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DIRECT METHODS FOR TRANSIENT STABILITY ANALYSIS
OF POWER SYSTEMS: RECENT RESULTS

by

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DIRECT METHODS FOR TRANSIENT STABILITY ANALYSIS OF POWER SYSTEMS: RECENT RESULTS ¹

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ABSTRACT

Transient stability analysis of a power system is concerned with the system's ability to remain in synchronism following a disturbance. In utility planning, transient stability is studied by numerical simulation. The long CPU run times for simulation preclude their use for on line security analysis. Interest has therefore shifted toward the Lyapunov direct method of stability analysis. This paper provides a critical review of research on direct methods since 1970.

Considerable progress has been made on both theoretical properties of energy functions and on criteria suitable for on-line implementation. Current theory provides a satisfactory treatment of voltage dependent reactive power demand, transfer conductances, and flux decay. However, it can not incorporate the exciter control loop which can create oscillations. Proposed on-line criteria appear to work very well on sample examples; but, they still lack rigorous justification.

Finally, recent work has shown that power systems can exhibit chaotic behavior. This surprising fact demonstrates that our understanding of the dynamics of power systems remains incomplete.

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1. The Transient Stability Problem

1.1. Introduction

Large disturbances occur frequently in power systems. Examples are sudden change in bus, load, or generation powers, or in the transmission system configuration due to faults and line switching. In the planning and operation of a power system, one is concerned with the ability of the system to serve load demand in the presence of disturbances. In the planning context this is called *reliability*, and in the operation context this is called *security* [58]. As traditional stringent reliability criteria for planning analysis become more and more difficult to meet due to economic and other considerations, the second line of defense, on-line security analysis, becomes ever more important to guarantee that quality of service to consumers is not unduly affected.

Immediately after a disturbance, one is concerned with the system's ability to remain in synchronism, that is to remain stable. This is called *transient stability*. A mathematical formulation of the problem of transient stability analysis is presented in Section 1.2. If the system can withstand the initial impact of the disturbance, one then is concerned with the power balance between generation and load demand in the network. This is the problem of steady-state *load flow* analysis. Both transient stability analyses and steady-state load flow studies are performed extensively in today's utility planning reliability analysis. However, only steady-state load flow is currently conducted in on-line security analysis [50].

Transient stability analysis is routinely performed in utility system planning. The industry standard for transient stability performance of a system is the ability to withstand any "possible but improbable" three-phase fault close to a generator with stuck breaker [21]. The method used for analysis is time-domain numerical simulation [49]. Computer programs that can handle two thousand buses and three hundred generators with detailed models are available. Typically ten to forty runs may be required for a plan. The CPU time for a typical run is around 15 minutes. Since the time interval of interest for transient stability analysis ranges from a few milliseconds to a few seconds, simulation is infeasible for on-line security analysis. An alternative approach is the (Lyapunov) direct method.

In Section 2 the direct method is introduced in the context of the classical model of a power system. In that section we point out the shortcomings of this model that arise from the simplifications made in regard to modeling transmission lines, loads, and generator dynamics.

Section 3 is devoted to three recent approaches designed to overcome limitations of the classical model that are due to its neglect of transfer conductances and flux decay. The Lyapunov functions proposed in these approaches also offer the advantage that they preserve network structure, thereby permitting a more precise accounting of the shifts in energy among system components during a transient.

Section 4 is devoted to approximations proposed to make the direct methods implementable on-line. These approximations are inspired by the theoretical advances discussed in Section 3. However, they are justified exclusively on the basis of relatively few and simple simulations.

Section 5 is concerned with two recent studies that illuminate certain complexities of the transient stability problem without being directly motivated by it. One study shows that the introduction of exciter control can create an oscillation that reduces the stability region. The second study shows that the classical model can exhibit chaotic behavior.

Finally, Section 6 summarizes current research bottlenecks and suggestions for future research directions.

This paper places more emphasis on theoretical advances since 1970. Practical considerations and experience are discussed and reported in [35].

1.2. A mathematical formulation

As stated before, transient stability analysis is concerned with the system's ability to remain in synchronism following a major disturbance such as short-circuit caused by lightning. Protective relays are placed strategically throughout the system to detect faults and to trigger the opening of circuit breakers to isolate the fault. Therefore, the power system can be considered as going through changes in configuration in three stages, from pre-fault, faulted, to post-fault systems. The pre-fault system is in a stable steady state. The fault occurs (e.g., a short circuit) and the system is then in the faulted condition before it is cleared by the protective system operation. The transient stability problem is the study of the stability of the post-fault system.

Mathematically the problem can be stated as follows.¹ We consider a dynamical system that can be described by a set of three differential equations:

$$\dot{x}(t) = f_I(x(t)), \quad -\infty < t < t_F \quad (1.1I)$$

$$\dot{x}(t) = f_F(x(t)), \quad t_F \leq t < t_P \quad (1.1F)$$

$$\dot{x}(t) = f(x(t)), \quad t_P \leq t < \infty. \quad (1.1)$$

The underlying idea is as follows. $x(t)$ is the vector of state variables of the system at time t . At some time t_F the system undergoes a fault, that is, there is a structural change. This is represented by the change in the dynamics governing the system behavior from f_I to f_F . Prior to the fault we have the *pre-fault* or initial dynamics, f_I .

The fault duration is confined to the time interval $[t_F, t_P)$. During this interval the system is governed by the *faulted* dynamics, f_F . At time t_P the fault is cleared and the system is henceforth governed by the *post-fault* dynamics f .²

In the pre-fault regime the system is at a known stable equilibrium, say x_I . This, in effect, eliminates the need to discuss (1.1I). We have instead,

$$\dot{x}(t) = f_F(x(t)), \quad t_F < t < t_P, \quad x(t_F) = x_I \quad (1.1F')$$

$$\dot{x}(t) = f(x(t)), \quad t_P < t < \infty. \quad (1.1)$$

At the end of the fault period the state of the system is

$$x_P := \Phi_F(x_I, t_P - t_F) \quad (1.2)$$

where $\Phi_F(x, t)$ is the state of the faulted system (1.1F) at time t if it is in state x at time 0.

Next, assume that the post-fault system (1.1) has a stable equilibrium x^s . That is $f(x^s) = 0$, and x^s is stable.³

The fundamental problem of transient stability can now be posed. Starting in the post fault initial state, x_P , will the post fault system converge to the equilibrium x^s ? More precisely, will

$$\lim_{t \rightarrow \infty} \Phi(x_P, t) = x^s ? \quad (1.3)$$

¹ This discussion is motivated by Pai [33].

² It may be the case that the clearing of the fault leads to a restoration of the pre-fault system. In that case $f = f_I$. However this is not necessary.

³ By stable we mean that all eigenvalues of the Jacobian matrix $\partial f / \partial x(x^s)$ have strictly negative real parts. This guarantees that x^s is asymptotically stable [32].

Here $\Phi(x, t)$ is the state of the post fault system (1.1) at time t if its state at time 0 is x .

There is a region of initial conditions in the state space from which trajectories converge to x^s . This is the attractor or the *region of stability* of x^s , denoted $A(x^s)$. By definition,

$$A(x^s) := \{x \mid \lim_{t \rightarrow \infty} \Phi(x, t) = x^s\}. \quad (1.4)$$

The question (1.3) can now be reformulated as: Does $x_P \in A(x^s)$?

This question immediately leads to two fundamental tasks at the level of theory and computation. From the general theory of differential equations we know that the stability region $A(x^s)$ is an open subset of the state space. Thus, from a topological viewpoint, the task is one of estimating the *boundary* of the stability region, $\partial A(x^s)$. More precisely, the theoretical task is

To characterize $\partial A(x^s)$ in terms of its geometric properties.

Suppose one has an adequate mathematical characterization of $A(x^s)$ and $\partial A(x^s)$. The stability problem is to determine whether or not the post fault initial state $x_P \in A(x^s)$. To be able to do this on-line requires an effective execution of two computational tasks. The first is

*To compute the post-fault initial state, x_P .*⁴

Recall that $x_P = \Phi_F(x_I, t_P - t_F)$. Now, in practice, the switching time or duration of the fault, namely, $\tau := t_P - t_F$, is used as a variable in stability analysis. Therefore, one must be able to estimate the faulted trajectory,

$$x_P(\tau) := \Phi_F(x_I, \tau), \quad (1.5)$$

by integration of the faulted dynamics (1.1F). This important task will not be discussed further below. The function Φ_F is analyzed in [25].

The second computational task is more central to the problem discussed here. Suppose that $x_I = x_P(0) \in A(x^s)$, that is, suppose the pre-fault equilibrium is in the stability region.⁵ See Figure 1. As the fault proceeds, the corresponding faulted trajectory $x_P(\tau)$ may eventually leave the stability region. Suppose this happens at time τ_C , so that

⁴ While it is reasonable to suppose that the pre-fault initial state, x_I , is known, we can not suppose the same for x_P . This is because the faulted system dynamics are not known in advance since we don't know which of the many possible faults will occur.

⁵ This is certainly the case if the pre-fault and post-fault systems are the same. In that case $x_I = x^s$.

$$x_P(\tau) \in A(x^s), \tau < \tau_C, \text{ and } x_P(\tau_C) \in \partial A(x^s). \quad (1.6)$$

[FIGURE 1 NEAR HERE]

The time τ_C is called the *critical clearing time*. If the fault duration $t_P - t_F < \tau_C$, then transient stability is maintained; if it exceeds the critical clearing time, the system will undergo transient instability.

Thus the second task is

To formulate on-line criteria to determine stability, i.e., to evaluate (1.6).

The fault duration $\tau := t_P - t_F$ is determined by the relay setting. In system operation, τ is a fixed parameter. But in system planning, stability analysis is used to help select relay settings, hence τ is treated as a variable. Since the result of stability analysis is binary -- stable or not -- which does not give much information, it is conventional in power systems practice to use the critical clearing time as a measure of stability. Note, however, that this is a very crude measure of the stability region.

2. Transient Stability For The Classical Model

In this section we study the boundary of the stability region and stability criteria for the classical model. Shortcomings due to modeling simplifications are also discussed.

2.1. The classical model

For an n generator system, in the classical model, the dynamics of the i th generator is governed by the equations:

$$\dot{\delta}_i = \omega_i, \quad (2.1a)$$

$$M_i \dot{\omega}_i = P_i^m - D_i \omega_i - P_i^e, \quad (2.1b)$$

$$P_i^e = E_i^2 G_i + \sum_{j \neq i}^{n+1} E_i E_j Y_{ij} \cos(\delta_i - \delta_j - \varphi_{ij}). \quad (2.1c)$$

Equations (2.1a), (2.1b) represent rotor mechanics: δ_i , ω_i are the angle and speed of the i th rotor; M_i and D_i are its inertia and damping constants; P_i^m is the constant mechanical input power and P_i^e is the electric output power. P_i^e is determined by the electrical network. The i th generator is modeled as a voltage

source⁶ of constant magnitude E_i and phase δ_i , driving (through a transient reactance) an electric network consisting of transmission lines and loads, modeled as constant impedances. Viewed as a n -port driven by the voltage sources $E_i \angle \delta_i$, this network is described by the reduced complex admittance matrix Y with coefficients $Y_{ij} \angle \varphi_{ij}$. See Figure 2. The real, positive, diagonal terms, G_{ii} , of Y are separated out as indicated. Equation (2.1c) gives the real power delivered to the electric network by the i th generator.

[FIGURE 2 NEAR HERE]

In the version above, node $n+1$ is an infinite bus, with $E_{n+1} = 1$ and $\delta_{n+1} = 0$. Thus the voltage at node $n+1$ serves as a synchronous reference.⁷

Equations (2.1) represent the post fault system. It is the counterpart of (1.1) above. There is a similar equation for the faulted system.

Suppose (2.1) has a stable equilibrium, $(\delta^s, \omega^s) = (\delta_1^s \dots \delta_n^s, 0, \dots, 0)$. (This is the counterpart of x^s in Section 1.2.) The first task is to estimate its stability region. To use the Lyapunov direct method for this, it is necessary to assume that the transfer conductances are zero, i.e.,

$$\varphi_{ij} = \frac{1}{2} \pi, \text{ all } i \neq j. \quad (2.2)$$

Using (2.2) in (2.1c), substituting into (2.1b), and denoting $P_i := P_i^m - E_i^2 G_{ii}$, yields a version of the classical model,

$$\dot{\delta}_i = \omega_i, \quad (2.3a)$$

$$M_i \dot{\omega}_i = P_i - D_i \omega_i - \sum_{j \neq i}^{n+1} E_i E_j Y_{ij} \sin(\delta_i - \delta_j). \quad (2.3b)$$

2.2. The boundary of the stability region

Theorem 1 characterizes the boundary of the stability region in terms of stable manifolds of the equilibrium points that lie on the boundary.

⁶ This is the voltage at a fictitious 'internal' generator bus or node. The generator *terminal* voltage $V_i \angle \vartheta_i$ is after the transient reactance. See Figure 2.

⁷ In the absence of an infinite bus one may use the so-called center of angle as reference. This creates no essential difference [34].

For convenience, let $x := (\delta, \omega)$ denote the state of the power system.

Theorem 1

Let x^s denote a post fault stable equilibrium, and let $A(x^s)$ and $\partial A(x^s)$ denote the corresponding region of stability and its boundary. Let $x_i, i = 1, 2, \dots$ be the equilibria lying on $\partial A(x^s)$. Let W_i^s (W_i^u) denote the stable (unstable) manifold⁸ of x_i . Then the x_i are unstable equilibria, and

$$\partial A(x^s) \subseteq \bigcup_i W_i^s. \tag{2.4}$$

Moreover, if $W_i^s \cap W_j^u = \phi$, for $x_i \neq x_j$, then

$$\partial A(x^s) = \bigcup_i W_i^s. \tag{2.5}$$

This is a "folk theorem". For example, Pai [34] seems to regard it as self-evident. Jovic [20] cites Shahshahani [47] as a reference. Tsolas, Arapostathis and Varaiya [52] provide a complete proof. Their proof utilizes the energy function developed below. The condition that $W_i^s \cap W_j^u = \phi$ holds 'generically'; hence (2.5) is true for almost all parameter values. See [45] for a counterexample for the simplest case, $n = 1$.

2.3. The direct method

Direct methods for transient stability analysis of power systems are based on Lyapunov theory. The early work on Lyapunov theory is presented in [19]. Subsequent research has concentrated on extending the quadratic Lyapunov functions designed for linear systems to systems involving certain class of memoryless nonlinearities. The work of Lure, Popov and others is presented in [54]. There are attempts to extend these methods to interconnected systems either through vector Lyapunov functions or through aggregation of Lyapunov functions for individual subsystems [29, 48]. These approaches have, however, proved to be conservative.

Although he did not employ the term, the first use of Lyapunov functions in power systems transient stability analysis can be traced to Aylett [6]. The first explicit use of Lyapunov functions in power systems, however, is due to Gless [18], El-Abiad and Nagappan [14]. A mathematically rigorous treatment was provided by Willems [56,57] in 1970. Since that date, scores of technical papers

⁸ W_i^s (W_i^u) is the set of all initial states from which trajectories converge to x_i as $t \rightarrow \infty$ ($t \rightarrow -\infty$).

have appeared in the literature. That work is systematically presented in Pai [34], and in survey papers by Fouad [16] and Ribbens-Pavella [37], so our summary is very brief.

Let (δ^s, ω^s) be a stable post fault equilibrium. Define the *energy function*⁹ $V(\delta, \omega)$ of the state

$$V(\delta, \omega) := \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 - \sum_{i=1}^n P_i (\delta_i - \delta_i^s) - \sum_{i < j}^{n+1} E_i E_j Y_{ij} [\cos(\delta_i - \delta_j) - \cos(\delta_i^s - \delta_j^s)]. \quad (2.6)$$

At the post fault equilibrium $V(\delta^s, \omega^s) = 0$. Differentiating $V(\delta, \omega)$ along the trajectories of (2.3) gives,

$$\frac{d}{dt} V(\delta, \omega) = \frac{\partial V}{\partial \delta} \dot{\delta} + \frac{\partial V}{\partial \omega} \dot{\omega} = - \sum_{i=1}^n D_i \omega_i^2 \leq 0. \quad (2.7)$$

The inequality (2.7) shows that V qualifies for use as a Lyapunov function since $V(\delta(t), \omega(t))$ must decline along the post fault trajectory. This observation is the basis of the following fundamental result.

Theorem 2

Let $v > 0$ be a constant such that on the boundary of the stability region one has $V(\delta, \omega) < v$. Then the connected component containing (δ^s, ω^s) of the set

$$R(v) := \{(\delta, \omega) \mid V(\delta, \omega) < v\} \quad (2.8)$$

is inside the stability region.

Two observations need to be made. First, Theorem 2 is not an immediate consequence of general Lyapunov theory since $V(\delta, \omega)$ does *not* become unbounded as $|\delta| \rightarrow \infty$. For a complete proof see [52].

Second, Theorem 2 provides an estimate of the stability region. To get the best estimate (the largest $R(v)$) one wants to choose v in (2.8) as large as possible. The largest value, v_{cr} , is

$$v_{cr} = \min \{V(\delta, \omega) \mid (\delta, \omega) \in \partial A(\delta^s, \omega^s)\}.$$

From (2.4) we deduce that

$$v_{cr} = \min_{usp} \{V(\delta^s, \omega^s)\} \quad (2.9)$$

⁹ The first term in (2.6) is in the form of kinetic energy, the remaining terms can be identified as potential energy. Hence V is called the energy function.

where the minimization is over all the unstable equilibrium points (uep) $(\delta^e, \omega^e) \in \partial A(\delta^s, \omega^s)$. The uep that achieves the minimum in (2.9) is called the *critical uep*. See Figure 3.

[FIGURE 3 NEAR HERE]

2.4. Stability criteria

Theorem 2 suggests a stability criterion according to the following three step procedure.

Step 1

Calculate the uep lying on the boundary $\partial A(\delta^s, \omega^s)$. Calculate v_{cr} by (2.9).

Step 2

Evaluate V along the faulted trajectory $(\delta(\tau), \omega(\tau))$. If $V(\delta(\tau), \omega(\tau)) < v_{cr}$, then $\tau < \tau_c$, and the system is stable.

Step 3

The value τ_{cr} of τ for which $V(\delta(\tau), \omega(\tau)) = v_{cr}$ provides an estimate of the true critical clearing time τ_c .

The main computational difficulty in implementing this procedure is the calculation of the uep in Step 1. Observe that (δ^e, ω^e) is an equilibrium of (2.3) if and only if $\omega^e = 0$ and

$$P_i = \sum_{j \neq i}^{n+1} E_i E_j Y_{ij} \sin(\delta_i^e - \delta_j^e), \quad i = 1, \dots, n.$$

These are essentially the real power flow equations and determining all the solutions is difficult, see Section 4.

In the second place, it is obvious from Figure 3 that the $R(v_{cr})$ is an *underestimate* of the stability region. Put differently, the boundary of the stability region, $\partial A(\delta^s, \omega^s)$, is poorly approximated by the surface of constant energy $\{V(\delta, \omega) = v_{cr}\}$. For the same reason τ_{cr} is an *underestimate* of the true critical clearing time τ_c .

A much improved estimate of τ_c can be obtained. From Theorem 1 we know that the boundary of the stability region is composed of the stable manifolds of the uep that lie on the boundary. For instance, in Figure 3, $\partial A(\delta^s, \omega^s) = W^s(\text{uep1}) \cup W^s(\text{uep2})$. Also uep3 is the critical uep, and so $v_{cr} < V(\text{uep}i)$ for $i = 1, 2$. Now consider the faulted trajectory $(\delta(\tau), \omega(\tau))$ as shown. It leaves the

stability region by exiting the boundary through $W^s(\text{uep2})$. Hence an estimate of the critical time is given by τ_{co} where

$$V(\delta(\tau_{co}), \omega(\tau_{co})) = V(\text{uep2}) =: v_{co}. \quad (2.10)$$

Since $v_{co} > v_{cr}$, this estimate will be superior to τ_{cr} obtained previously. The equilibrium uep2 is called the *controlling uep* corresponding to the faulted trajectory. Note that the controlling uep depends on the faulted trajectory, but the critical uep does not.

The significance of the controlling uep seems to have been observed first by Kakimoto, Ohsawa and Hayashi [23] in 1978. It forms the basis of most on-line stability criteria proposed since then. In general these criteria conform to the following two step procedure.

Step 1

From the faulted trajectory $(\delta(\tau), \omega(\tau))$ estimate the controlling uep $(\delta^{co}, \omega^{co})$. Calculate $v_{co} = V(\delta^{co}, \omega^{co})$.

Step 2

The value τ_{co} given by (2.10) gives an estimate of the critical clearing time τ_c .

Criteria proposed by various researchers differ in the way they estimate the critical and controlling uep [4,5,38]. See Section 4.

2.5. Limitations of the classical model

The classical model (2.3) makes unwarranted simplifications in its formulation of the transmission lines and loads on the one hand, and of the generators on the other [35].

The load is viewed as a constant impedance, and the network of transmission lines and loads is then reduced to an n -port as seen from the generator internal buses. This has two shortcomings:

- (1) *It precludes consideration of reactive power demand and voltage variation at load buses;*
- (2) *Reduction of the network leads to loss of network topology and hence precludes study of transient energy shifts among different components of the network.*

In order to obtain the crucial inequality (2.7) it is necessary to ignore transfer conductances in the reduced admittance matrix, i.e., one assumes that $\varphi_{ij} = \frac{1}{2} \pi$ (see (2.2)). The transfer conductance, $Y_{ij} \cos \varphi_{ij}$, arises from the load

impedances and from conductances in the transmission lines. The latter can often be neglected.¹⁰ However, it is impermissible to neglect the transfer conductance arising from load impedances. The difficulty is compounded by the fact that no one has estimated the magnitude and direction of the bias due to neglect of transfer conductances.¹¹ Thus the classical model has an additional shortcoming:

- (3) *Neglect of transfer conductances leads to a bias in stability estimates of unknown magnitude and direction.*

With respect to generator models the principal deficiency is:

- (4) *Neglect of flux decay and exciter control may be unacceptable simplifications.*

3. Structure Preserving Energy Functions

Three models have recently been proposed to overcome some of the shortcomings of the classical model (2.3) and its associated energy function (2.6). The principle breakthrough has been in their treatment of loads. The load can be modeled more realistically and is no longer seen as a constant impedance, the network is not reduced, and so the identity of the load nodes is not destroyed. Because the network structure is maintained the associated Lyapunov functions are called 'structure preserving' or 'topological' energy functions. The modeling of generator dynamics is improved by inclusion of flux decay. However, the exciter control loop is still absent from these models.

3.1. The Bergen-Hill model

In this model [9], at each load node the voltage is assumed constant and there is a demand for real power equal to a constant plus an amount proportional to the instantaneous frequency deviation at the node.

As in the classical model, nodes $1, \dots, n$ are generator internal buses, node $n+1$ is the infinite bus, viewed as voltage sources, $E_i \angle \delta_i$. There are m load

¹⁰ The total loss in a power system, including machines, transmission lines, transformers, and distribution system, is about 7 per cent. Transfer conductances are often permitted by resorting to faulted trajectory-dependent energy function, see, e.g. [4, 34]. However, this procedure does *not* yield Lyapunov functions and its use for stability analysis is suspect.

¹¹ It is believed that the presence of transfer conductances should increase stability, i.e., that neglecting them leads to conservative estimates. This is not always true. It is easy to give examples in which transfer conductances *reduce* the stability region. See, for example, [1.30].

nodes labeled $n+2, \dots, n+m+1$. At the k th load node there is a demand for real power, $P_k^d + D_k \omega_k$, where P_k^d and $D_k > 0$ are constants and ω_k is the instantaneous frequency deviation. This power is delivered at the voltage $V_k \angle \varphi_k$, with V_k fixed. Thus $\omega_k = \dot{\varphi}_k$.

The m load nodes include the generator terminal buses in addition to the 'regular' load nodes. Each generator's terminal bus is connected to its internal bus by its transient reactance which is treated as a lossless transmission line. All the transmission lines are assumed lossless.¹² Let $Y = jB$ be the node admittance matrix seen from the $n+m+1$ nodes. For $i \neq j$, $B_{ij} \geq 0$ denote the susceptance of the line connecting nodes i and j .

Below indices i, j stand for generator nodes and the infinite bus, and k, l stand for load nodes. In place of the classical model (2.3) one gets,

$$\dot{\delta}_i = \omega_i, \quad i = 1, \dots, n, \quad (3.1a)$$

$$M_i \dot{\omega}_i = P_i - D_i \omega_i - \sum_{\substack{j=1 \\ j \neq i}}^{n+1} E_i E_j B_{ij} \sin(\delta_i - \delta_j) - \sum_{l=n+2}^{n+m+1} E_i V_l B_{il} \sin(\delta_i - \varphi_l), \quad i = 1, \dots, n, \quad (3.1b)$$

$$0 = P_k^d - D_k \dot{\varphi}_k - \sum_{j=1}^{n+1} V_k E_j B_{kj} \sin(\varphi_k - \delta_j) - \sum_{\substack{l=n+2 \\ l \neq k}}^{n+m+1} V_k V_l B_{kl} \sin(\varphi_k - \varphi_l), \quad k = n+2, \dots, n+m+1. \quad (3.1c)$$

Equation (3.1) gives the dynamics for the $(2n+m)$ -dimensional state $(\delta_1 \dots \delta_n, \omega_1 \dots \omega_n, \varphi_{n+2} \dots \varphi_{n+m+1})$. Equation (3.1c) gives the real power balance at the load nodes.¹³

Define the function

$$\begin{aligned} W_1(\delta, \omega, \varphi) := & \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 - \sum_{i=1}^n P_i \delta_i - \sum_{k=n+2}^{n+m+1} P_k^d \varphi_k \\ & - \sum_{i < j}^{n+1} E_i E_j B_{ij} \cos(\delta_i - \delta_j) \\ & - \sum_{i=1}^{n+1} \sum_{k=n+2}^{n+m+1} E_i V_k B_{ik} \cos(\delta_i - \varphi_k) \end{aligned}$$

¹² This assumption is much more reasonable than (2.2).

¹³ Real power injected into the network carries, by convention, a positive sign. Therefore, P_i in (3.1b) is usually positive, and P_k^d in (3.1c) is usually negative.

$$- \sum_{k < l}^{n+m+1} V_k V_l B_{kl} \cos(\varphi_k - \varphi_l). \quad (3.2)$$

Let $(\delta^s, \omega^s, \varphi^s)$ be a stable post fault equilibrium. Define the energy function

$$V(\delta, \omega, \varphi) := W_1(\delta, \omega, \varphi) - W_1(\delta^s, \omega^s, \varphi^s). \quad (3.3)$$

Then $V(\delta^s, \omega^s, \varphi^s) = 0$. Differentiating $V(\delta, \omega, \varphi)$ along the trajectories of (3.1) gives, after some manipulation,

$$\frac{d}{dt}V = \frac{\partial V}{\partial \delta} \dot{\delta} + \frac{\partial V}{\partial \omega} \dot{\omega} + \frac{\partial V}{\partial \varphi} \dot{\varphi} = - \sum_{i=1}^n D_i \omega_i^2 - \sum_{k=n+2}^{n+m+1} D_k \dot{\varphi}^2 \leq 0.$$

This is the counterpart of the crucial inequality (2.7). With it one can prove the analogs of Theorems 1 and 2 as in [52], and propose stability criteria based either on the critical uep or the controlling uep.

The structure preserving energy function (3.3) overcomes the defect due to neglect of transfer conductances. However, the assumption of constant voltage magnitudes and the neglect of reactive power demand at the load nodes are still unrealistic. These are removed in the next two models.

3.2. The Narasimhamurthi-Musavi model

In this model [31], at the k th load node there is a demand for real power equal to $P_k^d + D_k \dot{\varphi}_k$ as before. In addition, there is a demand for reactive power $Q_k^d(V_k)$ given as a function of the voltage magnitude at that load. The voltage magnitude V_k is not fixed. The machine model is as before. Using the same notation as above leads to a model consisting of equations (3.1a-c), and (3.1d):

$$\begin{aligned} Q_k^d(V_k) = & - \sum_{i=1}^{n+1} V_k E_i B_{ki} \cos(\varphi_k - \delta_i) \\ & - \sum_{l=n+2}^{n+m+1} V_k V_l B_{kl} \cos(\varphi_k - \varphi_l), \quad k = n+2, \dots, n+m+1. \end{aligned} \quad (3.1d)$$

This equation says that the reactive power flows into node k must sum to zero, a consequence of Kirchoff's laws.

The introduction of the algebraic 'constraint' (3.1d) raises the question about defining the solutions of this model. It is assumed that the m equations (3.1d) can be solved continuously for the m voltage magnitudes $V_{n+2}, \dots, V_{n+m+1}$ in terms of δ and φ . One substitutes these values into (3.1a-c) to obtain a differential equation involving only the state vector $(\delta, \omega, \varphi)$. It is also possible to define a solution via singular perturbation, see [12].

Define the function

$$W_2(\delta, \omega, \varphi, V) := W_1(\delta, \omega, \varphi) - \frac{1}{2} \sum_{k=n+2}^{n+m+1} V_k^2 B_{kk} - \sum_{k=n+2}^{n+m+1} \int_1^{V_k} \frac{Q_k^d(v)}{v} dv,$$

where W_1 is as in (3.2). Let $(\delta^s, \omega^s, \varphi^s, V^s)$ be a stable equilibrium of (3.1a-d). Define the energy function

$$V(\delta, \omega, \varphi) := W_2(\delta, \omega, \varphi, V) - W_2(\delta^s, \omega^s, \varphi^s, V^s). \quad (3.4)$$

Here $V = (V_{n+2}, \dots, V_{n+m+1})$ is a function of (δ, φ) given implicitly by (3.1d). Differentiating (3.4) along the trajectories of (3.1a-d) gives,

$$\frac{d}{dt} V = \frac{\partial W_2}{\partial \delta} \dot{\delta} + \frac{\partial W_2}{\partial \omega} \dot{\omega} + \frac{\partial W_2}{\partial V} \dot{V} = - \sum_{i=1}^n D_i \omega_i^2 - \sum_{k=n+2}^{n+m+1} D_k \dot{\varphi}_k^2 \leq 0. \quad (3.5)$$

Once again one can obtain the analogs of Theorems 1 and 2 following the argument in [52].

3.3 The Tsolas-Arapostathis-Varaiya model

So far, the generator internal voltage magnitudes, the E_i , were assumed fixed. This means that the flux linkages (which are proportional to E_i) are constant during the transient following a disturbance. The model in [52] permits flux decay.¹⁴

To describe that model it is necessary to identify explicitly the generator terminal nodes. (Above, those nodes were included in the m load nodes.) In the rest of this section $i, j = 1, \dots, n$ will be used for the generator internal and terminal nodes, and $k, l = n+2, \dots, n+m+1$ will denote regular load nodes.

The i th generator's terminal voltage is denoted $V_i \angle \vartheta_i$, and its internal voltage is denoted $E'_{iq} \angle \delta_i$. The latter is the quadrature axis voltage. (It replaces $E_i \angle \delta_i$ above.) The infinite bus voltage is $V_{n+1} \angle \vartheta_{n+1}$, $V_{n+1} = 1$, $\vartheta_{n+1} = 0$. The model for generator i is

$$\dot{\delta}_i = \omega_i, \quad (3.6a)$$

$$M_i \dot{\omega}_i = P_i - D_i \omega_i - P_i^s, \quad (3.6b)$$

$$T'_{doi} \dot{E}'_{qi} = - \frac{x_{di}}{x'_{di}} E'_{iq} + \frac{(x_{di} - x'_{di})}{x'_{di}} V_i \cos(\delta_i - \vartheta_i) + E_{F_i}, \quad (3.6c)$$

$$P_i^s = \frac{E'_{iq} V_i \sin(\delta_i - \vartheta_i)}{x'_{di}} + \frac{V_i^2 \sin[2(\delta_i - \vartheta_i)] (x'_{di} - x_{qi})}{2x_{qi} x'_{di}}. \quad (3.6d)$$

¹⁴ For an earlier attempt to incorporate flux decay see [24].

Equations (3.6a), (3.6b) give the rotor dynamics. Equation (3.6c) is the flux decay equation for a one-axis model. The electrical power P_i^e injected into the network is given by (3.6d). All the terms in these equations that have not been defined previously such as T'_{doi} , x_{di} , etc. are generator constants. For details see [52].

The model is completed by the power flow equations at the generator terminal and load nodes. The real power flow equations are

$$P_i^e = \sum_{j \neq i}^{n+1} V_i V_j B_{ij} \sin(\vartheta_i - \vartheta_j) + \sum_{l=n+2}^{n+m+1} V_i V_l B_{il} \sin(\vartheta_i - \varphi_l), \quad i = 1, \dots, n. \quad (3.7a)$$

$$0 = P_k^d - D_k \dot{\varphi}_k - \sum_{j=1}^{n+1} V_k V_j B_{kj} \sin(\varphi_k - \vartheta_j) - \sum_{l \neq k}^{n+m+1} V_k V_l B_{kl} \sin(\varphi_k - \varphi_l), \quad k = n+2, \dots, n+m+1. \quad (3.7b)$$

The reactive power flow equations are

$$0 = \frac{x'_{di} + x_{qi}}{2x_{qi}x'_{di}} V_i^2 - \frac{E'_{iq} V_i \cos(\vartheta_i - \delta_i)}{x'_{di}} - \frac{V_i^2 \cos[2(\vartheta_i - \delta_i)](x'_{di} - x_{qi})}{2x_{qi}x'_{di}} - \sum_{j=1}^{n+1} V_i V_j B_{ij} \cos(\vartheta_i - \vartheta_j) - \sum_{l=n+2}^{n+m+1} V_i V_l B_{il} \cos(\vartheta_i - \varphi_l), \quad i = 1, \dots, n, \quad (3.8a)$$

$$Q_k^d(V_k) = - \sum_{j=1}^{n+1} V_k V_j \cos(\varphi_k - \vartheta_j) - \sum_{l=n+2}^{n+m+1} V_k V_l \cos(\varphi_k - \varphi_l), \quad k = n+2, \dots, n+m+1. \quad (3.8b)$$

It is assumed that the algebraic equations can be solved for V and ϑ in terms of δ , φ , and E'_q . Substitution into (3.6a-c), (3.7b) gives a differential equation in the state vector $(\delta, \omega, \varphi, E'_q)$.

Define the function

$$\begin{aligned} W_3(\delta, \omega, \varphi, E'_q, V, \vartheta) := & \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 - \sum_{i=1}^n P_i \delta_i - \sum_{k=n+2}^{n+m+1} P_k^d \varphi_k \\ & - \sum_{i < j}^{n+1} V_i V_j B_{ij} \cos(\vartheta_i - \vartheta_j) - \sum_{i=1}^{n+1} \sum_{k=n+2}^{n+m+1} V_i V_k B_{ik} \cos(\vartheta_i - \varphi_k) \\ & - \sum_{k < l}^{n+m+1} V_k V_l B_{kl} \cos(\varphi_k - \varphi_l) - \frac{1}{2} \sum_{k=n+2}^{n+m+1} V_k^2 B_{kk} - \sum_{k=n+2}^{n+m+1} \int_1^{V_k} \frac{Q_k^d(v)}{v} dv \\ & - \frac{1}{2} \sum_{i=1}^{n+1} V_i^2 \left[B_{ii} + \frac{(x'_{di} - x_{qi}) \{ \cos 2(\delta_i - \vartheta_i) - 1 \}}{2x_{qi}x'_{di}} \right] - \sum_{i=1}^{n+1} \frac{E'_{iq} V_i \cos(\delta_i - \vartheta_i)}{x'_{di}} \end{aligned}$$

$$+ \frac{1}{2} \sum_{i=1}^{n+1} E'_{iq}{}^2 \frac{x_{di}}{x'_{di}(x_{di} - x'_{di})} - \sum_{i=1}^{n+1} \frac{E'_{iq} E_{Fi}}{x_{di} - x'_{di}} \quad (3.9)$$

The last two terms are contributed by the flux decay equation (3.6c). If we assume $x'_{di} = x_{qi}$, then the expression simplifies as in [10].

Let $(\delta^s, \omega^s, \varphi^s, E'_q{}^s, V^s, \vartheta^s)$ be a stable equilibrium and define the energy function

$$V(\delta, \omega, \varphi, E'_q) := W_3(\delta, \omega, \varphi, E'_q, V, \vartheta) - W_3(\delta^s, \omega^s, \varphi^s, E'_q{}^s, V^s, \vartheta^s). \quad (3.10)$$

Then, along the trajectories of (3.6)-(3.8) one has

$$\begin{aligned} \frac{d}{dt} V &= \frac{\partial V}{\partial \delta} \dot{\delta} + \frac{\partial V}{\partial \omega} \dot{\omega} + \frac{\partial V}{\partial \varphi} \dot{\varphi} + \frac{\partial V}{\partial E'_q} \dot{E}'_q \\ &= - \sum_{i=1}^{n+1} D_i \omega_i^2 - \sum_{k=n+2}^{n+m+1} D_k \dot{\varphi}_k^2 - \sum_{i=1}^{n+1} \frac{T'_{doi}}{x_{di} - x'_{di}} \dot{E}'_q{}^2 \leq 0. \end{aligned} \quad (3.11)$$

The additional term in (3.11) compared with (3.5) equals the dissipation in the field winding [52]. The inequality (3.11) can be used to prove the analogs of Theorems 1 and 2.

The energy function (3.10) gives a qualitative improvement over the classical function (2.6): it does not neglect transfer conductances due to load impedance, it permits voltage dependent reactive power demand, and it incorporates flux decay. Theorems 1 and 2 indicate how these energy functions can be used to derive stability criteria.

Second, the improved energy functions are more difficult to compute than the classical function. To appreciate this, note that V in (3.10) is defined in terms of the post fault system, but it must be evaluated along the *faulted* trajectory. Suppose the faulted trajectory $(\delta(t), \omega(t), \varphi(t), E'_q(t), V(t), \vartheta(t))$ is obtained by numerical integration. To evaluate V at each step we must calculate

$$W_3(\delta(t), \omega(t), \varphi(t), E'_q(t), V_P(t), \vartheta_P(t)),$$

where the post fault values $V_P(t)$ and $\vartheta_P(t)$ are obtained in terms of $\delta(t)$, $\varphi(t)$ and $E'_q(t)$ by solving the algebraic equations (3.6d), (3.7a), and (3.8a-b).

4. On-Line Stability Criteria

In this section we review the main ideas underlying proposals for on-line implementation of stability criteria based on direct methods.

4.1. Approximating uep

In Section 2.4 we saw how Theorems 1 and 2 relate energy functions to transient stability. To formulate on-line stability criteria requires, in addition, the specification of this relation in a computable form. The criteria entail a comparison of the energy along the faulted trajectory with the energy at the critical or controlling uep. Hence the first step is to calculate the relevant uep. That depends on the power system model. In this section we consider the classical model.

From (2.3) we note that (δ, ω) is an equilibrium if and only if $\omega = 0$ and

$$P_i = \sum_{j \neq i}^{n+1} E_i E_j Y_{ij} \sin(\delta_i - \delta_j), \quad i = 1, \dots, n. \quad (4.1)$$

These are n equations in the n variables δ_i (recall that $\delta_{n+1} = 0$). Since the equations are 2π -periodic in δ_i we may limit the search for equilibria to the region¹⁵

$$-\pi < \delta_i \leq \pi, \quad i = 1, \dots, n. \quad (4.2)$$

Consider the task of finding the critical uep. By definition (2.9), it is the equilibrium on the stability boundary with the least energy. Thus to determine the critical uep one must first find all the uep. To appreciate the complexity of that problem take the special case $P_i = 0$, all i , so that (4.1) simplifies to

$$0 = \sum_{j \neq i}^{n+1} E_i E_j Y_{ij} \sin(\delta_i - \delta_j), \quad i = 1, \dots, n. \quad (4.3)$$

Each of the 2^n vectors δ with $\delta_i = 0$ or π satisfies (4.3). It is also true that $\delta^s = 0$ is stable while the $2^n - 1$ other equilibria are unstable, see [3].

On the other hand, for $n = 1$, (4.1) reduces to

$$P_1 = E_1 E_2 Y_{12} \sin \delta_1.$$

For $|P_1| < E_1 E_2 Y_{12}$ this has exactly two equilibria δ^s and δ^u with δ^s stable, δ^u unstable and $\delta^s + \delta^u = \pi$ or $-\pi$.

Based on these two special cases it has been conjectured that in general (4.2) possesses a unique stable solution, δ^s , and $2^n - 1$ uep δ^u near the vectors $\delta^a = (\delta_1^a, \dots, \delta_n^a)$ of the form $\delta_i^a = \delta_i^s$ or $\delta_i^a + \delta_i^s = \pi$ or $-\pi$. This suggests the following

¹⁵ This is not strictly legitimate unless the intersection of the stability region with the hyperplane $\omega = 0$ is contained in the region (4.2). That is true for $n = 1$; for $n > 1$ it is plausible but unproved. Note that the energy function V is not periodic in δ_i .

on-line criterion.

Step 1

Use Newton-Raphson or any other appropriate scheme to solve (4.1) for the uep δ^a starting from each of the $2^n - 1$ approximate uep δ^a as initial points.

Step 2

For each of the $2^n - 1$ uep δ^a obtained this way evaluate $V(\delta^a)$. The uep with minimum V is an estimate of the critical uep.

For $n \geq 10$ say, Step 1 becomes computationally prohibitive, and one must reduce the search. This might be done as follows. To each approximate uep one can associate a 'mode of instability'. For example, if $\delta^a = (\pi - \delta_1^s, \pi - \delta_2^s, \delta_3^s, \dots, \delta_n^s)$, we can say that δ^a corresponds to an instability due to the acceleration of generators 1 and 2 while generators 3, . . . , n remain in synchronism with the infinite bus. Researchers have noted that in most simulations transient instability occurs when a single generator accelerates or 'separates' from the rest of the system.¹⁶ In terms of Theorem 1 this means that the faulted trajectory leaves the stability region through the stable manifold of a uep near one of the n approximate uep $\delta^{ai} = (\delta_1^s, \dots, \pi - \delta_i^s, \dots, \delta_n^s)$. Hence in Step 1 above it is enough to consider the n starting points $\delta^{a1}, \dots, \delta^{an}$. A further approximation would be to assume that the energy at the approximate uep and the corresponding true uep are sufficiently close [36]. This would avoid Step 1 altogether, and one would estimate the critical energy by

$$v_{cr} = \min \{ V(\delta^{ai}, 0) \mid i = 1, \dots, n \}. \quad (4.4)$$

It seems that the estimate given by (4.4) is conservative for large n . To get a better estimate one should study the faulted trajectory to determine the controlling uep. According to the preceding paragraph the controlling uep is likely to be near one of the approximate uep $\delta^{a1}, \dots, \delta^{an}$. Ribbens-Pavella and her colleagues [38-41] have argued that the 'correct' approximate uep can be obtained as follows. First, find the generator i that has the fastest acceleration immediately after the fault starts. Second, calculate the time τ_i when the energy along the faulted trajectory $(\delta(\tau), \omega(\tau))$ reaches the energy equal to that of the

¹⁶ For the structure preserving energy functions (3.3) and (3.4), other rules have been proposed to reduce the number of approximate uep that need to be considered. These rules use the network structure to predict the set of generators most likely to separate from the rest of the system [9,31]. However, the rules have not been validated theoretically or through simulation.

approximate uep $\delta^{\alpha i}$,

$$V(\delta(\tau_i), \omega(\tau_i)) = V(\delta^{\alpha i}, 0).$$

Third, find the generator j that has the fastest acceleration at time τ_i . Then $\delta^{\alpha j}$ is the correct approximate uep; and the controlling uep is estimated to be the one near $\delta^{\alpha j}$. That estimate can be obtained by say a Newton-Raphson scheme to solve (4.3) starting at $\delta^{\alpha j}$ [40]. A slightly different choice for $\delta^{\alpha j}$ is given in [41].

The proposals discussed above can be implemented on-line. Unfortunately, they all lack theoretical justification. To illustrate the gap between our theoretical understanding and these proposals consider the belief that (4.3) has 2^n solutions. Bailleul and Byrnes [7] give an example that has a *continuum* of solutions. Their best provable estimate is that for almost all values of the 'parameters' $E_i E_j Y_{ij}$, the number of solutions is between 2^{n-1} and 2^{2n} . Moreover, as the P_i in (4.1) change, solutions merge and new solutions are created as discussed in [3]. Another belief implicit in these proposals is that the post fault system has a unique stable equilibrium. However, in [52] there is an example with two stable equilibria. Also, [3] contains an example of (4.1) in which there is *no* stable equilibrium but which does have an unstable equilibrium.

4.2. Approximating the stability boundary: PEBS

Another proposal has been made by Kakimoto et al [23] and more extensively pursued by Athay et al [4,5]. The basic idea underlying the 'Potential Energy Boundary Surface (PEBS)' goes back to the classical equal area criterion [2].

The energy function (2.6) can be decomposed as $V(\delta, \omega) = V_K(\omega) + V_P(\delta)$. $V_K(\omega) := \frac{1}{2} \sum M_i \omega_i^2$ is the kinetic energy and $V_P(\delta)$ is the potential energy. Then (2.3) can be written as

$$\dot{\delta}_i = \omega_i, \tag{4.5a}$$

$$M_i \dot{\omega}_i = -D_i \omega_i - \frac{\partial V_P}{\partial \delta_i}(\delta). \tag{4.5b}$$

Also, $(\delta^e, \omega^e=0)$ is an equilibrium of (4.5) if and only if $\frac{\partial V_P}{\partial \delta}(\delta^e) = 0$, i.e., δ^e is an extremal of V_P . It is not difficult to see [46] that the equilibrium is stable if and only if δ^e is a local minimum of $V_P(\delta)$. Consider now the gradient system

$$\dot{\delta} = -\frac{\partial V_P}{\partial \delta}(\delta). \tag{4.6}$$

From the preceding remark we conclude that δ^s is an equilibrium (stable equilibrium) of (4.6) if and only if $(\delta^s, 0)$ is an equilibrium (stable equilibrium) of (4.5).

Now let $(\delta^s, 0)$ be the post fault stable equilibrium of (4.5), and let $\partial A(\delta^s)$ denote the boundary of the stability region. This is a surface in R^{2n} of the state (δ, ω) . δ^s is also a stable equilibrium of (4.6). The boundary of its stability region, $\partial P(\delta^s)$, is a surface in R^n . The set $\partial P(\delta^s)$ is called PEBS.¹⁷

We can now give a version of the PEBS procedure.

Step 1

Calculate the faulted trajectory $(\delta(t), \omega(t))$. Suppose $\delta(t)$ crosses the PEBS at time τ . Evaluate the corresponding potential energy $v_P := V_P(\delta(\tau))$.

Step 2

Estimate the critical clearing time τ_C by the smallest time¹⁸ τ_P when the *total* energy equals v_P ,

$$V(\delta(\tau_P), \omega(\tau_P)) = v_P.$$

Several comments are in order. First, the intuition behind the procedure is that v_P is the maximum energy (along the faulted trajectory) that can be converted into potential energy before $\delta(t)$ reaches PEBS. Hence, if the fault is cleared before τ_P , all the kinetic energy can be converted into potential energy before the faulted trajectory exits the stability region. This intuition is valid for $n = 1$ and we get the equal area criterion.

Second, to implement Step 1 on-line the faulted trajectory must be calculated faster than real time. Several schemes have been proposed for doing this [4,22,23]. In addition, there must be a quick way of deciding when $\delta(\tau)$ reaches PEBS. One popular approximation is to take τ to be the first time that $V_P(\delta(\tau))$ reaches a local maximum or $\frac{d}{d\tau}V_P(\delta(\tau)) = 0$.

Third, a variation of the PEBS procedure would be to substitute for V_P the potential energy of the subsystem consisting of one or more generators (and associated transmission lines) that are most likely to separate from the rest of the system as in Vittal [55] or Michel et al [28].

¹⁷ This is a precise definition. Whether it agrees with the 'constructive definitions' in [4,23] is a matter of interpretation.

¹⁸ From Step 1 it follows that $\tau_P < \tau$.

Fourth, while PEBS was originally proposed for the classical model, its applicability depends only on the decomposition of the total energy into kinetic and potential energy. A look at the structure preserving energy functions proposed in Section 3 shows that they all have this form. It should therefore be possible to obtain PEBS procedures for them as well. This would be particularly valuable since transfer conductances and voltage dependent loads are better handled by these energy functions. For a very recent study that exploits this idea, see Sastri [45].

Finally it must be emphasized that theoretical justification for the PEBS procedure is lacking.¹⁹ According to our formulation PEBS is the stability boundary of the gradient system (4.6). Its relation with the stability boundary of the actual system (4.5) has not been theoretically investigated. At the very least one would like to know whether the equilibria of (4.6) that lie on PEBS correspond to the equilibria of (4.5) that lie on its stability boundary.

In conclusion, the proposals for on-line criteria reviewed here have been shown to be promising on simulations of systems of varying size. They differ in terms of amount of computation required and in the accuracy with which they can estimate the critical clearing time. A much more rigorous test would be to see how well the stability region itself is estimated.²⁰ Unfortunately none of the proponents of these schemes have attempted such a test.

5. Exciter Control and Chaotic Motion

Two recent studies illuminate the complex behavior of power systems. The first shows that the introduction of an exciter loop can create an oscillatory instability that reduces the transient stability region. The second exhibits the possibility of chaotic motion in the classical model without damping. These studies are reviewed below.

¹⁹ It is often claimed that the PEBS procedure does guarantee so-called 'first swing' stability. This stability concept is very imprecise, and the alleged proofs of the claim are logically unsound.

²⁰ One can get an *exact* series representation for the stable manifolds characterizing the stability boundary [42]. This series can be computed recursively. However, no numerical tests have been conducted. See also [62] for a first order approximation.

5.1. Exciter control

Of the three controllers of a generating unit (boiler control, governor, and exciter) only the exciter acts sufficiently fast to be of concern in transient stability.²¹ As mentioned before, the absence of the exciter loop is the only serious deficiency in the machine models of Section 3.

To appreciate the effect of the exciter consider a single generator connected to an infinite bus by a lossless transmission line whose inductance is X . In the model of Section 3.3 take $n = 1$, denote by $E'_q \angle \delta$ and $E \angle 0$ the generator and infinite bus voltages respectively, assume $x'_d = x_q$, to get the model

$$\dot{\delta} = \omega, \quad (5.1a)$$

$$M \dot{\omega} = P - D \omega - \frac{E'_q E}{x'_d + X} \sin \delta, \quad (5.1b)$$

$$T'_{do} \dot{E}'_q = -\frac{x_d}{x'_d} E'_q + \frac{x_d - x'_d}{x'_d + X} (E'_q - E \cos \delta) + E_F \quad (5.1c)$$

In Section 3.3 it was assumed that the field voltage E_F is fixed. If an exciter is included, then E_F is its output and can no longer be considered constant. If one considers the IEEE Type 1 excitation system, then E_F can be modeled as the output of a third order nonlinear system,

$$\dot{z} = f(z; K), \quad (5.2a)$$

$$E_F = h(z). \quad (5.2b)$$

Here $z \in R^3$ is the state of the exciter, f and h are nonlinear functions whose particular form is not important for our purposes (for details, see [1]). The parameter K in (5.2a) is the exciter amplifier gain.

For quite some time it was known that this system can exhibit oscillations in theory [8,13] and in practice [53] for realistic values of K . The oscillations are suggested by linear analysis. Let $x^g(K) = (\delta^g, \omega^g=0, E'^g_q, z^g)$ be an equilibrium for a given value of K . Let $\Lambda(K)$ be the set of eigenvalues of the system linearized about $x^g(K)$. It was observed that for $K < K_C$, $\Lambda(K)$ is contained in the open left-half plane, and for $K > K_C$ a pair of eigenvalues cross the imaginary axis into the right-half plane at a frequency $\omega_C \neq 0$. Thus for $K < K_C$, $x^g(K)$ is a stable equilibrium, and it is unstable for $K > K_C$. Moreover, one can predict an

²¹ The protective relays in the system will act well before the governor and boiler control to prevent damage due to transient stability.

oscillation of frequency near ω_c when K is near the critical value K_c .

To characterize the oscillation in more detail the nonlinear system (5.1)-(5.2) must be examined directly. This was first done by Abed and Varaiya [1]. Based on the theory of Hopf bifurcation²² they study the system behavior in a small neighborhood of $x^s(K_c)$ and for $|K-K_c|$ small. They show that

(1) For $K < K_c$, there is a unique *unstable* limit cycle near the stable equilibrium $x^s(K)$ whose amplitude is of order $(K-K_c)^2$;

(2) For $K > K_c$, there is no limit cycle near the unstable equilibrium $x^u(K)$.

The effect of the excitation system on the transient stability region was clarified by Tsolas [51]. His findings are conveniently summarized in Figure 4 which displays the projections of the trajectories of (5.1)-(5.2) onto the $\delta-\omega$ plane. In that figure $(\delta^s, \omega^s=0)$ is the (projection of the) normally stable equilibrium x^s ; $(\delta^u, \omega^u=0)$ is a uep with $\delta^u \sim \pi-\delta^s$. These equilibria repeat at 2π intervals and the figure shows the one at $(\delta^u-2\pi, \omega^u=0)$.

[FIGURE 4 NEAR HERE]

(1) For $\underline{K} < K < K_c$ (Figure 4b) there is a unique unstable limit cycle around the stable equilibrium $(\delta^s, 0)$; the limit cycle forms the boundary of the stability region. The amplitude and the period of the limit cycle (hence the stability region) grow as K is decreased to \underline{K} . At $K = \underline{K}$ the limit cycle touches $(\delta^u, 0)$ and it is destroyed.

(2) For $K < \underline{K}$ (Figure 4a) there is no limit cycle and the behavior is qualitatively similar to the case where there is no exciter.

(3) For $K > K_c$ (Figure 4c), $(\delta^s, 0)$ becomes unstable as predicted by linear analysis.

These results show that the excitation system can *reduce* the transient stability region. The oscillatory behavior also suggests it may be difficult to estimate the stability region by Lyapunov functions.

²² An accessible account of Hopf bifurcation is available in [27].

5.2. Chaotic motion in the classical model

For $n = 1$ and zero damping the classical model (2.3) reduces to

$$\dot{\delta} = \omega, \quad (5.3a)$$

$$M \dot{\omega} = P - \alpha \sin \delta, \quad (5.3b)$$

where $\alpha := E_1 E_2 Y_{12}$. This is also the equation for a pendulum with driving force P and for $\alpha > |P|$, as will be assumed here, its behavior is described by the phase portrait of Figure 5.

[FIGURE 5 NEAR HERE]

There is a Lyapunov-stable equilibrium $x^s = (\delta^s, \omega=0)$ and a saddle point $x^u = (\delta^u, \omega=0)$ with $\alpha \sin \delta^s = P$, $\delta^u = \pi - \delta^s$. Since there is no damping, the energy function

$$V(\delta, \omega) = \frac{1}{2} M \omega^2 - P (\delta - \delta^s) - \alpha [\cos \delta - \cos \delta^s]$$

is constant along trajectories. If the energy is sufficiently small the trajectory forms a closed orbit (limit cycle) around x^s . The amplitude and period of this orbit increase with the energy until it reaches a critical value ν_c when the orbit touches x^u and its 'period' becomes infinite. (For energy larger than ν_c there is no closed orbit.) Observe that this infinite period orbit, $W(x^u)$, is simultaneously the stable and unstable manifold of x^u which is therefore called a homoclinic point.

Now consider (5.3) with initial condition $x \in W(x^u)$ and replace the constant force P by P plus a small periodic force $p(t)$ with some period T ,

$$\dot{\delta} = \omega, \quad (5.4a)$$

$$M \dot{\omega} = P + p(t) - \alpha \sin \delta. \quad (5.4b)$$

This small periodic force creates chaotic behavior in (5.4) in a neighborhood of $W(x^u)$. In particular, it can be shown that (5.4) has periodic orbits of period kT for arbitrary large values of k . For a detailed proof see Kopell and Washburn [26].

The periodic term $p(t)$ can be created as follows. In place of the single machine (5.3) consider a coupled system of two lossless generators one of which has a small inertia constant and the other has a very large inertia. The small machine will have a negligible effect on the large machine. If the latter has a

small initial energy its trajectory will exhibit a small limit cycle. Through the transmission line coupling the two machines this limit cycle will exert a small periodic force on the small machine. The behavior of the small machine will then be similar to that of (5.4). A complete analysis is given in [26].

If instead of a single large machine we had several large machines, each of which had a limit cycle, then the system can exhibit an even more complex chaotic motion called Arnold diffusion. That observation is due to Salam, Marsden and Varaiya [43,44].

The results above depend crucially on the assumption of zero damping. From (2.7) we can see that in the classical model with positive damping there can be no limit cycle since the energy must be strictly decreasing except at equilibrium. As a consequence chaotic motion cannot exist either. Nevertheless, for small time intervals (certainly of intervals of interest in transient stability), the trajectories of a classical model with small damping will be close to those with zero damping. Hence the results above do show that during a transient the power system trajectories can appear chaotic or random. Of course, in [26,43] chaotic motion has been exhibited in relatively contrived examples, and it would be worth knowing whether realistic examples can also have such behavior.

6. Future Directions

Since 1978, progress in direct methods has been rapid and has reached the stage that one can foresee their adoption for on-line stability analysis.²³ This progress has also revealed new bottlenecks in on-line algorithms and in theory. This section is devoted to articulating these barriers and to offering suggestions for further research. It begins with algorithms.

The new structure preserving energy functions reviewed in Section 3 need to be reconsidered with a view towards on-line implementation. Initially, this may take the form of inventing ways of limiting the number of equilibria to be investigated to obtain a good approximation to the critical and controlling uep. In particular, the preservation of the network structure in these functions should provide a basis for a good prediction of the likely ways in which the

²³ We are ignoring the necessity for rapid and reliable detection of faults. That is a prerequisite for on-line methods and is a conceptually separate issue which also needs much more research.

system would separate to create loss of synchronism. The concept of 'saturating cutsets' introduced in [9,31] may ultimately prove valuable in this regard. The apparent success of the PEBS algorithm for the classical energy function suggests that it can fruitfully be reformulated for the new functions as well, and a recent study confirms this promise [45].

The algorithms proposed for on-line implementation were inspired by advances in theory. However, these algorithms are justified almost exclusively on the basis of a small number of simulation studies. That is insufficient to inspire adoption of these algorithms in real-world systems and more work should be done in several directions. First, the algorithms should be tested on more realistic system models. But to do this adequately it seems to us not enough only to study critical clearing times which is the current practice. One should try to compare the stability boundary estimated by these algorithms with the true stability boundary.²⁴ Second, there is a need to develop a set of standardized test problems that can serve as a means of comparing different algorithms.²⁵ Third, it is urgent to direct effort to providing theoretical justification for proposed algorithms. For example, one should relate PEBS to the true stability boundary.

Theoretical research should be focused on several questions. First, the only major remaining shortcoming with respect to machine models is the absence of the excitation system. The voltage-dependent reactive power loads that can now be incorporated in energy functions is a significant advance. But the modeling of real power loads should be improved. Second, current results propose energy functions for a fixed set of loads. In practice, loads are changing continuously. A major advance in the theory would be to treat loads parametrically and derive corresponding parametric energy functions. This would help in estimating changes in the stability boundary as a function of changes in loading conditions. Such results would also more closely tie the analyses of transient stability and security [11,17,59-61]. The study reported in [3] might suggest ideas for a useful parametrization. The geometric viewpoint in [7] may also prove fruitful. Finally, the studies discussed in Section 5 reveal that our understanding of the global behavior of power systems is still woefully incomplete.

²⁴ The boundary is a surface in the state space, whereas the critical clearing time is simply a real valued function of the state. Thus a very poor estimate of the boundary can still yield a good estimate of the clearing time.

²⁵ This suggestion is motivated by the practice in mathematical programming.

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List of Figures

1. The critical clearing time τ_c
2. The electric network for the classical model
3. The critical and controlling uep
4. Effect of excitation control
5. Phase portrait of lossless machine

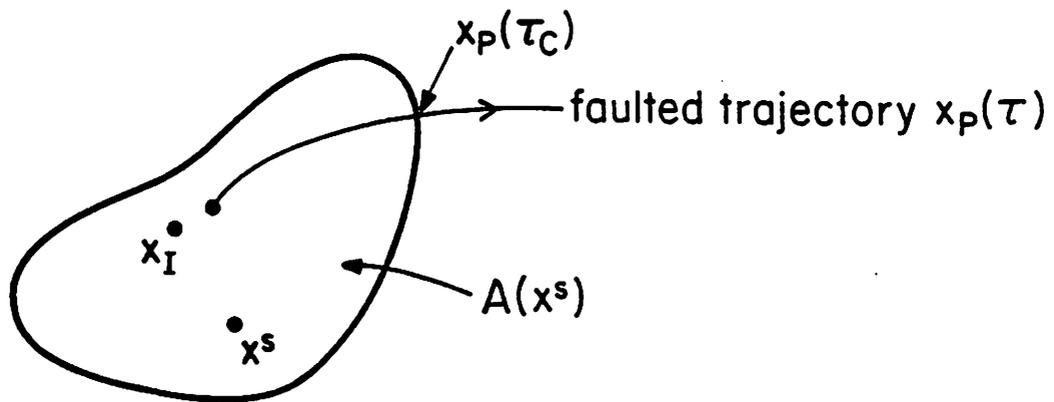


Figure 1

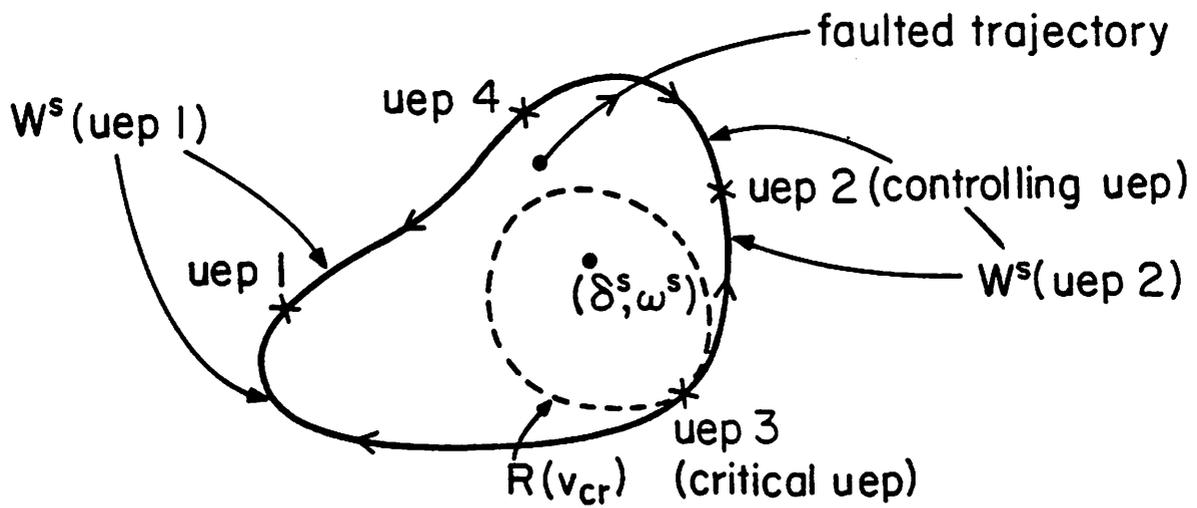


Figure 3

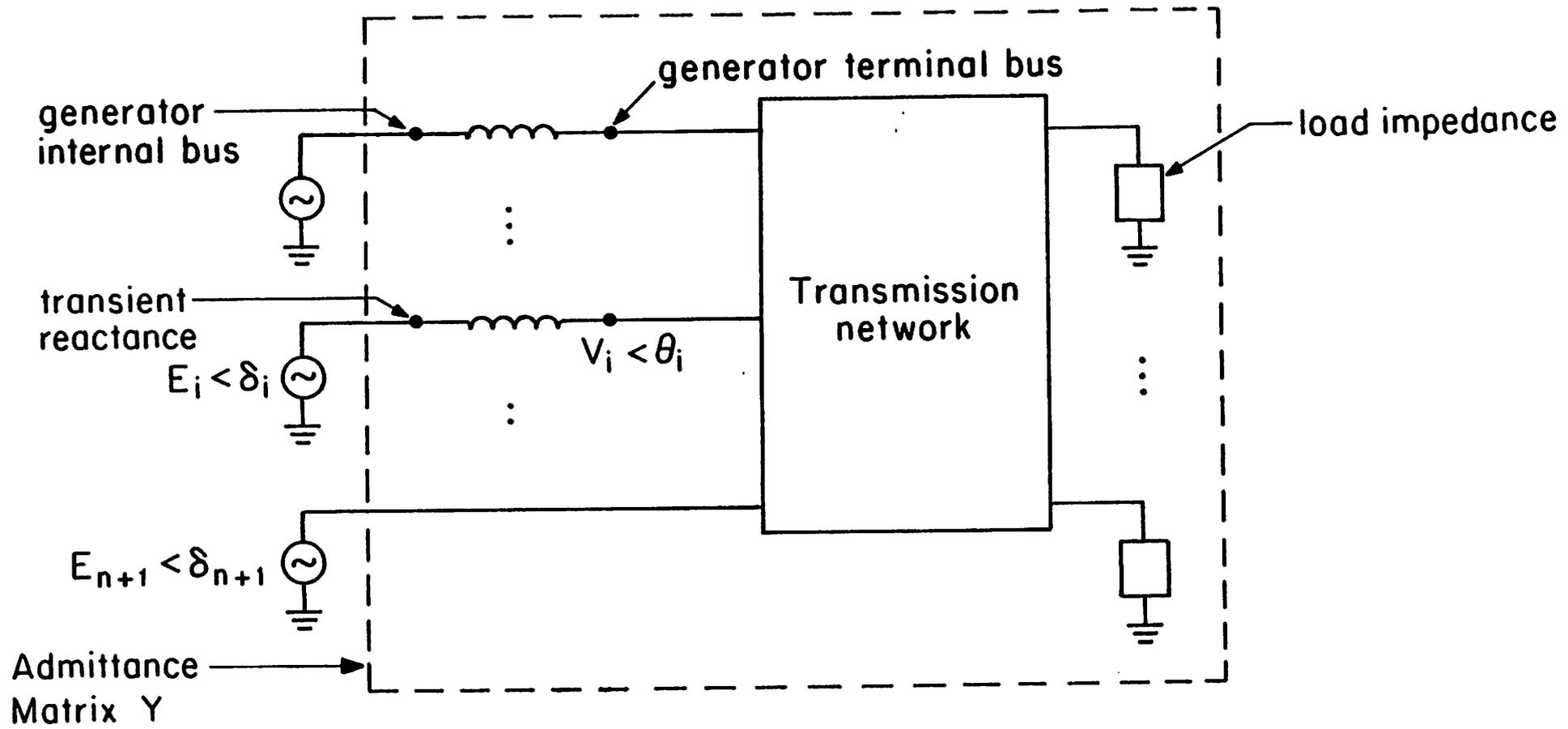


Figure 2

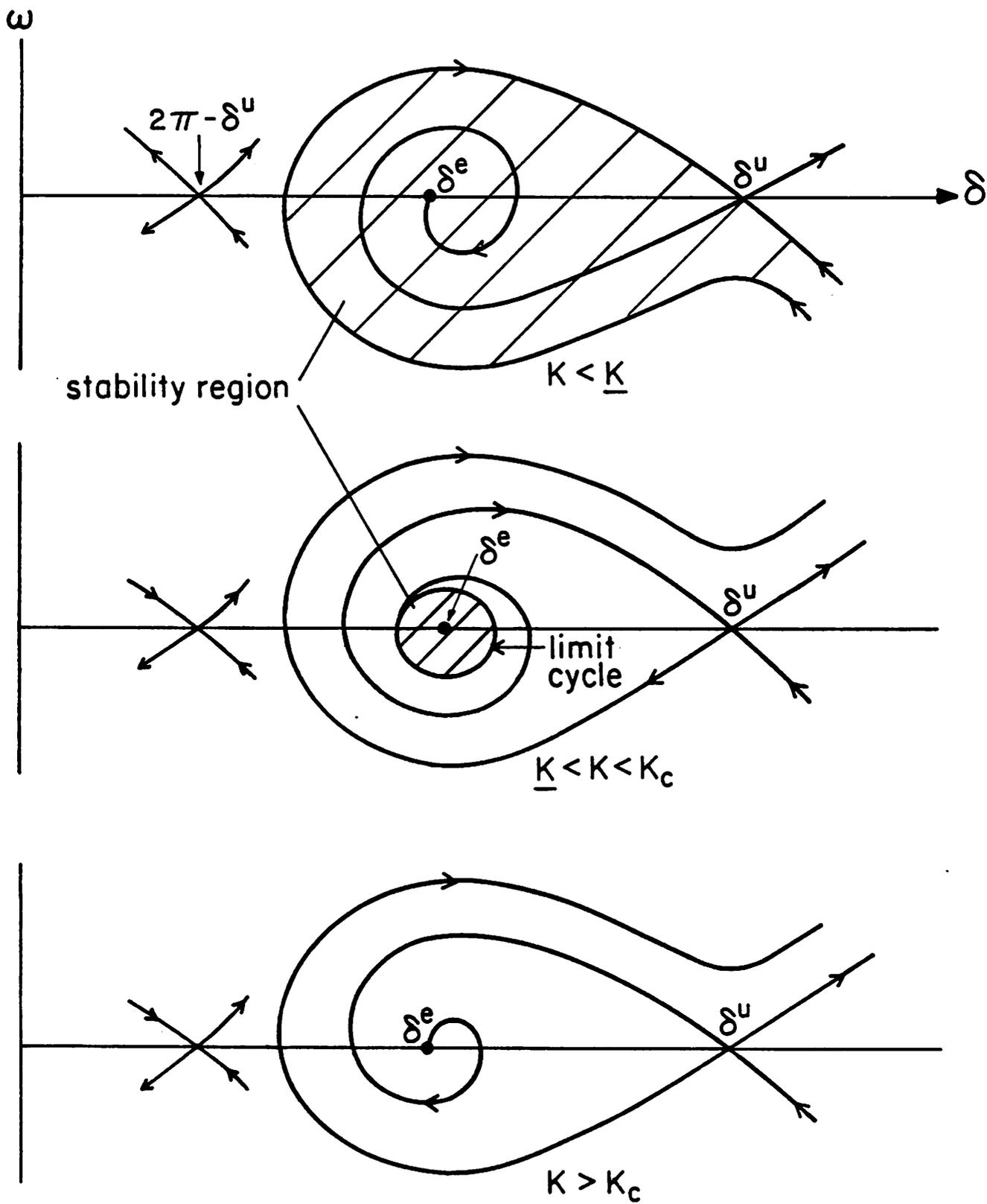


Figure 4

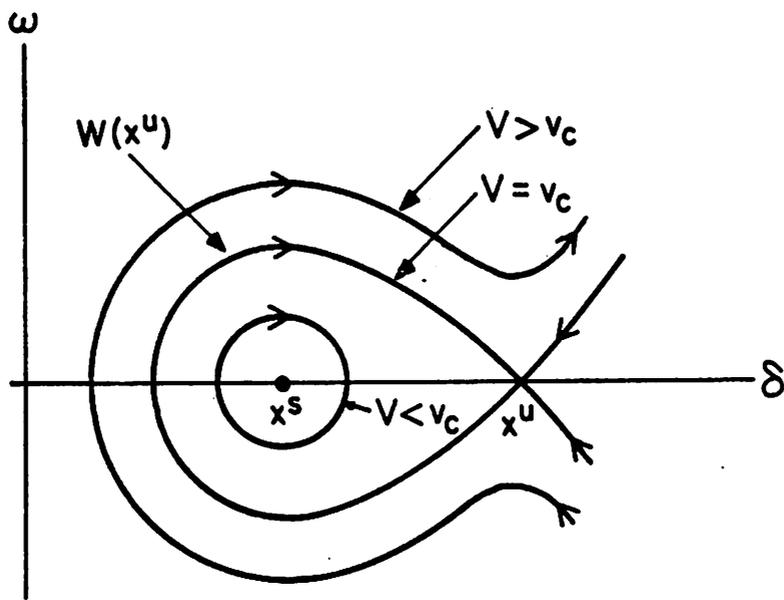


Figure 5