THE DRIFT CONE MODE AND THE RADIAL LOCALIZATION PROBLEM: A REVIEW

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A summary is given of the research recently completed and currently in progress by the plasma computational group on the drift cone mode and its radial localization (Part I). A survey is made of previous research on the local approximation and its justification (Part II), the linear theory of the drift cone mode (Part III), the two-temperature instability and warm plasma stabilization of the drift cone mode (Part IV), and the nonlinear theory of the drift cone mode (Part V), with 94 references.

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I. Research on the Drift Cone Mode and Radial Localization in the Plasma Computational Group

The current research in the plasma computational group was motivated by results of the 2X and 2XII mirror plasma confinement experiments. Although the drift cone mode was predicted to have a linear growth rate greater than the ion cyclotron frequency[1] for the parameters of the plasma in these experiments, and this instability was expected (on the basis of quasilinear theory) to cause the ions to be lost in a few ion transit times[2], the 2X and 2XII plasmas were in fact found to be quite stable, with ion loss times of several hundred ion transit times, sometimes close to the collisional loss rate.[3,4] Since the linear theory[1] was based on a "local approximation" which assumed that the ion Larmor radius a_i was much smaller than the radial scale length R_p, while for the 2XII experiment the ion Larmor radius $a_i \sim R_p$ (Ref. 4), the possibility arose that a linear theory correctly including the large a_i/R_p would predict linear stability for the 2XII. Even if this were not true, it was still desirable to have a linear theory valid for $a_i \sim R_p$, as a starting point for quasilinear and nonlinear studies (including computer simulations[5,6]). Accordingly, a method was developed by A. B. Langdon in 1969–70 for finding the normal modes of a Vlasov plasma slab with sinusoidally varying density without assuming $a_i \sim R_p$, and preliminary numerical calculations were made using this method by Langdon, C. K. Birdsall and D. Fuss between 1970 and 1972. The author completed this project between 1973 and 1976, making a number of changes, mostly involving improvement of the numerical techniques to make the numerical calculations practical. It became apparent from the numerical results that the normal modes were often well-localized even with $a_i \sim R_p$. It was found that the essential requirement for a local dispersion relation was not $a_i << R_p$ but $kR_p >> 1$ (where k is the wave number). A local dispersion relation, valid when $a_i \sim R_p$, was developed and shown to give numerical results in good agreement with the nonlocal theory of Langdon when $kR_p >> 1$. All of this work is described in Ref. 7.

In 1975, results of the 2XIIB experiment indicated that the presence of warm Maxwellian ion component might account for the lack of drift cone instability in the quiet mode of operation, and that such a component, deliberately introduced, could stabilize the drift cone mode when it was present, in the noisy mode[8]. Since previous studies of the effect of a warm Maxwellian component were incomplete (usually consisting of numerical solutions for particular sets of parameters, as in Ref. 9), a systematic study was made of the dispersion relation of the drift cone mode with all possible density and temperature ratios of warm to hot plasma. This was done using the usual local approximation with $a_i << R_p$. The results are given in Ref.10. At the time Ref.10 was written, the term "cool" was used to describe the Maxwellian component, but the term "warm" appears to have become standard nomenclature, and is used in this review.

H.L. Berk pointed out to the author that electromagnetic effects, in particular the term ω_{pe}^2/k^2c^2 discussed by Callen and Guest[11], could significantly affect the results of Ref. 10 for the moderately large values of β and the small values a_i/R_p expected in a mirror confined plasma in a fusion reactor. Calculations of the minimum amount of warm plasma needed to stabilize the drift cone mode, including some of the effects of finite β, were made by H.L. Berk and the author, and are described in Ref. 12.

The results of Refs. 10 and 12 assume $a_i \ll R_p$, and hence may not be valid for the 2XIIB experiment. Recently we have calculated the amount of warm Maxwellian plasma necessary to stabilize the drift cone mode in a hot loss cone plasma with $a_i \sim R_p$, using the method developed in Ref. 7 for a plasma slab with sinusoidal density gradient. The preliminary results are given in Ref. 13.

II. The Local Approximation and its Justification

The local approximation is a method for calculating linear growth rates or other behavior of a non-uniform plasma, without finding the normal modes exactly. Usually the perturbed quantities are assumed to be independent of the radial coordinate x or to have a dependence like $\exp(i\,k_x x)$. All unperturbed quantities, such as the density, density gradient, temperature etc., are evaluated at a particular position x_0, which may be chosen arbitrarily or according to some formula (e.g. the point of steepest density gradient).

Since the early 1960's, the local approximation has been widely used for studying Vlasov plasmas which are non-uniform in a direction perpendicular to a magnetic field. This survey will be concerned only with those authors who have considered the conditions under which the local approximation is valid, not with those who have only used it as a "black box." Similarly, we will not be concerned with the many authors who have solved radial normal mode problems without reference to the local approximation.

The earliest use of the local approximation seems to have been by Tserkovnikov[14], in 1957. In studying the electrostatic stability of a cylindrical Vlasov plasma with temperature and density gradients in an axial magnetic field, and radial electric field, he found the linear dispersion relation, $\omega(k_z)$ by assuming a radial wave number $k_r \gg R_p^{-1}$, declaring that in this case boundary conditions could be ignored and unperturbed quantities, e.g. density, temperature and their gradients, could be considered constant over many radial wavelengths. He then found that ω was independent of k_r as long as $k_r a_i \ll 1$. Thus the dispersion relation

$\omega(k_z)$ applied to any perturbation $\phi(r)$ of the proper width, as long as $a_i \ll R_p$. Tserkovnikov did not address himself to the questions of what such a perturbation $\phi(r)$ would do on a time scale much greater than ω^{-1}, or whether unstable normal modes (or any normal modes) had $\phi(r)$ of the proper width.

The first of these questions was considered by Krall and Rosenbluth[15] in 1962. They use the local approximation, i.e. set $k_x=0$ and consider the density and density gradients to be uniform, in studying the drift cyclotron instability in a plasma with a magnetic field gradient and finite electron temperature (i.e. $k_y a_e \sim 1$). They then use the linearized Vlasov-Poisson equation to solve for $\phi(x,t)$ as an initial value problem, [with $\phi(x,0)$ = const.] assuming uniform gradients of density and magnetic field. $\phi(x,t)$ is found to vary for many oscillation periods as $\exp[i\omega_{loc.}(x)t]$, where $\omega_{loc.}(x)$ is the local approximation frequency, provided $k_y R_B \gg 1$, where R_B is the radial scale length of the magnetic field [the density scale length does not enter here because $\omega_{loc.}(x)$ does not depend very much on the density in the limit considered; $\omega_{loc.}$ does depend on the density gradient, but the density gradient is independent of x]. However, this important conclusion is not stated explicitly, but rather the result is given in terms of $k_y a_e$ and a_i/R_p.

A comparison between the local approximation results and the radial normal modes is made by Rosenbluth, Rostoker and Krall[16]. In dealing with finite Larmor radius corrections [i.e. $(k_y a_i)^2 \sim \omega/\omega_{ci} \ll 1$) to the Rayleigh-Taylor instability, they first consider a plane slab with uniform density gradient and use the local approximation, which they justify by saying that $k_y R_p \gg 1$. In this calculation $k_y a_i \ll 1$, so $a_i/R_p \ll 1$. They then consider a cylindrical plasma with Gaussian density profile, with $a_i/R_p \ll 1$ and $k_y a_i \ll 1$, but no longer assume $k_y R_p \gg 1$ (in this case, the role of $k_y R_p$ is played by the azimuthal mode number m). The Vlasov-Poisson equation becomes a differential equation (because $a_i \ll R_p$) which is solved exactly for the normal modes $\phi(r)$ in terms of Whittaker functions. For large azimuthal mode number, the dispersion relation approaches that of the plane slab case using the local approximation.

A similar calculation of the normal modes of the drift cone and drift cyclotron instabilities (with $\omega \approx \ell\omega_{ci}$) in a cylindrical plasma was made by Shima and Fowler[17]. They reduce the integral equation to a differential equation by assuming $k_\perp R_p \gg 1$, and find the normal modes in terms of Bessel functions. They also assume $a_i \ll R_p$, although this is not essential to their calculation. Stability thresholds and growth rates are found which agree in order of magnitude with results found by other authors for these instabilities[1,18] using the local approximation.

Well-localized normal modes were found in Refs. 16 and 17, while they were not found in Ref. 15 because the density gradient varied with position in Refs. 16 and 17. The question of under what conditions localized normal modes can be found was considered by Galeev[19,20], by Silin[21], by Hoh[22], by Krall and Rosenbluth[23], and by Berk et al.[9] If there is an appropriate "small parameter", which is usually taken to be a_i/R_p [although $(k_y R_p)^{-1}$ will also do, and is used in ref. 22], then the integral Vlasov-Poisson equation can be approximated by a differential equation, viz. the Schrödinger equation in a potential well $Q(x,\omega)$. The small parameter also guarantees that the energy levels of the potential well are close together, so that WKB techniques can be used to solve the Schrödinger equation. Localized normal mode solutions exist in a real-valued potential $Q(x)$ only if the potential has two turning points, which allow WKB solutions with real k_x between them. In general, the potential and its turning points will have complex values, so the condition for local normal modes to exist is a little more complicated, and involves the behavior of the Stokes lines in the complex plane, as explained in Refs. 9 and 19-23. However, the general idea is the same as for a real-valued potential, viz. $Q(x,\omega)$ must be "concave." The most well-localized normal modes will occur for ω such that the turning points are close together [if $Q(x,\omega)$ is concave and sufficiently "smooth"]. In this case the potential well can be approximated by a parabola, and the Schrödinger equation becomes a Weber-Hermite equation. The most well-localized mode will then be centered around a point x_0 such that $\partial Q(x_0)/\partial x = 0$, and will be well-localized compared to scale length R_p, but broad compared to the small parameter[21] (a_i in Ref. 21). As pointed out in Ref. 23, the local approximation at $x = x_0$ will then give results very similar to a full normal mode analysis. The requirement for the validity of the local approximation given by Hoh[22], $kR_p >> 1$, is more fundamental than those given in Refs. 9, 19-21, 23, $a_i << R_p$, or by Pearlstein[24], $\lambda_D << R_p$; it is assumed in all these papers that $k_\perp R_p >> 1$.

Although the local approximation at $x = x_0$ will always give the frequency of some normal mode if $k_\perp R_p >> 1$ and $Q(x)$ is concave, it need not be true that this normal mode is the most unstable one. It is pointed out in Ref. 7 that a more unstable mode might exist that is intrinsically non-local, e.g. the negative energy loss cone mode of Berk et al.[25]. In most cases with $k_y R_p >> 1$ and $Q(x)$ concave, however, the most unstable mode is well localized, and hence is well described by the local approximation at $x=x_0$.

Several authors have confirmed this general result by finding exact normal mode solutions for various models. Pearlstein[24] used a density profile proportional to tanh (x/R_p) and found good agreement between the exact normal modes, the solutions to the Weber-Hermite eq., and the local approximation, when $\lambda_{De} << R_p$ (for the "universal instability" drift waves he was considering, $k\lambda_{De} \sim 1$). Chen[26] found normal mode solutions for resistive drift waves in a cylindrical plasma column with Gaussian density profile. When the azimuthal wave number was large, the radial normal modes tended to be well localized. Cordey et al.[27] numerically found normal mode solutions for the drift cone instability in a plane plasma slab of constant density gradient and finite width (bounded by vacuum on the low density side and by a conducting wall on the high density side). These solutions, found for particular values of the parameters, were in good agreement with the local approximation results. Batchelor and Davidson[28] found the normal modes for the fluid lower hybrid drift instability in a cylindrical column with either a gaussian density profile or a density which is constant out to a sharp cutoff. In the latter case, the scale length was zero at the surface of the plasma so of course the local approximation broke down, but with a gaussian density profile the normal modes were in good agreement with the local approximation for large azimuthal mode number $(k_y R_p >> 1)$.

In all of the references just cited (24,26-28), it is assumed that $a_i << R_p$, so it is still not clear that this is not needed in order for the local approximation to be valid. (The assumption $a_i << R_p$ is not made explicitly in Ref. 28, but it follows from the assumption $\omega/k_y >> v_i$, since for this instability ω/k_y must be less than the relative drift velocity of the ions and electrons due to the centrifugal force, which is $v_d \sim R_p \omega_E^2/\omega_{ci}$, where ω_E is the rotation rate of the plasma; and $\omega_E \lesssim \omega_{ci}$ is necessary for equilibrium.)

We now describe some papers which do not assume $a_i << R_p$. Bajaj and Krall[29] found the normal modes of the ion acoustic instability in a plane shock wave with density profile proportional to $1-\eta \tanh (x/R_p)$. The results were in good agreement with the local approximation (at $x = 0$, where the modes were localized) if $\lambda_{De} << R_p$ (note that $k_y\lambda_{De} \sim 1$ for the most unstable mode); otherwise they were more stable than predicted by the local approximtion. In this case there was no assumption made that $a_i << R_p$; indeed a_i does not enter into the problem at all, since even the equilibrium changes on a time scale short compared to ω_{ci}^{-1}.

In a paper by Jungwirth[30], the assumption $a_i << R_p$ is dispensed with in a rather artificial way by considering a cylindrical plasma column in which the ion guiding centers are all located very near the center of the column, so that the radial position of an ion is strongly correlated with its velocity. If R_p is defined as the scale length of guiding centers, then $a_i >> R_p$ in this plasma; however, at any given position, there is very little spread in the velocities of the ions, so if we move to a frame rotating with the ions at ω_{ci}, the ions will be nearly cold, $a_i << R_p$, and the usual local approximation can be used. This method will not work if $a_i \sim R_p$, only if $a_i >> R_p$.

Davidson[31], using essentially the same techniques as those described in Ref. 7 (the authors were not aware of Davidson's work until after Ref. 7 was completed), developed a method for finding the electrostatic normal modes of a rotating Vlasov plasma column in a uniform axial magnetic field, without any ordering of a_i/R_p. (Davidson planned to apply this method to θ-pinch instabilities, since θ-pinches typically have $a_i \sim R_p$; see Refs. 32-34. A similar method was proposed by Lewis and Symon[35] and used for studying screw-pinch instabilities with $a_i \sim R_p$.) This method was used to study the lower hybrid drift instability in an isothermal Maxwellian plasma with gaussian density profile. For $k_\perp R_p \gg 1$, the radial normal mode potentials were proportional to Bessel functions; for moderately large azimuthal mode number, the fastest growing modes were well localized in the radial direction, and had frequencies close to what would be predicted by the local approximation at that radial position. For somewhat larger azimuthal mode numbers, the most unstable modes tended to be localized very close to the conducting wall which was located at a radius of a few R_p. [This was not surprising since the density gradient $(1/n)(dn/dx)$ and rotation velocity of the plasma both increased monotonically with radius; hence there was really no point x_0 away from the wall where $\partial Q/\partial x = 0$ in the WKB approximation. Similar behavior was seen in Ref. 26.] It can be seen from Ref. 31 and from the results in Ref. 7 that the crucial requirement for the local approximation to agree with a radial normal mode analysis is $k_\perp R_p \gg 1$, not $a_i \ll R_p$.

A few experiments have measured the radial localization of drift waves. Hendel and Yamada[36] found that drift-cyclotron instabilities (with axial current and $k_z \neq 0$) in a Q-machine are localized first near the edge of the plasma, where $(1/n)(dn/dx)$ is greatest (and hence, where $\partial Q/\partial x = 0$). As they saturate at this radius, they begin to develop further in, and eventually they are spread over the whole plasma independent of radial position. Simonen[37] found the absolute amplitude of the drift cone instability in the 2XIIB mirror experiment only weakly dependent on radial position, but found that the frequency was lower (~0.5 ω_{ci}, as opposed to ~0.9ω_{ci}) at the edge of the plasma. Since the reduction in frequency below the cyclotron frequency is probably due to nonlinear effects[38] it seems plausible that here too the instability originated at the edge of the plasma where $(1/n)(dn/dx)$ is greatest, and later spread toward the center (where nonlinear effects had not yet become important at the time of the observations).

As shown in Refs. 9, 19-23, localized normal modes cannot exist, even though $k_\perp R_p >> 1$, if Q(x) is "convex", i.e. if the Stokes lines behave the wrong way as they go to infinity in the complex plane. This doesn't happen if there is only a density gradient, but it does happen if the magnetic field has enough shear. At first it was assumed that no normal modes could exist if Q(x) was convex, and that this provided a criterion for shear stabilization of the "universal" drift instability.[20,23] However, Pearlstein and Berk[39] showed that even if Q(x) is convex, a normal mode of the universal drift instability can be constructed which has the boundary conditions that energy is being convected outward as $x \to \pm\infty$, rather than that $\phi(x) \to 0$ as $x \to \pm\infty$. (Eventually the energy is absorbed at large x by ion Landau damping but this is not explicitly included in the model.) At first, these modes are still unstable, but with a still larger amount of shear they are stabilized. A more careful treatment of these modes, giving essentially the same result, was done by Rosenbluth and Catto.[40] The idea had been suggested by Coppi et al.[41] before Pearlstein and Berk[39], but was not used then as the basis for a calculation.

Several authors[41-43] have noted that a normal mode analysis is not the most appropriate way to study a non-uniform plasma if the local growth rate γ is much greater than v_g/R_p where $v_g = \partial\omega/\partial k$ is the group velocity of a localized pulse or wave packet. In such a plasma, a pulse will grow exponentially large (and saturate nonlinearly) before it has crossed the plasma, and hence before it has time to "know" whether the normal modes of the plasma are stable or not. Since the local approximation describes the behavior of a pulse for many wave periods if the pulse radial width is much greater than a wavelength, but much less than R_p, the condition $k_\perp R_p >> 1$ is a condition for the local approximation to agree with a wave packet description (as well as being a condition for it to agree with a normal mode description). If $\gamma >> v_g/R_p$, the local approximation is useful even if Q(x) is not concave, and even if it is evaluated at a point where $\partial Q/\partial x \neq 0$. (If $\partial Q/\partial x = 0$, then a pulse will stay put and will not convect away; if Q(x) is concave, it will not spread away; hence these are the conditions for a pulse to be a normal mode.) Indeed, Mikhailovskii[44] gives $\gamma >> v_g/R_p$ as the condition of validity for the local approximation, and this idea is also implicit in the justification for the local approximation given in Ref. 15.

A wave packet approach was used to study the shear stabilization of the universal drift mode by Coppi et al.[41] and by Rutherford and Frieman.[42] It was found that the requirement $\gamma \lesssim v_g/R_p$ gave a condition for stability on the same order as that found later by Pearlstein and Berk[39]; Coppi et al.[41] suggested that such unstable pulses could be made up of a superposition of normal modes which remained constant in absolute amplitude (rather than vanishing) for $x \to \pm\infty$. Chen[43] used wave packets to study shear stabilization of resistive drift waves, and Briggs and Lau[45] used this approach to study collisionless trapped particle instabilities in tokomaks.

Lau and Briggs[46] refined the wave packet approach by taking into account spreading and acceleration of wave packets, in addition to convection. They found that spreading was more important than convection in the case of shear stabilization of the universal drift instability, and in fact the normal mode stability criterion of Pearlstein and Berk[39] was equivalent to requiring that a wave packet spread fast enough to make up for its growth, so that its local amplitude would decrease with time. On the other hand, there were still cases (collisionless trapped particle modes) where $\gamma \gg v_g/R_p$ for pulses, so that a wave packet approach was more appropriate physically than a normal mode approach; and still other cases (negative energy loss cone mode) where wave packets were stable but normal modes were unstable, so that the instability was intrinsically nonlocal and could not be described by wave packets. Recently Manheimer[46a] has shown that even such "intrinsically nonlocal" instabilities can be analyzed by wave packets if terms proportional to $\partial^3 Q(x,\omega)/\partial x^2 \partial\omega$ are not ignored (i.e. if the sign of the wave energy is allowed to change across the width of the wave packet).

III. The Drift Cone and Drift Cyclotron Modes - Linear Theory

Most of the research we will survey in this section used the usual local approximation, assuming $a_i << R_p$.

The drift cyclotron instability in a Maxwellian plasma with $\gamma \simeq \ell\omega_{ci}$ and $\gamma << \omega_{ci}$ was first discussed by Mikhailovskii and Timofeev[18]. The drift cone instability seems to have been discovered independently by three different groups at about the same time. Rosenbluth and Post[47] briefly mention the possibility of such a mode, without giving any quantitative details, in a 1965 paper. Mikhailovskii[48] gives a dispersion relation for the drift cone mode using a ring distribtuion $f_i(v_\perp) = (2\pi v_0)^{-1} \delta (v_\perp - v_0)$, and points out that any loss cone distribution will be unstable to a similar mode. However, he uses straight-line ion orbits, valid when $\gamma >> \omega_{ci}$, so he does not find a stability threshold. Shima and Fowler[17] take the opposite limit $\gamma << \omega_{ci}$ and $\omega \simeq \ell\omega_{ci}$, so they do get an order of magnitude stability threshold, but don't get an exact value since they only include one cyclotron harmonic at a time. A precise stability threshold, including all cyclotron harmonics (and assuming $ka_i >> 1$, appropriate for $a_i/R_p << 1$) was found by Post and Rosenbluth[1], who also used collisional equilibrium loss-cone ion distributions rather than a ring distribution or a distribution of the form $v_\perp^2 \exp(-v_\perp^2/2v_i^2)$ (used in Ref. 17). The term "drift-cone" was apparently first used in a 1966 paper by Mihailovskii.[49] which also included the first use I have been able to find of an ion distribution made up of subtracted Maxwellians, $f_i(v_\perp) \propto \exp(-v_\perp^2/2v_{Hot}^2) - \exp(-v_\perp^2/2v_{hole}^2)$. (Such a distribution is much simpler to work with than an exact collisional equilibrium distribution and gives essentially the same results; in any case, the ion distributions in many real mirror confinement experiments are nowhere near collisional equilibrium.) Tang et al.[50] point out that such a distribution is close to a collisional equilibrium distribution for mirror ratio $R = v_{Hot}^2/v_{hole}^2 >> 1$. Comparisons between collisional equilibrium distributions, subtracted Maxwellain distributions, and distributions of the form $v_\perp^{2j} \exp(-v_\perp^2/2v_i^2)$, are made for smaller R by Moir.[51]

Aamodt[52] dropped the assumption that $ka_i >> 1$ and found stabilization of the drift cone mode with moderately low mirror ratio and $a_i/R_p \sim 1$, but pointed out that the local approximation was only marginally valid in this case. Cordey et al.[53] showed that this stabilization did not occur if the ion density gradient (neglected by Post and Rosenbluth[1] and by Aamodt[52]) were included. As pointed out by Mikhailovskii[50] the ion density gradient is more important than the loss cone when $(a_i/R_p)(v_{Hot}/v_{hole}) \gtrsim 1$, in which case the drift cone instability goes over into the drift cyclotron instability of a Maxwellian plasma. This condition is marginally satisfied for the loss cone plasma in the 2XIIB experiment.

Numerical calculations of the drift cone linear dispersion relation for various densities, density gradients, and mirror ratios, but limited to $\gamma \lesssim \omega_{ci}$, were made by Lindgren, Langdon and Birdsall[55].

Bhadra[56] as well as Krall and Fowler[57] showed that magnetic field curvature of the minimum-B type could stabilize the drift cyclotron mode since it would result in a relative drift of the ions and electrons $(T_i + T_e)c/eB_0$ in the direction opposite to the diamagnetic drift due to the density gradient. Even a small reduction in the drift could make the slope of the ion distribution function negative at the phase velocity.

This stabilization is made even greater by the effect of the field gradient ∇B on the electron convection term $\frac{\omega_{pe}^2}{R_p k \omega \omega_{ce}}$. Because the electron convection velocity $cE_\perp/B_0$ varies with x, it gives rise to an additional electron density variation of the same sign as that produced by the electron density gradient, and the phase velocity is increased, pushing it further into the range where the slope of the ion distribution is negative. These effects were believed to be important in toriodal octupole experiments by Ohkawa and Yoshikawa[58] in which the drift cyclotron mode may have been observed. For the drift cone instability Cordey et al.[27,53] showed that the field curvature is de-stabilizing, since an increase in the phase velocity and drift velocity put the phase velocity in a region where the slope of the ion distribution is even more positive.

Baldwin[59] showed that fanning of the magnetic field could stabilize modes which were unstable only over narrow-ranges of $k_\perp$, such as the Dory-Guest-Harris[60] or drift cyclotron ($\gamma << \omega_{ci}$) modes. However, fanning was found not to have much effect on the drift cone mode, which is unstable over a broad range of $k_\perp$.[61]

Finite β introduces effects associated with the field gradient $\nabla B/B = -(\beta/2)\ \nabla p/p$ which is present even in the absence of field curvature, as well as electromagnetic effects due to the coupling of the electrostatic drift waves with transverse modes. The effect of the field gradient on the electrons, discussed by Cordey et al.[27] in studying field curvature effects, is to multiply the electron convection term $(\omega_{pe}^2/k_y\omega\omega_{ce}R_p)$ by a factor $1 - 2(\nabla B/B)(n/\nabla n)$, or $1 + \beta$ in the absence of field curvature and temperature gradients. The effect of ∇B on the ion terms is discussed in Refs. 12, 50 and 62. The effect is not very great if $a_i/\nabla B/B << 1$.

The electromagnetic effects were found by Callen and Guest[11], and result in a term $\omega_{pe}^4/\omega_{ce}^2 k^2 c^2$, for $k_\parallel = 0$. This term has a simple physical explanation; it results from the fact that the electrons, although tied to the magnetic field lines (because $\omega \ll \omega_{ce}$ and $ka_e \ll 1$) can still move perpendicular to the field by pulling the field lines with them. In fact $\omega_{pe}^4/\omega_{ce}^2 k^2 c^2$ is equal to $1/k^2\lambda_B^2$, where λ_B is the Debye length of a massless fluid with pressure $B_0^2/8\pi$, a charge density $n_e e$ and a γ of 2. The effect of this term is to reduce the phase velocity. For $\beta \to 0$, the pressure $B_0^2/8\pi$ is very large (compared to other pressures in the problem) and this "magnetized electron fluid" is incompressible, so does not contribute to the dispersion relation.

For the drift cyclotron instability of a Maxwellian plasma, the ∇B effect on the electrons and the electromagnetic term $\omega_{pe}^4/\omega_{ce}^2 k^2 c^2$ nearly cancel out, so finite β has little effect on this instability, to the extent that the vacuum term and the effect of ∇B on the ions can be ignored. For the drift cone mode, the electromagnetic term is much more important, and is a stabilizing influence since it decreases the phase velocity. Tang et al[50] and Mikhailovskii[63] showed that this term is important if $\beta \gtrsim (v_{Hot}/v_{hole})^{2/3}(\omega_{ci}^2/\omega_{pi}^2 + m_e/m_i)^{1/3}$, and that in this case the minimum density gradient needed for instability is much greater than for $\beta \to 0$.

The effects of finite length on the drift cone mode and other flute modes (i.e. modes which require $k_\parallel \simeq 0$ to be unstable) are discussed in Refs. 62, 64-66. If the plasma were in contact with conducting plates at the ends of the machine, then only modes with $k_\parallel L = n\pi$ would be possible, and if $L \lesssim v_e/\omega$ these modes would all be stabilized by electron Landau damping (or more precisely, electron bounce resonance damping). Because of the ambipolar potential, however, the plasma is not directly in contact with the ends of the machine, but is surrounded by vacuum. In this circumstance normal modes can exist when $L \lesssim v_e/\omega$ for which the perturbed potential on a given field line is very nearly constant over the length of the plasma (i.e. $\partial\phi/\partial z \sim \omega k_\perp/\omega_{pe} \ll \omega/v_e \lesssim 1/L$) out to a point where the plasma density is sufficiently low that $k_\perp \lambda_D \gtrsim 1$, at which point the potential drops quickly to its value at the wall. (This "flute approximation" requires, of course, that $k_\perp\lambda_D \ll 1$ in the center of the plasma. If $k_\perp\lambda_D \gg 1$ throughout the plasma, then the perturbation sees the plasma as essentially vacuum, and only modes with $k_\parallel L = n\pi$ can exist. This might be enough to stabilize the short wavelength mode discussed in Ref. 12, even without the stabilizing mechanism discussed there.)

IV. Warm Plasma Stabilization and the Two Temperature Instability

Many authors have suggested that warm plasma filling or partially filling the loss cone might stabilize the drift cone mode; this suggestion has been made both as a means of getting rid of an observed instability, and as an explanation for the drift cone mode not being observed in experiments where it was expected. The warm plasma could come from an injected stream, or from charge exchange with cold neutrals present in the plasma (perhaps unintentionally) or from diffusion of hot ions into the loss cone.

Post and Rosenbluth[1] suggested warm plasma as a means of stabilizing the drift cone mode in a reactor. Galeev[2] calculated that in a machine of reasonable length, the instability would grow to level where ions were being diffused into the loss cone at the same rate as they would be lost out the ends of the machine once they were in the loss cone if the loss cone were just full enough for the mode to be marginally stable (i.e. quasilinear saturation). Only if the machine were unreasonably short would ions be lost out the ends so quickly that the wave would have to grow to a high enough level to saturate nonlinearly, i.e. by wave-wave coupling. (Actually, this calculation was done for the high frequency convective loss cone mode, but similar considerations would apply to the drift cone mode). If it is assumed that a large fraction of the ions have to be in the loss cone for marginal stability, it follows that the lifetime of plasma in a reasonably long mirror machine should not be more than a few ion bounce times. The fact that mirror confined plasmas have been observed to have much longer lifetimes[3,4] has motivated the search for a nonlinear mechanism to saturate the drift cone mode at a lower amplitude; these will be discussed in Sec. V of this chapter. However, Baldwin, Berk and Pearlstein[67] have recently shown that quasilinear saturation is consistent with the experimental observations if it is realized that only a small fraction of the ions need to be in the loss cone for marginal stability in present experiments, becasue most of the ions outside the loss cone have $v_\perp \gg v_\parallel$ (since they are injected that way, and electron drag is more important than pitch angle scattering when $T_e \ll T_i$; an experimental measurement of the pitch angle distribution of ions is given in Ref. 68). This also increases the ion transit time, which is $L/v_\parallel \gg L/v_i$.

If the plasma in the loss cone comes from an initially cold component, the plasma may also exhibit an ion two-temperature instability, in addition to the drift cone mode. Since the ion two-temperature mode is unstable when the warm component is too cool or too dense, and the drift cone stability occurs in the opposite limits, it is not immediately clear that both modes can be stabilized at once, without filling in the loss cone completely, so that the ion perpendicular velocity distribution function is monotonically decreasing. That this can be done with only a small amount of warm plasma (much less than that needed to completely fill the loss cone) is the most important result of Ref. 10.

The ion two-temperature instability was first discussed by Pearlstein, Rosenbluth and Chang[69] (although an earlier paper by Hall, Heckrotte and Kammash[70] dealt with a related instability requiring $k_\parallel \neq 0$ and $\omega_{pi} \ll \omega_{ci}$). They considered a cold component of much greater density than a hot loss cone component, with no density gradient. This corresponds to regime III in Ref. 10. They found stability when $n_{cold}/n_{hot} \gtrsim (\omega_{ci}^2/\omega_{pi}^2 + m_e/m_i)^{-1/2}$, unless the lower hybrid frequency happens to be very close to an ion cyclotron harmonic; this is condition (3) is the discussion of regime III in Sec. V of Ref. 10.

Pearlstein[71] extended this study to $n_{warm}/n_{hot} \lesssim 1$ and to $T_{warm} > 0$, and found stability when $n_{warm}/n_{hot} \lesssim (T_{warm}/T_{hot})^{3/2}$; this corresponds to the boundary between regimes II and V in Ref. 10. A similar result was obtained by Guest, Farr and Dory[72].

Farr and Budwine [73] numerically obtained stability thresholds for a cold component and a hot loss cone component with distribtuion function proportional to $v_\perp^{2j}\exp(-v_\perp^2/2v_i^2)$, varying j, n_{cold}/n_{hot}, and ω_{pi}/ω_{ci}. They found (as did Tataronis and Crawford[74] for the corresponding electron instability) that for a ring distribtuion of hot ions ($j \to \infty$), instability persists for n_{hot}/n_{cold} arbitrarily small (though at small growth rates) if the lower hybrid frequency is close to an ion cyclotron harmonic. Mynick, Birdsall and Gerver[75] found the corresponding stability thresholds using a warm Maxwellian component of varying temperature in place of the cold component, and varying j and n_{warm}/n_{hot}, but taking $\omega_{\ell h}/\omega_{ci} \to \infty$ so that straight line ion orbits could be used. A numerical study of the stability threshold for each cyclotron harmonic, in a hot loss cone plasma with a cold component, was made by Brossier et al.[76]. They found a threshold for the first harmonic which corresponds to the boundary between regimes II and III in Ref. 10, except that they use ω_{pi} only a few times ω_{ci}, instead of $\omega_{pi} >> \omega_{ci}$ as in Ref. 10. On the basis of this calculation, noise at the ion cyclotron harmonics in a mirror experiment was identified as an ion two-temperature instability.

The ion two-temperature instability with finite β effects (the electromagnetic $\omega_{pe}^2 k^2/c^2$ correction of ref. 11) was considered by Gomberoff and Cuperman[77], who made some analytic and numerical calculations applicable to the magnetosphere, using $n_{cold} \sim n_{hot}$, straight-line ion orbits, and loss cones of the form $v_\perp^{2j}\exp(-v_\perp^2/\omega_\perp^2)$. This instability was also considered more briefly by Mikhailovskii[63], and a related instability with $k_z \neq 0$ was studied by Coroniti et al.[78]

Crume et al.[79] did a computer simulation (1½ D, with particles) of a two temperature instability and found saturation when the cold particles were coherently heated by the wave to a sufficient temperature so that the distribtuion function would be linearly stable. This supports the idea that cold plasma in a mirror machine could be heated by the instability until a marginally stable distribution (on the boundary between regimes II and V in Ref. 10) were reached.

Several studies have been done of the use of warm plasma to stabilize the drift cone mode without introducing a two-temperature instability. Post[80] and Berk et al.[9] showed analytically that a warm loss cone component of the same mirror ratio as the hot loss cone component increases the critical density gradient for the drift cone mode by a "stabilization factor" which is, however, never much greater than one if the two-temperature mode is stable. Refs. 80 and 9 also include numerical studies of the effect of a Maxwellian warm component and found some values of the parameters for which the critical density gradient was greatly increased by a fairly small amount of warm plasma. However, most of these cases involved high mirror ratio plasmas in which the loss cone was filled in completely. In these studies the warm component was sometimes of a different species than the hot component; a warm hydrogen component would have four times less energy than a warm deuterium component of the same plasma frequency and thermal velocity, and might be advantageous to use in a reactor.

Moir[51] calculated the amount and temperature of warm plasma needed to stabilize loss cone instabilities, but assumed that the total distribution function had to be monotonic (a sufficient but not necessary condition for stability).

Brossier, Girard, and Hennion[81] did a numerical study of $k_{\parallel} = 0$ modes in a plasma composed of hot loss cone and warm Maxwellian ion components, with a density gradient. As in Refs. 80 and 9, for the stable cases considered the loss cone is almost completely filled in, and the conclusion drawn is that warm plasma is de-stabilizing if $T_{warm} \ll T_{hot}$, but stabilizing if $T_{warm} \sim T_{hot}$.

Fowler[82] showed that a small amount of warm plasma, not filling in the loss cone, could stabilize the drift cone mode if $n_{warm}/n_{hot} \gtrsim (T_{warm}/T_{hot})^{3/2}$; this is the boundary between regimes IV and V in Ref. 10. However, this is also approximately the condition for the ion two-temperature mode to be unstable. Because the condition for stabilizing the drift cone mode was not calculated exactly but only approximately, it is not clear from Ref. 82 that a regime like regime V in Ref. 10 exists at all, i.e. it could be that regime IV borders regime II. This would mean that stability of both the drift cone and ion two-temperature modes is possible only by completely filling in the loss cone, i.e. in regime VII.

For a plasma with $a_i/R_p \sim v_{hole}/v_{hot}$, like that of the 2XIIB and other mirror experiments, regime V barely exists, and it is necessary to completely fill in the loss cone in order to stabilize the drift cone mode. This requirement for stability was assumed in the quasilinear model of Baldwin, Berk and Pearlstein,[67] explaining the much lower amplitude of waves and longer plasma lifetime when a cold stream was injected. The amount of warm plasma needed to completely fill the loss cone is $n_{warm}/n_{hot} \sim (v_{hole}/v_{hot})^2$. Hence, in the absence of a stream, the drift cone mode should remain at a level so that the hot ions are diffused into the loss cone in a time $(v_{hot}/v_{hole})^2$ times an ion bounce time L/v_{hole} (since $v_{\parallel}$ for a typical ion in the loss cone is v_{hole}). With cold ions present (either from an injected stream or background gas) of density $n_{cold}/n_{hot} \geq (v_{hole}/v_{hot})^2$, it is only necessary for the waves to grow to an amplitude great enough to diffuse the cold ions to a velocity $v_{warm} \sim v_{hole}$ in a flight time. Since the diffusion rate in velocity space goes as $1/v_{\perp}^3$ for $v_{\perp} \gg \omega/k_{\perp} \sim v_{hole}$, the loss time for hot ions due to this amplitude of waves is about $(v_{Hot}/v_{hole})^5$ times an ion bounce time, considerably greater than without the cold ion source. In fact, this diffusion loss time is greater than the electron drag time for 2XIIB parameters, so the ion loss is dominated by electron drag when there is a cold ion source. The good agreement between the evolution of ion temperature and density in the experiment[8] and in a numerical calculation based on this quasilinear model[83] suggest that the drift cone mode does saturate by quasilinear diffusion, at least when there is a cold source present. Stabilization of the drift cone mode with an ion stream is also seen in the PR-7 by Ioffe et al.[84] (Saturation by electron trapping[85], discussed in Sec. V, is also consistent with the experiments in the absence of a cold source.)

V. Nonlinear Saturation of the Drift Cone Mode

Several other mechanisms for saturation of the drift cone mode have been proposed over the years, in addition to quasilinear diffusion into the loss cone.

Mikhailovskii[49] used some kind of strong turbulence theory involving wave-wave interaction to find a spatial diffusion coefficient for the drift cone mode $D_x \sim \gamma/k^2 \sim a_i^2\, v_{hole}/R_p$ (for the $\omega \sim \omega_{ci}$ mode, which has the highest γ/k^2) which implies a spatial diffusion time $\tau_x \sim (R_p/a_i)^3 (v_{Hot}/v_{hole})\, \omega_{ci}^{-1}$ for the plasma to diffuse a distance R_p. There is also diffusion in velocity space, and the time required for an ion to diffuse into the loss cone is $\tau_v \sim \tau_x (a_i/R_p)^{\frac{1}{2}} (v_{Hot}/v_{hole})^{\frac{1}{2}}$. Thus spatial diffusion is more important than velocity space diffusion precisely in those cases where the drift cone mode becomes a drift cyclotron mode ($a_i/R_p > v_{hole}/v_{Hot}$), i.e., when the ion density gradient is more important than the loss cone in driving the instability. (Note, however, that the calculation is not necessarily valid for $a_i/R_p \gg v_{hole}/v_{Hot}$.) This calculation was made assuming that ions are lost immediately when they reach the loss cone, thus precluding quasilinear saturation. Galeev[2] showed that in practice the ions would be lost only after an ion bounce time, L/v_i, and that quasilinear saturation would occur before nonlinear saturation except for very short machines.

Because $a_i/R_p \sim v_{hole}/v_{Hot}$ for the 2XIIB plasma, diffusion across the density gradient ought to be comparable in importance to diffusion into the loss cone; if the mode saturates by diffusion into the loss cone, then the density gradient ought to be significantly reduced by spatial diffusion at the same time. Sperling[86] considered quasilinear diffusion across the density gradient by the drift cyclotron mode.

Several authors[79,87-89] have looked at resonance broadening or trapping saturation of the Dory-Guest-Harris[60] and ion two-temperature modes in a uniform plasma, obtaining differing results.

A nonlinear disruption of the ambipolar potential by the perturbed potential near the ends of the plasma was suggested by Baldwin et al.[64] This would allow some electrons to flow to the ends of the machine, destroying the flute approximation (which assumed the plasma was isolated from the ends) and allowing electron bounce resonance damping. Ioffe et al.[84] consider (but reject) the possibility that an ion stream might stabilize the drift cone mode not by filling in the loss cone, but by end conduction.

Baldwin et al.[61] postulated an electron Langmuir mode (with $\omega \sim \omega_{pe} \sim \omega_{ce}$) driven unstable by electron anisotropy due to the mirror ratio and the ambipolar potential, saturating when the spatial diffusion rate reached γ/k^2. This electron spatial diffusion rate would be greater than γ/k^2 for the drift cone mode, and thus stabilize it, for $\omega_{pe}{}^2/\omega_{ce}{}^2 > 0.4$ and $a_i/R_p \lesssim 0.5$, accounting for the apparent stability of the drift cone mode in the 2XII. This mechanism was proposed because it was felt that no mechanism involving ion diffusion could account for the stability of the 2XII since the ion lifetime was several hundred bounce periods. However, once it was realized that $v_{Hot} \gg v_{hole}$ for the 2XII plasma, and that the velocity diffusion coefficient was proportional to $1/v_\perp{}^3$ for $v_\perp \gg v_{hole}$, it was possible to saturate the drift cone by quasilinear diffusion of cold ions in the loss cone, while diffusion of the hot ions with $v_\perp \gg v_{hole}$ was negligible, and the mechanism involving electron diffusion was no longer necessary. However, Liu and Aamodt[38] note that this electron diffusion could in any case account for the fact that the drift cone mode has never been observed at $\omega \gg \omega_{ci}$, even if it cannot stabilize the drift cone mode completely. This applies also to the short wavelength mode discussed in Ref. 12.

Aamodt[85] suggested that electron trapping could be the most important saturation mechanism of the drift cone mode when there is no cold ion source. The amplitude of the wave required for such trapping was found to be in fairly good agreement with the amplitude observed in the "noisy" mode of operation (i.e. with no ion stream and free of gas) for several different mirror machines with different parameters, viz. 2XII, 2XIIB, PR-6 and PR-7. The trapped electrons move around the equipotentials of the wave in their E x B motion (see Figure 3 in Ref. 7). Hence the trapping amplitude depends on the radial width of the normal modes, and the results of Ref. 7 and 13 should be useful in determining the trapping amplitude more exactly than in Ref. 85.

Another stabilizing mechanism was discussed by Ioffe et al.,[84] who noted that electron cyclotron resonance heating (ECRH) of electrons in the PR-7 plasma with $\omega_{pe} < \omega_{ce}$ resulted in stabilization of the drift cone mode. Since such heating results mostly in increasing the perpendicular velocity of the electrons, the authors suggested that the heated electrons became magnetically trapped, locally decreasing the ambipolar potential, and electrostatically trapping ions in the loss cone, thus stabilizing the drift cone mode. Aamodt[90] considers applying this method of stabilization to a fusion reactor.

We conclude this section by describing two nonlinear de-stabilizing mechanisms. First, Kanaev and Yushmanov[91] note that once the drift cone instability develops, it may lead to heating of the electrons through some unspecified nonlinear process. This in turn increases the ambipolar potential, which increases the size of the hole in the ion distribution, which increases the growth rate of the drift cone mode. The increase of the ambipolar potential, electron temperature, and phase velocity of the drift cone mode with time are actually observed in the PR-6 experiment. This cycle continues until some mechanism, such as ion diffusion or electron trapping, saturates the drift cone mode. The authors suggest that without this positive feedback effect the drift cone mode might not be observed at all, or would saturate at a lower amplitude. The connection between electron temperature and the drift cone mode growth rate is supported by experiments in which electrons are heated by ECRH (at $\omega_{pe} > \omega_{ce}$, resulting in isotropic heating) and the drift cone amplitude increases earlier than it would without ECRH.

Liu and Aamodt[38] consider a parametric instability in which a negative energy drift cone mode of finite amplitude with frequency slightly less than ω_{ci} decays nonlinearly into two positive energy waves of lower frequency. (The nonlinearity is in the motion of the electrons in the direction of the density gradient; the ion motion and the electron polarization drift are still treated linearly.) For PR-7 parameters, this parametric instability has a growth rate of 0.1 ω_{ci}, but is stable if the plasma is azimuthally asymmetric. The authors note that the PR-7 plasma[92] is azimuthally asymmetric when first injected; at this time, a drift cone mode grows linearly, but remains nearly monochromatic with frequency near ω_{ci}, and does not cause anomalous ion loss (because it is monochromatic).

Once the plasma has become axisymmetric (due to ∇B drifts) the parametric instability becomes unstable, and the wave spectrum broadens, spreading to frequencies of 0.5 ω_{ci} and below, and ion losses occur. Similar behavior is observed in the 2XIIB plasma[37]. The drift cone mode is unstable linearly, but this nonlinear effect may be crucial in its causing ion diffusion, which requires a finite bandwidth. Chu, Hendel and Simonen[93] have found experimental evidence for a similar nonlinear broadening of the spectrum of low frequency ($\omega \ll \omega_{ci}$, $k_z \neq 0$) drift waves in a Maxwellian Q-machine plasma.

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