DIVISION OF LABOR AND THE DISTRIBUTION OF INCOME

by

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ABSTRACT

Earlier work dealing with the effects on intersectoral migration of increased productivity is extended in the light of Vislie's paper. Specifically included is the possibility of different wages in different sectors. Such differences, and changes in such wage differences, may lead to redistributions of income, which in turn open up possibilities of demand changes. Disequilibrium analysis is the principal analytical tool used, and it is suggested that not only the earlier work of Simon and Baumol but also the more recent studies by Harris-Todaro, Houthakker and Vislie can be integrated into the conceptual framework here developed.

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1. Introduction

Jon Vislie's response to our paper (1977) on the division of labor in society is interesting and stimulating. His paper contains a number of attractive features. First, and foremost, he introduces an assumption that the growth rate of wages in each sector is a pre-determined variable. In Vislie's paper, the predetermination occurs through government control over the growth in wages, but one might also consider, at any one point in a temporal sequence, the predetermination occurring as the result of union-employer negotiations. Although we have serious reservations about the formal way in which Vislie introduces this growth-rate variable, we believe that the basic idea is a very fertile one, and we shall use it in this paper to extend both his and our own earlier results. Second, he derives an expression for the sectoral growth rate of employment -- see equation (7) in Vislie's paper -- which is interesting and quite rich in its analytical contents. Third, some of his conclusions are provocative -- see, in particular, the verbal discussions related to his equations (10) and (11). Indeed, they have provoked us into preparing this response to Jon Vislie's comment.[1] But first, and for the benefit in particular of readers who are not familiar with our original paper, allow us to summarize very briefly the main features of the economic model we used and the main results that we obtained.

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[1] Vislie claims that his paper is much "simpler" than ours. We certainly agree that our original paper was more formal, more abstract, than his -- but whether that makes it less "simple" is an open question. For one thing, there was no counterpart in our paper to the long and cumbersome derivation of Vislie's central equation (7) -- sensibly appended to his paper. For another, and of greater significance, Vislie is forced to use very restrictive assumptions. For example, Vislie uses linear production functions, whereas we only required concavity. His demand functions have no theoretical basis, as Vislie himself admits, whereas much more general demand functions were used in our framework.

Our paper was steeped in a classical tradition, taking an early paper by Herbert Simon as its starting-point. It assumed flexibility of wages and complete mobility of labor among sectors. Our methodological approach was to use a form of disequilibrium analysis, whereby we investigated under what conditions excess demands would arise in the sectors of the economy. In turn, and under our assumptions, labor would migrate away from a sector with negative excess demand and into a sector with positive excess demand. A key result in our analysis determined the <u>direction</u> of labor migration on the basis of a relationship between labor productivities and price elasticities in the sectors. From this we demonstrated that the conclusion drawn by Simon about migration from rural to urban areas could be obtained as a special case. In another application, we showed in the context of "unbalanced growth" studied more recently by Baumol, under what condition labor would move towards the so-called "progressive" sector. (We also obtained a number of other results, particularly concerning shifts in relative prices, but they are of lesser relevance here, since Vislie's work does not address itself to any of those issues.)[2] Now, Vislie using a different model, and with more restrictive underlying assumptions, reaches the same Simon-Baumol type of conclusions as we did. However, when he introduces a new assumption about differing wage-growth rates between the sectors, he finds that <u>exactly the opposite results may occur</u>.

What we shall set out to do in this follow-up study is to explore whether these seemingly contradictory results can be obtained within our

[2] We have discovered some printing errors in our paper. On p.187, line 7, from the bottom, the second inequality should read: "$e_{jj} < -1$" (rather than "$e_{ij} < -1$", as printed). On p.189, eq.(3.7), right-hand side, should read: "$\bar{x}_i + o(\Lambda_1,\Delta_2)$."

own earlier disequilibrium framework and, thus, whether a satisfactory theoretical explanation can be found to comprehend these "rather strange" results, as Vislie refers to them.

In order to retain comparability between Vislie's results and whatever conclusions we can draw in the following exploration, we shall use <u>either</u> the same assumptions as he uses, <u>or</u> more general assumptions in which case his results should come out as special cases. The major generalization that we shall introduce -- and we believe this is a very important extension of both his and our own earlier work -- concerns effects on demand by way of changes in the distribution of income.

2. Production, labor, wages and prices

As in Vislie, assume that supplies of the products are provided through production functions of the form $S_i = A_i L_i (1 + \gamma_i)$, $i = 1,2$ where S_i is the quantity produced and competitively supplied in sector i; L_i is the quantity of labor employed in sector i; γ_i denotes the change in labor productivity and A_i is a positive constant. For convenience where Vislie used exp $\gamma_i t$ we have used $(1 + \gamma_i)$. Here, and without loss of generality, we can choose units such that $A_i = 1$ for all i. Hence, initially: $S_i = L_i$ and $p_i = w_i$; $i = 1,2$, where p_i denotes the unit price of product i and w_i denotes the unit wage paid in sector i.

Consider then a new labor allocation vector L_i^N, i=1,2 and a <u>new wage</u> vector

$$w_i^N = (1 + g_i) w_i \qquad i = 1,2 \tag{1}$$

where g_i is a government-controlled parameter.

From (1) and following Vislie's assumption of marginal cost pricing, we obtain a <u>new price</u> vector

$$p_i^N = (1 + \gamma_i)^{-1} w_i^N \qquad (2)$$

3. <u>Demand and supply</u>

At the new wage, w_j^N, $j = 1,2$, and the new prices p_i^N, $i = 1,2$, a consumer's demand for commodity i is:

$$d_i^j = d_i^j(p_1^N, p_2^N, w_j^N) \qquad i,j = 1,2 \qquad (3)$$

Hence, the aggregate new demand for commodity i will be:

$$D_i^N = L_1^N d_i^1 (p_1^N, p_2^N, w_1^N) + L_2^N d_i^2 (p_1^N, p_2^N, w_2^N) \qquad i = 1,2 \qquad (4)$$

The corresponding new supplies will be:

$$S_i^N = (1 + \gamma_i) L_i^N \qquad i = 1,2 \qquad (5)$$

4. <u>Excess demand analysis</u>

From (4) and (5), the aggregate excess demand, X_i, for each commodity is obtained:

$$X_i^N = D_i^N - S_i^N \qquad i = 1,2 \qquad (6)$$

<u>Lemma 1</u> $\quad p_1^N X_1^N + p_2^N X_2^N = 0 \qquad (7)$

<u>Proof</u> $\quad p_1^N X_1^N + p_2^N X_2^N = L_1^N\{p_1^N d_1^1(p_1^N,p_2^N,w_1^N) + p_2^N d_2^1(p_1^N,p_2^N,w_1^N)$
$-p_1^N(1+\gamma_1)L_1^N\} + L_2^N \{p_1^N d_1^2(p_1^N,p_2^N,w_2^N) + p_2^N d_2^2(p_1^N,p_2^N,w_2^N)$
$-p_2^N(1+\gamma_2)L_2^N\}$

Now

$$p_1^N d_1^j(p_1^N,p_2^N,w_j^N)+p_2^N d_2^j(p_1^N,p_2^N,w_j^N) = w_j^N \qquad j = 1,2$$

Hence:

$$p_1^N X_1^N + p_2^N X_2^N = L_1^N \{w_1^N-(1+\gamma_1)p_1^N\} + L_2^N \{w_2^N - (1+\gamma_2)p_2^N\} \qquad (8)$$

Substituting from (2) into (8) gives (7).

Define the excess demand per capita:

$$x_i^N = \frac{X_i^N}{L_1^N+L_2^N} \quad \text{and} \quad \ell_i^N = \frac{L_i^N}{L_1^N+L_2^N}, \text{ with } \ell_1^N + \ell_2^N = 1$$

So:

$$x_i^N = \ell_1^N d_i^1(p_1^N, p_2^N, w_1^N) + \ell_2^N d_i^2(p_1^N, p_2^N, w_2^N) - (1+\gamma_i)\,\ell_i^N \qquad i = 1,2 \tag{9}$$

As before and merely transforming to per capita units:

$$p_1^N x_1^N + p_2^N x_2^N = 0 \tag{10}$$

Lemma 2 $\quad x_1^N \gtrless 0 \Leftrightarrow x_2^N \lessgtr 0$

Proof $\quad$ Trivial from (10).

Substituting from (1) and (2) into (9), and concentrating for the moment on sector 1:

$$x_1^N = \ell_1^N d_1^1\left(\frac{1+g_1}{1+\gamma_1}\cdot w_1,\ \frac{1+g_2}{1+\gamma_2}\cdot w_2,\ (1+g_1)w_1\right) + \ell_2^N d_1^2\left(\frac{1+g_1}{1+\gamma_1}\cdot w_1,\ \frac{1+g_2}{1+\gamma_2}\cdot w_2,\ (1+g_2)w_2\right) - (1+\gamma_1)\ell_1^N \tag{11}$$

Now, let us assume that in the initial situation before any wage or productivity change, that is, where $g_1 = \gamma_1 = 0$, if $\ell_1^N = \ell_1^*$ there exists no excess demand:

$$0 = \ell_1^* d_1^1(w_1, w_2, w_1) + \ell_2^* d_1^2(w_1, w_2, w_2) - \ell_1^* \tag{12}$$

Next, we calculate the per capita excess demand, x_1^N, at this initial labor allocation (ℓ_1^*, ℓ_2^*) given the new prices and wages, p_i^N, w_i^N, $i = 1,2$, and the new supply, $(1+\gamma_1)\ell_1^*$:

$$x_1^N = \ell_1^*\{d_1^1((1+g_1-\gamma_1)w_1,\ (1+g_2-\gamma_2)w_2,\ (1+g_1)w_1) - (1+\gamma_1)d_1^1(w_1, w_2, w_1)\}$$

$$+ \ell_2^*\{d_1^2((1+g_1-\gamma_1)w_1,\ (1+g_2-\gamma_2)w_2,\ (1+g_2)w_2) - (1+\gamma_1)d_1^2(w_1, w_2, w_2)\}$$

$+ 0(\gamma_1, \gamma_2)$, where we have used the initial condtion $p_i = w_i$, and the approximation: $(1+\gamma_i)^{-1}(1+g_i) \approx 1+g_i-\gamma_i$, $\ i = 1,2$.

5. Interpretation

Finally, some algebraic manipulation of the above expression gives us the following:

$$x_1^N = -(\ell_1^* d_1^1 + \ell_2^* d_1^2)\gamma_1 + (\ell_1^* d_1^1 E_{11}^1 + \ell_2^* d_1^2 E_{11}^2)(g_1 - g_2 - \gamma_1 + \gamma_2) + \ell_1^* d_1^1 E_{10}^1 (g_1 - g_2 - \gamma_1 + \gamma_2) + \ell_1^* d_1^1 E_{10}^1 \gamma_1 + \ell_2^* d_1^2 E_{10}^2 \gamma_2 + 0_1(\gamma_1, \gamma_2), \tag{13}$$

where we have used E_{11}^j to denote the own-price elasticity of demand for commodity 1, among consumers with wage j, and E_{10}^j to denote the corresponding income elasticity of demand.

Following the same procedure to find an expression for the per capita excess demand for the second commodity, we get:

$$x_2^N = -(\ell_1^* d_2^1 + \ell_2^* d_2^2)\gamma_2 + (\ell_1^* d_2^1 E_{22}^1 + \ell_2^* d_2^2 E_{22}^2)(g_2 - g_1 + \gamma_1 - \gamma_2) + \ell_2^* d_2^1 E_{20}^1 (g_2 - g_1 + \gamma_1 - \gamma_2) + \ell_1^* d_2^1 E_{20}^1 \gamma_1 + \ell_2^* d_2^2 E_{20}^2 \gamma_2 + 0_2(\gamma_1, \gamma_2), \tag{14}$$

There is a structural affinity between these two expressions, on the one hand, and Vislie's central equation (7), on the other. In order to bring out the similarities, as well as the differences, let us take a careful look at each component term; since (13) and (14) are completely symmetrical, we focus on (13).

$\underline{-\gamma_1(\ell_1^* d_1^1 + \ell_2^* d_1^2)}$

The interpretation of this first term goes as follows. The parenthetical expression shows, as can be seen from eq.(12), above, the initial per capita supply of commodity 1; multiplying it by the (growth) rate of productivity change in sector 1, γ_1, leads to the change in supply (per capita) which results if the initial labor allocation is maintained. For productivity <u>increases</u>, ($\gamma_1 > 0$), this term is always <u>negative</u>, i.e., it reduces the excess demand for commodity 1.

$\underline{(\ell_1^* d_1^1 E_{11}^1 + \ell_2^* d_1^2 E_{11}^2)(g_1 - g_2 - \gamma_1 + \gamma_2)}$

To facilitate the interpretation of this term, note that the factor $(g_1 - g_2 - \gamma_1 + \gamma_2)$ can be rewritten as $\Delta(p_1/p_2)[p_1/p_2]^{-1}$, i.e., it expresses the relative change in the price ratio of the two commodities. In turn, such changes are determined by the sectoral wage-growth rates and productivity growth rates, as can be seen from our equations (1) and (2).

The other factor in the term measures the weighted (absolute) change in per capita demand for commodity 1 resulting from a relative change in the price of commodity 1.

Taken together, the two factors measure a kind of substitution or price effect, namely the change in the per capita demand for commodity 1 which results from the relative change in the price ratio. The resulting change in the excess demand is positive, if the price of commodity 1 falls in relation to the price of commodity 2.

$\underline{\ell_1^* d_1^1 E_{10}^1 (g_1 - g_2 - \gamma_1 + \gamma_2)}$

This term can be interpreted as a redistribution-of-income effect on the demand for commodity 1. It arises whenever there is a (positive or negative) change in the price ratio p_1/p_2.

As above, the second factor can be rewritten as $\Delta(p_1/p_2)[p_1/p_2]^{-1}$. Suppose this factor is positive. That means that the income earned in sector 1, per unit of product, increases -- at the expense of income earned in sector 2. By way of the factor $\ell_1^* d_1^1 E_{10}^1$ this (redistributive) income gain is partially used to increase the demand for commodity 1. Thus, the resulting change in the excess demand is positive, if the price of commodity 1 rises in relation to the price of commodity 2.

$\underline{\ell_1^* d_1^1 E_{10}^1 \gamma_1 + \ell_2^* d_1^2 E_{10}^2 \gamma_2}$

These two terms taken together can be described as a real-income effect on the demand for commodity 1. Suppose that both γ_1 and γ_2 are positive. Then, the economy produces more of both commodities (recall that the labor allocation is left unchanged). In turn, this raises the (real) incomes of the wage-earners in both sectors. By way of the respective income elasticities, E_{10}^1 and E_{10}^2, these gains in real income are partially channelled into new demands for commodity 1. The effect that these two terms exert on the excess demand for "normal" commodities (thus disregarding "inferior" goods) is always of the same sign as the productivity changes; for example, if labor productivity were to fall in a sector, there would be a negative real-income effect on the excess demand.

6. Direction of labor migration

We can now consider the four cases treated by Vislie.

Case 1 $\gamma_1 = \gamma_2 = \gamma, \quad g_1 = g_2.$

The excess demand (13) for this case can be expressed as

$$x_1^N = \gamma\{\ell_1^* d_1^1 (E_{10}^1 - 1) + \ell_2^* d_1^2 (E_{10}^2 - 1)\} + 0(\gamma).$$

If the demand for the output of sector 1 (the non-food sector in Simon's model) is income elastic for both income groups, $E_{10}^1 > 1$, $E_{10}^2 > 1$, then $x_1^N > 0$ (and so $x_2^N < 0$ by Lemma 2), and there is an excess demand for labor in sector 1. This is Simon's result and conforms with Vislie's and our earlier result. Observe, however, that if the two incomes are very different, $w_1 >> w_2$ say, so that $E_{10}^2 < 1$ and if ℓ_2^* is sufficiently large then x_1^N may well be negative, which would be contrary to the earlier results.

Case 2 $\gamma_1 = \gamma > 0$, $\gamma_2 = 0$, $g_1 = g_2$

The expression (13) now simplifies as

$$x_1^N = -\gamma\{\ell_1^* d_1^1(1 + E_{11}^1) + \ell_2^* d_1^2(1 + E_{11}^2)\}$$

Hence if $E_{11}^1 < -1$, $E_{11}^2 < -1$ i.e., the demand for the output of the "progressive" sector 1 is elastic, then there is an excess demand for labor in this sector. Conversely, if the demand for this sector is inelastic, then labor migrates to the non-progressive sector 2. This is in conformity with Baumol, Vislie and our earlier result. Observe again that if the incomes of the two groups differ considerably then the "expected" result might not hold.

This is a good place to note that Baumol's paper has been criticized on the grounds that he assumes that wages in both sectors are identical, $w_1 = w_2$, and that he ignores income effects (see Birch and Cramer (1968), Lynch and Redman (1968) and Keren (1972).) Here we permit $w_1 \neq w_2$ and since we are considering a general equilibrium setting, income effects are automatically included in (13). The "robustness" of Baumol's result is remarkable.

Case 3 $\gamma_1 = \gamma_2 = \gamma > 0$, $g_1 \neq g_2$

For this case, the excess-demand relationship will take the form:

$$-\gamma(\ell_1^* d_1^1 + \ell_2^* d_1^2 - \ell_1^* d_1^1 E_{10}^1 - \ell_2^* d_1^2 E_{10}^2)$$

$$+ (\ell_1^* d_1^1 E_{11}^1 + \ell_2^* d_1^2 E_{11}^2)(g_1 - g_2) + \ell_1^* d_1^1 E_{10}^1 (g_1 - g_2)$$

Here, the first term is identical to the expression obtained in Case 1, above. As we saw from the discussion of that case, this term is positive for $E_{10}^1 > 1$, $E_{10}^2 > 1$, i.e., where both income groups exert income-elastic demands for commodity 1. But whereas in that case the first term was also the only term, we now have two additional terms in the expression for excess demand. The second term represents what we in the interpreation of our more general case, above, called a substitution or price effect. (Recall that the factor $(g_1 - g_2)$, here, is a special case of $(g_1 - g_2 - \gamma_1 + \gamma_2)$, since we have taken $\gamma_1 = \gamma_2$.) It can be described as the change in per capita demand for commodity 1 which results from the relative change in the price ratio, for the two commodities. This term is very close, in its contents, to the last term in Vislie's eq.(7) -- and that is precisely the term which leads Vislie to draw a seemingly anomalous conclusion, in the discussion of his eq.(10). He suggests that for g_1 sufficiently large in relation to g_2, this term (which is negatively signed) will out-weigh the influence of the first term: Labor will migrate from urban to rural areas, the opposite of Simon's result. In the terms of our conceptual framework, this possible effect can now be simply explained as follows. If g_1 is much larger than g_2, it means that prices rise sharply in sector 1 (the non-food sector in Simon's model). This will curtail consumer demand for the products of sector 1 -- in turn leading to an outflow of labor from this sector and into the other sector.

The third term in the expression for excess demand is what we in the general discussion, above, called a redistribution-of-income effect. Using Vislie's anomalous-case assumption once again, this positively signed term reinforces the first term in the excess-demand expression, hence reduces the likelihood of occurrence of the Vislie special case.

As an example of the significance of this term in interpreting real-world phenomena, take the case of the so-called "modern" sector in a developing economy. If that sector (corresponding to sector 1, here) is sufficiently "disarticulated", it can to a large extent generate its own demand, thereby further widening the gap between, say, wages in the "modern" and the "traditional" sectors.

<u>Case 4</u> $\gamma_1 = \gamma > 0, \gamma_2 = 0 \quad g_1 \neq g_2$

For this case, no less than four of the five terms in the general expression of excess demand are retained:

$$x_1^N = -\gamma(\ell_1^* d_1^1 + \ell_2^* d_1^2) + (g_1 - g_2 - \gamma)(\ell_1^* d_1^1 E_{11}^1 + \ell_2^* d_1^2 E_{11}^2)$$
$$+ (g_1 - g_2 - \gamma)\ \ell_1^* d_1^1 E_{10}^1 + \ell_1^* d_1^1 E_{10}^1 \gamma + 0(\gamma)$$

Algebraically, this can be simplified to

$$x_1^N = -\gamma[\ell_1^* d_1^1(1 + E_{11}^1) + \ell_2^* d_1^2(1 + E_{11}^2)]$$
$$+ (g_1 - g_2)(-\ell_1^* d_1^1 E_{12}^1 + \ell_2^* d_1^2 E_{11}^2) + 0(\gamma)$$

Here, again, we see that for sufficiently large g_1, the second term which would then be negatively signed, could outweigh the (positively signed) first term. Thus, the exceptional case of <u>reverse</u> movement of labor, as compared to case 2, above, could arise. However, in this simplified form there does not seem to exist any easy interpretation (or explanation) of the second term. Hence, to comprehend the excess-demand possibilities for <u>case 4</u>, it seems best to take recourse to our general case discussion, above, and hence to couch the interpretation in terms of our substitution effect, redistribution-of-income effect and real income effect. As before, then, the real income effect on excess demand would be positive (for $\gamma > 0$), whereas the signs of the other two effects would be determined by the direction of the price change, i.e., the sign of $(g_1 - g_2 - \gamma)$.

7. Dynamical Models

From (11) we note that there is a unique labor allocation which results in zero excess demand given by

$$0 = \ell_1^N d_1^1\left(\frac{1+g_1}{1+\gamma_1} w_1, \frac{1+g_2}{1+\gamma_2} w_2, (1+g_1)w_1\right) + \ell_2^N d_1^2\left(\frac{1+g_1}{1+\gamma_1} w_1, \frac{1+g_2}{1+\gamma_2} w_2,\right.$$

$$\left.(1+g_2)w_2\right) - (1+\gamma_1)\ell_1^N,$$

which gives

$$\frac{\ell_1^N}{\ell_2^N} = \{d_1^2\left(\frac{1+g_1}{1+\gamma_1} w_1, \frac{1+g_2}{1+\gamma_2} w_2, (1+g_2)w_2\right)\}\ \{1 + \gamma_1 - d_1^1\left(\frac{1+g_1}{1+\gamma_1} w_1,\right.$$

$$\left.\frac{1+g_2}{1+\gamma_2} w_2, (1+g_1)w_1\right)\}^{-1} = f(g_1,g_2,\gamma_1,\gamma_2), \text{ say.} \tag{15}$$

At this labor allocation ratio both product and labor markets clear.

Now suppose that $\gamma_i(t)$ is the exogenously given productivity increase and let $g_i(t)$ be the exogenously specified wage increase. Assume, as Vislie does, that labor migrates in such a way that excess demand is driven to zero. (This may equally be labeled a 'full employment' assumption.) Then the labor allocation must be governed by

$$\frac{\ell_1(t)}{\ell_2(t)} = \frac{L_1(t)}{L_2(t)} = \frac{L_1(t)}{L(t)-L_1(t)} = f(g_1(t), g_2(t),\gamma_1(t),\gamma_2(t)), \tag{16}$$

where $L_i(t)$ is the labor in sector i and $L(t)$ is the exogenously given total labor force. When represented as a differential equation, (16) is a generalization of Vislie's equation (7).

The assumption that full employment obtains in the face of unequal wages rests on accepting the notion that labor migrates only in response to employment opportunities and not in response to wage differences. If we think of sectors 1 and 2 as non-farm and farm or urban and rural sectors respectively and assume that urban wages are higher, it is likely that workers move to the city in search of higher wage jobs even if employment opportunities in the city are restricted. This would result in unemployment in the city. In this case, in equation (16) $L_i(t)$ is the labor force employed in sector i, and if $\hat{L}_i(t)$ is the actual supply of labor force in sector i, then

$$L_1(t) < \hat{L}_1(t), \quad L_2(t) = \hat{L}_2(t)$$

How are $L_1(t)$, $\hat{L}_1(t)$ related? One possibility, following Harris and Todaro (1970), is to say that migration into the city continues until the point where the expected urban wage becomes equal to the rural wage, that is,

$$w_1(t) \frac{L_1(t)}{\hat{L}_1(t)} = w_2(t) \tag{17}$$

Here $\frac{L_1(t)}{\hat{L}_1(t)}$ is the probability of finding a job in the city and so the first term in (17) is the expected urban wage rate. Equations (16), (17) yield a model very similar to that of Harris and Todaro.

In Vislie's model full employment is maintained in the face of wage rigidity by restricting labor mobility. In the Harris-Todaro model wage rigidity together with labor mobility leads to unemployment in the high-wage sector. Houthakker (1976) adopts a position in between these two. He takes the supply of labor $(L_1(t), L_2(t))$ to be momentarily fixed in each period. Wages in this period then adjust to obtain a full employment equilibrium. In terms of (16) suppose we take sector 1 wages as numeraire i.e., $w_1(t) \equiv 1$ so that $g_1(t) \equiv 0$. Given $L_1(t)$, $L_2(t)$, $\gamma_1(t)$, $\gamma_2(t)$, (16) is solved for $g_2(t)$.

$$\frac{L_1(t)}{L_2(t)} = f(0, g_2(t), \gamma_1(t), \gamma_2(t))^{3}. \tag{18}$$

[3] This is clearly not a convenient expression of the full employment equilibrium in order to evaluate $g_2(t)$. For that purpose, Houthakker's formulation is more suitable.

The equilibrium wage rate $(w_1(t), w_2(t)) = (1, w_2(1+g_2(t))$ is assumed to prevail in the interval $(t,t+\Delta t)$. Labor now migrates in response to this new wage so that

$$(L_1(t+\Delta t), L_2(t+\Delta t)) = \phi(w_1(t), w_2(t), L_1(t), L_2(t)). \tag{19}$$

Houthakker adopts a particular form for the migration function ϕ:

$$L_1(t+\Delta t) - L_1(t) = \mu L_1(t) L_2(t) (w_1(t) - w_2(t)).$$

$$L_2(t+\Delta t) - L_2(t) = -\mu L_1(t) L_2(t) (w_1(t) - w_2(t)).$$

where the constant $\mu > 0$ controls the magnitude of migration. Having obtained from (19) the new labor supplies at time $t+\Delta t$, we return to (18) to obtain the new wages, and the process is repeated. This is Houthakker's model of growth and income distribution.

8. Conclusion

In the light of Vislie's paper, we have here extended the conceptual format of our earlier work on division of labor (1977). The new features have enabled us to analyze the economic effects of two principal forms of shock or exogenous change, namely technological changes (captured as sectoral productivity changes) and wage changes. The effects of such changes on the allocation of labor between sectors have been traced by way of their effects on prices and incomes. The labor markets have been modelled in a sufficiently flexible way to permit analysis of quite different situations. These range from the classical and neo-classical case of complete wage equalization among sectors to cases of wage rigidity or of highly restricted labor mobility.

The analytical results have reproduced the conclusions of our earlier paper, as well as the results of such seemingly diverse papers as Baumol (1967), Harris-Todaro (1970), and Houthakker (1976). Moreover, a theoretically satisfying explanation has been provided for the seemingly anomalous results on labor migration obtained by Vislie (1978).

One limitation that the present paper shares with Vislie (1978), arises from the rigid relationship here assumed between a wage change and a (consequent) price change. In further work that we are presently pursuing, this rigidity will be removed.

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