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REVIEW OF THE LOWER HYBRID DRIFT INSTABILITY  
AND ITS SATURATION MECHANISMS

by

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***ABSTRACT***

A survey is given of previous research on the linear theory of the lower hybrid drift instability (Sec. I). A summary is made of research recently completed as well as research currently in progress on the nonlinear theory and computer simulations of the lower hybrid drift instability (Sec. II).

## I. The Lower Hybrid Drift Instability - Linear Theory

In the past several years there has been considerable interest in the lower hybrid drift instability (Mikhailovskii and Tsypin, 1966; Krall and Liewer, 1971), since it may play an important role in the anomalous transport processes of theta pinches and field-reversed configurations. The lower hybrid drift instability may also account for small scale F region irregularities in the equatorial ionosphere (Huba et al., 1978; Huba and Ossakow, 1979). The existence of this mode was first shown by Mikhailovskii and Tsypin (1966) using the cold-fluid hydrodynamic equations. The instability is caused by a coupling of ion plasma or lower hybrid oscillations to drift waves in a nonuniform plasma.

Krall and Liewer (1971) first treated this mode through the Vlasov equation with an assumption  $k_y a_e \ll 1$  in a slab geometry, where  $a_e$  is the electron Larmor radius. The usual local approximation was used for the profiles of the magnetic field, density and temperature. In their model the plasma supports an ambipolar electric field  $E_0$  in the density gradient direction. It was assumed that  $E_0$  drives an electron current with velocity  $cE_0/B_0$ , but that the ions are not affected by the zeroth order electric or magnetic fields. This assumption restricts their analysis to frequencies well above the ion cyclotron frequency and also restricts the application of their theory to experiments where some feature prevents the electrons and ions from acquiring equal  $\bar{E} \times \bar{B}$  drifts. For example, in theta pinch experiments, the ions do not acquire a drift because of the time scales involved; it is observed that the time scale on which the ambipolar  $E$  field forms and excites the lower hybrid drift instability which then decays is shorter than the ion cyclotron period. This same assumption has been adopted by most of the recent papers. They found that the zero beta plasma is unstable to a flutelike wave, propagating across the field, even when  $T_e < T_i$ . Their condition for this strong instability is that the particle drifts and currents not be too weak:  $v_e v_E \geq c_s^2$ , where  $v_E$  is the  $\bar{E} \times \bar{B}$  drift velocity,  $v_e$  is the electron diamagnetic drift velocity due to the inhomogeneity in the electron density, temperature and magnetic field, and  $c_s$  is the ion sound speed.

Davidson and Gladd (1975) dropped the assumption  $k_y a_e \ll 1$ , kept the low beta limit

and moved  $v_E$  from the large-drift-velocity regime ( $v_E \gg v_{ii}$ ) to the low-drift-velocity regime ( $v_E \ll v_{ii}$ ) where  $v_{ii}$  is the ion thermal velocity. Four additional important features of the lower hybrid drift instability were found. First, the wave has negative energy whenever  $\text{sgn } \omega/k_y = \text{sgn } v_e(k_y)$ . Hence, instability can exist even though  $(\partial F_i/\partial v_y)_{v_y=\omega/k_y} < 0$ , where  $F_i$  is the ion velocity distribution function. Second, the growth rate measured in units of  $\omega_{ih} = \omega_{pi}/(1+\omega_{pe}^2/\omega_{ce}^2)^{1/2}$ , is substantial even when  $v_E/v_{ii} \leq 1$ . Third, for fixed  $v_E/v_{ii}$ , the maximum growth rate is an increasing function of  $T_e$ , since the electron diamagnetic current is correspondingly larger. Finally, the wavelength that maximizes the growth rate generally satisfies  $k_y a_e \approx 1$ . For  $k_y a_e \ll 1$ ,  $\omega_{pe}^2/\omega_{ce}^2 \gg 1$ , the condition for  $\gamma > \omega_{ci}$  requires  $L_n/a_i < (\sqrt{\pi}/2)^{1/2} (m_i/m_e)^{1/4}$  (Davidson and Gladd, 1975; McBride and Hamasaki, 1978), where  $a_i$  is the ion Larmor radius and  $L_n$  is the density scale length.

The influence of finite plasma beta effects on the lower hybrid drift instability, including the effects of transverse electromagnetic perturbations ( $\delta B \neq 0$ ), and resonant and non-resonant  $\nabla B_0$  electron-orbit modifications, was first investigated by Davidson et al. (1976, 1977). The net effect of finite plasma beta is to reduce the maximum growth rate  $\gamma_m$  of the lower hybrid drift instability. However, the details depend on plasma parameters. In the regime where  $T_e \approx T_i$  and  $v_E \approx v_{ii}$ , finite beta electromagnetic effects are destabilizing for all  $k_y$ , whereas the finite beta effects associated with resonant  $\nabla B_0$  orbit modifications, which lead to a significant decrease in the electron diamagnetic drift velocity, are stabilizing for all  $k_y$ . The two finite-beta effects combine to give a net reduction in maximum growth rate  $\gamma_m$ . In the cold electron ( $T_e \ll T_i$ ), low drift velocity regime ( $v_E \ll v_{ii}$ ), the finite beta electromagnetic effects are destabilizing for small  $k_y$ , and stabilizing for large  $k_y$ , whereas the finite beta effects associated with nonresonant  $\nabla B_0$  orbit modifications are stabilizing for small  $k_y$  and destabilizing for large  $k_y$ . Combining these two finite beta effects reduces the maximum growth rate by a factor  $(1+\beta_i/2)^{-1/2}$  relative to the value obtained when  $\beta_i=0$ . Finally, except in the limit of  $T_e/T_i \rightarrow 0$ , a critical value for the local plasma beta ( $\beta_{cr}$ ) was found, such that the lower hybrid drift instability is completely stabilized ( $\gamma < 0$ ) for  $\beta > \beta_{cr}$ .

The influence of magnetic shear on the lower hybrid drift instability has been studied (Krall, 1977; Gladd et al., 1977; Davidson et al., 1978; McBride and Krall, 1978), including application to microstability properties of a number of experiments. It was found that sufficiently strong magnetic shear can completely stabilize the lower hybrid drift instability for  $ka_e \leq 1$  (Krall, 1977; Gladd et al., 1977; Davidson et al., 1978). However, computer simulations showed that shear reduced the growth rate and saturation level of the instability, but that complete stabilization of the mode did not occur (Winske, 1978). A general condition for shear stabilization of the lower hybrid drift modes was given as  $L_s < L_n(a_i/L_n + L_n/a_i)$  for all values of  $v_E$  and  $\omega/k_z v_{te} \gg 1$  (Krall, 1977).  $L_s$  is the magnetic field scale length. Including finite electron gyroradius ( $ka_e \geq 1$ ), McBride and Krall (1978) found that the low-density systems ( $\omega_{pe} < \omega_{ce}$ ) are much easier to stabilize with shear than are higher-density systems ( $\omega_{pe} > \omega_{ce}$ ).

All the theoretical work mentioned above have been based on the local approximation of the gradients. Batchelor and Davidson (1976) performed a nonlocal stability analysis in which the effects of global equilibrium properties, finite radial geometry, and the presence of a conducting wall were included in their calculations. Their analysis was under assumptions that (1) the ion thermal velocity is much less than the wave phase velocity, and (2) the characteristic instability wavelength is larger than a thermal electron Larmor radius. In this regime, the lower hybrid drift instability does not depend on resonant-particle effects and can be described by a macroscopic model. They showed that nonlocal and local theories are in agreement for the fastest growing mode. A fully kinetic, nonlocal, matrix dispersion equation for electrostatic perturbations about a spatially nonuniform cylindrical plasma equilibrium was derived by Davidson (1976). His analysis was carried out for radially confined rigid-rotor equilibria and based on the assumption of equilibrium charge neutrality. The nonlocal structure of the lower hybrid drift instability in a reversed field configuration was investigated by Huba et al. (1980) by using kinetic theory. Their calculation includes electromagnetic effects and  $\nabla B$  electron orbit modifications, and ignores the electrons' ambipolar  $\bar{E}_0 \times B$  drift velocities. They found

that the fundamental mode is well localized away from the neutral line. Higher order modes, however, have growth rates comparable to the fundamental mode and are much more global. Recently, we have worked out a nonlocal analysis of the lower hybrid drift instability in the low drift velocity regime by using kinetic theory and considering only electrostatic perturbations in slab geometry. We found that the lower hybrid drift instability is a negative energy wave driven by resonant ions whenever  $\text{sgn } \omega/k_y = \text{sgn } v_e(k_y)$  as predicted by the local theory. Furthermore, it was discovered that the lower hybrid drift wave with a finite  $k_x$  value will propagate to regions where the electron drift velocity  $v_E$  equals the wave phase velocity ( $\omega/k_y$ ), and be stabilized by these resonant electrons. Our nonlocal theory and local theories are also in agreement for the most unstable mode. Finally, we also found that higher order modes have growth rates comparable to the fundamental mode and are more global.

## II. Saturation Mechanisms and Transport of the Lower Hybrid Drift Instability

The saturation of the lower hybrid drift instability has been studied by a number of authors. Saturation mechanisms include ion quasilinear flattening (Davidson, 1978), current relaxation (Davidson and Gladd, 1975; Davidson, 1978; Davidson and Krall, 1977; Chen and Birdsall, 1980; Myra and Aamodt, 1980), ion trapping (Winske and Liewer, 1978; Chen and Birdsall, 1980), electron resonance broadening (Huba and Papadopoulos, 1978; Gary and Sanderson, 1979; Gary, 1980), electron trapping (Drake and Lee, 1980) or electron  $\bar{E} \times \bar{B}$  trapping (Drake and Huba, 1981; Chen et al., 1981) and nonlinear frequency shift (Chen and Cohen, 1981; Ishihara and Hirose, 1981). The quasilinear evolution of the lower hybrid drift instability was investigated by Davidson (1978) for the electron drift velocities ( $v_d$ ) less than the ion thermal velocity. It was shown that current relaxation ( $v_d \rightarrow 0$ ) and plateau formation ( $\partial F_i / \partial v_y \rightarrow 0$ ) in the ion velocity distribution function are generally competing processes for stabilization. The corresponding saturation levels were given analytically. If the initial drift velocity  $v_d(t=0)$  lies in the range

$$(45\sqrt{\pi} m_e / 8 m_i)^{1/3} v_{ii} \approx v_{ii}/7 < v_d(t=0) < v_{ii} ,$$

then it may be energetically favorable for stabilization to occur through current relaxation, that is, forcing  $v_d \rightarrow 0$ . If

$$v_d(t=0) < v_{ii}/7 \approx (45\sqrt{\pi} m_e / 8 m_i)^{1/3} v_{ii} ,$$

then plateau formation will be completed before  $v_d$  relaxes to zero. The instantaneous heating rates and rate of momentum transfer during the current relaxing were estimated in Davidson and Gladd (1975), Davidson (1978), and Davidson and Krall (1977). It was shown that the lower hybrid drift instability can result in substantial resistivity and plasma heating. Saturation of the lower hybrid drift instability via current relaxation in the low drift velocity regime was observed by us in our one dimensional particle hybrid simulations when  $v_d$  was allowed to vary in time self-consistently during the wave growth (1980), and our two dimensional electrostatic particle simulations (1981). Our two dimensional simulations also showed that plateau formation in density  $n(x)$  occurs locally after saturation in the region where electrons have the larg-



est drift velocities. Myra and Aamodt (1980) showed that reducing of the relative electron- ion drift velocity causes saturation of the lower hybrid drift instability in their guiding-centers-on-axis model. However, Drake et al., (1981) showed that in a finite beta plasma, the particle drifts and magnetic field are coupled and, consequently, the anomalous dissipation of the particle drift energy and magnetic energy also linked. It was found that the magnetic free energy can be substantially larger than the particle drift energy and can effectively act as a free energy source to drive the instability. Therefore, current relaxation will not occur in the high beta plasma.

If cross-field drift velocities are kept constant in time, our one dimensional electrostatic particle hybrid simulations (1980) showed that the lower hybrid drift instability is stabilized by ion trapping. It was found that the saturation level predicted by quasilinear theory (Davidson, 1978) is the saturation level of the most unstable mode for ion trapping. In deriving the saturation level for plateau formation, Davidson began with an energy conservation equation and the only real invocation of quasilinear theory seems to be the specification that saturation occurs when the ion velocity distribution function has been "flattened" around the mode phase velocity. Such flattening could in principle be due to a variety of causes besides the usual quasilinear diffusion, for example, trapping. Furthermore, he assumed that the spectrum is sufficiently peaked about  $k^2 = k_m^2$  (the wavenumber corresponding to maximum growth for the initial equilibrium conditions) that  $1 + k^2/k_m^2 \approx 2$  is a good approximation in the integrand in his calculations. This was equivalent to a single mode assumption, in which only the most unstable mode exists. Finally, the trapping frequency corresponding to the saturation level predicted by quasilinear theory is larger than the growth rate and hence larger than the bandwidth  $\Delta\omega \approx \gamma$  as well. Therefore, ion trapping will be the saturation mechanism when the fastest growing mode is dominant, and the saturation level predicted by Davidson will be the saturation level due to ion trapping. Ion trapping was also observed by Winske and Liewer (1978) in their two dimensional electromagnetic particle simulations with  $v_d$  larger than  $v_{th}$ .

Electron resonance broadening has also been considered as a possible saturation mechan-

ism for high beta plasma (Huba and Papadopoulos, 1978; Gary and Sanderson, 1979; Gary, 1980). Physically, the significance of this nonlinearity is due to the fact that a substantial number of electrons can attain a  $\nabla B$  drift velocity comparable to the phase velocity of the wave in finite beta plasmas. Over a broad range of parameters, the saturation energy is less than that expected if stabilization occurred through either current relaxation or ion trapping. This stabilization mechanism allows a steady state turbulent spectrum to develop which is crucial to several plasma phenomena such as magnetic field line reconnection (Vasyliunas, 1975) and collisionless shock waves (Biskamp, 1973). However, this nonlinear phenomenon has not been observed in any simulation studies. This may be due to the unrealistic mass ratio and small number of particles used in simulations.

In the previous quasilinear investigation of heating by the lower hybrid drift instability, only perpendicularly propagating waves were considered. Ions, which behave as if they are completely unmagnetized (since  $\omega \gg \omega_{ci}$ ), resonantly interact with the wave and exchange momentum and energy. The electrons, however, which are tightly bound to the magnetic field lines ( $\omega \ll \omega_{ce}$ ) are nonresonant and therefore undergo no irreversible energy or momentum exchange (neglecting  $\nabla B$  resonances which lead to electron resonance broadening). The quasilinear electron "heating" previously calculated simply results from the coherent sloshing of the electron velocity distribution function in the lower hybrid drift waves and is completely reversible. Drake (1980, 1981) examined the electron dynamics in a single large amplitude, low frequency ( $\omega \ll \omega_{ce}$ ) wave propagating perpendicular to  $\bar{B}$ . At small wave amplitude, the electron motion in the electric field is simply given by the usual  $\bar{E} \times \bar{B}$  and polarization drifts and is accurately described by the quasilinear theory. It was found that above a threshold, the electron cyclotron motion is strongly modified and the electron motion becomes stochastic (Drake and Lee, 1980). In two dimensional calculations (Drake and Huba, 1981), it was found that when electron  $\bar{E}_x \times \bar{B}$  drift velocity greater the wave phase velocity, i.e.,  $e\phi/T_i > (k_x L_n)^{-1}$ , this electron  $\bar{E} \times \bar{B}$  trapping can cause the end of wave growth. Then irreversible electron transport can take place. An unstable lower hybrid drift mode propagating along the diamagnetic

drift velocity direction  $y$  was found to decay into off angle modes with  $k_x \approx k_y$ , which saturates as they  $\bar{E} \times \bar{B}$  trap the electrons. The finite  $k_x$  decay modes therefore act as an intermediary which both transports electrons irreversibly and saturates the original instability. The off angle modes with  $k_x \approx k_y$ , have been observed both in experiments (Fahrbach et al., 1979;) and in our two dimensional particle simulations (1981).

Finally, saturation of the instability can be due to the frequency modulation when the amplitude of the wave is not very small. We (1981) derived the nonlinear dielectric response of the lower hybrid drift instability in the low drift velocity regime by solving a coupled Vlasov-Poisson equation of a single unstable mode self-consistently. The nonlinear temporal evolution of the mode was calculated analytically. It was found that a finite perturbation of the ion orbits leads to a nonlinear frequency shift that reduces the mode frequency and has a weak stabilizing effect on the lower hybrid drift instability. The nonlinear frequency shift does not seem to be a potent saturation mechanism in a collisionless plasma, but may be more relevant when there are ion-ion collisions. Ishihara and Hirose (1981) derived the nonlinear dispersion for the hydrodynamic lower hybrid drift instability ( $v_d \gg v_{th}$ ), which is characterized by the time dependent complex frequency and the renormalized ion susceptibility. They found that when the instability is saturated by the frequency modulation, the electron drift velocity slows down by 30% of its initial value.

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