PRICING AND SITE SELECTION WITH MARKET IMPERFECTIONS: HOTELLING REVISITED

by

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Memorandum No. UCB/ERL M79/17

15 March 1979

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Research sponsored in part by the National Science Foundation
Grant SOC76-80333 administered by the Electronics Research Laboratory,
College of Engineering, University of California, Berkeley, CA 94720.

"(A) landlord or realtor who can determine the location of future stores, expecting to absorb their profits in the sales value of the land, has a motive for making the situation as unsymmetrical as possible..."
H. Hotelling (1929)

This year marks the fiftieth anniversary of Harold Hotelling's "Stability in Competition". It is recognized as having provided one of the intellectual sparks that led to the monopolistic-competition revolution; Kenneth Boulding, for one, used it as a basis for enunciating "the principle of minimum differentiation". In the realm of location theory, Paul Samuelson saw in this paper "a worthy successor to the special models of Thünen, Weber, and Fetter and a worthy forerunner of the Lösch-Christaller hexagonal patterns of two-dimensional location" (Samuelson, 1960). All told, Hotelling's paper has attracted a remarkable number of students over the years, and it continues to inspire new analysis and results, as illustrated in the recent work by Eaton and Lipsey (1975) and Gannon (1972).

Yet, one aspect of the paper appears to have been neglected in the literature: Suppose that a landowner or realtor can control the location of each firm. How would he or she determine the sites for the firms' locations? This problem was indeed posed by Hotelling, as seen from the quotation above, but he provided no formal analysis and his brief statement about the problem raises questions rather than answers them. For example, what does it mean, when Hotelling writes that the landowner has a motive for making the locational pattern of firms "as unsymmetrical as possible?" How does the spatial pattern -- and its economic implications -- vary, as the number of firms varies? If the landowner chooses sites for the firms so as to maximize the rent which can be extracted, and if the firms -- given these rents -- choose prices so as to maximize their profits, do equilibrium solutions always exist? How are the costs borne by consumers affected by the landowner's optimizing decisions?

In the following we analyze these and some related questions by considering the case of a rent-maximizing landlord who determines the locations for two, three, or four sellers in a one-dimensional market, either bounded or unbounded. We also examine the case where the selection of sites is regulated by a government agency. In the spirit of Hotelling's original paper, we rule out monopoly, as well as the possibility of collusion among the sellers, in the product market. Our results suggest that the landlord has an incentive to make the number of firms admitted to the market as small as possible and that, independent of the number of firms to which the landowner allows entry, he or she will always choose a locational pattern such that every firm has only a one-sided market. As will be shown, this pattern has the implication of forcing <u>all</u> consumers to travel in the same direction to purchase the good, leading to higher "social" costs than alternative patterns. Differently expressed, the imperfection in the factor market ("site market") induces behavior which makes the product market as imperfect as law or custom will permit.

Hotelling's Analysis

First, let us go back to the origin and see briefly how Hotelling formulated and solved his problem. The essential assumptions underlying Hotelling's analysis were as follows: 1) The buyers of a homogeneous product are uniformly distributed (constant density) along a line of length L. 2) The buyers' demand for the product is completely inelastic and such that one unit of the product is demanded per unit length of the line (and per unit of time). 3) Each buyer incurs a transportation cost in bringing the product from the seller's location to his or her destination. This transportation cost is \$ c per unit distance and per unit of product. 4) There are two sellers of the product, X and Y. They compete with each other on the basis of two means of competition ("policy variables"), namely price -- p_x and p_y -- and location -- measured as the distances, a and b, to the endpoints of the line (the market). Relocation costs are disregarded. A buyer chooses a seller on the basis of delivered price (mill price plus transport cost). 5) The two firms are sales-maximizers. Since Hotelling ignores production costs, this is equivalent to profit-maximizing. 6) The two firms apply their policy variables using a behavioral rule known as "zero conjuctural variation", meaning that each firm varies its price and location without considering any (price or locational) reaction from the other firm. 7) There is no uncertainty, in the sense that both sellers and all buyers know all there is to know about the market. 8) Collusion between the firms and cut-throat-competition -- in a meaning to be defined more precisely below -- are ruled out.

Using a and x to denote the market segments of firm X, and b and y to denote the market segments of firm Y, the sum of these four segments equals the total market L (see Fig. 1):

$$a + x + b + y = L \tag{1}$$

The market boundary between the two firms is where delivered prices are equal:

$$p_x + cx = p_y + cy \tag{2}$$

By means of equations (1) and (2), expressions of the profits, π_x and π_y, are then obtained:

$$\pi_x = p_x(a+x) = p_x(L+a-b)/2 - p_x^2/2c + p_x p_y/2c \tag{3}$$

$$\pi_y = p_y(b+y) = p_y(L-a+b)/2 - p_y^2/2c + p_x p_y/2c \tag{4}$$

Profit-maximization, with respect to own price, then requires:

$$p_x = c(L+(a-b)/3) \tag{5}$$

$$p_y = c(L-(a-b)/3) \tag{6}$$

Substituting for p_x and p_y into (3) and (4) yields:

$$\pi_x^* = (L+(a-b)/3)^2 c/2 \tag{7}$$

$$\pi_x^* = (L-(a-b)/3)^2 c/2 \tag{8}$$

Beginning with an initial pair of prices, both firms will iteratively and alternately adjust their pricing until equations (5) and (6) are satisfied. Hotelling imposed one restriction on this price-adjustment process, namely:

$$p_x + c(x+y) > p_y > p_x - c(x+y) \tag{9}$$

In words, the difference in prices must be less than the cost of transporting the good between the two firms' locations. Thus, each firm presumes that should its delivered price at the competitor's location be less than the latter's mill price, a price war would result. As a consequence, either firm's entire market might be wiped out, and this behavior is avoided.

Further, as re-location costs are assumed negligible, each firm has a motive to choose a location such that its protected hinterland, i.e. market segment a or b, is maximized. The final equilibrium positioning has both firms located within an arbitrarily small distance, ε, of the market center and charging the same price.

This clustering at the center, Hotelling claimed, was of broad generality, explaining the homogeneity of many goods and services, be they brands of cider or political platforms.

Hotelling's paper was truly seminal. Many have followed in his footsteps, and they have extended his model and modified his assumptions. However, one disappointing, even disturbing, feature in this Hotelling-related literature is that it has shown a certain lack of robustness and a certain lack of "universality" in Hotelling's result. For example, when the number of firms in the market is extended to three, no equilibrium exists, and with four firms they tend to locate off the center of the market and in pairs.

Monopoly control of the location of the two firms

In the following we shall examine, how Hotelling's "clustering-of-firms" result holds up, when one important institutional change is introduced. Namely: we shall assume that a landlord owns all the sites in the market and thereby can determine the locations of firms. The landowner invites bids from prospective firms for predetermined sites and gives entry to the market to those firms which offer the highest bids.

Looking at Hotelling's problem from this particular angle, will allow us to analyze the "coupling" of two markets, namely a factor market -- in this instance the market for sites (being controlled by a monopolist-landowner) -- and a product market, in which the firms sell their goods to consumers/customers.

The landlord seeks to maximize either the sales value of the land or the rent extractable from the firms. Accordingly, the landlord chooses the optimal number of firms, allowed entry, as well as their locations. The firms, in turn, maximize their profits by competing pricewise. Thus, Hotelling's problem is here recast into a two-step optimization problem.

Beginning with two firms, X and Y, their total profits and therefore the base of the extractable rents are:

$$\pi_x + \pi_y = c(L^2+((a-b)/3)^2) \tag{10}$$

As Hotelling noted, aggregate profits increase with the term $(a-b)^2$. Recalling the meaning of a and b, one can see that (10) is maximized for {a=L,b=0} or, equivalently, for {a=0,b=L}. In other words, the landlord should locate both firms at one end of the market. However, tracing back, it can be shown that this solution cannot satisfy both the optimal price conditions (5) and (6) and the price-difference restriction (9). Hence, the expression (10) must be maximized subject to (9) as a constraint. The solution yields {a=3L/5,b=0} or, equivalently, {a=0,b=3L/5}. Some further calculation shows that the firm located at one endpoint of the market will have to be more aggressive in its pricing and charge a lower price than the other firm, in order to maximize its profit. These results are shown graphically in <u>Figure 2a</u>.

As seen, each firm has a one-sided market, and the entire market [0,L], is spatially organized <u>as if</u> it were a one-sided market, with all customers travelling in the same direction to purchase the product. Thus, the monopolist-landlord having control over the factor market, structures the product market as if there were a single firm located at one end of the market.

Since the aggregate consumer costs of transportation are sensitive to the locations of the firms, it is of interest to compare these costs for different cases. For instance, within the market segment a, the total transportation costs add up to a $ca^2/2$ (from $\int_0^a ct\, dt$). Through analogous integration across the other three market segments, one finds that the landowner's optimal solution gives rise to higher costs for the consumers than the duopoly case considered by Hotelling. On the other hand, if a government agency were to regulate the locations and place the two firms at the quartiles, $\{a=0.25L, b=0.75L\}$, consumer costs of transportation would be reduced by at least 50 percent, in comparison with either the landowner case (see also Figures 2a. and c.) or the duopoly case (see Figure 2b). The numerical results are summarized in Table 1.

Monopoly control of the locations of three or more firms

Consider next the case where the landlord permits three firms to enter the market. In order to simplify the notation, we redefine some of the variables as shown in Figure 3 and in equations (11a) through (11d), below. For example, the market segments for firm Y are y_1 and y_2, and its distance from the leftmost market boundary, a_2, is its location variable. Equations (12) and (13) represent the equal delivered-price conditions that define the boundaries between adjacent firms. Finally, the inequalities (14) and (15) state the continuity restrictions which correspond to the earlier relation (9).

$$a_1 = x_1 \tag{11a}$$

$$a_2 - a_1 = x_2 + y_1 \tag{11b}$$

$$a_3 - a_2 = y_2 + z_1 \tag{11c}$$

$$L - a_3 = z_2 \tag{11d}$$

$$p_x + cx_2 = p_y + cy_1 \tag{12}$$

$$p_y + cy_2 = p_z + cz_1 \tag{13}$$

$$p_x + c(x_2+y_1) > p_y > p_x - c(x_2+y_1) \tag{14}$$

$$p_y + c(y_2+z_1) > p_z > p_y - c(y_2+z_1) \tag{15}$$

The landlord will now seek to locate the three firms in such a pattern that their aggregate profits are maximized, since that positioning will enable the landlord to extract the highest possible rents from the firms. The optimization problem is:

$$\text{Max.}\pi = \underset{a_1,a_2,a_3}{\text{Max.}} \{p_x(x_1+x_2)+p_y(y_1+y_2)+p_z(z_1+z_2)\} \tag{16}$$

subject to (11), (12), (13), (14), and (15).

The optimal solution is depicted in Figure 4a. The close similarity with the two-firm solution is noteworthy. Once again, each firm has a one-sided market and all customers must travel in the same direction to purchase the good. Furthermore, the total market has the appearance <u>as if</u> it contained one firm -- rather than three.

The numerical results are summarized in <u>Table 1</u>. As expected, the aggregate profits for the three firms and the aggregate transportation costs resulting for the consumers are now considerably lower than in the two-firm case. With the "invisible hand" performing better, the need for public regulation is lessened. A regulation is now also more complicated than it was in the duopoly case. All the regulatory agency had to do, in the latter case, was to locate the two firms such that all market segments were of equal length. With three or more firms in the market, there exists a certain asymmetry in the sense that interior firms compete for customers at both market boundaries, whereas the two exterior firms have only one such competitive boundary. Thus, it can be expected that an interior firm would charge a lower price, in optimum, than an exterior firm with a protected hinterland would do.

Such pricing behavior has been ruled out in the relevant literature, where equal prices have typically been assumed. Hence, if a regulatory agency, anticipating equal pricing among all three firms, selects locations to equalize market segments, the firms will respond with the following profit-maximizing prices: $p_x = p_z = 0.555$ cL, and $p_y = 0.444$ cL (see Figure 4.b). If on the other hand, the agency correctly anticipates the sellers' reactions, it will spread the firms more widely, thereby gaining a marginal saving in transportation costs as well as average price for consumers (Figure 4.c). Regardless of which behavioral assumption is used, government regulation of the monopolist-landlord reduces consumer transportation costs by more than half, as shown in Table 1.

We shall conclude this section by considering the case where the landlord permits four firms to enter the market. In addition to X, Y, and Z, we now add a firm W, with the following modification and new equations:

$$a_4 - a_3 = z_2 + w_1 \tag{11d}$$

(modified)

$$L - a_4 = w_2 \tag{11e}$$

$$p_z + cz_2 = p_w + cw_1 \tag{17}$$

$$p_z + c(z_2+w_1) > p_w > p_z - c(z_2+w_1) \tag{18}$$

For this case, the solution of the landlord's site selection problem is given in Figure 5a, and the corresponding government-regulated spatial patterns are shown in Figures 5b and c. We find the same remarkable regularity in the conclusions. The landlord arranges the firms in a locational pattern such that each firm has a one-sided market, and the whole market is also one-sided, as if it were served by one firm located at one endpoint. Likewise, we find parallels in the government-regulated situation. Regulation, as before, leads to substantial relative reductions in total transportion costs and in total profits (see Table 2). If the agency correctly anticipates that the interior firms will charge lower prices than the two exterior firms, then it will locate all four firms somewhat closer to the endpoints of the market than it would if it made the myopic (and incorrect) assumption of anticipating equal prices.

In the traditional case (no landlord), the equilibrium "competitive" pattern has the four firms paired at the first and third quartiles. If a rent-maximizing landlord were to pair firms, giving the impression of preserved competition, he or she would locate firms as shown in Figure 5d (one pair at 0.167 L and the other at 0.833 L). However, as shown by Table 2 this would sharply reduce the landowner's rent potential.

The circular case

The equilibrium configuration for the landlord case is consistently invariant with changes in the number of sellers in the product market -- a result that does not conform to the traditional Hotelling literature. The only general result in the Hotelling model is that the exterior firms must be paired with a second firm in any equilibrium, a result directly due to the boundedness of the market.[1] Given the asymmetrical nature of the equilibrium in the landlord case, it is important to question whether the boundedness of the market is again crucial to the results. With a minor qualification, the answer is no.

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[1]Eaton and Lipsey (1975) prove that given equal prices there exists no equilibrium for three firms, a unique equilibrium for two, four, or five firms, and multiple equilibria for six or more firms.

The critical feature of an unbounded market is that every firm faces competition at the boundary of both market segments, i.e. there exist no protected hinterlands. This case can effectively be studied by considering the market to be the circumference of a circle. The only change required to the Hotelling model is the addition of an equation of delivered prices defining the boundary between the a and b segments of the market (Figure 1):

$$p_x + ca = p_y + cb \tag{2a}$$

Otherwise, the analysis is identical to our previous examination. In the two-firm case total profits are independent of the location of the firms, and the landlord is then indifferent with respect to the locational pattern. This parallels the results in the two-sellers-only model. For three and four firms, however, the asymmetry does reappear in the optimal locational pattern for the landlord. There is an infinity of equilibria around the circle but the spacing of the firms is unique. Arbitrarily, then, the 12'o'clock position is defined as the origin, and the first firm, C, is located at that point. In the three-firm case, the landlord would then locate the second and third firms at 0.125 L and 0.875 L, respectively (Table 3). The half-market to the right of the vertical axis mirrors the half-market to the left of the axis but within each half-market, a structure similar to Figure 2a obtains. In the four-firm case, the symmetry around the vertical axis repeated, and within each half-market the landlord locates the firms such that each has only a one-sided market and such that all consumers travel in the same direction to purchase the good. Thus, the only effect of the unboundedness of the circular market is that the landlord locates the firms as asymmetrically as possible within each of the two (symmetric) half-markets, rather than in the market as a whole.

Because there are no exterior firms, there is no distinction between the myopic and the correctly anticipating government regulator. Nonetheless, regulatory action still drastically reduces the social costs relative to the costs incurred if there is an unregulated landlord. The decrease in potential rent, however, is much less than in the circular case.

Conclusion

The starting-point for this paper was Hotelling's classical study of duopolistic competition in a spatial market. To Hotelling's assumptions we added one more, namely that the locations of firms were determined by a monopolist-landlord. We also extended Hotelling's case of two firms to three and four firms. The specific locational assignments chosen by the optimizing landlord and the pricing behavior exhibited by the profit-maximizing firms, as shown in our results, appear counterintuitive. For example, in the two-firm case, why should one of the firms be located at a point three fifths from the end of the market? However, both the locational pattern and the pricing behavior fit a framework in which the landlord has a motivation to structure the product market to be as imperfect as feasible. Thus, each firm finds itself having a one-sided market in equilibrium. At the same time, the total market will also appear one-sided, in the sense that each buyer must travel in the same direction as every other buyer to purchase the good. The results hold up in all three cases examined -- with two, three, and four firms in the market. This gives support for a conjecture that the results are universally valid, i.e., independent of the number of firms entering the market. At the same time, a comparison between the three cases shows that the landowner has a very strong incentive to give entry to the smallest number of firms which the law will permit; in our case that number is two. Differently expressed, the paper illustrates how imperfect competition in the factor market imposes just so much imperfection in the product market that law or custom will permit.

References

Chamberlin, E. H., The Theory of Monopolistic Competition, Cambridge, Harvard University Press, 1932.

__________________, "The Product as an Economic Variable," Quarterly Journal of Economics, 67 (1953) p. 1-29.

Devletoglou, N., "A Dissenting View of Duopoly and Spatial Competition," Economica 32, (May 1965) p. 140-160.

Eaton, B. C., "Spatial Competition Revisited," Canadian Journal of Economics 2 (May 1972) p. 268-278.

____________, "Free Entry in One-Dimensional Models: Pure Profits and Multiple Equilibria," Journal of Regional Science, 16 (1) April 1976, p. 21-33.

____________ and R. G. Lipsey, "The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition," Review of Economic Studies 42 (1975) p. 27-49.

Gannon, C. A., "Consumer Demand, Conjectural Interdependence and Location Equilibria in Simple Spatial Duopoly," Papers, Regional Science Association 28 (1972) p. 83-107.

Grace, H. "Professor Samuelson on Free Enterprise and Economic Efficiency: A Comment," Quarterly Journal of Economics, v. 81 (May 1970) p. 337-340.

Hotelling, H., "Stability in Competition," Economic Journal 39 (March 1929) p. 41-57.

Lerner, A. and Singer, H., "Some Notes on Duopoly and Spatial Competition," Journal of Political Economy 45 (April 1937) p. 145-188.

Samuelson, P., "The Monopolistic Competition Revolution," in Monopolistic Competitive Theory: Studies in Impact, R. Kuenne, ed., New York, Wiley & Sons, 1967.

____________, "Harold Hotelling as a Mathematical Economist," The American Statistician, 14 (June 1960) p. 21-25.

____________, "Reply," The Quarterly Journal of Economics, 81 (May 1970), p. 341-345.

Smithies, A., "Optimal Location in Spatial Competition," Journal of Political Economy 49 (June 1941) p. 423-439.

	Two Firms			Three Firms		
	Hotelling's Duopoly	Landlord	Gov't Regulation	Landlord	Myoptic Gov't Regulation	Optimal Gov't Regulation
a_1	.5L	0	.25L	0	.167L	.145L
a_2	.5L	.4L	.75L	.143L	.5L	.5L
a_3	-	-	-	.571L	.833L	.855L
Total Profits	$1.0cL^2$	$1.04cL^2$	$1.0cL^2$	$.592cL^2$	$.506cL^2$	$.504cL^2$
Total Transport Cost	$.25cL^2$	$.26cL^2$	$.125cL^2$	$.194cL^2$	$.090cL^2$	$.089cL^2$
Total Consumer Cost	$1.25cL^2$	$1.30cL^2$	$1.125cL^2$	$.786cL^2$	$.596cL^2$	$.593cL^2$

Table 1

Four Firms

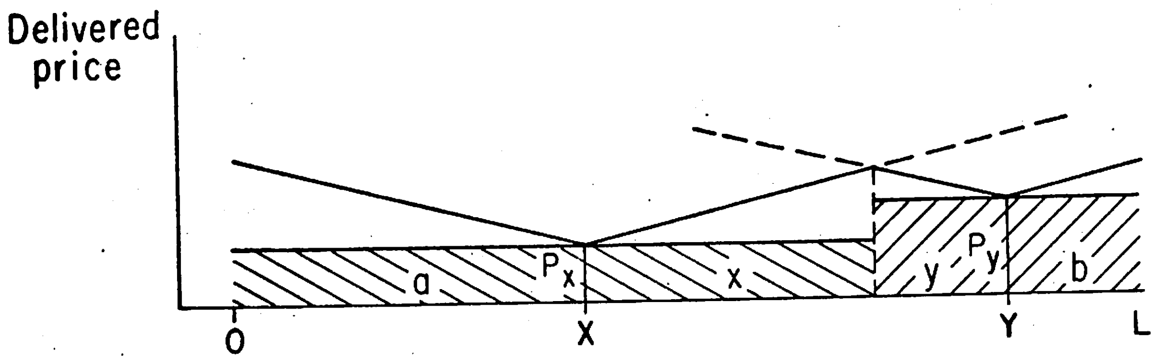
	Competitive Equilibrium	Landlord	Myoptic Regulation	Optimal Regulation	Apparent Competiton
a_1	.25L	0	.125L	.103L	.167L
a_2	.25L	.063L	.375L	.353L	.167L
a_3	.75L	.25L	.625L	.647L	.833L
a_4	.75L	.625L	.875L	.897L	.833L
Total Profits		$.465cL^2$	$.340cL^2$	$.337cL^2$	$.333cL^2$
Total Transport Cost		$.160cL^2$	$.068cL^2$	$.066cL^2$	$.140cL^2$
Total Consumer Cost		$.625cL^2$	$.408cL^2$	$.403cL^2$	$.470cL^2$

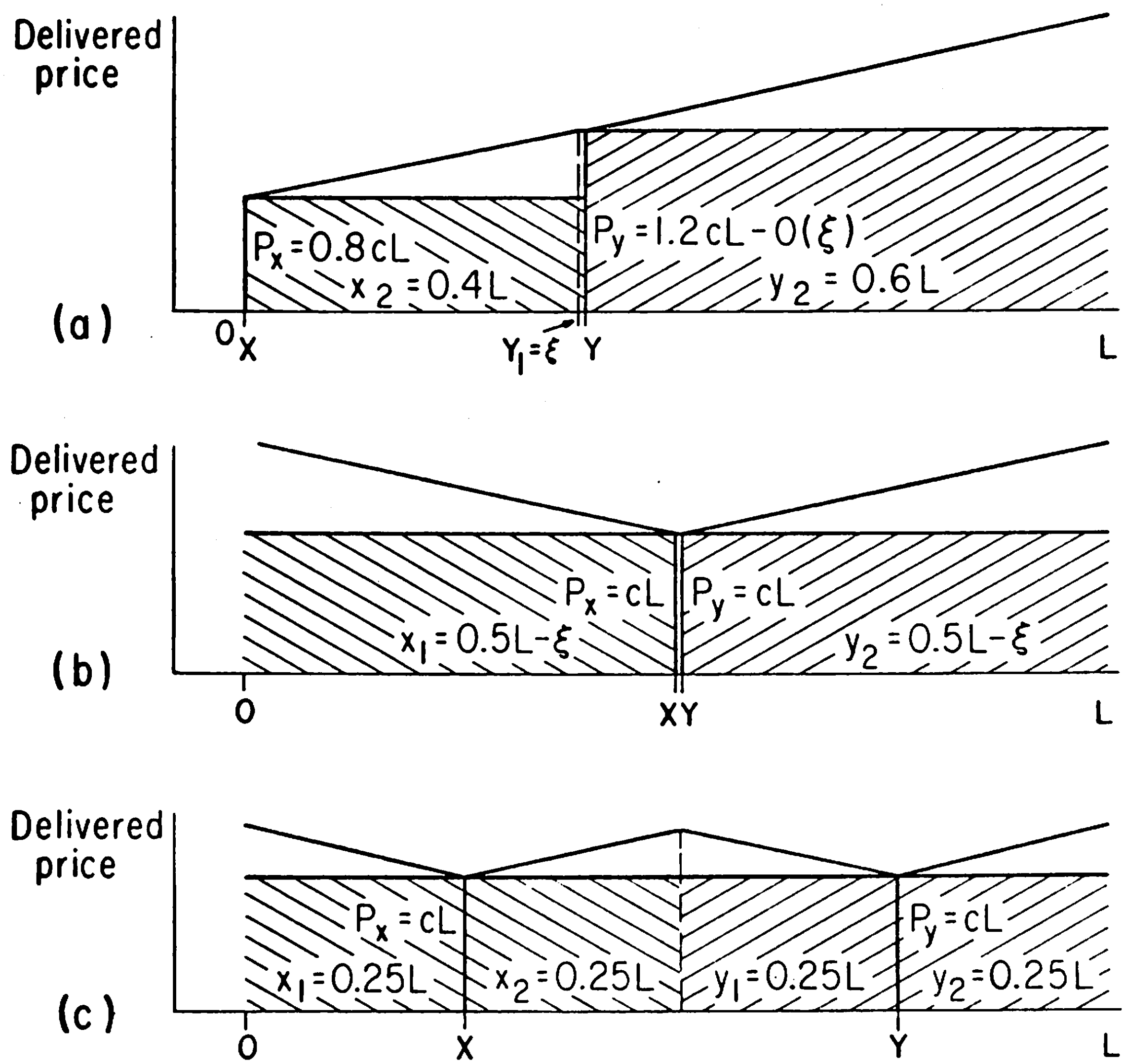
Table 2

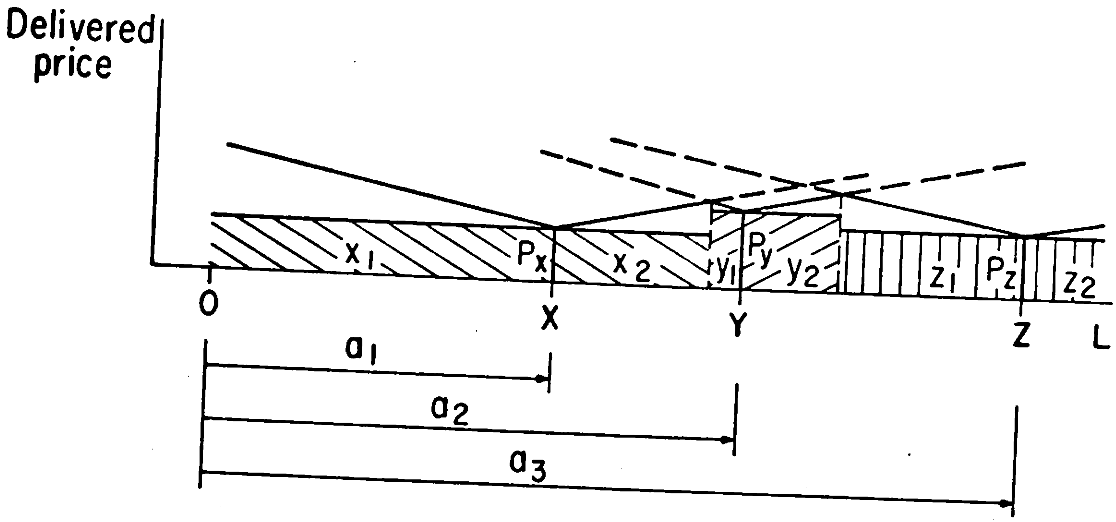
	Three Firms		Four Firms	
	Landlord	Gov't Regulation	Landlord	Gov't Regulation
a_1	0	0	0	0
a_2	.125L	.333L	0	.25L
a_3	.875L	.667L	.167L	.5L
a_4	-	-	.833L	.75L
Total Profit	$.344cL^2$	$.333cL^2$	$.278cL^2$	$.250cL^2$
Total Transport Cost	$.156cL^2$	$.083cL^2$	$.139cL^2$	$.063cL^2$
Total Consumer Cost	$.500cL^2$	$.416cL^2$	$.417cL^2$	$.313cL^2$

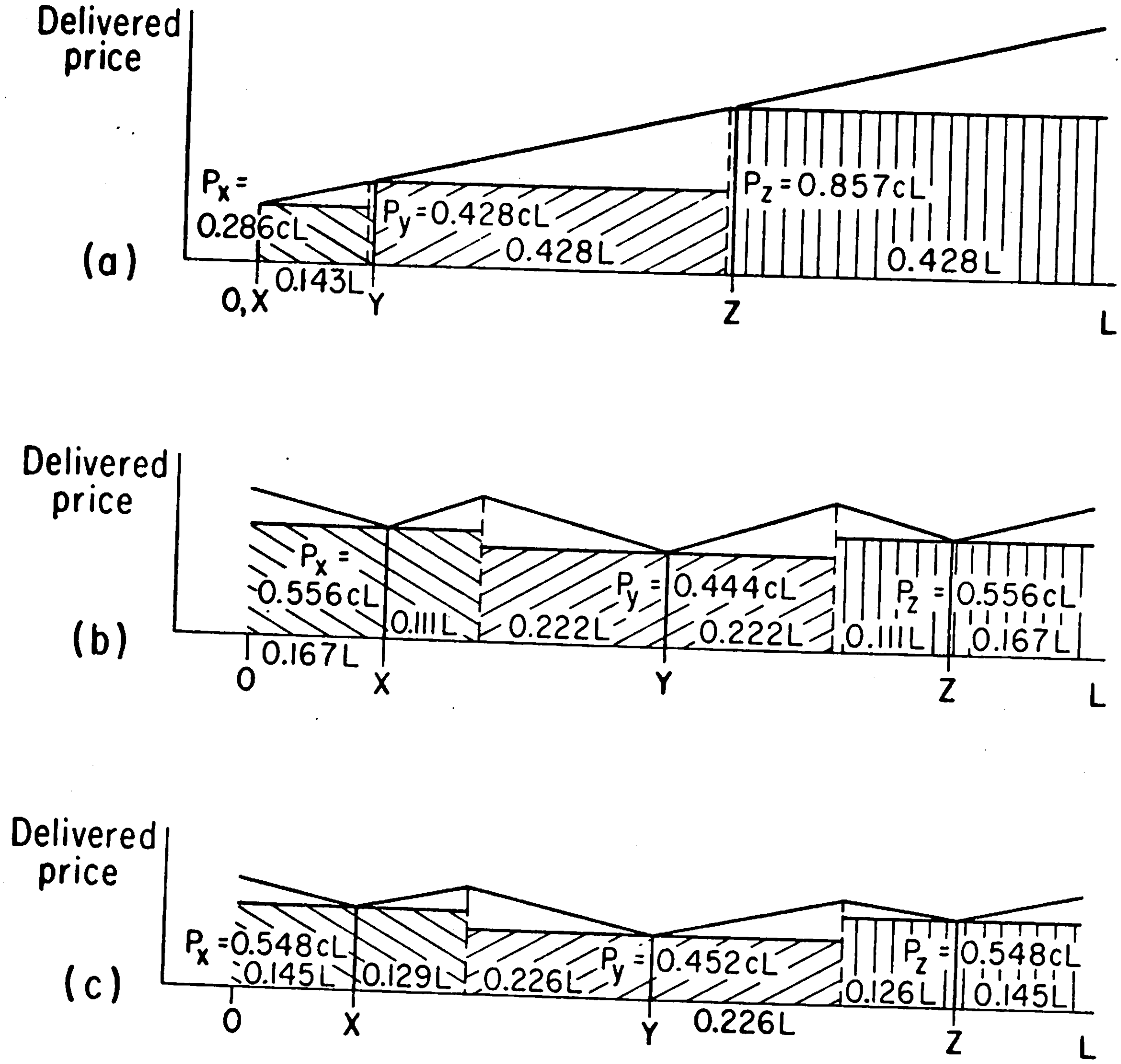
Table 3

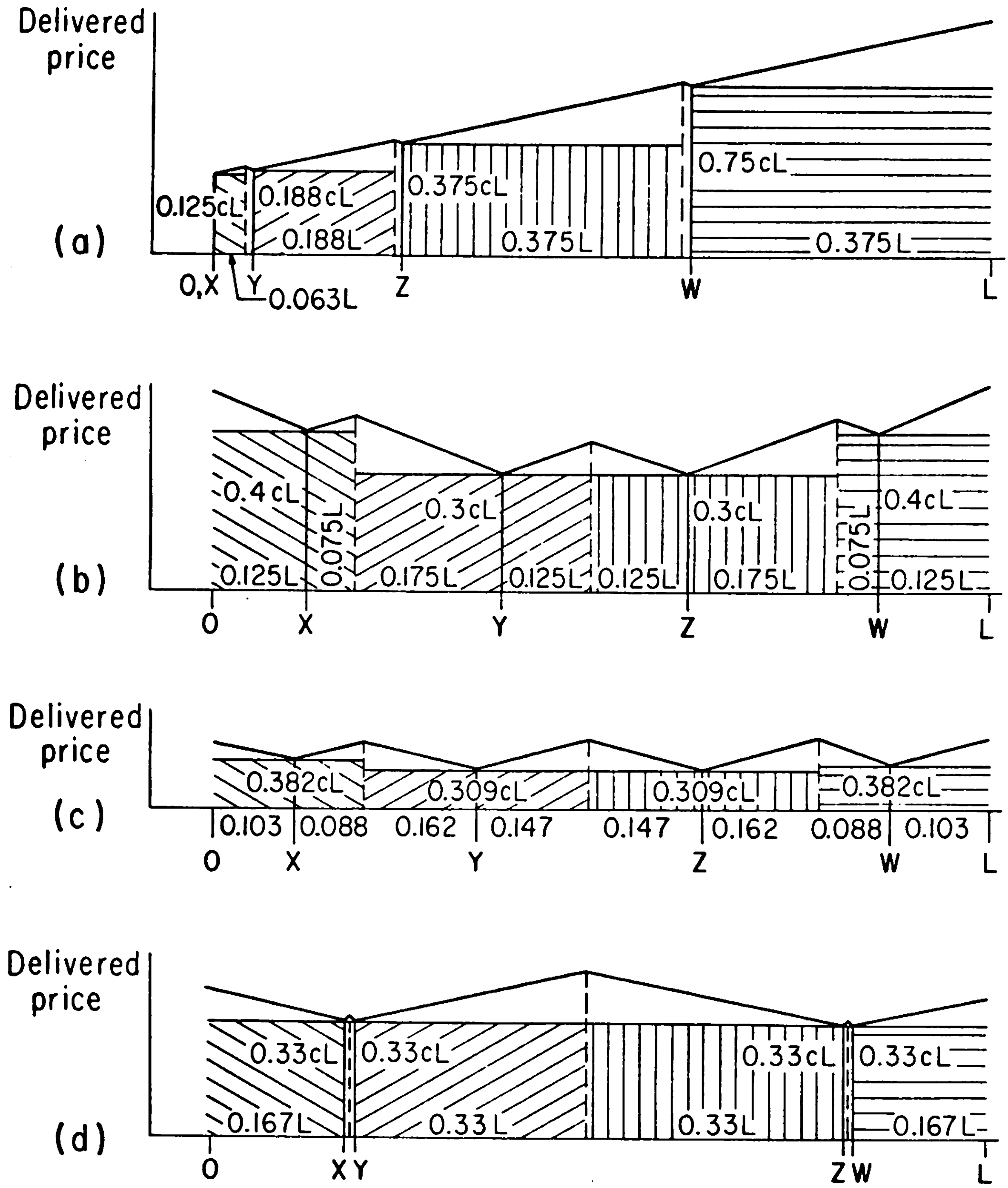
Circular Market


Delivered
price
a
Px
x
y
Py
b
O
X
Y
L


Delivered price
$P_x = 0.8cL$
$x_2 = 0.4L$
$P_y = 1.2cL - O(\xi)$
$y_2 = 0.6L$
(a)
O
X
$Y_1 = \xi$
Y
L
Delivered price
$P_x = cL$
$x_1 = 0.5L - \xi$
$P_y = cL$
$y_2 = 0.5L - \xi$
(b)
O
XY
L
Delivered price
$P_x = cL$
$x_1 = 0.25L$
$x_2 = 0.25L$
$y_1 = 0.25L$
$P_y = cL$
$y_2 = 0.25L$
(c)
O
X
Y
L


Delivered price
x1
Px
x2
y1
Py
y2
z1
Pz
z2
O
X
Y
Z
L
a1
a2
a3


Delivered price
$P_x =$ 0.286cL
P_y=0.428cL
P_z=0.857cL
0.143L
0.428L
0.428L
(a)
0,X
Y
Z
L
Delivered price
P_x = 0.556cL
P_y = 0.444cL
P_z = 0.556cL
0.111L
0.222L
0.222L
0.111L
0.167L
0.167L
(b)
O
X
Y
Z
L
Delivered price
P_x =0.548cL
P_y = 0.452cL
P_z = 0.548cL
0.145L
0.129L
0.226L
0.126L
0.145L
0.226L
(c)
O
X
Y
Z
L



Delivered price
(a)
0.125cL
0.188cL
0.188L
0.375cL
0.375L
0.75cL
0.375L
0,X
Y
0.063L
Z
W
L
Delivered price
(b)
0.4 cL
0.125L
0.075L
0.3cL
0.175L
0.125L
0.125L
0.3cL
0.175L
0.075L
0.4cL
0.125L
O
X
Y
Z
W
L
Delivered price
(c)
0.382cL
0.309cL
0.309cL
0.382cL
0.103
0.088
0.162
0.147
0.147
0.162
0.088
0.103
O
X
Y
Z
W
L
Delivered price
(d)
0.33cL
0.167L
0.33cL
0.33L
0.33L
0.33cL
0.33cL
0.167L
O
X Y
Z W
L