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PRECISIATION OF HUMAN COMMUNICATION
VIA TRANSLATION INTO PRUF

by

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Precisiation of Human Communication via Translation into PRUF

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Abstract

It is suggested that communication between humans--as well as between humans and machines--may be made more precise by the employment of a meaning representation language PRUF which is based on the concept of a possibility distribution. A brief exposition of PRUF is presented and its application to precisiation of meaning is illustrated by a number of examples.

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1. Introduction

Of the many ways in which natural languages differ from synthetic languages, one of the most important relates to ambiguity. Thus, whereas synthetic languages are, for the most part, unambiguous, natural languages are maximally ambiguous in the sense that the level of ambiguity in human communication is usually near the limit of what is disambiguable through the use of an external body of knowledge which is shared by the parties in discourse.

Although vagueness and ambiguity¹ can and do serve a number of useful purposes, there are many cases in which there is a need for a precisiation of meaning not only in communication between humans but also between humans and machines. In fact, the need is even greater in the latter case because it is difficult, in general, to provide a machine with the extensive contextual knowledge base which is needed for disambiguation on the syntactic and semantic levels.

The traditional approach to the precisiation of meaning of utterances in a natural language is to translate them into an unambiguous synthetic language --which is usually a programming language, a query language or a logical language such as predicate calculus. The main limitation of this approach is that the available synthetic languages are nowhere nearly as expressive as natural languages. Thus, if the target language is the first order predicate calculus, for example, then only a small fragment of a natural language would be amenable to translation, since the expressive power of

¹As is pointed out in [81], ambiguity, vagueness and fuzziness are not coextensive concepts. Specifically, a proposition, p , is fuzzy if it contains words with fuzzy denotations, e.g., $p \triangleq$ Ruth has dark skin and owns a red Porsche. A proposition, p , is vague if it is both fuzzy and ambiguous in the sense of being insufficiently specific. For example, the proposition $p \triangleq$ Ruth lives somewhere near Berkeley is vague if it does not characterize the location of residence of Ruth with sufficient precision. Thus, a proposition may be fuzzy without being vague, and ambiguous without being fuzzy or vague.

first order predicate calculus is extremely limited in relation to that of a natural language.

To overcome this limitation what is needed is a synthetic language whose expressive power is comparable to that of natural languages. A candidate for such a language is PRUF [81]--which is a meaning representation language for natural languages based on the concept of a possibility distribution [80].

In essence, a basic assumption underlying PRUF is that the imprecision which is intrinsic in natural languages is possibilistic rather than probabilistic in nature. With this assumption as the point of departure, PRUF provides a system for translating propositions or, more generally, utterances in a natural language into expressions in PRUF. Such expressions may be viewed as procedures which act on a collection of relations in a database--or, equivalently, a possible world--and return possibility distributions which represent the information conveyed by the original propositions.²

In what follows, we shall outline some of the main features of PRUF and exemplify its application to precisiation of meaning. As a preliminary, we shall introduce the concept of a possibility distribution and explicate its role in PRUF.³

²As will be seen in Sections 2 and 3, PRUF is a language in a somewhat stretched sense of the term. Basically, it is a translation system in which only the simpler procedures may be represented as expressions in PRUF. For the description of complex procedures, PRUF allows the use of any suitable mathematically oriented language.

³In our exposition of the concept of a possibility distribution and the relevant parts of PRUF we shall draw on the definitions and examples in [79], [81] and [82].

2. Possibility and Meaning

A randomly chosen sentence in a natural language is almost certain to contain one or more words whose denotations are fuzzy sets, that is, classes of objects in which the transition from membership to nonmembership is gradual rather than abrupt. For example:

Hourya is very charming and intelligent.

It is very unlikely that inflation will end soon.

In recognition of his contributions, Mohammed is likely to be promoted to a higher position

in which the italics signify that a word has a fuzzy denotation in a universe of discourse.

For simplicity, we shall focus our attention for the present on canonical propositions of the form "N is F," where N is the name of an object, a variable or a proposition, and F is a fuzzy subset of a universe of discourse U. For example:

$$p \triangleq \text{John is very tall} \quad (2.1)$$

$$q \triangleq X \text{ is small}$$

$$r \triangleq (\text{John is very tall}) \text{ is not quite true}$$

where:

In p, $N \triangleq \text{John}$, and very tall is a fuzzy subset of the interval [0,200] (with the height assumed to be measured in centimeters).

In q, $N \triangleq X$ and small is a fuzzy subset of the real line.

In r, $N \triangleq \text{John is very tall}$, and not quite true is a linguistic truth-value [77] whose denotation is a fuzzy subset of the unit interval.

Now if X is a variable taking values in U, then by the possibility distribution of X, denoted by Π_X , is meant the fuzzy set of possible

values of X , with the possibility distribution function $\pi_X: U \rightarrow [0,1]$ defining the possibility that X can assume a value u . Thus,

$$\pi_X(u) \triangleq \text{Poss}\{X = u\} \quad (2.2)$$

with $\pi_X(u)$ taking values in the interval $[0,1]$.

The connection between possibility distributions and fuzzy sets is provided by the

Possibility Postulate. In the absence of any information about X other than that conveyed by the proposition

$$p \triangleq X \text{ is } F, \quad (2.3)$$

the possibility distribution of X is given by the possibility assignment equation

$$\Pi_X = F. \quad (2.4)$$

This equation implies that

$$\pi_X(u) = \mu_F(u) \quad (2.5)$$

where $\mu_F(u)$ is the grade of membership of u in F , i.e., the degree to which u fits one's subjective perception of F .

As a simple illustration, consider the proposition

$$p \triangleq X \text{ is SMALL} \quad (2.6)$$

where SMALL is a fuzzy set defined by⁴

⁴We use uppercase symbols to differentiate between a term, e.g., small, and its denotation, SMALL. The notation

$$F = \mu_1/u_1 + \dots + \mu_n/u_n \quad (2.9)$$

which is employed in (2.6) signifies that F is a collection of fuzzy singletons μ_i/u_i , $i = 1, \dots, n$, with μ_i representing the grade of membership of u_i in F . More generally, F may be expressed as $F = \sum_i \mu_i/u_i$ or $F = \int_U \mu_F(u)/u$. (See [78] for additional details.)

$$\text{SMALL} = 1/0 + 0.8/2 + 0.6/3 + 0.4/4 + 0.2/5 . \quad (2.7)$$

In this case, the possibility assignment equation corresponding to (2.5) may be expressed as

$$\Pi_X = 1/0 + 0.8/2 + 0.6/3 + 0.4/4 + 0.2/5 \quad (2.8)$$

with Π_X representing the possibility distribution of X . In this case--and more generally--the proposition $p \triangleq N$ is F will be said to translate into the possibility assignment equation

$$\Pi_X = F$$

where X is a variable that is explicit or implicit in N . To express this connection between p and the corresponding possibility assignment equation, we shall write

$$N \text{ is } F \rightarrow \Pi_X = F . \quad (2.10)$$

When X is implicit rather than explicit in N , the possibility assignment equation serves, first, to identify X and, second, to characterize its possibility distribution. For example, in the proposition

$$p \triangleq \text{Clara has dark hair} \quad (2.11)$$

X may be expressed as

$$X = \text{Color}(\text{Hair}(\text{Clara}))$$

and the possibility assignment equation reads

$$\Pi_{\text{Color}(\text{Hair}(\text{Clara}))} = \text{DARK} . \quad (2.12)$$

Before proceeding further in our discussion of the relation between possibility and meaning, it will be necessary to establish some of the basic

properties of possibility distributions. A brief exposition of these properties is presented in the following.

Joint, Marginal and Conditional Possibility Distributions

In the preceding discussion, we have assumed that X is a unary variable such as color, height, age, etc. More generally, let $X \triangleq (X_1, \dots, X_n)$ be an n -ary variable which takes values in a universe of discourse $U = U_1 \times \dots \times U_n$, with X_i , $i = 1, \dots, n$, taking values in U_i . Furthermore, let F be an n -ary fuzzy relation in U which is characterized by its membership function μ_F . Then, the proposition

$$p \triangleq X \text{ is } F \quad (2.13)$$

induces an n -ary joint possibility distribution

$$\Pi_X \triangleq \Pi(X_1, \dots, X_n) \quad (2.14)$$

which is given by

$$\Pi(X_1, \dots, X_n) = F. \quad (2.15)$$

Correspondingly, the possibility distribution function of X is expressed by

$$\begin{aligned} \pi(X_1, \dots, X_n)(u_1, \dots, u_n) &= \mu_F(u_1, \dots, u_n), \quad u \triangleq (u_1, \dots, u_n) \in U \\ &= \text{Poss}\{X_1 = u_1, \dots, X_n = u_n\}. \end{aligned}$$

As in the case of probabilities, we can define marginal and conditional possibilities. Thus, let $s \triangleq (i_1, \dots, i_k)$ be a subsequence of the index sequence $(1, \dots, n)$ and let s' denote the complementary subsequence $s' \triangleq (j_1, \dots, j_m)$ (e.g., for $n = 5$, $s = (1, 3, 4)$ and $s' = (2, 5)$). In terms of such sequences, a k -tuple of the form $(A_{i_1}, \dots, A_{i_k})$ may be

expressed in an abbreviated form as $A_{(s)}$. In particular, the variable $X_{(s)} = (X_{i_1}, \dots, X_{i_k})$ will be referred to as a k-ary subvariable of $X \triangleq (X_1, \dots, X_n)$, with $X_{(s')} = (X_{j_1}, \dots, X_{j_m})$ being a subvariable complementary to $X_{(s)}$.

The projection of $\Pi_{(X_1, \dots, X_n)}$ on $U_{(s)} \triangleq U_{i_1} \times \dots \times U_{i_k}$ is a k-ary possibility distribution denoted by

$$\Pi_{X_{(s)}} \triangleq \text{Proj}_{U_{(s)}} \Pi_{(X_1, \dots, X_n)} \quad (2.16)$$

and defined by

$$\pi_{X_{(s)}}(u_{(s)}) \triangleq \sup_{u_{(s')}} \pi_X(u_1, \dots, u_n) \quad (2.17)$$

where $\pi_{X_{(s)}}$ is the possibility distribution function of $\Pi_{X_{(s)}}$. For example, for $n = 2$,

$$\pi_{X_1}(u_1) \triangleq \sup_{u_2} \pi_{(X_1, X_2)}(u_1, u_2)$$

is the expression for the possibility distribution function of the projection of $\Pi_{(X_1, X_2)}$ on U_1 . By analogy with the concept of a marginal probability distribution, $\Pi_{X_{(s)}}$ will be referred to as a marginal possibility distribution.

As a simple illustration, assume that $n = 3$, $U_1 = U_2 = U_3 = a + b$ or, more conventionally, $\{a, b\}$, and $\Pi_{(X_1, X_2, X_3)}$ is expressed as a linear form

$$\Pi_{(X_1, X_2, X_3)} = 0.8aaa + 1aab + 0.6baa + 0.2bab + 0.5bbb \quad (2.18)$$

in which a term of the form $0.6baa$ signifies that

$$\text{Poss}\{X_1 = b, X_2 = a, X_3 = a\} = 0.6 .$$

To derive $\Pi_{(X_1, X_2)}$ from (2.18), it is sufficient to replace the value of X_3 in each term in (2.18) by the null string Λ . This yields

$$\begin{aligned}\Pi_{(X_1, X_2)} &= 0.8aa + 1aa + 0.6ba + 0.2ba + 0.5bb \\ &= 1aa + 0.6ba + 0.5bb\end{aligned}$$

and similarly

$$\begin{aligned}\Pi_{X_1} &= 1a + 0.6b + 0.5b \\ &= 1a + 0.6b\end{aligned}$$

An n-ary possibility distribution is particularized by forming the conjunction of the propositions "X is F" and "X_(s) is G," where X_(s) is a subvariable of X. Thus,

$$\Pi_X[\Pi_{X(s)} = G] \triangleq F \cap \bar{G} \quad (2.19)$$

where the right-hand member denotes the intersection of F with the cylindrical extension of G, i.e., a cylindrical fuzzy set defined by

$$\mu_{\bar{G}}(u_1, \dots, u_n) = \mu_G(u_{i_1}, \dots, u_{i_k}), \quad (u_1, \dots, u_n) \in U_1 \times \dots \times U_n. \quad (2.20)$$

As a simple illustration, consider the possibility distribution defined by (2.18), and assume that

$$\Pi_{(X_1, X_2)} = 0.4aa + 0.9ba + 0.1bb.$$

In this case,

$$\begin{aligned}\bar{G} &= 0.4aaa + 0.4aab + 0.9baa + 0.9bab + 0.1bba + 0.1bbb \\ F \cap \bar{G} &= 0.4aaa + 0.4aab + 0.6baa + 0.2bab + 0.1bbb\end{aligned}$$

and hence

$$\Pi_{(X_1, X_2, X_3)}[\Pi_{(X_1, X_2)} = G] = 0.4aaa + 0.4aab + 0.6baa + 0.2bab + 0.1bbb .$$

There are many cases in which the operations of particularization and projection are combined. In such cases it is convenient to use the simplified notation

$$X_{(r)} \Pi[\Pi_{X(s)} = G] \quad (2.21)$$

to indicate that the particularized possibility distribution (or relation) $\Pi[\Pi_{X(s)} = G]$ is projected on $U_{(r)}$, where r , like s , is a subsequence of the index sequence $(1, \dots, n)$. For example,

$$X_1 \times X_3 \Pi[\Pi_{(X_3, X_4)} = G]$$

would represent the projection of $\Pi[\Pi_{(X_3, X_4)} = G]$ on $U_1 \times U_3$. Informally, (2.21) may be interpreted as: Constrain the $X_{(s)}$ by $\Pi_{X(s)} = G$ and read out the $X_{(r)}$. In particular, if the values of $X_{(s)}$ --rather than their possibility distribution--are set equal to G , then (2.21) becomes

$$X_{(r)} \Pi[X_{(s)} = G] .$$

We shall make use of (2.21) and its special cases in Section 3.

If X and Y are variables taking values in U and V , respectively, then the conditional possibility distribution of Y given X is induced by a proposition of the form "If X is F then Y is G " and is expressed as $\Pi_{(Y|X)}$, with the understanding that

$$\pi_{(Y|X)}(v|u) \triangleq \text{Poss}\{Y = v | X = u\} \quad (2.22)$$

where (2.22) defines the conditional possibility distribution function of Y given X . If we know the distribution function of X and the conditional

distribution function of Y given X , then we can construct the joint distribution function of X and Y by forming the conjunction ($\wedge \triangleq \min$)

$$\pi_{(X,Y)}(u,v) = \pi_X(u) \wedge \pi_{(Y|X)}(v|u) . \quad (2.23)$$

Translation Rules

The translation rules in PRUF serve the purpose of facilitating the composition of the meaning of a complex proposition from the meanings of its constituents. For convenience, the rules in question are categorized into four basic types: Type I: Rules pertaining to modification; Type II: Rules pertaining to composition; Type III: Rules pertaining to quantification; and Type IV: Rules pertaining to qualification.

Remark. Translation rules as described below relate to what might be called focused translations, that is, translation of p into a possibility assignment equation. More generally, a translation may be unfocused, in which case it is expressed as a procedure which computes the possibility of a database, D , given p or, equivalently, the truth of p relative to D . A more detailed discussion of these issues will be presented at a later point in this section.

Modifier rule (Type I). Let X be a variable which takes values in a universe of discourse U and let F be a fuzzy subset of U . Consider the proposition

$$p \triangleq X \text{ is } F \quad (2.24)$$

or, more generally,

$$p = N \text{ is } F \quad (2.25)$$

where N is a variable, an object or a proposition. For example,

$$p \triangleq \text{Lucia is young} \quad (2.26)$$

which may be expressed in the form (2.24), i.e.,

$$p \triangleq \text{Age(Lucia) is young} \quad (2.27)$$

by identifying X with the variable Age(Lucia) .

Now, if in a particular context the proposition X is F translates into

$$X \text{ is } F \rightarrow \Pi_X = F \quad (2.28)$$

then in the same context

$$X \text{ is } mF \rightarrow \Pi_X = F^+ \quad (2.29)$$

where m is a modifier such as not, very, more or less, etc., and F^+ is a modification of F induced by m . More specifically: If $m = \text{not}$, then $F^+ = F' = \text{complement of } F$, i.e.,

$$\mu_{F^+}(u) = 1 - \mu_F(u), \quad u \in U. \quad (2.30)$$

If $m = \text{very}$, then $F^+ = F^2$, i.e.,

$$\mu_{F^+}(u) = \mu_F^2(u), \quad u \in U. \quad (2.31)$$

If $m = \text{more or less}$, then $F^+ = \sqrt{F}$, i.e.,

$$\mu_{F^+}(u) = \sqrt{\mu_F(u)}, \quad u \in U. \quad (2.32)$$

As a simple illustration of (2.31), if SMALL is defined as in (2.7), then

$$X \text{ is very small} \rightarrow \Pi_X = F^2 \quad (2.33)$$

where

$$F^2 = 1/0 + 1/1 + 0.64/2 + 0.36/3 + 0.16/4 + 0.04/5 .$$

It should be noted that (2.30), (2.31) and (2.32) should be viewed as default rules which may be replaced by other translation rules in cases in which some alternative interpretations of the modifiers not, very and more or less may be more appropriate.

Conjunctive, Disjunctive and Implicational Rules (Type II). If

$$X \text{ is } F \rightarrow \Pi_X = F \quad \text{and} \quad Y \text{ is } G \rightarrow \Pi_Y = G \quad (2.34)$$

where F and G are fuzzy subsets of U and V , respectively, then

$$(a) \quad X \text{ is } F \text{ and } Y \text{ is } G \rightarrow \Pi_{(X,Y)} = F \times G \quad (2.35)$$

where

$$\mu_{F \times G}(u,v) \triangleq \mu_F(u) \wedge \mu_G(v) . \quad (2.36)$$

$$(b) \quad X \text{ is } F \text{ or } Y \text{ is } G \rightarrow \Pi_{(X,Y)} = \bar{F} \cup \bar{G} \quad (2.37)$$

where

$$\bar{F} \triangleq F \times V , \quad \bar{G} \triangleq U \times G \quad (2.38)$$

and

$$\mu_{\bar{F} \cup \bar{G}}(u,v) = \mu_F(u) \vee \mu_G(v) . \quad (2.39)$$

$$(c) \quad \text{If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \Pi_{(Y|X)} = \bar{F} \circ \bar{G} \quad (2.40)$$

where $\Pi_{(Y|X)}$ denotes the conditional possibility distribution of Y given X , and the bounded sum \circ is defined by

$$\mu_{\bar{F} \circ \bar{G}}(u,v) = 1 \wedge (1 - \mu_F(u) + \mu_G(v)) . \quad (2.41)$$

In stating the implicational rule in the form (2.40), we have merely chosen one of several alternative ways in which the conditional possibility distribution $\Pi_{(Y|X)}$ may be defined, each of which has some advantages and disadvantages depending on the application. Among the more important of

these are the following [1], [41], [62]:

$$(c_2) \text{ If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \Pi_{(Y|X)} = \bar{F}' \cup G \quad (2.42)$$

$$(c_3) \text{ If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \Pi_{(Y|X)} = F \times G \cup F' \times V \quad (2.43)$$

$$(c_4) \text{ If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \pi_{(Y|X)}(v|u) = 1 \text{ if } \mu_G(v) \geq \mu_F(u) \quad (2.44)$$

$$= \frac{\mu_G(v)}{\mu_F(u)} \text{ otherwise}$$

$$(c_5) \text{ If } X \text{ is } F \text{ then } Y \text{ is } G \rightarrow \pi_{(Y|X)}(v|u) = 1 \text{ if } \mu_G(v) \geq \mu_F(u) \quad (2.45)$$

$$= \mu_G(v) \text{ otherwise}$$

Quantification Rule (Type III). If $U = \{u_1, \dots, u_N\}$, Q is a quantifier such as many, few, several, all, some, most, etc., and

$$X \text{ is } F \rightarrow \Pi_X = F \quad (2.46)$$

then the proposition "QX is F" (e.g., "many X's are large") translates into

$$\Pi_{\text{Count}(F)} = Q \quad (2.47)$$

where $\text{Count}(F)$ denotes the number (or the proportion) of elements of U which are in F . By the definition of cardinality of F , if the fuzzy set F is expressed as

$$F = \mu_1/u_1 + \mu_2/u_2 + \dots + \mu_N/u_N \quad (2.48)$$

then

$$\text{Count}(F) = \sum_{i=1}^N \mu_i \quad (2.49)$$

where the right-hand member is understood to be rounded-off to the nearest integer. As a simple illustration of (2.47), if the quantifier several is defined as

$$\text{SEVERAL} \triangleq 0/1 + 0.4/2 + 0.6/3 + 1/4 + 1/5 + 1/6 + 0.6/7 + 0.2/8 \quad (2.50)$$

then

$$\begin{aligned} \text{Several } X\text{'s are large} &\rightarrow \prod_{i=1}^N \mu_{\text{LARGE}}(u_i) & (2.51) \\ &= 0/1 + 0.4/2 + 0.6/3 + 1/4 + 1/5 + 1/6 + 0.6/7 + 0.2/8 \end{aligned}$$

where $\mu_{\text{LARGE}}(u_i)$ is the grade of membership of the i^{th} value of X in the fuzzy set LARGE .

Alternatively, and perhaps more appropriately, the cardinality of F may be defined as a fuzzy number, as is done in [79]. Thus, if the elements of F are sorted in descending order, so that $\mu_n \leq \mu_m$ if $n \geq m$, then the truth-value of the proposition

$$p \triangleq F \text{ has at least } n \text{ elements} \quad (2.52)$$

is defined to be equal to μ_n , while that of q ,

$$q \triangleq F \text{ has at most } n \text{ elements} , \quad (2.53)$$

is taken to be $1 - \mu_{n+1}$. From this, then, it follows that the truth-value of the proposition r ,

$$r \triangleq F \text{ has exactly } n \text{ elements} , \quad (2.54)$$

is given by $\mu_n \wedge (1 - \mu_{n+1})$.

Let $F\downarrow$ denote F sorted in descending order. Then (2.52) may be expressed compactly in the equivalent form

$$\text{FGCount}(F) = F\downarrow \quad (2.55)$$

which signifies that if the fuzzy cardinality of F is defined in terms of

(2.52), with G standing for greater than, then the fuzzy count of elements in F is given by $F\downarrow$, with the understanding that $F\downarrow$ is regarded as a fuzzy subset of $\{0,1,2,\dots\}$. In a similar fashion, (2.53) leads to the definition

$$\text{FLCount}(F) = (F\downarrow)' - 1 \quad (2.56)$$

where L stands for less than and subtraction should be interpreted as translation to the left, while (2.54) leads to

$$\text{FECCount}(F) = (F\downarrow) \cap ((F\downarrow)' - 1)$$

where E stands for equal to. For convenience, we shall refer to FGCount , FLCount and FECCount as the FG cardinality, FL cardinality and FE cardinality, respectively. The concept of FG cardinality will be illustrated in Example 9, Section 3.

Remark. There may be some cases in which it may be appropriate to normalize the definition of FECCount in order to convey a correct perception of the count of elements in a fuzzy set. In such cases, we may employ the definition

$$\text{FENCCount}(F) = \frac{\text{FECCount}(F)}{\text{Max}_n(\mu_n \wedge (1 - \mu_{n+1}))}. \quad (2.57)$$

Truth Qualification Rule (Type IV). Let τ be a linguistic truth-value, e.g., very true, quite true, more or less, true, etc. Such a truth-value may be regarded as a fuzzy subset of the unit interval which is characterized by a membership function $\mu_\tau: [0,1] \rightarrow [0,1]$.

A truth-qualified proposition, e.g., "It is τ that X is F ," is expressed as " X is F is τ ." As shown in [79], the translation rule for such propositions is given by

$$X \text{ is } F \text{ is } \tau \rightarrow \Pi_X = F^+ \quad (2.58)$$

where

$$\mu_{F^+}(u) = \mu_\tau(\mu_F(u)) . \quad (2.59)$$

As an illustration, consider the truth-qualified proposition

Susana is young is very true

which by (2.58), (2.59) and (2.31) translates into

$$\Pi_{\text{Age}}(\text{Susana}) = \mu_{\text{TRUE}}^2(\mu_{\text{YOUNG}}(u)) . \quad (2.60)$$

Now, if we assume that

$$\mu_{\text{YOUNG}}(u) = \left(1 + \left(\frac{u}{25}\right)^2\right)^{-1}, \quad u \in [0, 100] \quad (2.61)$$

and

$$\mu_{\text{TRUE}}(v) = v^2, \quad v \in [0, 1]$$

then (2.60) yields

$$\Pi_{\text{Age}}(\text{Susana}) = \left(1 + \left(\frac{u}{25}\right)^2\right)^{-4}$$

as the possibility distribution of the age of Susana.

A more general type of translation process in PRUF which subsumes the translation rules given above is the following.

Let $\mathcal{D} = \{D\}$ denote a collection of databases, with D representing a generic element of \mathcal{D} . For the purposes of our analysis, D will be assumed to consist of a collection of possibly time-varying relations. If R is a constituent relation in D , then by the frame of R is meant the name of R together with the names of its columns (i.e., attributes). For example, if a constituent of D is a relation labeled POPULATION whose tableau is comprised of columns labeled Name and Height, then the frame

of POPULATION is represented as $\text{POPULATION} \parallel \text{Name} \mid \text{Height} \mid$ or, more simply, as $\text{POPULATION}[\text{Name}; \text{Height}]$.

If p is a proposition in a natural language, its translation into PRUF can assume one of three--essentially equivalent--forms.⁵

- (a) $p \rightarrow$ a possibility assignment equation
- (b) $p \rightarrow$ a procedure which yields for each D in \mathcal{D} the possibility of D given p , i.e., $\text{Poss}\{D \mid p\}$
- (c) $p \rightarrow$ a procedure which yields for each D in \mathcal{D} the truth-value of p relative to D , i.e., $\text{Tr}\{p \mid D\}$

Remark. An important implicit assumption about the procedures involved in (b) and (c) is that they have a high degree of what might be called explanatory effectiveness, by which is meant a capability to convey the meaning of p to a human (or a machine) who is conversant with the meaning of the constituent terms in p but not with the meaning of p as a whole. For example, a procedure which merely tabulates the possibility of each D in \mathcal{D} would, in general, have a low degree of explanatory effectiveness if it does not indicate in sufficient detail the way in which that possibility is arrived at. On the other extreme, a procedure which is excessively detailed and lacking in modularity would also have a low degree of explanatory effectiveness because the meaning of p might be obscured by the maze of unstructured steps in the body of the procedure.

The equivalence of (b) and (c) is a consequence of the way in which the concept of truth is defined in fuzzy logic [77], [2]. Thus, it can readily be shown that, under mildly restrictive assumptions on \mathcal{D} , we have

⁵It should be noted that (b) and (c) are in the spirit of possible-world semantics and truth-conditional semantics, respectively. In their conventional form, however, these semantics have no provision for fuzzy propositions and hence do not provide a sufficiently expressive system for our purposes.

$$\text{Tr}\{p|D\} = \text{Poss}\{D|p\}$$

which implies the equivalence of (b) and (c).

The restricted subset of PRUF which we have discussed so far is adequate for illustrating some of the simpler ways in which it may be applied to the precisiation of meaning. We shall do this in the following section.

3. Precisiation of Meaning - Examples

There are two distinct and yet interrelated ways in which PRUF provides a mechanism for a precisiation of meaning of propositions. First, by expressing the meaning of a proposition as an explicitly defined procedure which acts on the fuzzy denotations of its constituents; and second, by disambiguation--especially in those cases in which what is needed is a method of differentiation between the nuances of meaning.

In what follows, we shall illustrate the techniques which may be employed for this purpose by several representative examples, of which Examples 6, 7, 8 and 9 relate to cases in which a proposition may have two or more distinct readings. Whenever appropriate, we consider both focused and unfocused translations of the given proposition.

Example 1

$$p \triangleq \text{John is very rich} \quad (3.1)$$

Assume that the database, D , consists of the following relations

$$\text{POPULATION}[\text{Name};\text{Wealth}] \quad (3.2)$$

$$\text{RICH}[\text{Wealth};\mu]$$

in which the first relation, POPULATION, tabulates the wealth, Wealth_i , of each individual, Name_i , while the second relation, RICH, tabulates the degree, μ_i , to which an individual whose wealth is Wealth_i is rich.

Unfocused translation: First, we find John's wealth, which is given by

$$\text{Wealth}(\text{John}) = \text{Wealth} \text{POPULATION}[\text{Name} = \text{John}] \quad (3.3)$$

Second, we intensify RICH to account for the modifier very by squaring RICH,⁶

⁶If the frame of RICH is RICH[Wealth; μ] then the frame of RICH² is RICH²[Wealth; μ^2], which signifies that each μ in RICH is replaced by μ^2 . This representation of very rich is a consequence of the translation rule (2.14).

and substitute $Wealth(John)$ into $RICH^2$ to find the degree, δ , to which John is very rich. This yields

$$\delta = (\mu_{RICH}[Wealth =_{Wealth} POPULATION[Name = John]])^2 . \quad (3.4)$$

Finally, on equating δ to the possibility of the database, we obtain

John is very rich

$$\rightarrow \pi(D) = (\mu_{RICH}[Wealth =_{Wealth} POPULATION[Name = John]])^2 . \quad (3.5)$$

Focused translation: On interpreting the given proposition as a characterization of the possibility distribution of the implicit variable $Wealth(John)$, we are led to the possibility assignment equation

$$\text{John is very rich} \rightarrow \Pi_{Wealth(John)} = RICH^2 - \quad (3.6)$$

which implies that

$$\text{Poss}\{Wealth(John) = u\} = (\mu_{RICH}(u))^2 \quad (3.7)$$

where μ_{RICH} is the membership function of the fuzzy set $RICH$, with u ranging over the domain of $Wealth$.

Example 2

$$p \triangleq \text{Hans is much richer than Marie} \quad (3.8)$$

We assume that the database, D , consists of the relations

$$POPULATION[Name;Wealth] \quad (3.9)$$

and

$$MUCH RICHER[Wealth1;Wealth2;u]$$

in which μ is the degree to which an individual who has $Wealth1$ is much richer than one who has $Wealth2$.

Unfocused translation: Proceeding as in Example 1, we arrive at

Hans is much richer than Marie \rightarrow

$$\pi(D) = \mu \text{ MUCH RICHER}[\text{Wealth} \text{ POPULATION}[\text{Name} = \text{Hans}]; \text{Wealth} \text{ POPULATION}[\text{Name} = \text{Marie}]] . \quad (3.10)$$

Focused translation:

Hans is much richer than Marie \rightarrow

$$\Pi(\text{Wealth}(\text{Hans}), \text{Wealth}(\text{Marie})) = \text{MUCH RICHER} \quad (3.11)$$

which implies that

$$\text{Poss}\{\text{Wealth}(\text{Hans}) = u_1, \text{Wealth}(\text{Marie}) = u_2\} = \mu \text{ MUCH RICHER}(u_1, u_2) \quad (3.12)$$

Example 3

$$p \triangleq \text{Vera is very kind} \quad (3.13)$$

In this case, we assume that kindness is not a measurable characteristic like height, weight, age, wealth, etc. However, we also assume that it is possible to associate with each individual his/her index of kindness on the scale from 0 to 1, which is equivalent to assuming that the class of kind individuals is a fuzzy set KIND, with the index of kindness corresponding to the grade of membership in KIND.

Unfocused translation: Assume that D consists of the single relation

$$\text{KIND}[\text{Name}; \mu] \quad (3.14)$$

in which μ is the degree of kindness of Name. Then

$$\text{Vera is very kind} \rightarrow \pi(D) = (\mu \text{ KIND}[\text{Name} = \text{Vera}])^2 . \quad (3.15)$$

Focused translation: A special type of possibility distribution which we need in this case is the unitor, \perp , which is defined as

$$\pi_{\perp}(v) = v, \quad 0 \leq v \leq 1. \quad (3.16)$$

In terms of the unitor, then, we have

$$\text{Vera is very kind} \rightarrow \Pi_{\text{Kindness(Vera)}} = \perp^2 \quad (3.17)$$

which implies that

$$\text{Poss}\{\text{Kindness(Vera)} = v\} = v^2, \quad 0 \leq v \leq 1. \quad (3.18)$$

This follows at once from (3.15), since

$$\text{Kindness(Vera)} = \mu_{\text{KIND[Name = Vera]}}. \quad (3.19)$$

Example 4

$$p \triangleq \text{Brian is much taller than most of his close friends} \quad (3.20)$$

Unfocused translation: For the purpose of representing the meaning of p , we shall assume that D is comprised of the relations

$$\text{POPULATION[Name;Height]} \quad (3.21)$$

$$\text{FRIENDS[Name1;Name2;\mu]}$$

$$\text{MUCH TALLER[Height1;Height2;\mu]}$$

$$\text{MOST}[\rho;\mu]$$

In the relation FRIENDS , μ represents the degree to which an individual whose name is Name2 is a friend of Name1 . Similarly, in the relation MUCH TALLER , μ represents the degree to which an individual whose height is Height1 is much taller than one whose height is Height2 . In MOST ,

μ represents the degree to which a proportion, ρ , fits the definition of MOST as a fuzzy subset of the unit interval.

To represent the meaning of p we shall express the translation of p as a procedure which computes the possibility of D given p . The sequence of computations in this procedure is as follows.

1. Obtain Brian's height from POPULATION. Thus,

$$\text{Height}(\text{Brian}) = \text{Height}^{\text{POPULATION}}[\text{Name} = \text{Brian}] .$$

2. Determine the fuzzy set, MT, of individuals in POPULATION in relation to whom Brian is much taller.

Let Name_i be the name of the i^{th} individual in POPULATION. The height of Name_i is given by

$$\text{Height}(\text{Name}_i) = \text{Height}^{\text{POPULATION}}[\text{Name} = \text{Name}_i] .$$

Now the degree to which Brian is much taller than Name_i is given by

$$\delta_i = \mu \text{MUCH TALLER}[\text{Height}(\text{Brian}), \text{Height}(\text{Name}_i)]$$

and hence MT may be expressed as

$$\text{MT} = \sum_i \delta_i / \text{Name}_i , \quad \text{Name}_i \in \text{Name}^{\text{POPULATION}}$$

where $\text{Name}^{\text{POPULATION}}$ is the list of names of individuals in POPULATION, δ_i is the grade of membership of Name_i in MT, and \sum_i is the union of singletons δ_i / Name_i .

3. Determine the fuzzy set, CF, of individuals in POPULATION who are close friends of Brian.

To form the relation CLOSE FRIENDS from FRIENDS we intensify FRIENDS by squaring, as in Example 1. Then, the fuzzy set of close friends of Brian is given by

$$CF = \mu_{\times \text{Name}_2} \text{FRIENDS}^2[\text{Name}_1 = \text{Brian}] .$$

4. Form the count of elements of CF:

$$\text{Count}(CF) = \sum_i \mu_{CF}(\text{Name}_i)$$

where $\mu_{CF}(\text{Name}_i)$ is the grade of membership of Name_i in CF and \sum_i is the arithmetic sum. More explicitly

$$\text{Count}(F) = \sum_i \mu_{\text{FRIENDS}}^2(\text{Brian}, \text{Name}_i) .$$

5. Form the intersection of CF and MT, that is, the fuzzy set of those close friends of Brian in relation to whom he is much taller.

$$H \triangleq CF \cap MT .$$

6. Form the count of elements of H

$$\text{Count}(H) = \sum_i \mu_H(\text{Name}_i)$$

where $\mu_H(\text{Name}_i)$ is the grade of membership of Name_i in H and \sum_i is the arithmetic sum.

7. Form the ratio

$$r = \frac{\text{Count}(MT \cap CF)}{\text{Count}(CF)}$$

which represents the proportion of close friends of Brian in relation to whom he is much taller.

8. Compute the grade of membership of r in MOST

$$\delta = \mu_{\text{MOST}}[\rho = r] .$$

The value of δ is the desired possibility of D given p . In terms of the membership functions of FRIENDS, MUCH TALLER and MOST, the value of δ is given explicitly by the expression

$$\delta = \mu_{\text{MOST}} \left\{ \frac{\sum_i \mu_{\text{MT}}(\text{Height}(\text{Brian}), \text{Height}(\text{Name}_i)) \wedge \mu_{\text{CF}}^2(\text{Brian}, \text{Name}_i)}{\sum_i \mu_{\text{CF}}^2(\text{Brian}, \text{Name}_i)} \right\} . \quad (3.22)$$

Thus,

$$\text{Brian is much taller than most of his close friends} \rightarrow \pi(D) = \delta \quad (3.23)$$

where δ is given by (3.22).

Focused translation: From (3.23) it follows at once that

$$\begin{aligned} p \rightarrow \pi_{\text{Height}(\text{Brian})}(u) &\triangleq \text{Poss}\{\text{Height}(\text{Brian}) = u\} && (3.24) \\ &= \mu_{\text{MOST}} \left\{ \frac{\sum_i \mu_{\text{MT}}(u, \text{Height}(\text{Name}_i)) \wedge \mu_{\text{CF}}^2(\text{Brian}, \text{Name}_i)}{\sum_i \mu_{\text{CF}}^2(\text{Brian}, \text{Name}_i)} \right\} \end{aligned}$$

Example 5

$$p \triangleq \text{Lane resides in a small city near Washington} \quad (3.25)$$

Unfocused translation: Assume that the database consists of the relations

$$\text{RESIDENCE}[P.\text{Name}; C.\text{Name}; \text{Population}] \quad (3.26)$$

$$\text{SMALL}[\text{Population}; \mu]$$

$$\text{NEAR}[C.\text{Name}_1; C.\text{Name}_2; \mu]$$

In RESIDENCE, $P.\text{Name}$ stands for Person Name, $C.\text{Name}$ for City Name,

and Population for population of C.Name. In SMALL, μ is the degree to which a city whose population figure is Population is small. In NEAR, μ is the degree to which C.Name1 and C.Name2 are near one another.

The population of the city in which Lane resides is given by

$$\text{Population}^{\text{RESIDENCE}[P.\text{Name} = \text{Lane}]} \quad (3.27)$$

and hence the degree, δ , to which the city is small may be expressed as

$$\delta_1 = \mu^{\text{SMALL}[\text{Population}^{\text{RESIDENCE}[P.\text{Name} = \text{Lane}]}]} \quad (3.28)$$

Now the degree to which the city in which Lane resides is near Washington is given by

$$\delta_2 = \mu^{\text{NEAR}[C.\text{Name}1 = \text{Washington}; C.\text{Name}2 = C.\text{Name}^{\text{RESIDENCE}[P.\text{Name} = \text{Lane}]}]} \quad (3.29)$$

On forming the conjunction of (3.28) and (3.29), the possibility of D-- and hence the translation of p-- is found to be expressed by ($\wedge \triangleq \min$)

$$\text{Lane resides in a small city near Washington} \rightarrow \pi(D) = \delta_1 \wedge \delta_2 \quad (3.30)$$

where δ_1 and δ_2 are given by (3.28) and (3.29).

Focused translation: The implicit variable in this case may be expressed as

$$X \triangleq \text{Location}(\text{Residence}(\text{Lane})) \quad (3.31)$$

Thus, the goal of the focused translation in this case is the computation of the possibility distribution of the location of residence of Lane.

To illustrate the effect of choosing different databases on the translation of p, we shall consider two cases each of which represents a particular assumption concerning the relations in D.

First, we consider the simpler case in which the constituent relations in D are assumed to be:

$$\text{SMALL}[C.\text{Name};\mu] \quad (3.32)$$

$$\text{NEAR}[C.\text{Name1};C.\text{Name2};\mu] \quad (3.33)$$

In SMALL , μ is the degree to which the city whose name is $C.\text{Name}$ is small. In NEAR , μ is the degree to which cities named $C.\text{Name1}$ and $C.\text{Name2}$ are near each other.

From NEAR , the fuzzy set of cities which are near Washington is found to be given by

$$C.\text{Name1} \times_{\mu} \text{NEAR}[C.\text{Name2} = \text{Washington}] .$$

Consequently, the fuzzy set of cities which are near Washington and, in addition, are small is given by the intersection

$$\text{SMALL} \cap C.\text{Name1} \times_{\mu} \text{NEAR}[C.\text{Name2} = \text{Washington}] .$$

With this expression in hand, the focused translation of p may be expressed as

$$\text{Lane resides in a small city near Washington} \rightarrow \quad (3.34)$$

$$\Pi_{\text{Location}}(\text{Residence}(\text{Lane})) = \text{SMALL} \cap C.\text{Name1} \times_{\mu} \text{NEAR}[C.\text{Name2} = \text{Washington}] .$$

In the case to be considered next, the relations in D are assumed to be less directly related to the denotations of words in p than the relations expressed by (3.32) and (3.33). More specifically, we assume that D consists of the relations

$$\begin{aligned}
 & \text{LIST}[C.\text{Name};\text{Population}] && (3.35) \\
 & \text{DISTANCE}[C.\text{Name1};C.\text{Name2};\text{Distance}] \\
 & \text{SMALL}[\text{Population};\mu] \\
 & \text{NEAR}[\text{Distance};\mu]
 \end{aligned}$$

In LIST, Population is the population of C.Name. In DISTANCE, Distance is the distance between C.Name1 and C.Name2. In NEAR, μ is the degree to which two cities whose distance from one another is Distance are near each other. As for SMALL, it has the same meaning as in (3.32).

For our purposes, we need a relation which tabulates the degree to which each city in LIST is small. To this end, we form the composition⁷ of LIST and SMALL, which yields the relation

$$G \triangleq \text{SMALL}[C.\text{Name};\mu] \triangleq \text{LIST}[C.\text{Name};\text{Population}] \circ \text{SMALL}[\text{POPULATION};\mu] \quad (3.36)$$

in which μ is the degree to which C.Name is small. Actually, since LIST and SMALL are functions, we can write

$$\mu_G(C.\text{Name}) = \mu_{\text{SMALL}}(\text{Population}(C.\text{Name})) \quad (3.37)$$

in which the right-hand member of (3.37) expresses the degree to which a city whose population is Population(C.Name) is small.

Now from DISTANCE we can find the distances of cities in LIST from Washington. These distances are yielded by the relation

$$DC \triangleq \text{C.Name1} \times \text{Distance} \text{DISTANCE}[C.\text{Name2} = \text{Washington}] . \quad (3.38)$$

⁷If the membership functions of $R[X;Y]$ and $S[Y;Z]$ are expressed as $\mu_R(x,y)$ and $\mu_S(y,z)$, respectively, then the membership function of the composition of R and S with respect to Y is given by

$$\mu_{R \circ S}(x,z) = \sup_y (\mu_R(x,y) \wedge \mu_S(y,z)) .$$

Furthermore, on forming the composition of this relation with NEAR, we obtain the relation

$$H[C.Name;\mu] \triangleq \text{NEAR}[Distance;\mu] \circ C.Name1 \times \text{Distance} \text{DISTANCE}[C.Name2 = \text{Washington}] . \quad (3.39)$$

In $H[C.Name;\mu]$, μ represents the degree to which $C.Name$ is near Washington. More explicitly:

$$\mu_H(C.Name) = \mu_{\text{NEAR}}(\text{Distance of } C.Name \text{ from Washington})$$

in which the distance of $C.Name$ from Washington is obtained from DC by expressing the distance as a function of $C.Name$.

At this point, we have constructed from the given database the relations which were given initially in the previous case. With these relations in hand, the translation of p may be expressed compactly as

$$\Pi_{\text{Location}}(\text{Residence}(\text{Lane})) = G \cap H \quad (3.40)$$

where G and H are defined by (3.36) and (3.39), respectively.

Example 6

$$p \triangleq \text{Vivien is over thirty} \quad (3.41)$$

The literal reading of p may be expressed as

$$p_1 \triangleq \text{Age of Vivien is greater than thirty}$$

which translates into

$$p_1 \rightarrow \text{Age}(\text{Vivien}) > 30 . \quad (3.42)$$

In many cases, however, the intended meaning of p would be

$p_2 \triangleq$ Vivien is over thirty but not much over thirty .

In this case, the translation of p_2 into PRUF would be expressed as

$$\Pi_{\text{Age}(\text{Vivien})} = (30,100] \cap_{\text{Age1} \times \mu} \text{MUCH OVER}'[\text{Age2} = 30] \quad (3.43)$$

in which $(30,100]$ is the age interval $30 < u \leq 100$;

$\text{MUCH OVER}[\text{Age1}; \text{Age2}; \mu]$ is a relation in which μ is the degree to which Age1 is much over Age2 ; and $\text{MUCH OVER}'$ is the complement of MUCH OVER .

More explicitly, (3.43) implies that

$$\begin{aligned} \text{Poss}\{\text{Age}(\text{Vivien}) = u\} &= 0 && \text{for } u \leq 30 && (3.44) \\ &= 1 - \mu_{\text{MUCH OVER}}(u,30) , && \text{for } u > 30 . \end{aligned}$$

Example 7

$$p \triangleq \text{John is not very smart} \quad (3.45)$$

Assuming that D consists of the relation

$$\text{SMART}[\text{Name}; \mu]$$

in which μ is the degree to which Name is smart, the literal translation of p may be expressed as (see Example 3)

$$p \rightarrow \Pi_X = (\perp^2)' \quad (3.46)$$

where

$$X = \mu \text{ SMART}[\text{Name} = \text{John}]$$

and $(\perp^2)'$ is the complement of the square of the unitor.

However, if the intended meaning of p is

$$p_1 \triangleq \text{John is very(not smart)} \quad (3.47)$$

then the translation of p into PRUF would be

$$\Pi_X = (1')^2 . \quad (3.48)$$

Note that (3.48) implies that

$$\text{Poss}\{X = v\} = 1 - v^2 \quad (3.49)$$

whereas (3.46) implies that

$$\text{Poss}\{X = v\} = (1-v)^2 . \quad (3.50)$$

Example 8

$$p \triangleq \text{Naomi has a young daughter} \quad (3.51)$$

There are three distinct readings of p :

$p_1 \triangleq$ Naomi has only one daughter and her daughter is young

$p_2 \triangleq$ Naomi has one or more daughters of whom only one is young

$p_3 \triangleq$ Naomi has one or more daughters of whom one or more are young

Assume that D consists of the relation

$$\text{DAUGHTER}[M.\text{Name};D.\text{Name};\mu_{DY}]$$

in which $M.\text{Name}$ and $D.\text{Name}$ stand for Mother's name and Daughter's name, respectively, and μ_{DY} is the degree to which $D.\text{Name}$ is young.

The translation of p_1 , p_2 and p_3 may be expressed as follows

$$p_1 \rightarrow \pi(D) = \delta_1 \wedge \mu_1 \quad (3.52)$$

where

$$\delta_1 = 1 \text{ if Naomi has only one daughter, i.e., if}$$

$$\text{Count}(\text{D.Name} \text{ DAUGHTER}[\text{M.Name} = \text{Naomi}]) = 1$$

and $\delta_1 = 0$ otherwise;

and

$$\mu_1 \triangleq \text{the degree to which Naomi's daughter is young, i.e.,}$$

$$\mu_1 = \mu_{DY} \text{ DAUGHTER}[\text{M.Name} = \text{Naomi}] .$$

Turning to p_2 and p_3 , let the set of daughters of Naomi be sorted in descending order according to the degree of youth. For this set, then, let

$$\mu_i \triangleq \text{degree of youth of } i^{\text{th}} \text{ youngest daughter of Naomi.}$$

Now applying the concept of fuzzy cardinality (see (2.52)) to the set in question, we obtain at once

$$p_2 \rightarrow \pi(D) = \mu_1 \wedge (1 - \mu_2) \quad (3.53)$$

and

$$p_3 \rightarrow \pi(D) = \mu_1 . \quad (3.54)$$

Example 9

$$p \triangleq \text{Naomi has several young daughters} \quad (3.55)$$

In this case, we assume that D consists of the relations

$$\text{DAUGHTER}[\text{M.Name}; \text{D.Name}; \mu_{DY}]$$

and

$$\text{SEVERAL}[\text{N}; \mu]$$

in which the first relation has the same meaning as in Example 8, and μ in SEVERAL is the degree to which an integer N fits one's perception of several.

Furthermore, we assume that p should be read as p_3 in Example 8.

With these assumptions, the translation of p may be expressed compactly as

$$p \rightarrow \pi(D) = \sup((\geq \circ \text{SEVERAL}) \cap \text{FGCount}(\underset{D.\text{Name} \times \mu}{\text{DAUGHTER}}[M.\text{Name} = \text{Naomi}])) \quad (3.56)$$

where FGCount is defined by (2.55); $\sup F$ is defined by

$$\sup F \triangleq \sup_{u \in U} \mu_F(u) \quad (3.57)$$

where F is a fuzzy subset of U and μ_F is its membership function; and $\geq \circ \text{SEVERAL}$ is the composition of the relations \geq and SEVERAL , i.e., (see footnote)

$$\begin{aligned} \mu_{\geq \circ \text{SEVERAL}}(u) &= \mu_{\text{SEVERAL}}(n) \quad \text{for } n \leq n_{\max} \\ &= 1 \quad \text{for } n \geq n_{\max} \end{aligned}$$

$$n_{\max} \triangleq \text{smallest value of } n \text{ at which } \mu_{\text{SEVERAL}}(u) = 1 .$$

Intuitively, the composition of \geq and SEVERAL serves to precisiate the count expressed in words as "at least several." The intersection of $(\geq \circ \text{SEVERAL})$ and the FGCount of the daughters of Naomi serves to define the conjunction of "at least several" with the FGCount of daughters of Naomi; and the supremum of the intersection provides a measure of the degree of consistency of "at least several" with the FGCount in question.

As a concrete illustration of (3.56), assume that the fuzzy relation SEVERAL is defined as

$$\text{SEVERAL} \triangleq 0.5/2 + 0.8/3 + 1/4 + 1/5 + 0.8/6 + 0.5/0.7 . \quad (3.59)$$

Then

$$\underline{\geq} \circ \text{SEVERAL} = 0.5/2 + 0.8/3 + 1/4 + 1/5 + 1/6 + \dots . \quad (3.60)$$

Furthermore, assume that

$$D.\text{Name} \times_{\mu} \text{DAUGHTER}[M.\text{Name} = \text{Naomi}] = 1/\text{Eva} + 0.8/\text{Lisa} + 0.6/\text{Ruth} \quad (3.61)$$

so that

$$\text{FGCount}(D.\text{Name} \times_{\mu} \text{DAUGHTER}[M.\text{Name} = \text{Naomi}]) = 1/1 + 0.8/2 + 0.6/3 . \quad (3.62)$$

From (3.60) and (3.62), we deduce that

$$\begin{aligned} (\underline{\geq} \circ \text{SEVERAL}) \cap \text{FGCount}(D.\text{Name} \times_{\mu} \text{DAUGHTER}[M.\text{Name} = \text{Naomi}]) & \quad (3.63) \\ & = 0.5/2 + 0.6/3 \end{aligned}$$

and since

$$\text{sup}(0.5/2 + 0.6/3) = 0.6$$

we arrive at

$$\pi(D) = 0.6 \quad (3.64)$$

which represents the possibility of the given database given the proposition p .

4. Concluding Remark

The above examples are intended to illustrate the manner in which PRUF may be employed to precisiate the meaning of propositions expressed in a natural language. Such precisiation may be of use not only in communication between humans, but also--and perhaps more importantly--in communication between humans and machines.

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