COHERENCE AND ALERT STATES FOR

INTERCONNECTED POWER SYSTEMS

by

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Memorandum No. UCB/ERL M79/57

15 August 1979

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Abstract - A model of disturbances affecting power systems is proposed and linearized models of the pre- and post-disturbance dynamics are derived. These are used to study two topics: coherency and near coherency of a group of generators under multiple disturbances, and characterization of the alert region of the state space. The techniques used are motivated by the geometric theory of linear systems.

1. INTRODUCTION

We propose a model of some of the typical disturbances affecting an interconnected power system and derive pre- and post-disturbance models linearized around a base-case solution. These models are used to propose a coherency identification method and a characterization of the alert region. The analysis is based on the geometric theory for linear systems.

Security analysis is generally developed for a specified portion of an interconnected power system, called the internal or study system and which is often coincident with the domain of a particular operating utility, while details of the remainder or external portion of the system are considered only to the extent that they affect the study system. The external system is approximated by an "equivalent" of lower dimension, the aim is to realize valuable reduction in computational effort, and possibly in information requirements, without introducing an intolerable loss in accuracy. For purposes of transient stability analysis one seeks a "dynamic equivalent" that is, an equivalent system of lower order which nevertheless accurately reflects the short term impact of the external system (see, for example, [4], [5]).

The two approaches to the construction of dynamic equivalents which have been most developed are the modal reduction method [18] and the coherency method [10,13,22]. We shall be concerned with the latter. Both approaches use a linearized model to approximate the behavior of a nonlinear power system model in the neighborhood of a base-case solution. (Such a linearized model is presented in the next section.) The coherency method is founded on the empirical observation that, following a disturbance (line or generating unit outage or sudden change in load), certain groups of generators "swing together," that is, the generators in each group maintain nearly constant angular differences with each other. Each such coherent group can then be replaced by a single bus. The objective of the method is, therefore, to identify generators which are coherent with respect to single or multiple disturbances. The case of a single disturbance has been studied notably by Lee and Schweppe [10], Podmore [13], and by Wu and Narasimhamurthy [22]. In [10] a method for recognizing coherency is suggested based on the heuristic notions of electrical distance and symmetry, while [13] suggests examination for coherence within the solution obtained by numerical integration of the linearized model.

A mathematically rigorous necessary and sufficient condition for identifying strict coherency is presented in [22]. (Strict coherence means that the post-fault angular differences of generators in a coherent group are strictly constant.) Besides the fact that this condition is developed only for a single disturbance it is not useful for identifying "near" coherency. The first objective of this paper is to remedy these two deficiencies. In Section 3 we characterize strict coherence for multiple disturbances and in Section 4 the characterization is extended to near coherency. Furthermore we indicate algorithms for checking strict coherency which may be more suitable than the one proposed in [22].

Starting with the fundamental work of Dyliacco [6] and with significant clarification and elaboration by Debs and Benson [7], and Fink and Carlsen [8], discussion of security evaluation and emergency control is usually posed in terms of a qualitative partitioning of the state space as shown in Figure 3. In particular the alert or insecure state indicates reductions in reserve margins and an increased probability of disturbance which could result in violation of system inequality constraints such as the power-carrying capacity of a line or the generating capacity of a unit. The usefulness of this state description still remains heuristic, however, and most of the literature proposes ad hoc measures of security evaluation, while the very ambitious model of Blankenship and Fink [2] remains analytically intractable. The second objective of the paper is to propose a characterization of the alert region which is closer in spirit to earlier formulations in that it focuses on a deterministic evaluation of contingencies. The basic idea is this: We linearize the model around a base-case solution at which one or more lines and units are operating close to capacity and we consider one or more potential disturbances. We say that the system is in the alert state (with respect to the potential disturbances) if there does not exist any feedback law which isolates the lines and units under study from the disturbances. While the definition has two major deficiencies, relying on a linearized analysis and ignoring probabilistic considerations, the characterization seems computationally reasonable and it does take into account operating conditions. Section 5 is devoted to it. Some suggestions for future work are given in Section 6. The proofs are collected in the Appendix.

2. LINEARIZED MODEL OF PRE- AND POST-DISTURBANCE DYNAMICS

The model used for coherency recognition as well as for the characterization of the alert region has been used previously [13,22]. We introduce it below, making explicit the various assumptions.

2.1. Modeling Assumptions

A1. (Synchronous generators). The classical swing equation model represents the dynamics of a synchronous generator on departure from equilibrium. That is, for the ith generator,

$$M_i\Delta\dot{\omega}_i + D_i\Delta\omega_i = \Delta PM_i - \Delta PG_i \tag{2.1}$$

$$\Delta\delta_i = 2\pi f_0 \Delta\omega_i, \tag{2.2}$$

where

$M_i(D_i)$ = moment of inertia (damping constant),

$\Delta\omega_i(\Delta\delta_i)$ = departure of speed (rotor angle) from equilibrium value,

$\Delta PM_i(\Delta PG_i)$ = departure of mechanical input (electrical output) power

f_0 = synchronous frequency of power system.

A2. (Decoupled load flow). The power flow in the network of load and generator buses is modeled by linearized load flow equations in which real power and phase angles are decoupled from reactive power and voltage magnitude. (This assumption is valid for transmission systems with high reactance to resistance ratios [16].)

Let $\Delta PG \in R^g$ and $\Delta PL \in R^\ell$ be the vectors of real power injections into the g generator and ℓ load buses respectively, with injections into the network being positive by convention. Let $\delta \in R^g$ and $\theta \in R^\ell$ respectively represent the vector of phase angles at the generator and load buses. Then

$$\begin{bmatrix} \Delta PG \\ \Delta PL \end{bmatrix} = \begin{bmatrix} H_{gg} & H_{g\ell} \\ H_{\ell g} & H_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\theta \end{bmatrix} = H \begin{bmatrix} \Delta\delta \\ \Delta\theta \end{bmatrix}, \tag{2.3}$$

where H_{gg}, $H_{g\ell}$ etc. are matrices of appropriate dimensions. The matrix entries are partial derivatives, for example,

$$(H_{gg})_{ij} = \frac{\partial PG_i}{\partial\delta_j}, \quad (H_{\ell g})_{ij} = \frac{\partial PL_i}{\partial\delta_j}, \tag{2.4}$$

the derivatives being evaluated at the equilibrium angle of the jth generator, $\delta_j(0)$.

In the special case when the transmission line resistances are neglected, H is particularly simple,

$$(H_{gg})_{ii} = \sum_{j\neq i} Y^{gg}_{ij}\cos(\delta_i(0)-\delta_j(0)) + \sum_{k\neq i} Y^{g\ell}_{ik}\cos(\delta_i(0)-\theta_k(0)), \tag{2.5}$$

$$(H_{gg})_{ij} = Y^{gg}_{ij}\cos(\delta_i(0)-\delta_j(0)), \; i \neq j, \tag{2.6}$$

$$(H_{g\ell})_{ik} = Y^{g\ell}_{ik}\cos(\delta_i(0)-\theta_k(0)), \tag{2.7}$$

etc. Here Y^{gg}_{ij}, $Y^{g\ell}_{ik}$ are the admittances of the lines connecting the ith generator bus to the jth generator and the kth load buses respectively. Observe that in this special case H is a symmetric matrix of dimension $g + \ell$.

A3. (Disturbance model). Three kinds of disturbances are modeled, namely, (i) load shedding or gaining, (ii) generator dropping, and (iii) line switching. These are discussed in turn below.

(i) This is described as a change, ΔPL, in the vector of load power deviations, so that a change in the ith load is modeled as

$$\Delta PL(t) = (0,\ldots,0,\underset{\substack{\uparrow\\ i}}{1},\ldots,0)^T q(t), \; t \geq 0$$

where 1 appears in the ith position, T denotes transpose, and q(t) is a bounded real-valued function. For example, q(t) may be a switching function, q(0) = 0 and q(t) rising monotonically to a non-zero value depending upon the characteristics of the circuit breakers that may have tripped causing load shedding.

(ii) Generator dropping can be modeled as a change in load as well. To do this we regard each generator as a bus behind a transient reactance so that each generator bus is coupled to one load bus only, and there are no interconnections between generator buses. Then, the outage of the ith generator can be described by an increase in the load at, say, the kth load bus which is connected to the ith generator (through its transient reactance). This increase, $\Delta PL_k(t)$, $t \geq 0$, should equal the ith generator's pre-fault power input. The resulting dynamics of the ith generator are of course neglected. The validity of the proposed model can be seen from the fact that in the view of the remaining network the dropping of the ith generator is equivalent to an increase in the load at the kth bus.

(iii) The switching of a line connecting load buses i and j is described as a change in load at these buses of the amount of power being carried by the line at the time of the fault, t = 0, and a change in the matrix $H_{\ell\ell}$ of (2.3). For example, in the case of purely reactive lines,

$$\Delta PL = (0,\ldots,\underset{\substack{\uparrow\\ i}}{1},\ldots,\underset{\substack{\uparrow\\ j}}{-1},\ldots,0)^T Y^{\ell\ell}_{ij}\sin(\theta_i(0)-\theta_j(0))q(t),$$

while the post-disturbance matrix is

$$H'_{\ell\ell} = H_{\ell\ell} - \begin{array}{c} \\ \\ i\rightarrow \\ \\ j\rightarrow \\ \\ \\ \end{array}\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ -1 \\ \vdots \\ 0 \end{bmatrix} [0,\ldots,\underset{\substack{\uparrow\\ i}}{1},\ldots,\underset{\substack{\uparrow\\ j}}{-1},\ldots 0] Y_{ij}\cos(\theta_i(0) - \theta_j(0)). \tag{2.8}$$

The function q(t), $t \geq 0$ is, as before, a switching function with q(0) = 0 and $q(t) \rightarrow 1$ as $t \rightarrow \infty$.

2.2. A Unified Model After a Single Disturbance

Continuing (2.1), (2.2), (2.3) and the preceding discussion gives the following linear model after a single disturbance has occurred,

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -M^{-1}D & 0 & -M^{-1} & 0 \\ 2\pi f_0 I & 0 & 0 & 0 \\ 0 & H_{gg} & -I & H_{g\ell} \\ 0 & H_\ell & 0 & H'_{\ell\ell} \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta PG \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} M^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} PM(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d \end{bmatrix} q(t). \tag{2.9}$$

Here $\Delta\omega$, $\Delta\delta$ are in R^g, M and D are g-dimensional diagonal matrices with entries M_i, D_i, $\Delta PM = (\Delta PM_1,\ldots,\Delta PM_g)^T$, q(t) is real-valued and, as above,

$$H'_{\ell\ell} = H_{\ell\ell} + \mu dd^T,$$

with $\mu = 0$ and $d = (0,\ldots,1,\ldots,0)^T$ for a load change or generator outage, whereas $\mu \neq 0$ and $d = (0,\ldots,1,\ldots,-1,\ldots,0)^T$ for line switching.

Observe that (2.9) consists of $3g + \ell$ equations of which $\ell + g$ are algebraic which we assume are solvable.

A4. (Solvability of linearized load flow equations). The ℓ dimensional matrices $H'_{\ell\ell}$ and $H_{\ell\ell}$ are invertible.

For the case of lossless lines, the nature of this assumption is clarified by the following result of Tavora and Smith [17].

Proposition 2.1. Suppose the pre-distance equilibrium values $\theta_i(0)$, $\delta_i(0)$ satisfy the following conditions:

$$|\theta_i(0) - \theta_j(0)| \leq \frac{\pi}{2} \quad \text{when} \quad Y^{gg}_{ij} \neq 0,$$

$$|\theta_i(0) - \delta_j(0)| \leq \frac{\pi}{2} \quad \text{when} \quad Y^{g\ell}_{ij} \neq 0,$$

$$|\delta_i(0) - \delta_j(0)| \leq \frac{\pi}{2} \quad \text{when} \quad Y^{\ell\ell}_{ij} \neq 0.$$

Then $H_{\ell\ell}$ is invertible if and only if there exists no cutset of zero elements in the subnetwork of load buses formed by replacing each line of the original network by its incremental capacity to deliver power, i.e., the line between i and j is replaced by $Y_{ij}\cos(\theta_i(0) - \theta_j(0))$.

Proposition 2.2. (Linearized model after single disturbance.) Under A4, Eq. (2.9) can be simplified to yield

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{\delta} \end{bmatrix} = A' \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} + B\Delta PM(t) + e'q(t) \tag{2.10}$$

where

$$A' = \left[\begin{array}{c|c} -M^{-1}D & [M^{-1} \,\vdots\, 0] \begin{bmatrix} -I & H_{g\ell} \\ 0 & H'_{\ell\ell} \end{bmatrix}^{-1} \begin{bmatrix} H_{gg} \\ H_{\ell g} \end{bmatrix} \\ \hline 2\pi f_0 I & 0 \end{array}\right] \tag{2.11}$$

$$B = \begin{bmatrix} M^{-1} \\ 0 \end{bmatrix} \tag{2.12}$$

$$e' = \left[\begin{array}{c} [M^{-1} \,\vdots\, 0] \left[\begin{array}{c|c} -I & H_{g\ell} \\ \hline 0 & H'_{\ell\ell} \end{array}\right]^{-1} \begin{bmatrix} 0 \\ d \end{bmatrix} \\ \hline 0 \end{array}\right] \tag{2.13}$$

Proof. Follows using straightforward algebraic manipulations. ¤

The matrix A' and the vector e' are primed to emphasize that they can be computed only with post-fault data, namely $H'_{\ell\ell}$. The next proposition relates them to their pre-fault values A, e which are defined by replacing $H'_{\ell\ell}$ by its pre-fault value $H_{\ell\ell}$. To guarantee the nontriviality of e' we make the following assumption which is implied, for instance, by the dynamic stability of the load flow solution before the disturbance.

A5. (Nontriviality of disturbance.) $H^{-1}_{\ell\ell} \in \mathbb{R}^{\ell \times \ell}$ is positive definite (not necessarily symmetric).

Proposition 2.3. (Feedback equivalence.) The pre-fault pair (A,e) and the post-fault pair (A',e') are feedback equivalent, that is, there exist $\eta \in R^1_+$ and $\gamma \in R^{2g}$ such that

$$e' = e\eta \quad \text{and} \quad A' = A + e\gamma^T \tag{2.14}$$

Proof. See Appendix. ¤

The term "feedback equivalence" arises from the fact that (A',e') can be obtained from (A,e) by state feedback as shown in Fig. 1. This proposition will be critical in the study of coherence.

2.3. A Unified Model for Multiple Disturbances

Equation (2.9) generalizes readily for a set of p disturbances by regarding $q(t) \in R^p$ as a vector of switching functions, by replacing $d \in R^\ell$ a with a $\ell \times p$ matrix, again denoted by d, with columns $d_1,\ldots,d_p$, and by letting $H'_{\ell\ell} = H_{\ell\ell} + \mu_1 d_1 d_1^T + \ldots + \mu_p d_p d_p^T$. Under A4, the resulting system of equations can be simplified exactly as in Proposition 2.2 to obtain (2.10) with A', B, e' once given by the formulas (2.10), (2.11), (2.12). The only difference is that e' is now a matrix of dimension $2g \times p$, whereas earlier it was a vector. The proof of the next proposition is identical to that of the preceding one.

Proposition 2.4. The pairs (A,e) and (A',e') for the case of multiple disturbances are feedback equivalent, i.e., there exist matrices η of dimension $p \times p$ (positive definite) and γ of dimension $2g \times p$ such that (2.14) holds.

This extension to multiple disturbances of the single disturbance case considered in the literature (see e.g., [22,10]) is of more than minor interest since under environmental stress there is an increased likelihood of multiple outages or increases in load and the model above in which these contingencies are assumed to occur simultaneously may be a reasonable one.

3. COHERENCY UNDER MULTIPLE DISTURBANCES

3.1. Characterization of Coherency

Definition 3.1. A group of generators $I \subset \{1,\ldots,g\}$ is (strictly) coherent for a single disturbance if $\delta_i(t) - \delta_j(t)$ is constant for all $t \geq 0$, for all i,j in I; and it is coherent for a set of disturbances if the group is coherent for any linear combinations of these disturbances.

Fix a group of generators $I \subset \{1,\ldots,g\}$ and a set of p disturbances which define the matrices A', e' in (2.10). Evidently the group I is coherent if every pair (i,j) of generators, with i and j in I, is coherent. Suppose there are m such pairs. Form the $m \times 2g$ matrix C with rows $c_1,\ldots,c_m$:

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} \begin{array}{c} \\ \left[\begin{array}{c|cccccccccc} & & & i & & k & & j & & \ell & \\ & 0 & \cdots & 1 & \cdots & & & -1 & \cdots & 0 & \cdots \\ 0 & 0 & & & & 1 & \cdots & & & -1.0 & \cdots \\ & & & & & \vdots & & & & & \end{array}\right] \end{array} \tag{3.1}$$

Here the first g columns of C are identically zero, c_1, corresponds to the pair (i,j), c_2 to the pair (k,ℓ) etc. Thus

$$c_1 \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} = \Delta\delta_i - \Delta\delta_j, \quad c_2 \begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} = \Delta\delta_k - \Delta\delta_\ell, \text{ etc.}$$

For the post-fault pair of matrices A', e' let $\langle A'|e'\rangle$ denote the subspace of R^{2g} spanned by the columns of the matrices $e', A'e', \ldots, (A')^{2g-1}e'$. Similarly define $\langle A|e\rangle$ for the pre-fault pair A, e. Finally let Ker C be the null space of C i.e. $\text{Ker } C = \{x \in R^{2g} | Cx = 0\}$.

Theorem 3.1. The group I is coherent for the set of p disturbances if and only if

$$\langle A'|e'\rangle \subset \text{Ker } C. \tag{3.2}$$

Furthermore, $\langle A|e\rangle = \langle A'|e'\rangle$ so that coherency can also be characterized in terms of pre-fault data as

$$\langle A|e\rangle \subset \text{Ker } C \tag{3.3}$$

Proof. See Appendix. ⌑

As mentioned in the Introduction this theorem was proved for the case of a single disturbance by Wu and Narsimhamurthy [22]. It may be worth pointing out here that there are efficient algorithms for checking (3.2) or (3.3), especially in view of the structure of C. One of the most popular of these is due to Rosenbrock and Mayne [11], or Aplevich [1]. However both of these, as well as the algorithm proposed in [22], rely on Gauss elimination which is known to be numerically unstable for large matrices (see [15, p. 152]). Better methods of computation using singular value decomposition (see, e.g., [9]) are now available and, in particular, the Rosenbrock-Mayne procedure can be replaced by one due to Sastry [14].

3.2. Physical Implications of the Coherency Condition

Various measures of electrical distance to a disturbance have been proposed to explain coherence (see e.g., [10]). We relate one such measure to the characterization given above, confining the discussion to the case of lossless lines and a single disturbance of the load change type. The entries of the matrix H can be interpreted as admittances and are given by (2.5), (2.6), (2.7). Moreover, the incremental power injections corresponding to the load changes ($\Delta PL_i(t)$ at the ith load bus) are interpreted as current sources of the same magnitude. We now group the nodes of the network into two sets: the first, $\mathcal{N}_g$, consists of the generator nodes and the second, $\mathcal{N}_\ell$, consisting of the load nodes, as shown in Figure 2. If we now take the Norton equivalent of $\mathcal{N}_\ell$ with respect to $\mathcal{N}_g$, then the resulting equivalent current source at the generator nodes is given by $H_{g\ell}(H_{\ell\ell})^{-1}dq(t)$. This should be clear from the Ward reduction procedure [19] which led to Eq. (2.13). Let $J = H_{g\ell}(H_{\ell\ell})^{-1}dq(t)$ denote the resulting vector of injections at the generator buses. The quantity J_i is intuitively the electrical distance of the ith generator to the fault; its units, however, are power/admittance. The vector e' in (2.13) is related to J by

$$e' = (J_1/M_1,\ldots,J_g/M_g,0,\ldots,0)^T \in R^{2g}$$

so that its non-zero entries are the ratios of the electrical distance to the corresponding moments of inertia. The following result is almost obvious.

Proposition 3.1. (Necessary condition for coherence). If generators i and j are coherent for the single disturbance of the type described above then $J_i/M_i = J_j/M_j$.

Proof. See Appendix. ⌑

The equality of the electrical distance (weighted by the moment of inertia) is necessary for coherence. On the other hand the symmetry of the reduced electrical network is sufficient as seen next. Note that the admittance matrix of the Norton equivalent resulting from the procedure above is

$$\tilde{H}_{gg} = -H_{g\ell}(H_{\ell\ell})^{-1}H_{\ell g}.$$

Proposition 3.2. (Sufficient condition for coherence). If the interconnection pattern of generators i and j is symmetric i.e.,

$$M_i^{-1}(H_{gg}+\tilde{H}_{gg})_{ik} = M_j^{-1}(H_{gg}+\tilde{H}_{gg})_{jk},\ k = 1,\ldots,g$$

$$M_i^{-1}J_i = M_j^{-1}J_j,$$

and if there is equal damping i.e.,

$$M_i^{-1}D_i = M_j^{-1}D_j,$$

then i and j are coherent.

Proof. See Appendix. ⌑

We can conclude that to relate electrical distance to coherence it is necessary to "normalize" the former by the moment of inertia and, moreover, the relation is only necessary. On the other hand, symmetry, again appropriately ormalized, guarantees coherence but is not necessary.

4. NEAR COHERENCE UNDER MULTIPLE DISTURBANCES

For the purpose of constructing a dynamic equivalent it seems reasonable to demand only that the post-fault angular differences of a group of generators be nearly constant. Definition 3.1 needs to be relaxed accordingly.

Definition 4.1. A group of generators $I \subset \{1,\ldots,g\}$ is ε-coherent on $[0,T]$ for the disturbances $q(t) \in R^p$, $t \geq 0$, if

$$\left\{\sum_{i,j}[\delta_i(t)-\delta_j(t)-\delta_i(0)+\delta_j(0)]^2\right\}^{1/2} \leq \varepsilon\|q\| \tag{4.1}$$

where the summation is over all pairs i,j in I and $\|q\|^2 = \int_0^T |q(t)|^2 dt$ is the L_2-norm of the disturbance.

Before deriving conditions for ε-coherence some remarks on the definition may be helpful. First, notice that the magnitude of the disturbance, $\|q\|$, does not figure in Definition 3.1. This is to be expected since strict coherence demands that the "outputs," $\delta_i(t) - \delta_j(t)$, be completely decoupled from the disturbance q. Indeed if in (3.1) ε is set to zero, then it is easy to show that Definition 4.1 collapses to Definition 3.1. From this observation and Theorem 3.1, it may be expected that if, for the disturbance represented by A',e', $\langle A'|e'\rangle$ is "close" to Ker C, where $C = (0,\ldots,0,1,.,-1,0)$ corresponds to the pair of generators i,j as in (3.1), then the latter should be ε-coherent for appropriate ε. The burden of this section is to make this intuition precise. This turns upon getting the correct measure of distance between subspaces which we take up next.

4.1. Distance Between Subspaces

This is obtained using the notion of orthogonal projection. The following lemma is well-known (see, e.g., [15]).

Lemma 4.1. Let U be the nxm matrix whose orthogonal columns form a basis for a subspace $\subset R^n$. Then UU^T is the orthogonal projection operator form R^n onto $\mathcal{V}$.

Let $\mathcal{V}^\perp$ be the orthogonal complement of $\mathcal{V}$. Then x in R^n can be uniquely expressed as $x = v + w$ with v in $\mathcal{V}$ and w in $\mathcal{V}^\perp$. By Lemma 4.1 $v = UU^Tx$, and so $I - UU^T$ is the orthogonal projection operator from R^n onto $\mathcal{V}^\perp$.

Definition 4.1. Let $\mathcal{V}_1$, $\mathcal{V}_2$ be two subspaces of R^n and U_i matrices whose orthogonal columns span $\mathcal{V}_i$, $i = 1,2$. Then the distance from $\mathcal{V}_1$ to $\mathcal{V}_2$ is

$$d(\mathcal{V}_1,\mathcal{V}_2) = \max\{|x-U_2U_2^Tx|\,|\,x \in \mathcal{V}_1, |x|=1\} \tag{4.2}$$

where $|\cdot|$ is the Euclidean norm.

For any mxn matrix U let $|U|$ denote the matrix norm induced by the Euclidean norm i.e., $|U| = \max\{|Ux| \mid x \in R^n, x = 1\}$. Then (4.2) can be easily rephrased as follows,

$$d(\mathcal{V}_1,\mathcal{V}_2) = |(I-U_2U_2^T)U_1|, \qquad (4.3)$$

$$d(\mathcal{V}_2,\mathcal{V}_1) = |(I-U_1U_1^T)U_2|. \qquad (4.3)$$

To give an appreciation for the proposed definition we list some properties. First, in general, $d(\mathcal{V}_1,\mathcal{V}_2) \neq d(\mathcal{V}_2,\mathcal{V}_1)$ unless $\mathcal{V}_1$, $\mathcal{V}_2$ have the same dimension. Second, $0 \leq d(\mathcal{V}_1,\mathcal{V}_2) \leq 1$, and $d(\mathcal{V}_1,\mathcal{V}_2) = 1$ only if there is an x in $\mathcal{V}_1$ orthogonal to $\mathcal{V}_2$, whereas $d(\mathcal{V}_1,\mathcal{V}_2) = 0$ only if $\mathcal{V}_1 \subset \mathcal{V}_2$. The next property will be useful later.

Proposition 4.1. Let $\mathcal{V}$ be a subspace of R^n and C a pxn matrix such that $d(\mathcal{V},\text{Ker } C) = \varepsilon$; then

$$\max\{|Cx| \mid x \in \mathcal{V}, |x|=1\} \leq \varepsilon|C|. \qquad (4.5)$$

4.2. Characterization of ε-Coherency

We recall the definition and some properties of the "reachability grammian" of a linear system (see e.g., [3] or [14] for details).

Definition 4.2. The reachability grammian of a linear system

$$\dot{x} = A'x + e'q. \qquad (4.6)$$

where $x \in R^{2g}$, $q \in R^p$ and A', e' are matrices of appropriate size is the matrix

$$W(T) = \int_0^T \exp(tA')e'e'^T\exp(tA'^T)dt \qquad (4.7)$$

The utility of the definition stems from the following elementary proposition (see [14] or [12]).

Proposition 4.2. The set $\mathcal{R}(T)$ of all states of the linear system (4.6) reachable at time T, starting at 0 at time 0, and using inputs $q(\cdot)$ with L_2-norm $\|q\| \leq 1$ is given by

$$\mathcal{R}(T) = \{[W(T)]^{1/2}\eta \mid \eta \in R^{2g}, |\eta| \leq 1\}. \qquad (4.8)$$

(Here $[W(T)]^{1/2}$ is the positive semi-definite square root of W(T).)

Theorem 4.1. (Characterization of ε-coherence). The group I of generators is ε-coherent on [0,T] for disturbances $q(\cdot) \in R^p$ if

$$d(\langle A'|e'\rangle,\text{Ker } C) \leq \varepsilon|C|^{-1}[\rho(W(T))]^{-1/2}. \qquad (4.9)$$

Moreover, in terms of the pre-fault matrices A, e (4.9) is equivalent to

$$d(\langle A|e\rangle,\text{Ker } C) \leq \varepsilon|C|^{-1}[\rho(W(T))]^{-1/2}. \qquad (4.10)$$

(In (4.9) A', e' describes the post-fault system (2.9), C corresponds to the group I as in (3.1), and $\rho(W(T))$ is the largest eigenvalue of W(T).)

Proof. See Appendix. ¤

Note that in (4.10) W(T) is still given by (4.6), that is, it is the reachability grammian of the post-fault system. Thus, unlike the (strict) coherency condition, ε-coherency cannot be related in terms of pre-fault data alone. The next result gives a partial result in this direction.

Theorem 4.2. Let $\varepsilon > 0$, and suppose $d(\langle A|e\rangle, \text{Ker } C) = \delta$. Then the group I is ε-coherent on [0,T] where

$$T = \min\{\frac{1}{2|A'|}, \frac{\varepsilon^2}{2|C|^2\delta^2\rho(e'e'^T)}\}. \qquad (4.11)$$

Proof. See Appendix. ¤

The estimate (4.11) seems computationally more useful than that provided by (4.10) since the latter requires computation of the reachability grammian. Observe that for any fixed ε, δ and T are inversely related as is to be expected.

5. CHARACTERIZATION OF THE ALERT STATES

Recall that in the Introduction a base-case solution was said to be in the alert state if some of the system variables are operating close to their rated capacities i.e., some inequality constraints are close to being violated, and if the variables cannot be decoupled from some likely disturbances. To formalize this idea we first model the inequality constraints of interest.

5.1. A Model of Inequality Constraints.

Three kinds of constraints are considered. These are (i) thermal limits of a line, (ii) generating capacity, and (iii) maximum permissible frequency deviation of a generator. The corresponding system variables are discussed in turn below. We assume lossless lines.

(i) If P_{ij} is the power flowing through a lossless line connecting load buses i and j, $P_{ij} = Y_{ij}^{\ell\ell}\sin(\theta_i - \theta_j)$, and so, incrementally,

$$\Delta P_{ij} = Y_{ij}^{\ell\ell}\cos(\theta_i(0)-\theta_j(0))[0,\ldots,0,\underset{\substack{\uparrow\\ i}}{1},\ldots,\underset{\substack{\uparrow\\ j}}{-1},\ldots,0]\begin{bmatrix}\Delta\delta\\ \Delta\theta\end{bmatrix}$$

(ii) The base-case power output of the ith generator is

$$PG_i = \sum_{j\neq i} Y_{ij}^{gg}\sin(\delta_i(0)-\delta_j(0)) + \sum_{k\neq i} Y_{ik}^{gg}\cos(\delta_i(0)-\theta_k(0)),$$

so that, incrementally,

$$\Delta PG_i = \alpha\begin{bmatrix}\Delta\delta\\ \Delta\theta\end{bmatrix} \qquad (5.2)$$

where $\alpha \in R^{g+\ell}$ is the partial derivative of PG_i with respect to (δ,θ).

(iii) Finally, the frequency deviation $\Delta\omega_i$ is just one of the state variables corresponding to the ith generator by (2.1).

From the preceding we can conclude that any vector $y \in R^m$ whose components consist of some of the variables ΔP_{ij}, ΔPG_i, $\Delta\omega_i$ can be represented with a suitable $m\times(3g+\ell)$ matrix C as

$$y = C\begin{bmatrix}\Delta\omega\\ \Delta\delta\\ \Delta PG\\ \Delta\theta\end{bmatrix} \qquad (5.3)$$

Using (5.3) with (2.9) and assuming A4 we obtain for a set of likely disturbances the following linear system model:

$$\dot{x} = A'x + B\Delta PM(t) + e'q(t), \qquad (5.4)$$

$$y = Cx, \qquad (5.5)$$

where $x^T = (\Delta\omega^T,\Delta\delta^T)$. We are now ready to propose a definition of the alert region.

5.2. The Alert Region.

Suppose that at equilibrium some of the variables $P_{1i}(0)$, $PG_i(0)$ and $\omega_i(0)$ are close to their rated values. Let y be the vector of incremental changes of these variables, and let A', e' correspond to the likely disturbances. The power system is said to be in the alert state with respect to these disturbances if there is no state feedback law APM(t) = Fx(t) such that the disturbance q(t) is decoupled from y i.e., y(t) ≡ 0, ∀t ≥ 0.

Evidently the system is in the alert state if there is no feedback matrix F such that

$$C[\exp t(A'+BF)]e' = 0, \quad t \geq 0 \tag{5.6}$$

This is known as the "disturbance decoupling" problem and has been well studied (see Wonham [21].) The characterization of the alert state utilizes the next definition.

Definition 5.1. A subspace $\mathcal{V}' \subset R^{2g}$ is said to be A' mod B invariant if $A'\mathcal{V}' \subset \mathcal{V}' + Sp(B)$, where Sp(B) is the subspace spanned by the columns of B.

Theorem 5.1. ([21]) There exists a matrix F satisfying (5.6) if and only if $Sp\ e' \subset \mathcal{V}'(\mathrm{Ker}\ C)$ where $\mathcal{V}'(\mathrm{Ker}\ C)$ is the largest A' mod B subspace contained in Ker C.

As a Corollary we obtain the desired characterization.

Theorem 5.2. (Characterization of Alert States). The system is in the alert state with respect to disturgances q iff

$$Sp(e') \not\subset \bar{\mathcal{V}}'(\mathrm{Ker}\ C) \tag{5.7}$$

Moreover, in terms of the pre-fault matrices A, e (5.7) may be replaced by

$$Sp(e) \not\subset \bar{\mathcal{V}}(\mathrm{Ker}\ C) \tag{5.8}$$

where $\bar{\mathcal{V}}(\mathrm{Ker}\ C)$ is the largest A mod B invariant subspace contained in Ker C.

Proof. See Appendix. ∎

We close this section with a few remarks. First, observe that we permit the use of an arbitrary feedback matrix F to decouple the disturbance fom the relevant outputs y. This may be too conservative an estimate of the alert region if only some restricted class of feedback matrices are implementable in practice. On the other hand the decoupling represented by (5.6) may be too strict and it may be enough to require only that y(t) be "small" enough. One possibility in this direction is suggested by the recent work of Willems [20]. Finally a numerical procedure for calculating $\bar{\mathcal{V}}(\mathrm{Ker}\ C)$ has been proposed by Moore and Laub [23].

6. CONCLUDING REMARKS

Further work needs to be done in three areas:

(i) The actual formation of a dynamic equivalent of the external system once coherent groups of generators have been identified. Some work in this area has been reported by Podmore and Germond [24] and Wu and Narasimhamurthy [25]; but the results so far are preliminary.

(ii) The definitions of Section 5 can be extended to ε-alert states which may then be characterized by the non existence of an A' mod B invariant subspace containing Sp(e') (or of an A mod B invariant subspace containing Sp(e)) which is close to Ker C. However, estimates of the form of Theorems 4.1, 4.2 cannot be obtained by the techniques of Section 4. Different estimates are needed to make this intuition precise.

(iii) The relation between the present linearized or local analysis with the nonlinear or global analysis of the power system dynamics needs to be made. Preliminary research on the nonlinear analysis of coherence and the alert state using techniques of differential geometry seems to yield results which are either obvious or too restrictive. What seems to be needed is a way of "stitching" together the above local (linearized) analysis using the topological properties of the load flow.

ACKNOWLEDGEMENT

Research sponsored by the Department of Energy Contract ET-78-S-01-3387. The authors are grateful to A. Arapostathis, Prof. E. Wong and Prof. F. Wu for helpful comments.

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APPENDIX

Proof of Proposition 2.3. Define $f \in R^{2g}$ by $f^T = (0^T \vdots d^T)$. Then

$$[M^{-1}\ 0]\begin{bmatrix} -I & H_{g\ell} \\ 0 & H'_{\ell\ell} \end{bmatrix}^{-1} f = [M^{-1}\ 0]\left\{\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix} + \mu f f^T\right\}^{-1} f,$$

$$= [M^{-1}\ 0]\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1}\left\{I+\mu f f^T\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1}\right\}^{-1} f,$$

$$= [M^{-1}\ 0]\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1} f\left\{1+\mu f^T\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1} f\right\}^{-1}.$$

where, in the last equality, the identity $(I+PQ)^{-1} = P(I+QP)^{-1}$ is used. Define

$$\eta = \left\{1+\mu f^T\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1} f\right\}^{-1} = \{1+\mu d^T H_{\ell\ell}^{-1} d\}^{-1}$$

(The positivity of η is insured by A5.) It follows immediately from (2.13) that $e' = \eta e$. A similar algebraic manipulation shows that if γ is defined by $\gamma^T = \eta\gamma_1^T$, where

$$\gamma_1^T = \mu f^T\begin{bmatrix} -I & H_{g\ell} \\ 0 & H_{\ell\ell} \end{bmatrix}^{-1}\begin{bmatrix} H_{gg} \\ H_{\ell g} \end{bmatrix},$$

then $A' - A = e\gamma^T$, and the proof is complete.

Proof of Theorem 3.1. The coherency of I is equivalent to $C\ \exp(tA')e' = 0$, $t \geq 0$. It is well-known that the latter condition is equivalent to (3.2). Next, $e' = e\eta$, $A' = A + e\gamma^T$ and η is nonsingular. Hence $Ce' = 0$ if and only if $Ce = 0$. Also, since $CA'e' = CAe\eta + Ce\gamma^T e$ $\{Ce'=0, CA'e'=0\}$ if and only if $\{Ce=0, CAe=0\}$. Continuing in this manner reveals that (3.3) is equivalent to (3.2).

Proof of Proposition 3.1. Let $C = (0,\ldots,0,0,\ldots,1,\ldots,-1,\ldots,0)$ correspond to the pair (i,j) as in (3.1). Next, for the case of load shedding, we see from (2.13) and the definition of electrical distance that

$$e = \begin{bmatrix} M^{-1}H_{g\ell}H_{\ell\ell}^{-1}d \\ 0 \end{bmatrix} = (J_1/M_1,\ldots,J_g/M_g,0,\ldots,0)^T.$$

Hence, using (2.11), we see that

$$Ae = 2\pi f_0(0,\ldots,0,J_1/M_1,\ldots,J_g/M_g)^T.$$

Now, if i and j are coherent, then $\langle A|e\rangle \subset \operatorname{Ker} C$ and, in particular, $CAe = 0$, but $CAe = 2\pi f_0(J_i/M_i - J_j/M_j)$.

Proof of Proposition 3.2. Consider the equation

$$\begin{bmatrix} \Delta\dot\omega \\ \Delta\dot\delta \end{bmatrix} = A\begin{bmatrix} \Delta\omega \\ \Delta\delta \end{bmatrix} + eq(t)$$

where A and e are of the form given in (2.11) and (2.13). From the latter we can check the detailed equations

$$\Delta\dot\omega_i = \frac{D_i}{M_i}\Delta\omega_i + \sum_{k=1}^{g}\frac{1}{M_i}(H_{gg}+\tilde H_{gg})_{ik}\Delta\delta_k + \frac{J_i}{M_i}q(t).$$

$$\Delta\dot\omega_j = \frac{D_j}{M_j}\Delta\omega_j + \sum_{k=1}^{g}\frac{1}{M_j}(H_{gg}+\tilde H_{gg})_{jk}\Delta\delta_k + \frac{J_j}{M_j}q(t).$$

$$\Delta\dot\delta_i = 2\pi f_0\Delta\omega_i,$$

$$\Delta\dot\delta_j = 2\pi f_0\Delta\omega_j.$$

Hence if the interconnection pattern of i and j is symmetric it follows that $\Delta\delta_i(t) - \Delta\delta_j(t) \equiv \Delta\delta_i(0) - \Delta\delta_j(0)$, $t \geq 0$ and the two generators are coherent.

Proof of Theorem 4.1. Consider the linear system (4.5) where $x^T = (\Delta\omega^T, \Delta\delta^T)$. By Proposition 4.2,

$$\operatorname{Max}\{|x(t)|\,|\,0\leq t\leq T\} \leq \rho(W(T))^{1/2}\|q\|. \tag{A.1}$$

On the other hand since $x(t) \in \langle A'|e'\rangle$ we see from (4.4) and (4.8) that

$$\operatorname{Max}\{|x(t)|\,|\,0\leq t\leq T\} \leq \varepsilon\,\rho(W(T))^{-1/2}\operatorname{Max}\{|x(t)|\,|\,0\leq t\leq T\}, \tag{A.2}$$

The first half of the assertion follows upon combining (A.1) and (A.2). The equivalence of (4.8), (4.9) is immediate from the fact that $\langle A|e\rangle = \langle A'|e'\rangle$ since A,e and A',e' are feedback equivalent.

Proof of Theorem 4.2. Using $d(\langle A|e\rangle, \operatorname{Ker} C) = \varepsilon$ it follows from Theorem 4.1 that I is ε-coherent on [0,T] if

$$\rho(W(T))^{1/2} \leq \varepsilon|C|^{-1}\delta^{-1}. \tag{A.3}$$

From (4.6) we obtain the estimate

$$\rho(W(T)) \leq \int_0^T |\exp|tA'|^2|e'e'^T|dt. \tag{A.4}$$

Now, if $T|A'| \leq 1/2$ then

$$\max_{0 \leq t \leq T} |\exp tA'| = \max_{0 \leq t \leq T} |I + tA' + \tfrac{1}{2}(tA')^2 +, \ldots|$$

$$\leq (1 - T|A'|)^{-1} \leq 2 \qquad (A.5)$$

Using $|e'e'^T| = \rho(e'e'^T)$ and (A5) in (A4) we obtain

$\rho(W(T)) \leq 4T\,\rho(e'e'^T)$

Clearly, with $T = \min\left\{\frac{1}{2|A'|}, \frac{\varepsilon^2}{2|C|^2\delta^2\rho(e'e'^T)}\right\}$; (A.3) is satisfied so that the group I is ε-coherent on $[0,T]$.

Proof of Theorem 5.2. The only part of the theorem not obvious from the discussion so far is that (5.7) may be replaced by (5.8). From Proposition (2.4), $e' = e\cdot\eta$ and $A' = A + e\gamma^T$ with η nonsingular. Hence $Spe' = Spe$. We now establish that

$$Spe' \subset \bar{\mathcal{V}}'(\text{Ker } C) \iff Spe \subset \bar{\mathcal{V}}(\text{Ker } C) \qquad (A.7)$$

$\Rightarrow A'\,\bar{\mathcal{V}}'(\text{Ker } C) \subset \bar{\mathcal{V}}'(\text{Ker } C) + Sp(B)$. Hence,

$A\bar{\mathcal{V}}'(\text{Ker } C) + e\gamma^T\bar{\mathcal{V}}'(\text{Ker } C) \subset \bar{\mathcal{V}}'(\text{Ker } C) + Sp(B)$

But, since $Spe = Spe' \subset \bar{\mathcal{V}}'(\text{Ker } C)$; we have

$A\bar{\mathcal{V}}'(\text{Ker } C) \subset \bar{\mathcal{V}}'(\text{Ker } C) + Sp(B)$

Hence, $\bar{\mathcal{V}}'(\text{Ker } C)$ is also an A mod B invariant subspace in Ker C and $\bar{\mathcal{V}}'(\text{Ker } C) \subset \bar{\mathcal{V}}(\text{Ker } C)$. Hence, $Spe \subset \bar{\mathcal{V}}(\text{Ker } C)$

$\Leftarrow$ The converse follows exactly as above.

By negating (A.7) we obtain

$$Spe' \not\subset \bar{\mathcal{V}}'(\text{Ker } C) \iff Spe \not\subset \bar{\mathcal{V}}(\text{Ker } C).$$

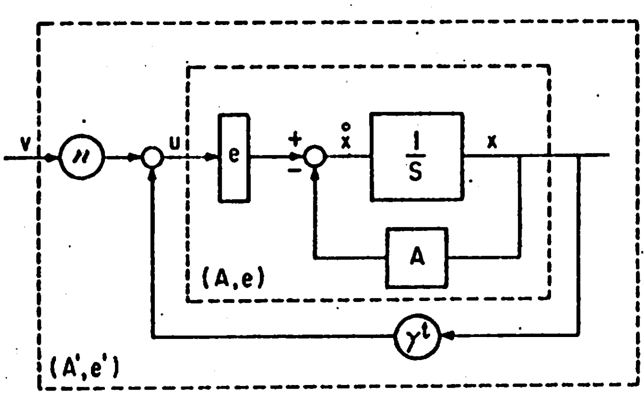
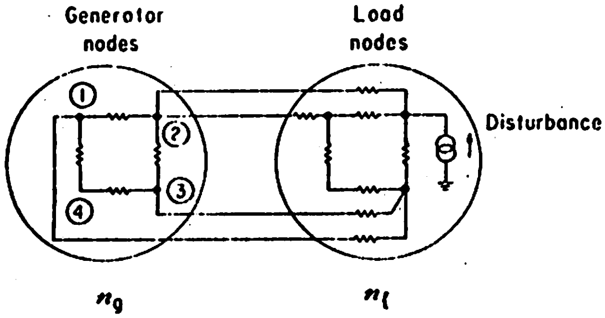
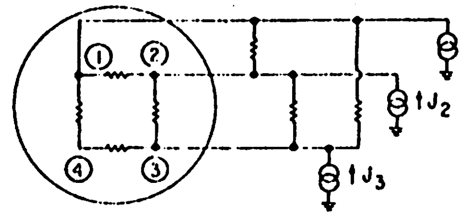


Figure 1. Feedback equivalence of (A,e) and (A',e').



(a)

Figure 2(a). Before Reduction



(b)

Figure 2(b). After Reduction

Figure 2. Resistive network analog of power system.

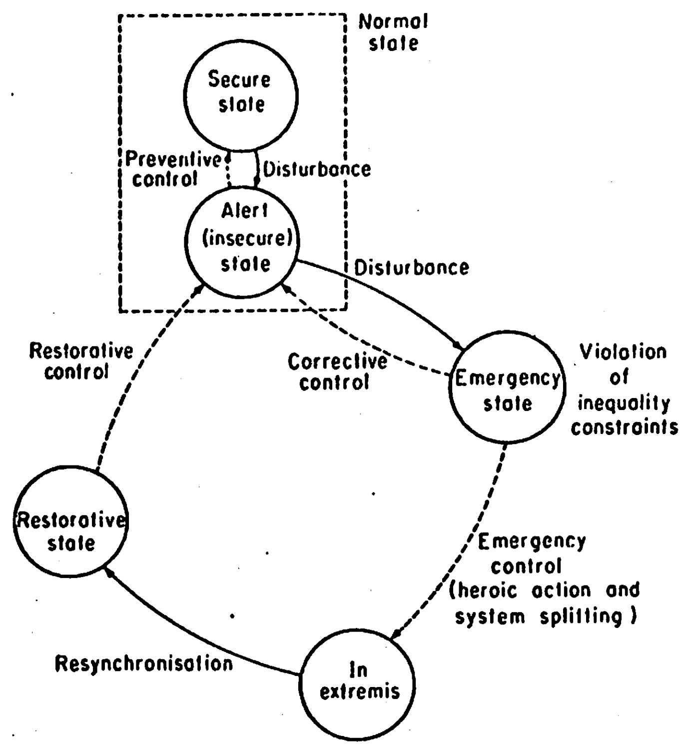


Figure 3. State space for emergency control (after Fink and Carlsen [17]).

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