PARAMETER IDENTIFICATION USING PRIOR INFORMATION

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TITLE PAGE

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ABSTRACT

We consider the problem of identifying some unknown gain parameters in a single input, single output transfer function which is written as the ratio of proper, stable transfer functions. This technique is a generalisation of currently available techniques of identification which do not use prior information. It usually involves the identification of fewer parameters and is faster in convergence and less susceptible to errors caused by the presence of unmodeled dynamics. Extensions to the discrete time systems and some multivariable systems are also covered.

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Parameter Identification Using Prior Information

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1.Introduction

We consider the problem of identifying a partially known single input, single output transfer function. The transfer function to be identified is represented as the ratio of proper, stable rational functions with unknown gain coefficients i.e. of the form

$$T(s)=\frac{g_0(s)+\sum_{i=1}^{m}\beta_i g_i(s)}{f_0(s)-\sum_{j=1}^{n}\alpha_j f_j(s)} \tag{1.1}$$

with the g_i's and f_j's known proper, stable rational functions and β_i, α_j s unknown.

Such transfer functions arise in several contexts, typically from the interconnection of several systems with unknown gains. Classical identification techniques such as those, for example of Luders and Narendra [10], Goodwin-Sin [11], Kreisselmeier [12] discuss the problem of identifying the numerator and denominator coefficients of a transfer function with no prior information. It is ,of course, clear that one could neglect the prior information embodied in the form of the transfer function (1.1) and identify the plant. However usage of the particular structure embodied in (1.1) may result in the identification of a fewer number of unknown parameters and faster convergence rates as we see in this paper. The framework of representing transfer functions as the ratio of proper, stable rational functions, proposed for example in [3,4], has proved useful in the H^∞ approach to linear control systems design and it has payoffs in our context as well in studying the effects of near pole-zero cancellations, unmodeled dynamics on the identification scheme. We will see the definite benefits of using an identifier incorporating prior information in terms of rapid

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convergence and smaller identification error.

The work reported in this paper was directly inspired by a recent Ph.D dissertation by Dasgupta [1,2] who considers transfer function of the form (1.1) with the g_i's and f_i's polynominals. The advantages of our representation occur in analysing the effects of unmodeled dynamics on the schemes.

The present paper covers continuous time and discrete time systems. For the discrete time case, we use different framework-we represent the transfer function to be identified as the ratio of polynomials in z^{-1}.

2 Parameter Identification for Some 'Partially Known' Continuous Time Systems

We consider the problem of identifying a class of 'partially known' single input, single output, proper, stable transfer functions of the form

$$T(s)=\frac{g_0(s)+\sum_{i=1}^{m}\beta_i g_i(s)}{f_0(s)-\sum_{j=1}^{n}\alpha_j f_j(s)} \tag{2.1}$$

Here the g_i's and f_j's are known, proper, stable rational functions in s and the β_i, α_j's are unknown, real parameters.

The identification problem is to identify β_i, α_j from input-output measurements of the system.

Remarks: 1) Transfer functions of the form (2.1) arise from the interconnection of proper, stable linear systems, with the unknown parameters representing the coupling or interconnection constants.

2) Classical transfer function identification, i.e. identification of a stable plant of the form

$$T(s)=N(s)/\,D(s)=\frac{\beta_1 s^{m-1}+\ldots+\beta_m}{s^n+\alpha_1 s^{n-1}+\ldots+\alpha_n} \tag{2.2}$$

with $m\leqslant n$ and α_i and β_j unknown, can be stated in terms of the set up of (2.1) by choosing

$$g_0(s)=0,\quad f_0(s)=\frac{s^n}{(s+\alpha)^r}$$

$$g_i(s)=\frac{s^{m-i}}{(s+\alpha)^n} \qquad i=1,..m$$

and

$$f_j(s)=-\frac{s^{n-j}}{(s+\alpha)^n} \qquad j=1....n$$

with $\alpha>0$ a positive, real number. Also if m is not known, we may set it equal to n. The parametrisation of transfer functions as the ratio of proper, stable rational functions is to our mind an interesting one in view of recent advances using this framework in the literature on robust(non-adaptive) linear control [3.4].

let y(s).u(s) denote the input and output to the plant of equation (2.1). (The initial conditions of the plant represent exponentially decaying terms which do not change any of the following discussions, as is well understood in the literature.) Then, after some rearrangements, we get

$$f_0 y - g_0 u = \sum_{j=1}^{n} \alpha_j f_j y + \sum_{i=1}^{m} \beta_i g_i u \tag{2.3}$$

Defining

$$z_0(s)=f_0(s)y(s)-g_0(s)u(s) \tag{2.4a}$$

$$h_j(s)=f_j(s)T(s) \qquad j=1....m \tag{2.4b}$$

$$h_{n+i}(s)=g_i(s) \qquad i=1....m \tag{2.4c}$$

and the unknown parameter vector θ_0 by

$$\theta_0^T=(\alpha_1,....\alpha_n,\beta_1,\ldots,\beta_m)$$

we get

$$z_0(s)=\theta_0^T \begin{bmatrix} h_1(s) \\ \vdots \\ h_{n+m}(s) \end{bmatrix} u(s) \tag{2.5}$$

The vector of signal $(h_1(s),....h_{n+m}(s))^T$ is denoted $w(s)$ and its Laplace inverse $w(t)$ so that in time domain (2.5) reads (again modulo decaying initial condition terms.)

$$z_0(t)=\theta_0^T(w(t)*u(t)) \tag{2.6}$$

(* stands for convolution.) By way of notation, we refer to w(t)*u(t) as z(t).

From the form of equation (2.6), it is easy to see how an estimator and equation can be derived. Let $\hat{\theta}(t)$ denotes the parameter estimate at time t. Then since z(t) is a vector of signals obtainable from the input and output by proper stable filtering, as seen from (2.4), we can construct the error

$$e(t)=\hat{\theta}(t)^T z(t)-z_0(t) \tag{2.7}$$

Using (2.7) and with $\phi(t)=\hat{\theta}(t)-\theta_0$ denoting the parameter error, we see that

$$e(t)=\phi^T(t)z(t) \tag{2.8}$$

Equation (2.8) is linear in the parameter error so that any one of a number of standard techniques for parameter update (see for eg [11]) may be used. We summarize two of the techniques here.

The least Squares Type Algorithm

The parameter update law is of the form (with $P(t)\in R^{(n+m)\times(n+m)}$)

$$\dot{\hat{\theta}}(t)=-P^{-1}(t)z(t)e(t) \tag{2.9}$$

$$\dot{P}(t)=z(t)z(t)^T \qquad P(0)=\alpha I>0 \tag{2.10}$$

It is well known (see, for eg [11]) that if z is persistently exciting i.e. there exists $\alpha_1, \delta>0$ such that

$$\int_s^{s+\delta} zz^T\,dt \geq \alpha_1 I \qquad \text{for any } s\in R_+ \tag{2.11}$$

then $\phi\to 0$ as $t\to\infty$. Of course, since z(t) is bounded we in fact have

$$\alpha_2 I \geq \int_s^{s+\delta} zz^T\,dt \geq \alpha_1 I \qquad \text{for any } s\in R_+ \tag{2.12}$$

The result of [7] can be used to give frequency domain conditions on u(t) to guarantee (2.12). First, we need the following identifiability condition.

I1 Identifiability Condition

The system (2.1) is said to be identifiable if for every choice of distinct (n+m) frequencies $\nu_1, \ldots, \nu_{n+m}$ the vectors $w(j\nu_i) \in C^{n+m}$ $(i=1,\ldots,n+m)$ are linearly independent

Comments: 1) From (2.5), it follows that if an input having (n+m) spectral lines were applied to the system, we would get

$$[z_0(j\nu_1),\ldots,z_0(j\nu_{n+m})]$$

$$=\theta_0^T [w(j\nu_1),\ldots,w(j\nu_{n+m})] diag(u(j\nu_1),\ldots,u(j\nu_{n+m})) \tag{2.13}$$

In turn, the identifiability condition implies that (2.13) has a unique solution for θ_0.

2) It is difficult to give a more concrete characterization of identifiability since the component of w(s) are proper, stable rational functions of different orders. An exception is the case of classical identification discussed in remark 2 (equation 2.2) in which case it has been shown in [7] that the identifiability condition holds if N(s) and D(s) are coprime polynominals.

Using the identifiability condition , we state the following fact easily derived from [7]:

Under the identifiability assumption I1, z is persistently exciting, i.e. it satisfies (2.12) if and only if the spectral measure of u is not concentrated on less than n+m points.

Thus, if there are at least as many frequencies in the input as there are unknown parameters, the parameter errors converge to zero. Of course, the least squares type algorithm (2.9),(2.10) shows rapid initial convergence with asymptotically slow adaptation (as P(t) gets large). Some form of resetting of P(t) or forgetting is introduced (as in Goodwin and Sin [11] pg.62), for example

$$\dot{P}(t) = -\alpha P(t) + z(t) z(t)^T \qquad P(0) = \alpha I > 0 \tag{2.19}$$

It is then easy to show that the convergence of the parameter error is exponential. It is important to note that forgetting is not used when z is not persistently exciting to keep P from going singular

Projection Type Algorithm

The update laws

$$\dot{\hat{\theta}}(t)=-z(t)e(t) \tag{2.15}$$

or

$$\dot{\hat{\theta}}(t)=-\frac{z(t)e(t)}{1+z(t)^T z(t)} \tag{2.16}$$

are referred to as projection type algorithms. They also yields exponential convergence when the input is sufficiently rich in the sense discussed above and the assumption I1 holds.

To illustrate the methods of this section, consider the following example

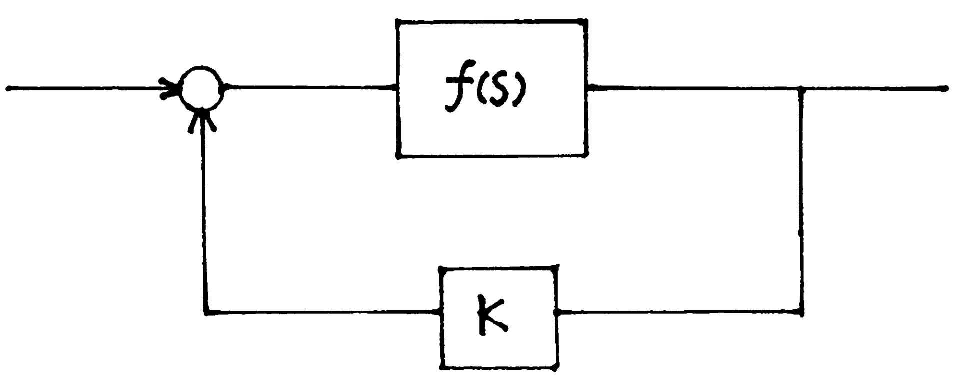

Fig. 2.1 The Plant of the form of equation 2.1.

In Fig. 2.1 above, f(s) is known (assumed to be $\frac{2s+2}{3s+5}$ for the simulations of Figures 2.2 and 2.3). The form of the closed loop transfer function is

$$\frac{f(s)}{1+kf(s)} = \frac{as+b}{s+c}$$

With the parameter k to be estimated. Figure 2.2 shows the parameter error for the algorithm with the projection type update law (the true value of k=1) and input u(t)=5 (only one spectral line is needed for identification).

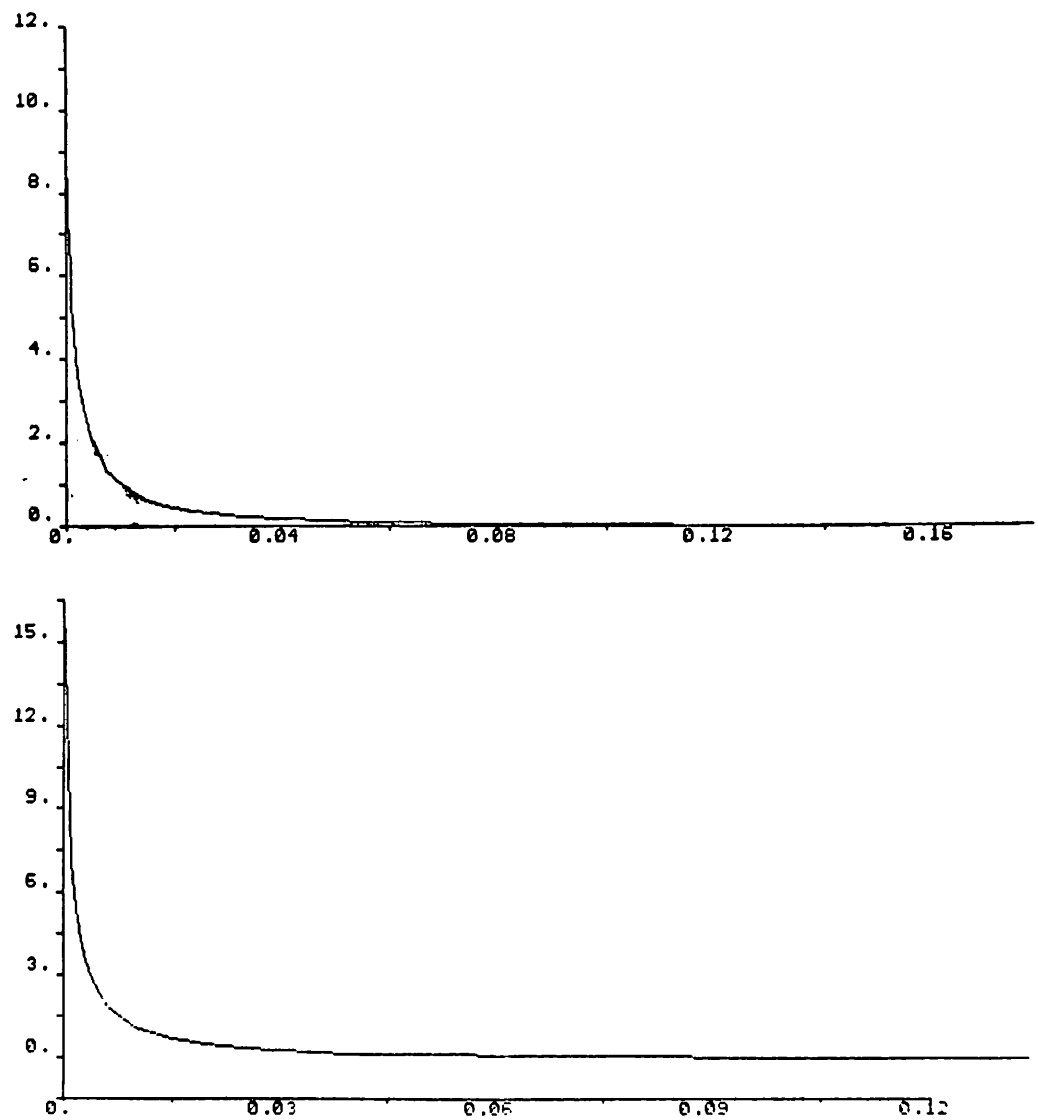

Fig. 2.2 Estimation errors of parameters a and b (above) and c (below) using prior information.

Identification of the closed loop plant without utilizing the structure of the system requires the estimation of three parameters a,b and c. Figure 2.3 shows the parameter errors for a, b and c using the input $u(t)=3+4\sin(4t)$. Note that the two inputs for figures 2.2 and 2.3 have the same energy. The input for Figure 2.3 is richer than that for Figure 2.2. However, the rate of convergence is much slower (by a factor of approximately 500) in Figure 2.3. In section 3, we will see that the scheme using prior information also has a larger robustness margin.

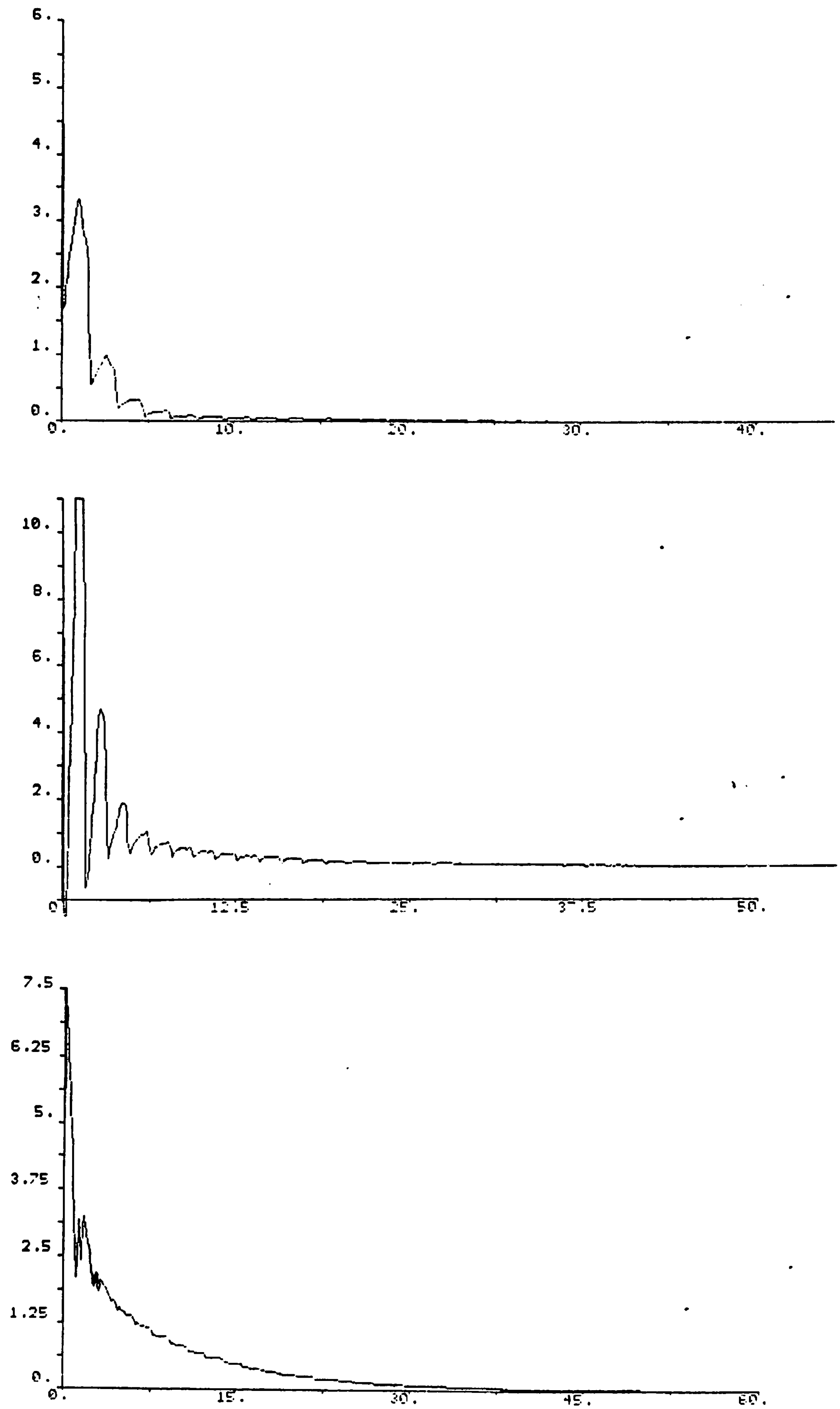

Fig. 2.3 Estimation errors of parameters a (upper), b (middle) and c (lower) without using prior information.

Spectral Analysis of The Convergence Rates

Though the identifiability condition I1 guarantees that the parameter errors converge to zero if and only if the support of the spectrum of input u has at least n+m points, it does not provide much insight into the connection between the spectral content of the input and the convergence rate. We will use averaging techniques as developed say in Fu, et al [8], to facilitate this analysis.

First consider the projection type algorithm (2.15) with slow update law (modeled by adaptation gain ϵ, a small positive number).

$$\dot{\phi}=-\epsilon z z^T \phi \tag{2.17}$$

Defining the averaged value of zz^T to be $R_z(0)$(see [7]) given by

$$R_z(0)=\lim_{T\to\infty}\frac{1}{T}\int_s^{s+T} zz^T\, dt \qquad \text{for } any\ s\in R_+ \tag{2.18}$$

(provided it exists- this in turn is guaranteed by assuming z to be stationary, see Fu,et al [8] for details.) We see that for ϵ small enough the dynamics of (2.17) (including rate of convergence up to the order of ϵ^2) are approximated by

$$\dot{\phi}_{av}=-\epsilon R_z(0)\phi_{av} \tag{2.19}$$

Noting that $R_z(0)$ is the integral of the spectral measure of z, we may rewrite an expression for $R_z(0)$ in terms of the input spectrum and the function w(s) as

$$R_z(0)=\int w(j\nu)S_u(d\nu)w^*(j\nu) \tag{2.20}$$

where $S_u(d\nu)$ stands for the spectral measure of u. Thus, the convergence rate of (2.19) is obtained to lie in an interval $[\lambda_{min}(R_z(0)),\lambda_{max}(R_z(0))]$. For optimum convergence, the spectrum of the input needs to be in the dominant part of $w(j\nu)w^*(j\nu)$. Of course, the expression (2.20) involves parameters of the unknown plant so that it is not easily approximated.

For the analysis of the slowed-down least squares algorithm, consider with z assumed to be persistently exciting

$$\dot{\phi}=-\epsilon P^{-1}zz^T\phi \tag{2.21}$$

$$\dot{P}=\epsilon(zz^T-\alpha P) \tag{2.22}$$

As before, we may approximate (2.21), (2.22) by the averaged system

$$\dot{\phi}_{av}=-\epsilon P^{-1}{}_{av} R_z(0)\phi_{av} \tag{2.23}$$

$$\dot{P}_{av}=\epsilon(R_z(0)-\alpha P_{av}) \tag{2.24}$$

Equation (2.24) may be explicitly integrated to give

$$P_{av}(t)=(P_{av}(0)-\frac{1}{\alpha}R_z(0))e^{-\epsilon\alpha t}+\frac{1}{\alpha}R_z(0) \tag{2.25}$$

In turn, using this in (2.23) and noting that $P_{av}(t)$ converges exponentially to $\frac{1}{\alpha}R_z(0)$, we see that the tail behavior of (2.23) is

$$\dot{\phi}_{av}=-\epsilon\alpha\phi_{av} \tag{2.26}$$

so that the tail convergence rate is a function of the forgetting factor α alone in the 'covariance' equation (2.22) and not the input spectrum!

Effect of Unmodeled Dynamics on Parameter Identification

The set up of the previous section used transfer functions of the form (2.1) with the f_i and g_i's known exactly. In practice, the f_i and g_i's will not be known exactly, but only approximately. In fact, the transfer functions used to approximate the f_i and g_j will generally be low order proper, stable rational functions(neglecting high frequency dynamics, and replacing near pole-zero cancellations by exact pole-zero cancellations). Thus, the identifier's model of the plant is of the form

$$\tilde{T}(s)=\frac{\tilde{g}_0+\sum_{i=1}^{m}\beta_i\tilde{g}_i}{\tilde{f}_0-\sum_{j=1}^{n}\alpha_j\tilde{f}_j} \tag{3.1}$$

where $\tilde{T}$ is a proper,stable transfer function and

$$|(\tilde{g}_i-g_i)(jw)|<\epsilon \qquad \text{for } all\ w \quad i=0....m \tag{3.2a}$$

$$|(\tilde{f}_j-f_j)(jw)|<\epsilon \qquad \text{for } all\ w \quad j=0....n \tag{3.2b}$$

We refer to $\tilde{g}_i-g_i$ as Δg_i in the sequel, similarly for Δf_j. For example, g_i may be of the form

$$\tilde{g}_i \frac{1}{v(s)} \frac{q(s)}{p(s)} \tag{3.3}$$

where $\frac{1}{v(s)}$ represents stable high frequency dynamics and $\frac{q(s)}{p(s)}$ represents near (stable) pole-zero cancellations.

The identifier uses the form (3.1) to derive the identifier for the true plant T(s) which is accurately described by (2.1). Consequently the transfer functions of (2.4) are replaced by

$$\tilde{z}_0(s) = \tilde{f}_0 Tu - \tilde{g}_0 u \tag{3.4a}$$

$$\tilde{h}_j(s) = \tilde{f}_j T \qquad j = 1, \ldots, n \tag{3.4b}$$

$$\tilde{h}_{n+i}(s) = \tilde{g}_i \qquad i = 1 \ldots m \tag{3.4c}$$

It is important to note that $\tilde{z}_0$ does not satisfy an equation of the form (2.5) i.e. it is not true that

$$\tilde{z}_0(s) = \theta_0^T \begin{bmatrix} \tilde{h}_1(s) \\ \vdots \\ \tilde{h}_{n+m}(s) \end{bmatrix} u(s)$$

Equation (2.5) is ,of course, still valid. The update law (least squares type) is now of the form (with $\tilde{z}_i(t) = \tilde{h}_i(t) * u(t), i = 1 \ldots n+m$)

$$\dot{\hat{\theta}} = -P^{-1} \tilde{z}(t) (\hat{\theta}^T(t) \tilde{z}(t) - \tilde{z}_0(t)) \tag{3.5}$$

$$\dot{P} = \tilde{z}\tilde{z}^T - \lambda P \qquad P(0) = \alpha I > 0 \tag{3.6}$$

We need an expression for $\tilde{z}_0$ in order to study this algorithm. For this purpose we note that

$$\tilde{z}_0(s) = z_0(s) + (\tilde{f}_0(s) - f_0(s)) T(s) u(s) - (\tilde{g}_0(s) - g_0(s)) u(s) \tag{3.7}$$

$$= z_0(s) + \Delta f_0 T(s) u(s) - \Delta g_0 u(s)$$

Also, we have

$$z(s) = \tilde{z}(s) - \begin{bmatrix} \Delta h_1 \\ \vdots \\ \Delta h_{n+m} \end{bmatrix} u(s) \tag{3.8}$$

Using (3.7) and (3.8) we see that equation (3.5) may be rewritten as

$$\dot{\hat{\theta}} = -P^{-1}\tilde{z}\tilde{z}^T(\hat{\theta}(t) - \theta_0) - P^{-1}\tilde{z}(t)\delta(t) \tag{3.9}$$

where $\delta(t)$ is the Laplace inverse of

$$\theta_0^T \begin{bmatrix} \Delta h_1(s) \\ \vdots \\ \Delta h_{n+m}(s) \end{bmatrix} u(s) - \Delta f_0(s)T(s)u(s) + \Delta g_0(s)u(s)$$

With $\hat{\theta}(t) - \theta_0 = \phi(t)$, the parameter error, the error dynamics are given by

$$\dot{\phi} = -P^{-1}\tilde{z}\tilde{z}^T\phi - P^{-1}\tilde{z}\,\delta(t) \tag{3.10}$$

$$\dot{P} = \tilde{z}\tilde{z}^T - \lambda P \qquad P(0) = \alpha I > 0 \tag{3.11}$$

The last term in equation (3.10) may be considered as a (state-dependent) driving term. If the undriven system is exponentially stable, then using the results of Bodson and Sastry [13], the driven system is stable as well. In turn, the undriven system is exponentially stable if and only if $\tilde{z}$ is persistently exciting i.e. (2.12) holds for $\tilde{z}$. We will give conditions, using the following two lemmas, on the persistent excitation of $\tilde{z}$ in the case when ϵ is small enough.

Lemma 3.1 Suppose that $z \in R^{n+m}$ is persistently exciting i.e.

$$\alpha_2 \geq \int_s^{s+\delta} zz^T \, dt \geq \alpha_1$$

for some $\alpha_1, \alpha_2, \delta > 0$ and for all s>0. Then, $z + \Delta z$ is also persistently exciting provided that

$$||\Delta z(\cdot)|| < (\alpha_1 / \delta)^{1/2} \tag{3.12}$$

Proof: $z + \Delta z$ is persistently exciting if for any $x \in R^{n+m}$ of unit norm

$$\alpha_2' \geq \int_s^{s+\delta} |x^T(z + \Delta z)|^2 \, dt \geq \alpha_1' \tag{3.13}$$

The upper bound on the integral in (3.13) is automatic for some α_2' simply because Δz is bounded. For the lower bound, we use the Minkowski inequality to get

$$\left(\int_s^{s+\delta} |x^T(z+\Delta z)|^2 dt\right)^{1/2} \geq \left(\int_s^{s+\delta} |x^T z|^2 dt\right)^{1/2} - \left(\int_s^{s+\delta} |x^T \Delta z|^2 dt\right)^{1/2}$$

$$\geq \alpha_1^{1/2} - \left(\int_s^{s+\delta} |\Delta z|^2 dt\right)^{1/2}$$

$$\geq \alpha_1^{1/2} - \delta^{1/2} sup\, |\Delta z(\cdot)| \quad (3.14)$$

The conclusion follows from (3.12).

To establish the norm bounded on error, we need the following lemma due to Doyle-Gohberg [6].

Lemma 3.2 If G(s) is a proper, n-th order stable rational function with Laplace inverse g(t), then

$$\int_0^{\infty} |g(t)|dt \leq 2n \sup_w |G(jw)| \quad (3.15)$$

Remark: Lemmas 3.2 and 3.2 are to be interpreted as follows:

1) Let $z(t)$ and $\Delta z(t)$ be the Laplace inverse of $(f_1 T, ..., f_n T, g_1, ... , g_m)^T u(s)$ and $(\Delta f_1 T .....\Delta f_n T, \Delta g_1, ... \Delta g_m)^T u(s)$ respectively. From (3.7) it follows that

$$\tilde{z} = z + \Delta z \quad (3.16)$$

If we assume that the true system in (2.1) satisfies the identifiability condition, then sufficient richness of the input u (in the sense of section 2) guarantees that $\tilde{z}$ is persistently exciting, provided that ϵ in equation (3.2) is small enough.

2) In practice, f_i and g_i are unknown. We may assume that the nominal plant $\tilde{T}$ satisfies the identifiability condition. In such a case, equation (3.16) still holds with $z(t)$ and $\Delta z(t)$ given by Laplace inverse of $(\tilde{f}_1 \tilde{T} . . . . , \tilde{f}_n \tilde{T}, \tilde{g}_1, ... , \tilde{g}_n)^T u(s)$ and $(\tilde{f}_1(T-\tilde{T}), ..., \tilde{f}_n(T-\tilde{T}), 0, ..., 0)^T u(s)$ respectively. Then, we get same result as in remark 1) above.

3) The classical identification can be thought of the special case of that in remark 2) as follows

$$T(s)=\frac{\beta_1 s^{m-1}+\ldots+\beta_m}{s^n+\alpha_1 s^{n-1}+\ldots+\alpha_n}\frac{1}{v(s)}\frac{q(s)}{p(s)}$$

$$=\tilde{T}(s)\frac{1}{v(s)}\frac{q(s)}{p(s)} \tag{3.17}$$

As in (3.3), $1/v(s)$ represents stable high frequency dynamics and $q(s)/p(s)$ represents near stable pole-zero cancellations. Then as pointed out in section 2,

$$g_i(s)=\frac{s^{m-i}}{(s+\alpha)^n}\frac{1}{v(s)}\frac{q(s)}{p(s)} \qquad i=1,\ldots,m$$

$$f_0=\frac{s^n}{(s+\alpha)^n},\quad f_j(s)=-\frac{s^{n-j}}{(s+\alpha)^n} \qquad j=1,\ldots,n$$

For the identifier, both $v(s)$ and $q(s)/p(s)$ are neglected and we have

$$\tilde{g}_i(s)=\frac{s^{m-i}}{(s+\alpha)^n} \qquad i=1\ldots m$$

$$\tilde{f}_j(s)=f_j(s) \qquad j=0,1,\ldots n$$

$|\Delta g_i|<\epsilon$ provided that cancellations are almost perfect and unmodeled dynamics occur at high enough frequencies.

From the form of $\delta(t)$ in (3.9) and lemma 3.2, it follows that there exists a K(m,T) depending only on sup|T(jw)| and m:=maximum order of Δf_i, Δg_j such that

$$\sup|\delta(\cdot)|\leq\epsilon K(m,T)\sup|u(\cdot)| \tag{3.18}$$

Under the condition that $\tilde{z}$ is persistently exciting, it follows that the parameter errors in (3.10) converge to a ball with radius of order ϵ(see for example, Bodson and Sastry [13]).

To end this section, let us consider the same example discussed in section 2 with $f(s)=\frac{2s+2}{s+3}\frac{s+5}{s+5.5}$ and $\tilde{f}(s)=\frac{2s+2}{s+3}$. The true closed loop transfer function is

$$T(s)=\frac{s+5}{s+5.1}\frac{0.667s+0.667}{s+1.73}$$

With the parameter k to be estimated, Figure 3.1 shows the parameter errors (for the projection type algorithm and input u(t)=5 same as in the no unmodeled dynamics case). It takes about 1 second to converge and the resulting closed loop transfer function is

$$P(s)=\frac{0.605s+0.605}{s+1.605}$$

The Bode plots of T(s) and P(s) are compared in Figure 3.3.

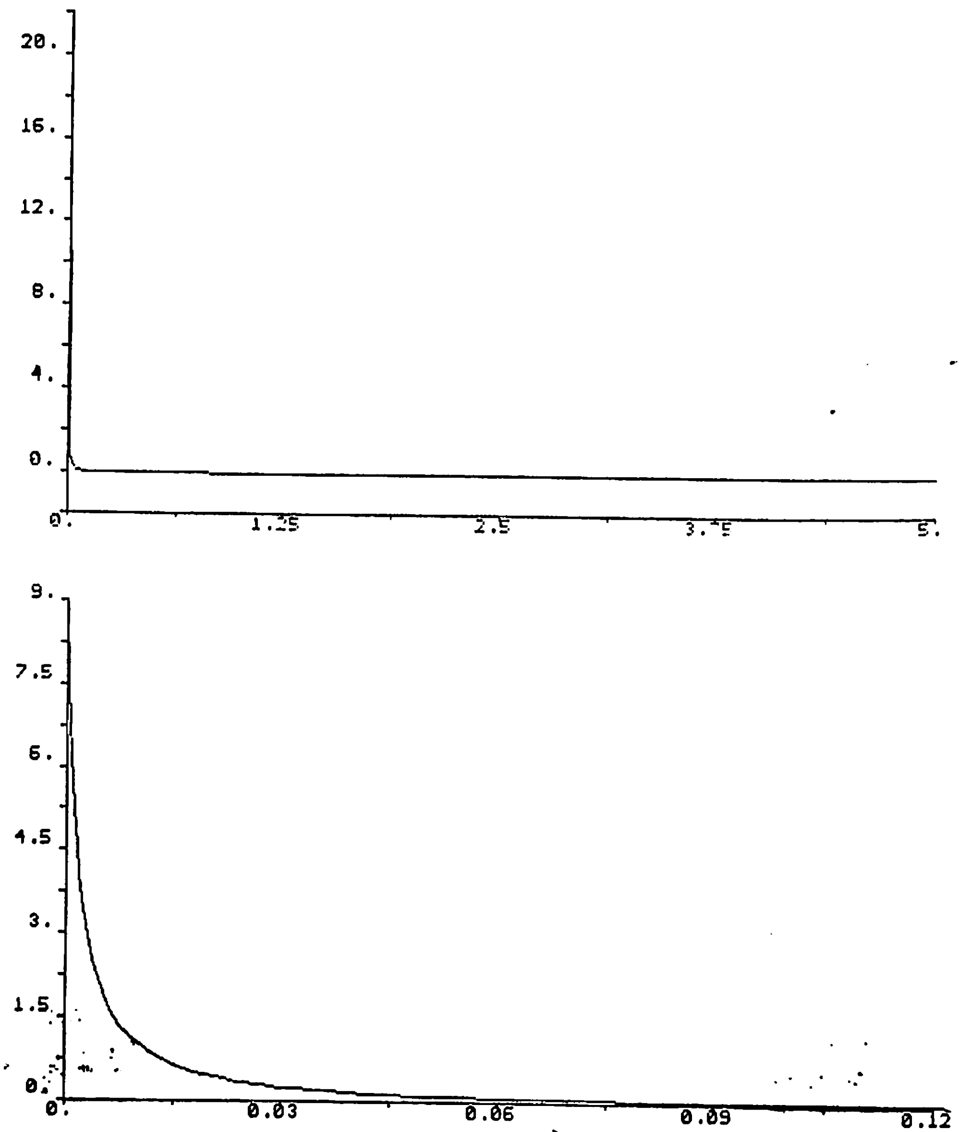

Fig. 3.1 Estimation errors of parameters a and b (above) and c (below) using prior information in the presence of unmodeled dynamics.

For the identification of the closed loop transfer function without utilizing the structure of the system, we have used input $u(t)=3+4\sin(4t)$ as in section 2. After 5000 seconds of simulation, the system does not converge. Figure 3.2 shows the estimation error of the parameter c.

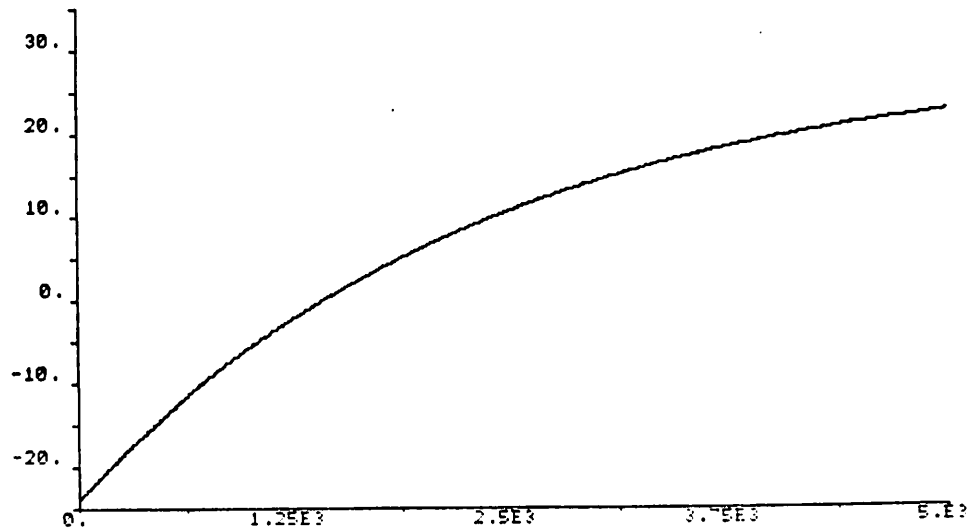

Fig. 3.2 Estimation error of parameter c without using prior information in the presence of unmodeled dynamics.

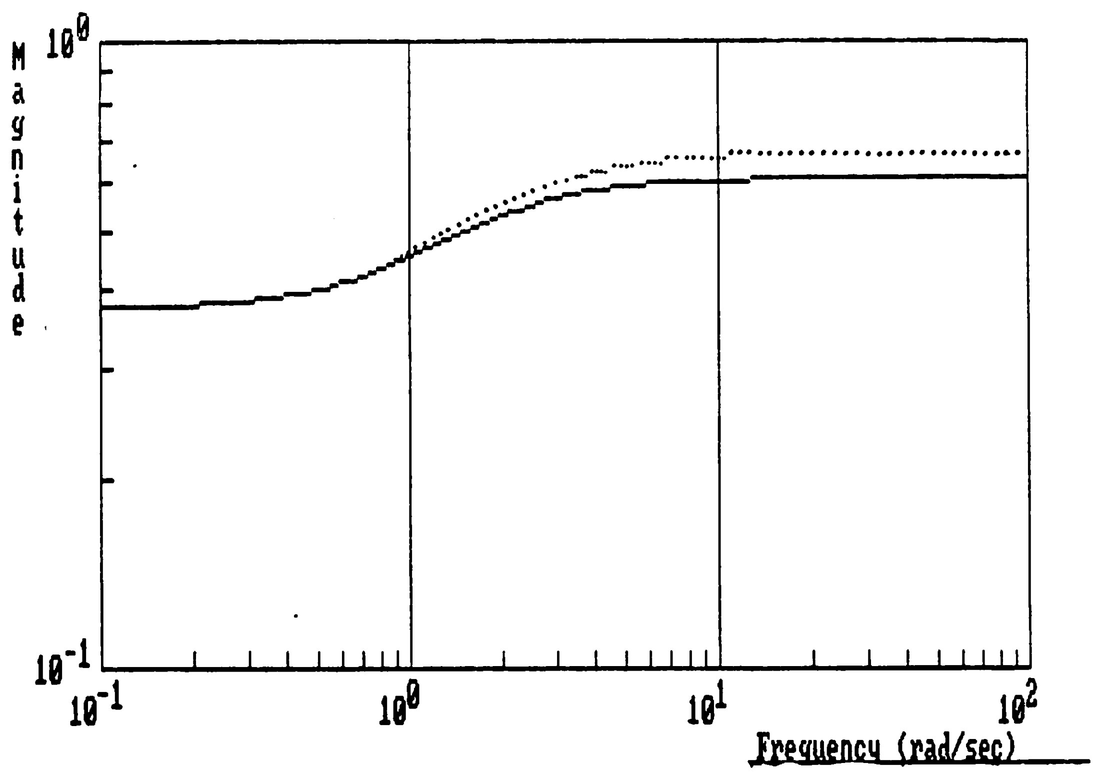

Fig. 3.3 The Bode plots of T(s) (····) and P(s) (--).

4 Discrete Time Systems

In this section ,we will deal with discrete time 'partially known' system. Unlike the continuous time case, we will use polynomials instead of rational functions for our analysis.

Consider the discrete time system described by

$$\frac{y(z^{-1})}{u(z^{-1})}=\frac{n_0(z^{-1})-\sum_{i=1}^{l} k_i n_i(z^{-1})}{d_0(z^{-1})-\sum_{i=1}^{l} k_i d_i(z^{-1})}=\frac{n(z^{-1})}{d(z^{-1})} \tag{4.1}$$

where k_i s are unknown parameters, $d_i(z^{-1})$ and $n_i(z^{-1})$ are known polynomials in the unit delay operator z^{-1}.

$$d_0(z^{-1})=1+d_{01}z^{-1}+...+d_{0n}z^{-n}$$

$$d_i(z^{-1})=d_{i1}z^{-1}+...+d_{in}z^{-n} \qquad i=1...l \tag{4.2}$$

$$n_i(z^{-1})=n_{i1}z^{-1}+...+n_{im}z^{-m} \qquad i=0.1...l \tag{4.3}$$

Assume that $n(z^{-1})$ and $d(z^{-1})$ are coprime.

Definition 4.1 A system of the form (4.1) with some unknown parameters is said to be identifiable if and only if there exist some inputs u(t) such that the unknown parameters can be uniquely determined based on input-output measurements.

Theorem 4.1 The necessary and sufficient condition for the system 4.1 to be identifiable is that the following matrix is full column rank.

$$D=\begin{vmatrix} d_{11} & d_{21} & d_{l1} \\ \cdot & \cdot & \cdot \\ d_{1n} & d_{2n} & d_{ln} \\ -n_{11} & -n_{21} & -n_{l1} \\ \cdot & \cdot & \cdot \\ -n_{1m} & -n_{2m} & -n_{lm} \end{vmatrix} \tag{4.4}$$

Proof; Rewrite 4.1 in the form

$$[d_0(z^{-1})y(z^{-1})-n_0(z^{-1})u(z^{-1})]=\sum_{i=1}^{l} k_i[d_i(z^{-1})y(z^{-1})-n_i(z^{-1})u(z^{-1})]$$

Define

$$h_i(z^{-1})=d_i(z^{-1})y(z^{-1})-n_i(z^{-1})u(z^{-1}) \qquad i=0,1...l \tag{4.5}$$

and also define the regressor vector $\phi(t)$ and parameter vector θ_0 by

$$\phi^T(t)=(y(t-1),...y(t-n),u(t-1),...u(t-m)) \tag{4.6}$$

$$\theta_0^T=(k_1,k_2...k_l) \tag{4.7}$$

Then the system can be written as

$$h_0(t)=\phi^T(t)D\theta_0 \tag{4.8}$$

The necessity may be readily seen, since if D is not full column rank, then any $\theta\in\theta_0+Null\ D$ will give the same transfer function. This situation corresponds intuitively to the case, in which there exists a $\theta=(k_1,...k_l)^T$ such that

$$\sum_{i=1}^{l}k_i n_i(z^{-1})\equiv\sum_{i=1}^{l}k_i d_i(z^{-1})\equiv 0$$

Now, we give the proof of sufficiency. By assumption, the sufficient richness of the input u(t) implies the persistent excitation of $\phi(t)$ [9], i.e. there exists $\alpha>0$ and $p\in Z_+$, such that

$$\sum_{t=t_0}^{t_0+p-1}\phi(t)\phi^T(t)\geqslant\alpha I \qquad \text{for } all\ t_0 \tag{4.9}$$

Thus, following inequality is obtained.

$$\left(\begin{bmatrix}\phi^T(t_0)\\ \vdots\\ \phi^T(t_0+p-1)\end{bmatrix}D\right)^T\begin{bmatrix}\phi^T(t_0)\\ \vdots\\ \phi^T(t_0+p-1)\end{bmatrix}D=D^T\sum_{t=t_0}^{t_0+p-1}\phi(t)\phi^T(t)D>0$$

By linear algebra, we know that the equation 4.8 has unique solution for θ_0. This completes the proof. □

For the general case, consider the system described by

$$\frac{y(z^{-1})}{u(z^{-1})}=\frac{n_0(z^{-1})-\sum_{i=1}^{i}\alpha_i n_i(z^{-1})}{d_0(z^{-1})-\sum_{j=1}^{k}\beta_j d_j(z^{-1})}=\frac{n(z^{-1})}{d(z^{-1})} \tag{4.10}$$

Assume that the notation and assumptions are same as in (4.1), then corollary 4.2 follows.

Corollary 4.2 The necessary and sufficient condition for the system (4.10) to be identifiable is that the following matrix is full column rank.

$$D=\begin{bmatrix} d_{11} & d_{k1} & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ d_{1n} & d_{kn} & 0 & 0 \\ 0 & 0 & -n_{11} & -n_{l1} \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & -n_{1m} & -n_{lm} \end{bmatrix}$$

Let us consider now another type of 'partially known' systems of the form

$$y(z^{-1})=(k_1\frac{n_1}{d_1}+...+k_l\frac{n_l}{d_l})u(z^{-1}) \tag{4.11}$$

where $n_i(z^{-1})$ and $d_i(z^{-1})$ are nonzero known coprime polynomials in the unit delay operator z^{-1}.

$$n_i(z^{-1})=n_{i1}z^{-1}+...+n_{im}z^{-m}$$

$$d_i(z^{-1})=1+d_{i1}z^{-1}+...+d_{in}z^{-n}$$

and the k_i's are unknown parameters.

Define

$$h_i(z^{-1})=\frac{n_i}{d_i}u(z^{-1}) \qquad i=1,2...l \tag{4.12}$$

Then it follows that

$$h_i(t)=-d_{i1}h_i(t-1)-...-d_{in}h_i(t-n)+n_{i1}u(t-1)+...+n_{im}u(t-m) \qquad i=1...l$$

so that

$$y(t)=(h_1(t),...,h_l(t))\begin{bmatrix} k_1 \\ \cdot \\ k_l \end{bmatrix}=\phi^T(t)D\theta_0 \tag{4.13}$$

with

$$\phi^T(t)=(h_1(t-1)...h_1(t-n)....h_l(t-1)...h_l(t-n),u(t-1)...u(t-m))$$

$$D=\begin{bmatrix} -d_{11} & 0 & 0 \\ \vdots & \vdots & \vdots \\ -d_{1n} & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & -d_{l1} \\ \vdots & \vdots & \vdots \\ 0 & 0 & -d_{ln} \\ n_{11} & n_{21} & n_{l1} \\ \vdots & \vdots & \vdots \\ n_{1m} & n_{2m} & n_{lm} \end{bmatrix} \tag{4.14}$$

Theorem 4.3 The necessary condition for system (4.11) to be identifiable is that the matrix D defined in (4.14) be of full column rank. The sufficient condition for the system (4.11) to be identifiable is that

$$rank\begin{bmatrix} \hat{d}_1 & 0 & & 0 & \hat{n}_1 \\ 0 & \hat{d}_2 & & 0 & \hat{n}_2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & \hat{d}_l & \hat{n}_l \end{bmatrix}=l \quad \text{for } all\ z \tag{4.15}$$

where

$$\hat{d}_i=z^n+d_{i1}z^{n-1}+...+d_{in} \tag{4.16}$$

$$\hat{n}_i=n_{i1}z^{m-1}+...+n_{im} \tag{4.17}$$

Proof: The proof of the necessary condition is very clear. Let us prove the sufficiency: A direct consequence of condition (4.15) is that sufficient richness of input u(t) implies the persistency of excitation of the regressor vector $\phi(t)$. Since

$$\phi(t)=A\,\phi(t-1)+bu(t) \tag{4.18}$$

where A and b are similar to those in [9], and the condition (4.15) guarantees the reachability of the system (4.18). Then persistency of excitation of $\phi(t)$ follows (see [9]). The rest of the proof is similar to the proof of theorem (4.1).

We now discuss multivariable extensions. Let us restrict ourself to the system described by

$$\begin{bmatrix} y_1(z^{-1}) \\ \vdots \\ y_p(z^{-1}) \end{bmatrix} = \begin{bmatrix} k_{11}\frac{n_{11}}{d_{11}} & . & k_{1l}\frac{n_{1l}}{d_{1l}} \\ \vdots & \vdots & \vdots \\ k_{p1}\frac{n_{p1}}{d_{p1}} & . & k_{pl}\frac{n_{pl}}{d_{pl}} \end{bmatrix} \begin{bmatrix} u_1(z^{-1}) \\ \vdots \\ u_l(z^{-1}) \end{bmatrix} \tag{4.19}$$

where $n_{ij}(z^{-1})$ and $d_{ij}(z^{-1})$ are known nonzero polynomials

$$n_{ij}(z^{-1}) = n_{ij}(1)z^{-1} + \ldots + n_{ij}(m)z^{-m} \tag{4.20}$$

$$d_{ij}(z^{-1}) = 1 + d_{ij}(1)z^{-1} + \ldots + d_{ij}(n)z^{-n} \tag{4.21}$$

k_{ij}'s are unknown parameters. Define

$$h_{ij}(z^{-1}) = \frac{n_{ij}}{d_{ij}} u_j(z^{-1}) \tag{4.22}$$

$$\phi_i^T(t) = (h_{i1}(t-1) \ldots h_{i1}(t-n) \ldots h_{il}(t-1) \ldots h_{il}(t-n),$$

$$u_1(t-1) \ldots u_1(t-m) \ldots u_l(t-1) \ldots u_l(t-m)) \tag{4.23}$$

and

$$D_i = \begin{bmatrix} -d_{i1}(1) & 0 \\ \vdots & \vdots \\ -d_{i1}(n) & 0 \\ \vdots & \vdots \\ 0 & -d_{il}(1) \\ \vdots & \vdots \\ 0 & -d_{il}(n) \\ n_{i1}(1) & 0 \\ \vdots & \vdots \\ n_{i1}(m) & 0 \\ \vdots & \vdots \\ 0 & n_{il}(1) \\ \vdots & \vdots \\ 0 & n_{il}(m) \end{bmatrix} \tag{4.24}$$

The system (4.19) can be written as

$$y_i(t) = \phi_i^T(t) D_i \theta_0^i \quad i = 1 \ldots p \tag{4.25}$$

with

$$\theta_0^i = (k_{i1}, k_{i2} \ldots k_{il})^T$$

Note that (4.25) is of the form of (4.13), so that we have following theorem resembling theorem 4.3.

Theorem 4.4 The sufficient condition for the system (4.19) to be identifiable is that

$$rank \begin{bmatrix} \hat{d}_{i1} & 0 & & 0 & \hat{n}_{i1} & 0 & & 0 \\ 0 & \hat{d}_{i2} & & 0 & 0 & \hat{n}_{i2} & & 0 \\ . & . & & . & . & . & & . \\ 0 & 0 & & \hat{d}_{il} & 0 & 0 & & \hat{n}_{il} \end{bmatrix} = l \quad \text{for } all\ z \text{ and } all\ i \tag{4.26}$$

where

$$\hat{d}_{ij} = z^n + d_{ij}(1)z^{n-1} + \ldots + d_{ij}(n) \tag{4.27}$$

$$\hat{n}_{ij} = n_{ij}(1)z^{m-1} + \ldots + n_{ij}(m) \tag{4.28}$$

Remarks; If $\hat{d}_{ij}$ and $\hat{n}_{ij}$ are coprime for all ij, then the condition 4.26 is satisfied.

5 Conclusion

We have introduced a method of identification of a class of 'partially known', stable, linear, time-invariant systems. We feel that our framework is particularly amenable to the study of the sensitivity of the schemes to the presence of unmodeled dynamics. This will prove to be particularly important when we device algorithms for the adaptive control of these 'partially known' systems.

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