -- more with the collective liabilities in the cities than with their collective assets. Rates of decay, as expressed in measures of crime, pollution, congestion and the like, appear to dominate the rates of growth and enhancement -- at least as far as easily available measures of the latter are concerned.

The observer will note that the most aggravating problems are found in the cities of the richest countries -- such as the United States -- and in the cities of the fastest growing economies -- such as Japan -- as well as in the large cities of the world's poor countries. A comparison between the cities of the ancient world and the cities of today, will reveal two differences which may help us understand some of the failures of the modern city.

First, the supreme military and political power held by the government of the ancient city-state contrasts sharply with the weakness of most of today's local governments. The composition of power in the modern city is usually very different. American cities provide perhaps the most conspicuous examples. It has been said that the pattern of city government established in the United States, particularly in the nineteenth century, "made possible the influence of every interest in city affairs except the public interest." The large business investors and the land speculators and builders not only shaped the physical form of the American city, they held the power. Samuel Bass Warner's scholarly study of the history of the city of Philadelphia was very aptly entitled "The Private City."

Second, the ancient city had control over access to its collective assets. The obvious example was the city wall: It not only enclosed and protected -- it defined. Those who had access to protection and to all or most of the collective assets, were clearly delimited from those who did not. In contrast, the modern city is usually an open city. Its existing stock of, tangible as well as intangible, collective assets have been accumulated by many generations of inhabitants. Many of the city's collective assets have capacity limits. But such capacity constraints are often diffuse, from the point of view of the individual user of the collective assets -- and especially diffuse from the point of view of an individual who is considering migration to the city. To gain access to these collective assets and to share in their use -- at little or no cost -- becomes an incentive for the individual to migrate to the city, especially in the poor countries, as well as an incentive for the establishment of new business enterprises, especially in the cities of the rapidly growing economies and in the rich countries.

CONCEPTS

In the previous sections, we have discussed, in breadth, the characteristics of collective assets and the role they have played in cities. In what follows, we shall attempt to gain some deeper understanding of the nature of these collective phenomena. For that purpose, we need to define the central concepts more carefully.

We have taken the view in this paper that "public goods," or collective services as we prefer to call them, are obtained from collective assets. These assets are collective in the sense that the services they provide are shared by two or more users or consumers. This property of sharing can be stated in more precise and formal language through the introduction of preference functions and availability constraints.

Thus, if each user is assumed to have a preference or utility function, we say that a particular good is "shared," whenever it simultaneously enters two or more users' preference functions as an argument. It need not enter in identically the same form, as we shall see shortly. If one user of a good excludes all others from consuming or using the good, we say that the asset that yields the good or service is an individual asset. Such exclusion may occur in different ways, for example through the act of consumption or through the exercise of property rights. If exclusion occurs through the act of consumption, then the availability of the service or asset at some particular point in time can be expressed through a constraint of the following kind:

$$(1) \qquad \Sigma\, c^i \leq c$$

Here, c^i denotes the quantity consumed or used by the individual i; c denotes the total supply of the service or asset; and the summation is over all i's. An illustration of (1) -- exclusion through the act of consumption -- is the use of gasolene in automobiles. In (1), both c^i and c may be scalars or vectors.

In the public finance literature there is heavy emphasis on the criterion of exclusion/no exclusion as the basis for defining public goods. This is rather confusing, because there may exist a great deal of exclusion in the presence of public goods. An important instance is through a lack of <u>access</u>. Although access can certainly be a multifaceted phenomenon, we shall here for simplicity consider the case where there is either access or no access at all. By access to an asset, we shall mean potential use of the asset.

Consider, then, a population of size h, at a particular point in time. In relation to a particular collective asset, denoted by x, we define two subsets of the population, called h_0 and h_1, such that

$$(2) \qquad h_0 \cup h_1 = h \qquad \text{and } h_0 \cap h_1 = \phi$$

Here, h_1 consists of all the individuals who have access to the collective asset, and h_0 consists of all the individuals who do not have such access.

Assuming that numbers can be meaningfully assigned to express quantities of the collective asset, we can then formulate availability constraints for the collective asset x.

$$(3) \qquad \begin{aligned} x^i &= x \quad , \quad \text{for all } i \text{ in } h_1 \\ x^i &= 0 \quad , \quad \text{for all } i \text{ in } h_0 \end{aligned}$$

Here, x^i denotes the quantity of the collective asset available to individual i, and x denotes the total quantity available of the collective asset. Examples of (3) would be the ancient city wall and the religion which protected the city-dwellers.

If the individual user of a collective asset has the option of varying the quantity of usage, the availability constraint (3) has to be reformulated as follows:

$$(4) \qquad \delta^i x^i \leq x$$

$$\text{with } \delta^i = 1 \quad \text{for all } i \text{ in } h_1$$

$$\delta^i = 0 \quad \text{for all } i \text{ in } h_0$$

Here, δ^i is a variable which signifies access/no access, with $\delta^i = 1$ denoting access, and with $\delta^i = 0$ denoting the lack of access. The symbol x^i can here be interpreted as the actual usage of the collective asset by an individual who does have access. Examples of (4) are radio and TV; the listener or viewer has the option of turning on or off the set.

In order to obtain services from a collective asset, the user must often apply individual assets. Depending on the nature of such "applications," the collective services that the user obtains can vary. For example, an individual who, by possession or otherwise, has access to a color TV set, obtains a somewhat different service than one who has a black-and-white set. An individual who has a radio with only AM reception, has no access at all to certain programs and stations.

These aspects of collective assets and their use can be formalized by means of preference or utility functions. Using the symbols U^i and U^j to denote the preference functions of two individuals, i and j, and the symbols c^i and c^j to denote their respective usage of individual assets, we write:

$$(5) \qquad \begin{aligned} U^i &= U^i(\underline{c}^i, f^i(\delta^i x^i, \bar{c}^i)) \ , \text{ with } \underline{c}^i + \bar{c}^i = c^i \\ U^j &= U^j(\underline{c}^j, f^j(\delta^j x^j, \bar{c}^j)) \ , \text{ with } \underline{c}^j + \bar{c}^j = c^j \end{aligned}$$

Here, the functions f^i and f^j express the fact that the user of a collective asset may have to allocate or apply some of his individual assets in order to obtain services from the collective asset. The functions f^i and f^j are the results of such applications or transformations, and they may vary among individual users, such as i and j. The symbols δ^i and x^i have the same interpretation as previously. Their variation may be within, or beyond, the control of the user. All the arguments of the functions in (5) can be scalar- or vector-valued.

So far, we have discussed the property of sharing which characterizes all collective assets. A much more complex -- and analytically more interesting -- property, associated with some collective assets, is that of sharing-and-interaction. A simple example would be congested traffic on a bridge. Here the collective asset is the bridge. It enables the individual user to cross the river. However, in order to use the bridge the individual may have to drive his automobile, and thus to apply one or more individual assets. As in the previous paragraph, there is then a function f^i which determines the bridge service that the user obtains. The difference, in this case of sharing-and-interaction, is that the argument x^i of the function f^i will now depend on the total allocation of individual assets (automobiles,etc.) by all other users of the bridge. Above some threshold level, at which congestion sets in, the value of x^i will vary negatively with this aggregated variable or set of variables. This kind of sharing-and-interaction is usually called a "public bad," and it is of course an important phenomenon in modern urban life. Since our interest in this paper lies more with public goods than with public bads, we shall however concentrate the following analysis to a case where the effects of interaction -- at least over a large and interesting interval -- move in a different direction.

ANALYSIS

As cities develop, in time and in space, there are obviously many forces and phenomena at work. This paper suggests that collective assets represent one such set of phenomena. In particular, it is suggested that the property of sharing-and-interaction associated with some collective assets, can contribute to our understanding of certain dynamic processes in the city. This dynamic aspect of collective assets will be pursued in the following theoretical analysis.

The collective asset we shall choose as setting for the analysis, is the Greek Agora, which we discussed in a previous section. Needless to say, by abstracting from all other forces which shaped the ancient Greek cities, the analysis cannot make claim to a very realistic portrayal. Still, it is presented with the hope that it may shed light on an essential element in the ancient city -- and in the modern city as well.

We consider a population consisting of h individuals or householders. The preferences of these individuals can be represented by utility functions U^i, $i = 1, \ldots h$. Each individual has an endowment of time, taken to be the same for all individuals (say, 24 hours per day). Each individual is assumed to allocate his or her time so as to maximize utility. If an individual wishes to visit the Agora, he will incur travel time, which will depend on how far he or she lives from the Agora. All individuals are assumed to have fixed locations. Each individual's utility function is assumed to be separable in the utility obtained from visits to the Agora and in the utility obtained from "everything else." Whether a particular individual will visit the Agora or not, depends on the utility he or she will derive from such visits as compared to the utility obtainable from other allocations of the available time. Thus, each individual will compare the following two expressions:

$$(6) \qquad U^i = U_1^i(T) \qquad , i = 1\ldots\ldots h$$

$$(7) \qquad U^i = U_1^i(T-T^i-dt) + U_2^i(x(t),dt), \; i = 1\ldots\ldots h$$

where T = the "endowment" of time available to the individual

T^i = the amount of travel time it takes for the i^{th} individual to visit the Agora, if he should desire to do so

dt = the "exposure-to-contact" time spent at the Agora (assumed to be the same for all individuals)

x(t) = the number of contact possibilities available at the Agora at time t

In (6) it is assumed that $U_2(0) = 0$. We also assume that, for each individual, U_1 and U_2 are concave and increasing. The individual will undertake the evaluations[1] of (6) and (7) at each point in time, say, at t, (t+dt).... The individual's optimization problem can also be expressed in terms of the dummy variable, δ^i, that we introduced in the previous section.

$$(8) \qquad \underset{\delta^i}{\text{Max}}\ U^i = U^i(T, \delta^i T^i, \delta^i x(t), \delta^i\ dt)$$

Here, δ^i can assume only the values 0 or 1. From (6) and (7), or from (8), we obtain the following relation:

$$(9) \qquad U_1^i(T) - U_1^i(T - T^i - dt) \gtreqless U_2^i(x(t), dt)$$

Thus, if -- at the time of evaluation -- the individual finds that the left-hand side of (9) is greater than the right-hand side, he will decide not to visit the Agora. If, contrariwise, the right-hand side of (9) is greater than or equal to the left-hand side, the individual will visit the Agora.

We can then conceive of two extreme situations, where either every individual visits the Agora, or nobody does. In between, there will be a very large number of possible permutations describing situations, in which some individuals will find it advantageous to visit the Agora, and in which the others will prefer not to visit there. Disregarding the two extreme or boundary situations mentioned, it is then reasonable to assume that at each point in time there will be at least one individual for whom (9) is an equation. Suppose that such an individual will make an evaluation at time (t+dt), and that the information available to him about the size of x (= the number of people he would have the opportunity to meet at the Agora) pertains to time t. We can thus rewrite (9) in the following way:

$$(10) \qquad U_1^i(T) - U_1^i(T - T^i(t+dt) - dt) = U_2^i(x(t), dt)$$

Equation (10) expresses an <u>indifference</u> relation. It says that individual i at time (t+dt) is indifferent between visiting and not visiting the Agora. We can also look at equation (10) in another way, namely as a functional relation between net available time ($T-T^i$) and the size of the Agora (size measured by x). A reasonable form for this functional relation is illustrated in the following diagram.

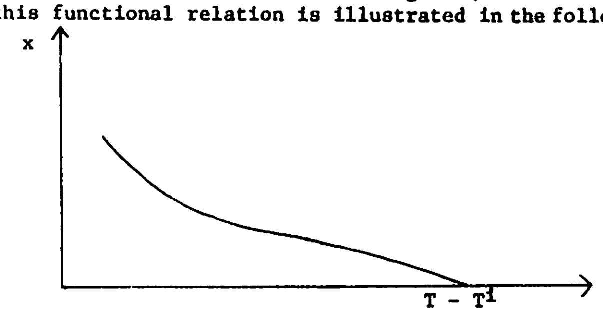

Diagram 1. Indifference relation between net available time and size of Agora

The Diagram can be interpreted as follows. If an individual has a large amount of net time available, implying that he or she lives close to the Agora, a small number of contact possibilities at the Agora is all that it takes to induce him or her to go there. The farther he lives from the Agora, the smaller is his net available time, and the greater is the necessary number of contacts required to induce him to visit the Agora. The curve in Diagram 1, in fact, corresponds to the functional form specified in our earlier paper, written together with C. Averous, where we used an exponential function for U_1 and a quadratic function for U_2. However, B. v. Rabenau and K. Stahl, two of our U.C. Berkeley students, have suggested that a somewhat different shape of the curve in Diagram 1 is more plausible — and, as it turns out, this alternative shape of the curve has some very interesting implications. The alternative form is illustrated in Diagram 2.

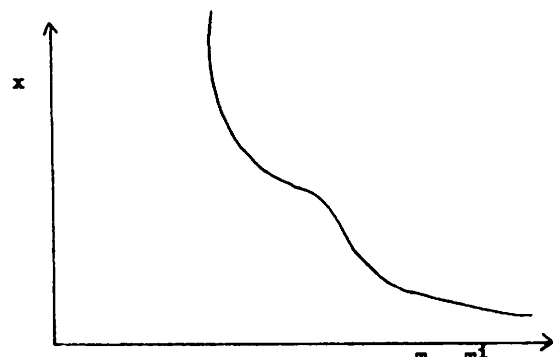

Diagram 2. Alternative shape of indifference relation

Before commenting on Diagram 2, we shall introduce yet another curve. We have stated that different individuals face different time outlays for visits to the Agora. Thus for each individual there is a net available time, which simply measures the difference between the individual's time endowment and the time outlay he would have to incur, should he decide to visit the Agora. For the aggregate of all individuals, there exists then a density function which for each possible amount of available time indicates the corresponding frequency of individuals. Through summation or integration (in the case of a continuous function), such a density function can be transformed into a cumulative function of the kind illustrated in Diagram 3.

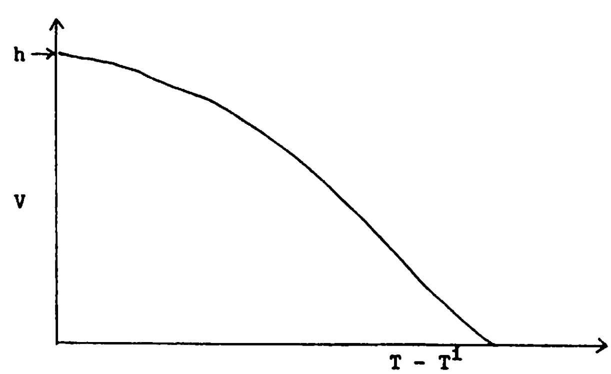

Diagram 3. Cumulative distribution of population w. r. to available time

This diagram shows the relationship between the amount of available time and the total number of individuals, V, who have at least the corresponding amount of time available. The curve is non-increasing, and it shows that as we move to the right there are fewer and fewer individuals with at least the corresponding amount of available time. Since every individual has only a finite amount of time available per period, the curve will intersect the horizontal axis.

Now, by superimposing Diagram 2 on Diagram 3, we obtain the picture illustrated in Diagram 4.

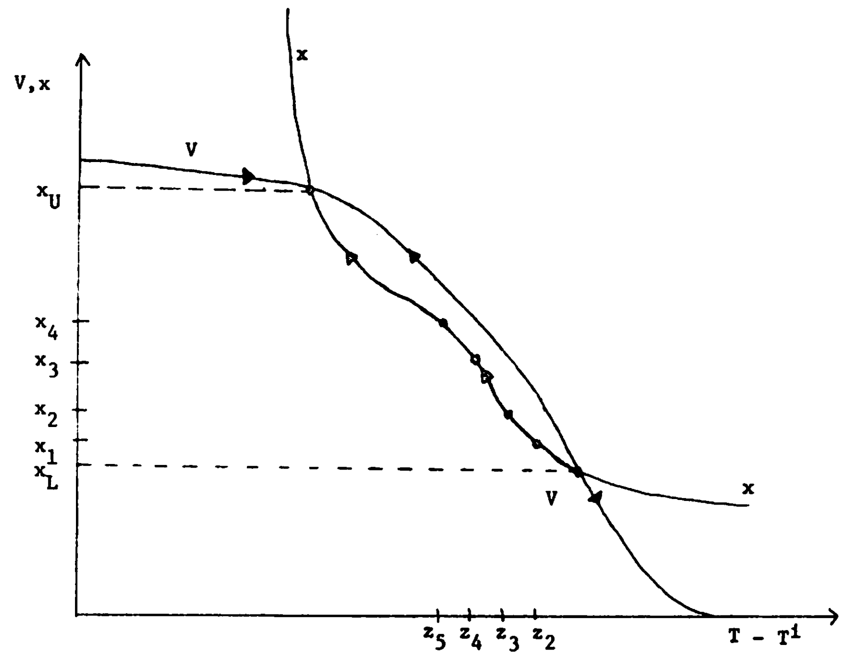

Diagram 4. Interaction between indifference relation and cumulative distribution

The purpose of this Diagram is to illustrate the dynamic process which we can associate with the ancient Greek Agora. Suppose that at a given point in time there are x_1 individuals who visit the Agora. Along the lines of our preceding discussion, there exists then a corresponding amount of available time -- denoted by z_2 on the horizontal axis -- such that some individual is indifferent between visiting and not visiting the Agora. However, from the cumulative distribution of all individuals -- the V-function in the Diagram -- we see that there are more than x_1 individuals who do in fact have at least that much time available. Over a time interval dt -- extending from t=1 to t=2 -- there will thus be an increment of new visitors, dx, who will begin to visit the Agora. Therefore, at the beginning of the next period there are x_2 persons who share in the Agora activities. Since each individual's utility function is assumed to be an increasing function of his or her contact possibilities -- measured as the number of individuals who attend activities at the Agora -- the increase in the number of visitors will be conducive for yet other individuals to join. In the Diagram, we see this by looking at the point (z_3,x_2) on the indifference relation -- the curve x. The meaning of that point is that with the higher number of visitors (x_2), it takes less available time than previously (z_2) in order for the indifference relation, between making a visit and not making a visit, to hold. In turn, the V-function shows that the number of individuals with that much net time available, is even greater than x_2. Thus, there will be a new increase in the number of visitors to x_3, and so on. This self-sustaining process of growth will continue until the point x_U is reached. Given the size of the total population considered, and given the shape of the cumulative distribution function and the shape and position of the indifference relation, there will thus be an upper limit to the size of the Agora. If, for some reason, the number of visitors were to increase beyond the upper level x_U, there would be some visitors who would be better off, according to their preference functions, by not visiting the Agora. Thus, a shrinkage or decay process would set in, with convergence back to the level of x_U. We can therefore say that the level x_U would represent a stable equilibrium.

The diagram depicts also a lower bound for x, namely the point x_L. As is seen from the diagram, that point is also an equilibrium point. However, x_L represents an unstable equilibrium. Any displacement upwards or downwards from x_L will induce further movements away from the equilibrium point.

This last observation leads to an important implication, namely that there exists some threshold level, below which the Agora cannot be sustained. If at some point in time, the number of visitors were to fall below the value x_L, some of the remaining visitors would prefer not to go there any longer, leading to a further contraction or decay, which in turn would induce still others to leave.

The Agora was an activity center of great richness and variety. As the number of participants in its various activities grew, the threshold levels associated with new activities could be surpassed, and new self-sustaining growth processes would be initiated. In relation to such complexities, it may seem that our assumptions and analyses have been unduly simplistic. Turning to the modern urban world, with its economic processes of increasing specialization and division of labor, the simplicity of the analyses seems even more apparent. Yet, that which we usually and loosely refer to as "the division of labor" contains elements of the same sharing-and-interaction property which we have analyzed in this paper. And that is indeed the deeper goal towards which we have been working -- to try to improve our understanding of the processes of specialization and division of labor in society. We have only just begun.

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