STABILITY OF A CLASS OF NONRECIPROCAL CELLULAR NEURAL NETWORKS

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Abstract

Cellular neural networks provide a new and powerful approach to neural computing. Each cellular neural network is uniquely defined by a template. Many useful templates for various applications, such as geometric pattern recognition, have been published. Not only local but even global pattern features can be recognized in real-time. This is one generic and remarkable property of cellular neural networks. If these networks are symmetric, i.e. if the feedback values between the cells are reciprocal then these networks and their circuit realizations are globally asymptotically stable. Practical circuit realizations, however, inevitably give rise to nonreciprocity. In this paper we show that, for a class of practically important templates (positive and opposite-sign templates), the stability property is assured even if the symmetry/reciprocity condition is not met. Moreover, the nonreciprocity allowed in our theorems is not restricted to small or local perturbations in an otherwise reciprocal circuit.

I. INTRODUCTION

Artificial analog "neural" networks compute in real-time. None of the digital realizations can compete in this respect (even for the new generation of massively parallel digital signal processors which can significantly outperform the conventional digital-logical approach). The recently invented cellular neural networks[1] are multidimensional generalizations of neural networks, and endowed with some additional properties. The recent applications of two-dimensional cellular neural networks are quite encouraging[2]. Moreover, some new analog templates[3,4] have revealed exciting new applications and great potentials for cellular neural networks.

A necessary condition for the proper operation of a neural network is that it be globally asymptotically stable within the dynamic range of prescribed inputs. The global asymptotic stability (convergence) of a subclass of cellular neural networks defined by symmetric templates has been proved in the original paper[1]. The symmetry condition means that the feedback coefficients (prescribed by the cloning templates) between any two cells are reciprocal in the sense that the corresponding values are the same; i.e. $A(i,j;k,l) = A(k,l;i,j)$.

In this paper we show that (i) unconditional global asymptotic stability is guaranteed for all cellular neural networks defined by positive templates (all feedback template values are positive, except some which are zero) without any symmetry requirement and (ii) for all cellular neural networks defined by templates having nonzero values only in the central row or column and having opposite signs relative to the central template element, a similar stability property as well as the mechanism of the dynamics, is shown. This latter class of templates can be characterized by templates having nonzero values in the central row or column with a pattern like $(s\ 2\ -r)$, where s and r are positive.

In Section II, the various forms of asymmetry/nonreciprocity and some classes of templates are rigorously defined. Section III contains the stability result for positive templates while in Section IV the stability properties and the structure of the dynamics of opposite sign-templates are presented. Some simulation results are given in Section V. Finally, some concluding remarks and conjectures are summarized in Section VI.

II. THE GENERAL FRAMEWORK AND TWO IMPORTANT CLASSES OF NON-SYMMETRIC TEMPLATES

The dynamical system equations associated with cellular neural networks are given in Reference 1. Here we recall only the main results. The extensions of these results for cellular neural networks of dimension more than two is straight-forward.

Consider the analog processing cell circuit, henceforth called a cell, as shown in Figure 1.a with only one nonlinear element whose characteristics is shown in Figure 1.b. This cell is located in the (i,j) position of a two-dimensional regular array of M x N cells. The r-neighbourhood $N_r(i,j)$ of a typical cell C(i,j) is defined as

$$N_r(i,j) = \{ C(k,l),\ \max(|k-i|, |l-j|) = r \text{ (integer)} \} \qquad (1)$$

An r=2 neighbourhood of a cell within a cell array consists of all those cells shown shaded in Figure 1.c.

In a typical cellular neural network, we have nonlinear elements with fixed characteristics, linear elements with fixed values (C, R_x, R_y and f), as well as elements with variable parameters associated with the feedback templates A(i,j;k,l), the feed-forward templates B(i,j;k,l) and the current source I. The space invariance of the templates A and B is assumed (i.e. all cells have the same A and B as their nominal template values).

The input pattern for a cellular neural network is the set of initial values $v_{xij}(0)$ across the capacitors and possibly the input voltage sources $v_{uij}(0)$.

For example[1], a black-and-white input picture can be coded as a binary image where 1 corresponds to a black pixel and -1 corresponds to a white pixel. In the case r=1 and

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} 10^{-3}; \qquad B=0; \qquad I=0;$$

the corresponding cellular neural network has been shown to operate as a line detector; i.e. after the transients has settled down the outputs of the cells (which tend to ± 1) will extract only the horizontal lines in the picture.

The dynamical system equations describing a cellular neural network are as follows

State equation:

$$C \frac{d v_{xij}}{dt} = -\frac{1}{R_x} v_{xij}(t) + \sum_{C(k,l) \in N_r(i,j)} A(i,j;k,l)\, v_{ykl}(t) + \sum_{C(k,l) \in N_r(i,j)} B(i,j;k,l)\, v_{ukl}(t) + I \qquad (2a)$$

Output equation:

$$v_{yij}(t) = 0.5\ (|v_{xij}(t)+1| - |v_{xij}(t)-1|) = f(v_{xij}) \qquad (2b)$$

For technical reasons, which reflects also the real circuit situations, the piecewise-linear funtion f can be approximated to within any precision by a smooth (C^1) sigmoid function which preserves the monotone-increasing character.

Input equation:

$$v_{uij} = E_{ij} \qquad (2c)$$

Constraint equations:

$$|v_{xij}(0)| \leq 1 \qquad (2d)$$

$$|v_{uij}| \leq 1 \qquad (2e)$$

Parameter assumptions:

$$A(i,j;k,l)=A(k,l;i,j) \quad \text{symmetry condition} \qquad (2f)$$

$$C > 0, \quad R_x > 0 \qquad (2g)$$

The assumption (2f) implies the perfect symmetry of the feedback-template values between any two cells within a neighbourhood. In the actual circuit realization this symmetry condition is equivalent to the reciprocity of the relevant feedback controlled current sources whose values are in turn determined by transistors and resistors used in their circuit implementation.

In the basic paper[1] several important theorems were proved concerning the qualitative and quantitative properties of cellular neural networks. An important result states that if the parameters satisfy the symmetry condition then Theorem 4 of Reference 1 guarantees that the circuit will be globally asymptotically stable. Moreover, if the condition

$$A(i,j;i,j) > 1/R_x \qquad (3)$$

is satisfied, then there is no oscillation (or chaos) and all the output magnitude values are greater than or equal to 1.

In real-life circuit realizations, however, condition (2f) can never be fulfilled exactly, in view of manufacturing imperfections and parameter variations due to aging. Fortunately, many experiments and simulations have shown that even non-perfectly symmetric cellular neural networks (and nonreciprocal circuits) do function properly: they are globally asymptotically stable and always tend to constant values. Nevertheless, it is desirable that a rigorous proof be given for this much larger class of cellular neural networks.

In this paper the stability properties of two classes of *nonsymmetric* templates are considered. First, let us define them precisely.

Definition 1. The class of *positive feedback templates* for cellular neural networks is defined by the dynamical equations (2) and (3), except (2f), and in addition the following two conditions must be satisfied

(i) $A(i,j;k,l) \geq 0$ for all $C(k,l) \in N_r(i,j)$ (4)

and (ii) the non-zero (hence positive) values of $A(i,j;k,l)$ are strategically located as to give rise to a *covering template* which we define below.

A template is called a *covering template* iff any two cells in the cell array can be connected by a sequence of cells with *strictly positive* template values.

For example, the template of Figure 2.a is a covering (positive) template while the template of Figure 2.b is not. A possible route connecting two given cells for the template in Figure 2.a is shown in Figure 2.c.

Definition 2. The class of *opposite-sign templates* is defined by template values satisfying the following structures and sign conditions[†]

$$A = \begin{bmatrix} 0 & s & 0 \\ 0 & p & 0 \\ 0 & -s & 0 \end{bmatrix} \quad \text{or} \quad A = \begin{bmatrix} 0 & 0 & 0 \\ s & p & -s \\ 0 & 0 & 0 \end{bmatrix}$$

where $p > 1/R_x$ and $s > 0$

Remark 1. We are interested in the global stability properties, irrespective of the local behaviour of the equilibrium points.

Remark 2. The symmetry condition (2f) is *not* equivalent to the symmetry condition of the Jacobian matrix of the state equation because the product of two symmetric matrices need not be symmetric.

III. STABILITY OF NONRECIPROCAL CELLULAR NEURAL NETWORKS WITH POSITIVE TEMPLATES

Quite a few feedback cloning templates of cellular neural networks[2] have nonnegative elements only (e.g. noise removal, line

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[†] The results in this paper can be easily generalized to include opposite-sign templates with the nonzero entries (s p -s) located in a diagonal direction (i.e. with slope equal to +1 or -1).

detector, etc.). In what follows we will show that under some mild restrictions they are generally globally asymptotically stable without requiring any symmetry condition. This is stated formally in the next theorem.

Theorem 1.

For any given input $\underset{\sim}{v}_u$ and initial state $\underset{\sim}{v}_x$, a cellular neural network defined by the dynamical system equations (2), with smooth function f(.), and having a positive feedback template, converges to stable equilibria without nonconstant periodic solutions, except possibly for a set of measure zero.

The proof contains three steps. First, we show that the state equation is cooperative and irreducible. Next, we claim that all solutions are within a bounded range (compact closure). Finally, applying a recent stability result[5,6] we show that the network converges to stable equilibria without nonconstant periodic solutions.

Before giving the proof, we pause to consider the following two definitions.

Definition 3. A dynamical system described by

$$\frac{d\ x_i(t)}{dt} = F_i(x_1, \ldots\ldots, x_n);\ i=1,2,\ldots n \qquad \text{i.e.}$$

$$\dot{\underset{\sim}{x}} = \underset{\sim}{F}(\underset{\sim}{x});\quad \underset{\sim}{F} \in C^1 \tag{5}$$

is said to be cooperative[5,6] iff the off-diagonal elements of the Jacobian matrix J=DF(x) are positive; i.e.

$$J_{ij} = \frac{\partial F_i}{\partial x_j} \geq 0 \qquad \text{for } i \neq j$$

Definition 4. A dynamical system described by equation (5) is irreducible iff for any distinct indices i and j a chain of indices $i=k_o, \ldots, k_m=j$ can be found such that

$$J_{k_r k_{r-1}} \neq 0,\ r=1,2,\ldots m.$$

Roughly speaking, irreducibility means that any cell output can affect, at least indirectly, the states of any other cell.

We are now ready to prove Theorem 1 in three steps.

Step 1.

Consider the state and output equations (2a) and (2b). Suppose, all the cell state variables (v_{xij}) are repacked into an MxN vector $\underset{\sim}{x}$ (ordered in any sequence, e.g. columnwise). The input and output variables v_{uij} and v_{yij} are packed into $\underset{\sim}{u}$ and $\underset{\sim}{y}$, respectively (using the same ordering as for $\underset{\sim}{x}$). Without loss of generality let $R_x=1$, $C=1$ and $A(i,j;k,l)>1$. Then the state equation assumes the form

$$\dot{\underset{\sim}{x}} = \underset{\sim}{F}(\underset{\sim}{x}) = -\underset{\sim}{x} + A'\underset{\sim}{f}(\underset{\sim}{x}) + B'\underset{\sim}{u} + \underset{\sim}{I} \tag{6}$$

where $f_i(\underset{\sim}{x})=f_i(x_i)=f(x_i)$, A' and B' contains the effects of the A and B templates of all cells while all elements of $\underset{\sim}{I}$ are associated with I. The off-the-center elements of the templates A and B will convert to the off-diagonal elements of A' and B', their location being dependent upon the corresponding order of the cell variables and the vectors $\underset{\sim}{x}$ and $\underset{\sim}{u}$. The diagonal elements of A' and B' contain the central values of the templates A and B, respectively.

Observe that the Jacobian matrix of equation (6) is given by

$$J = -U + A'P$$

where U is the unit matrix and P is a positive diagonal matrix whose diagonal entries are the positive slopes of f. Hence, all of the signs of the off-diagonal elements are determined solely by A' i.e. by A.

It is quite easy now to show that any cellular neural network, i.e. the dynamical system described by equation (6), which satisfies the conditions of Theorem 1 is cooperative. This follows from the fact that the off-diagonal elements of A' are the off-the-center elements of the A templates which are nonnegative.

The irreducibility property follows from the positive covering template hypothesis. Hence, choosing any two variables in x, i.e. any two cells, it is possible to find a sequence of variables, i.e. a sequence of cells, whose mutual partial derivatives are strictly positive.

Step 2.

It has been shown (Theorem 1 in Reference 1), without exploiting the symmetry condition, that all states in a cellular neural network are bounded for all time $t>0$. This bound v_{max} can be calculated in closed form (equation (3) in Reference 1). Hence, it can easily be stated that starting from $\underset{\sim}{x}(0)=\underset{\sim}{x}_o \in W^c$ ($W^c \subset R^n$ and satisfying the constraint condition (4)) all forward orbits are bounded, i.e. they remain within a bounded set $W \subset R^n$. Moreover, if condition (3) is satisfied, the magnitude of the stable equilibrium points is greater than 1. It follows from this fact and the piecewise-linear character of f that the number of stable equilibrium points (which is greater than 1) is finite.

Step 3.

Now, we have a cooperative, irreducible system with bounded forward orbits and with a finite number of stable equilibrium points. The following Lemma 1 is a summary of Theorems 1.7, 2.4, 4.1 and 4.4 of Reference 5.

Lemma 1.

Assume that the dynamical system of equation (5) is cooperative, irreducible and has bounded forward trajectories starting from $\underset{\sim}{x}_o \in W^c \subset R^n$. Under these conditions the following statements are true.

(a) The system can not have stable, nonconstant periodic solutions;

(b) Except for a set of measure zero, if $\underset{\sim}{x}_o \in W^c$ then $\underset{\sim}{x}(t)$ approaches the equilibrium set E as t goes to infinity;

(c) Assume E is countable then $\underset{\sim}{x}(t)$ converges to an equilibrium point $\underset{\sim}{p}$ ($\underset{\sim}{F}(\underset{\sim}{p})=\underset{\sim}{0}$) uniformly for almost all $\underset{\sim}{x}_o \in W^c$ ($\underset{\sim}{p}$ is called a trap).

Hence, the cellular neural network is stable in the sense stated in Theorem 1. Q.E.D.

Remark 3.

The templates $A=\begin{bmatrix}0 & 0 & 0\\ 1 & 2 & 1\\ 0 & 0 & 0\end{bmatrix}$ and $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 2 & 0\\ 0 & 1 & 0\end{bmatrix}$ are not positive templates because they violate condition (ii) of Definition 1 (this type of templates are used for edge enhancement). They are not irreducible because the state equations can be decomposed into M or N independent separate state equations, respectively (one row or one column of cells). Within these sets of independent dynamical systems, however, these templates do qualify as positive (covering) templates (all the cells in a row/column can be covered by the strictly positive template directions.). Hence, they are globally asymptotically stable in the sense of Theorem 1.

IV. THE DYNAMICS OF OPPOSITE-SIGN TEMPLATE SYSTEMS

The type of templates presented in the last remark is characterized by the property that nonzero elements occur only in one row or in one column. Clearly, this leads to a set of independent state equations because nonzero feedback comes only from the cells in one direction (the constant terms can arise only from the dc bias I, or from constant inputs feedforward through the templates B). In this case the state equation (6), again assuming $R_x=1$ and $C=1$, takes on a particularly simple form:

$$\dot{\underset{\sim}{x}} = -U\underset{\sim}{x} + A^1\underset{\sim}{f}(\underset{\sim}{x}) - \underset{\sim}{b} \tag{7}$$

where, in the case of the opposite-sign row-type template,

$$A = \begin{bmatrix} 0 & 0 & 0 \\ s & p & -s \\ 0 & 0 & 0 \end{bmatrix} \qquad p > 1, \quad s > 0$$

and where the matrix A^1 has the form

$$A^1 = \begin{bmatrix} p & -s & & & \\ s & p & -s & & \\ & s & & & \\ & & & & -s \\ & & & s & p \end{bmatrix} \quad ; \qquad \underset{\sim}{b} = \underset{\sim}{I} + \sum_{C(k,l) \in N_r(i,j)} B(i,j;k,l)\, u_{kl} \tag{8}$$

In (7) U denotes an n-dimensional unit matrix and $\underset{\sim}{b} \in R^n$, where n denotes the number of cells in each row. In the case of the column-type opposite-sign template, the <u>form</u> of the state equation (for a column of cells) is the same.

Let us now analyze the dynamics of the class of opposite-sign template cellular neural networks described by equation (7). Our following analysis depends crucially on the following lemma.

<u>Lemma 2.</u>

$$(A^1 - U) \in P \tag{9}$$

where P denotes the class of real nxn matrices with the property that all of their principal minors are strictly positive. One important property of these matrices is the fact that the real part of all of their eigenvalues are positive[7]. These matrices played a crucial role in ensuring the uniqueness of DC solutions of nonlinear circuits[8].

Observe that $(A^1 - U)$ has the same form as A^1 except that its diagonal elements are given by $q=p-1 > 0$ (instead of p).

<u>The proof</u> of Lemma 2 is as follows.

First, observe that any principal minor of order $n > 1$ has the same form as one of the successive principal minors of $(A^1 - U)$, or having a block-diagonal form containing blocks of this form. Hence, if all of the successive principal minors of $(A^1 - U)$ are positive, then $(A^1 - U) \in P$.

Next, we show that all successive principal minors of $(A^1 - U)$ are positive. The first, second and third principal minors are given respectively by:

$$\det^1 = q > 0, \qquad \det^2 = q^2 + s^2 > 0 \quad \text{and} \quad \det^3 = q\det^2 + s^2\det^1 > 0$$

Suppose, the principal minors up to the k-th order $(k > 2)$ are positive. Then the (k+1)st one is positive again. This is true because expanding the determinant along the first row, we obtain

$$\det^{k+1} = q\det^{k} + s^{2}\det^{k-1}$$

<u>Remark 4.</u> It is clear from the above proof that Lemma 2 is true even if the feedback template in (7) is generalized to the <u>non-symmetric</u> template

$$A = \begin{bmatrix} 0 & 0 & 0 \\ r & p & -s \\ 0 & 0 & 0 \end{bmatrix} \qquad p > 1, \quad r > 0, \quad s > 0,$$

In other words , the property $(A^{1} - U) \in P$ is valid <u>without</u> the additional assumption of "value symmetry", or "sign symmetry" for this special class of templates. What really matters is the <u>sign asymmetry</u> of the template values (in addition to the assumptions $p > 1$ and the special structure of the templates).

<u>Remark 5.</u> It is important in our following proof to realize that if a matrix belongs to the class P, then all of their eigenvalues have a positive real part[7].

Returning to our investigation of the dynamics let us classify the states of the cells according to the values assumed by their respective state variables v_{xij} or x_k. If a cell characterized by the k-th state variable x_k is in the range $|x_k| < 1$ i.e. $f(x_k) = x_k$ then it is said to be <u>operating in region α</u>. Conversely, if $|x_k| \geq 1$ the cell is said to be <u>operating in region β</u>.

The mechanism of the dynamics is characterized by the next theorem.

<u>Theorem 2</u>.

Let N be a cellular neural network with opposite-sign templates and hence described by the piecewise-linear state equation (7). Then the dynamics of N starting from any initial state satisfying the constraint equations (2d) and (2e) has the following properties.

(1) If all cells are operating in region α at any time $t = t_j$ then all eigenvalues of the associated Jacobian matrix have positive real parts at $t = t_j$.

(2) If all cells are operating in region β then all eigenvalues are -1, hence, they tend to decay to a constant.

(3) If there are b variables operating in region β, while the rest are operating in region α then the Jacobian matrix has b eigenvalues equal to -1, while all the other eigenvalues have a positive real part.

Remark 6. In view of Theorems 2 and 5 of Reference 1 a cellular neural network with opposite-sign templates and operating in region β must converge to a stable equilibrium point even if the network is nonreciprocal. Furthermore, if at least one cell is operating in region α then the circuit can not be in equilibrium because there is at least one cell whose dynamics is growing exponentially.

Proof of Theorem 2.

(1) If all cells are operating in region α, then equation (7) has the form

$$\dot{\underset{\sim}{x}} = -U\underset{\sim}{x} + A^1\underset{\sim}{x} - \underset{\sim}{b} = (A^1-U)\underset{\sim}{x} - \underset{\sim}{b} \qquad (10)$$

In this case the Jacobian matrix is equal to (A^1-U), which is a P-matrix in view of Lemma 2. It follows that all eigenvalues have positive real parts.

(2) If all cells are operating in region β then equation (7) has the form

$$\dot{\underset{\sim}{x}} = -U\underset{\sim}{x} + A^1\underset{\sim}{f}(\underset{\sim}{x}) - \underset{\sim}{b} = -U\underset{\sim}{x} - \underset{\sim}{b}' \qquad (11)$$

where $\underset{\sim}{b}' = \underset{\sim}{b} - A^1\underset{\sim}{f}(\underset{\sim}{x})$, $f_i(x_i) = 1$ or -1

In this case the Jacobian matrix is equal to $-U$. It follows that all eigenvalues are equal to -1. That is, all state variables are decaying to a constant until they are operating in this region.

(3) Suppose, there is one cell operating in region β (the k-th cell) while all the others are operating in region α. We show that one eigenvalue of the Jacobian matrix is equal to -1 while all the other have positive real parts. The general case when we have b variables operating in region β can be proved by a repetitive application of the above proof for the one variable case.

So, suppose the k-th variable is operating in region β and all the others are operating in region α. Equation (7) then becomes

$$\dot{\underset{\sim}{x}} = A^{\alpha\beta}\underset{\sim}{x} - \underset{\sim}{b}^k \qquad (12)$$

where

$$A^{\alpha\beta}=\left[\begin{array}{c|cccccccccc}
 & & & & & k-1 & k & k+1 & & & \\
\hline
 & q & -s & & & & & & & & \\
 & s & q & -s & & & & & & & \\
 & & s & . & . & & & & & & \\
 & & & . & . & -s & & & & & \\
k-1 & & & & s & q & 0 & & & & \\
k & & & & & s & -1 & -s & & & \\
k+1 & & & & & & 0 & q & -s & & \\
 & & & & & & & s & . & . & \\
 & & & & & & & & . & . & -s \\
 & & & & & & & & & s & q
\end{array}\right] ; \qquad b^k=\left[\begin{array}{c} b_1 \\ b_2 \\ . \\ b_{k-2} \\ b_{k-1}-sh \\ b_k+ph \\ b_{k+1}+sh \\ b_{k+2} \\ . \\ b_n \end{array}\right]$$

and where $q=p-1$; $h=f(x_{k+1})= 1$ or -1 ..

The Jacobian matrix $J=A^{\alpha\beta}$ and its eigenvalues are the roots of the determinant of $(\lambda U-A^{\alpha\beta})$. Expanding the determinant along the k-th column, we obtain

$$\det(\lambda U_n-A^{\alpha\beta}) = \det(\lambda U_{k-1}-A_1)(\lambda+1)\det(\lambda U_{n-k}-A_2) \qquad (14)$$

Since one eigenvalue is equal to -1 and the matrices A_1 and A_2 have the same form as A^1-U (with different order) it follows from Lemma 2 that they are P-matrices, and hence all their eigenvalues have positive real parts. Q.E.D.

V. SOME SIMULATION RESULTS

To investigate how changes in the numerical values in a template affect the steady state response, we simulated several examples numerically using the same PWLSPICE program which was used in Reference 2. The following three examples are instructive in that they reveal some interesting dynamic behaviours. In all examples (motivated by Reference 4) the basic parameters were the same[2] (relative units, C, R_x, etc.). In each example , 20 cells per row are processed. The numerical values at each sampled time step (a snap shot) are color coded as follows:

yellow -1, orange 0, red 1 and blue is the background.

The successive rows from the top to the bottom are the subsequent transient values starting from the initial condition corresponding to the pixel values of the picture to be processed and taking on the values at discrete time steps (snap shots) in the interval 0-10 μs. The three examples are shown on Figure 3, using the same $b_i=0.5$ and the same initial picture.

<u>Example 1</u>. Consider first the opposite-sign template

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

The resulting transient is shown on Figure 3.a as a sequence of snap shots. Notice that this circuit is stable.

<u>Example 2</u>. Consider next the template

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

where the absolute values of the off-the-center entries are different. The resulting transient is shown in Figure 3.b. Although the result is different from that of Example 1 this circuit is stable nevertheless.

<u>Example 3</u>. Finally, consider the template

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0.8 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

where the conditions $A(i,j;i,j) > 1/R_x$ and $(A^1-U) \in P$ are barely violated. The resulting transient is shown in Figure 3.c. Observe that this time, the circuit oscillates.

VI. CONCLUDING REMARKS

Cellular neural networks with appropriate choice of templates can solve, among other things, local and global pattern recognition problems. The global stability of these networks has been proved earlier for the symmetric/reciprocal cases where the feedback values between the different cells within a neighbourhood are the same in both directions. In this paper we have shown that, at least for some interesting classes of templates, this symmetry/reciprocity condition is in general not necessary for global stability. Moreover, our conditions are robust in the sense that they require neither precise template-value relation, nor a closeness to some prescribed values. On the other hand, we have also seen examples such that violating some basic conditions would give rise to oscillations.

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FIGURE CAPTIONS

Figure 1. The cell circuit and its neighbourhood in a cell array

a. The cell circuit. Indices u, x and y refer to the input, state and output, respectively; v denotes the node-to-datum voltage, E and I refer to the independent voltage and current sources. I_{xy} denotes the current sources controlled by the output voltages of the cells in the r-neighbourhood and I_{xu} denotes the current sources controlled by the input voltages of the cells in the r-neighbourhood.

b. Characteristics of the single nonlinear element (a voltage-controlled current source) in the cell circuit.

c. An r=2 neighbourhood in part of a cell array

Figure 2. Illustration of a positive template covering.

Figure 3. Transient response of three simulation studies:

a. Example 1
b. Example 2
c. Example 3

FIGURES

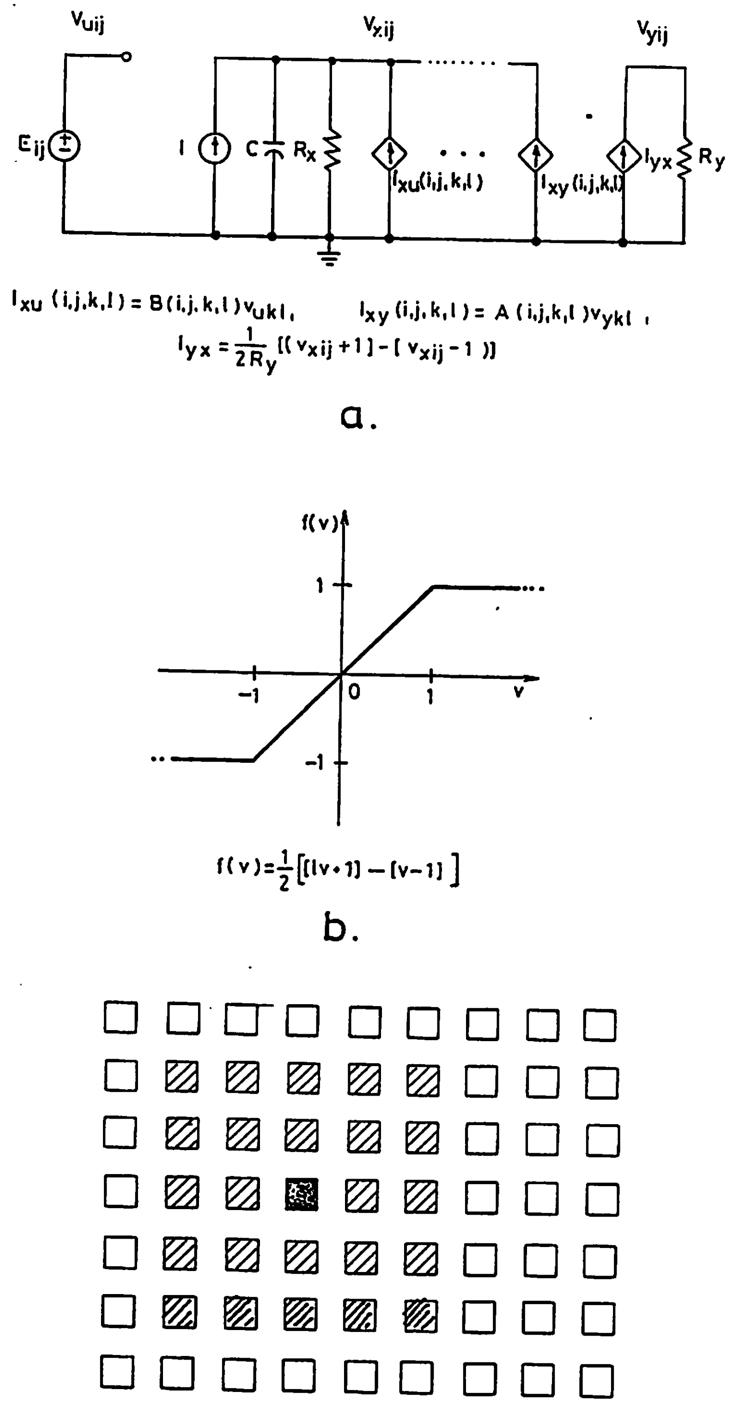


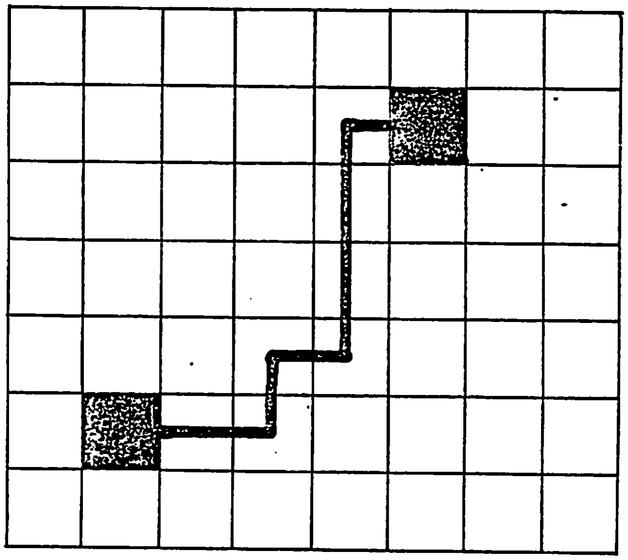
FIGURE 1.

0	1	0
1	2	1
0	1	0

a.

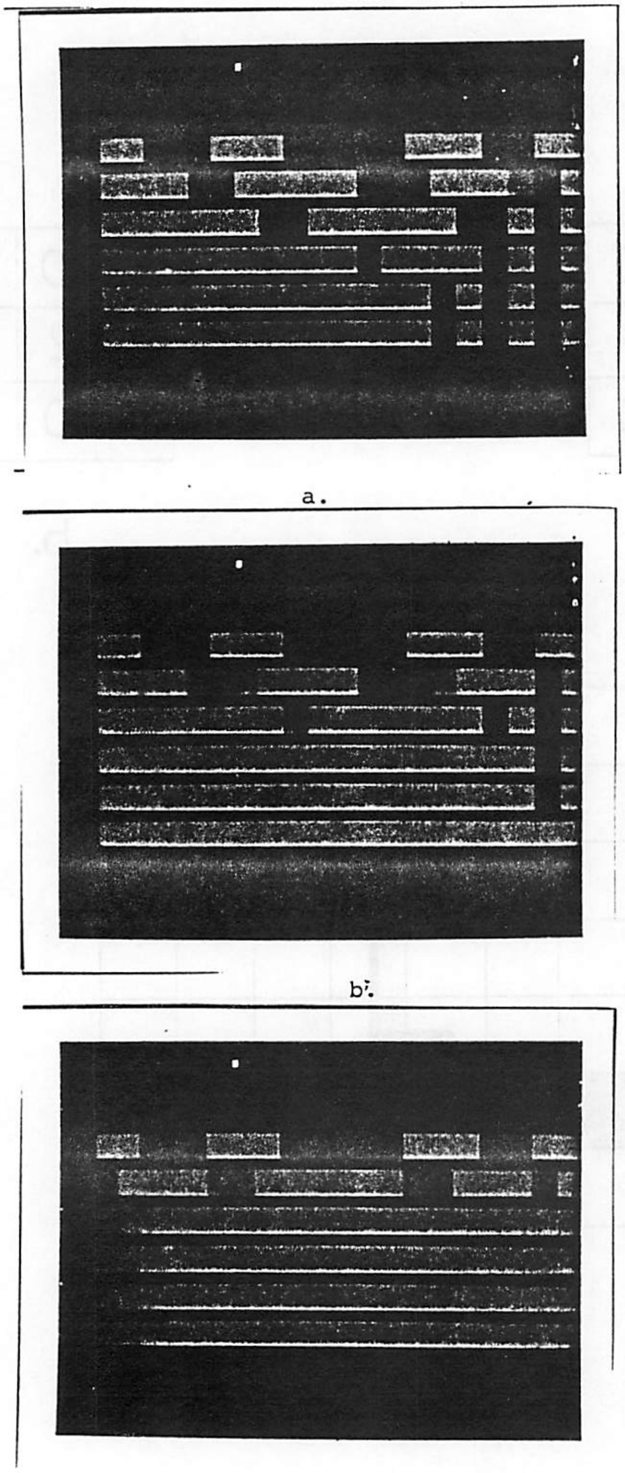
0	0	0
0	2	1
0	0	0

b.



c.

FIGURE 2.



a.

b.

c.

Figure 3.

Footnotes

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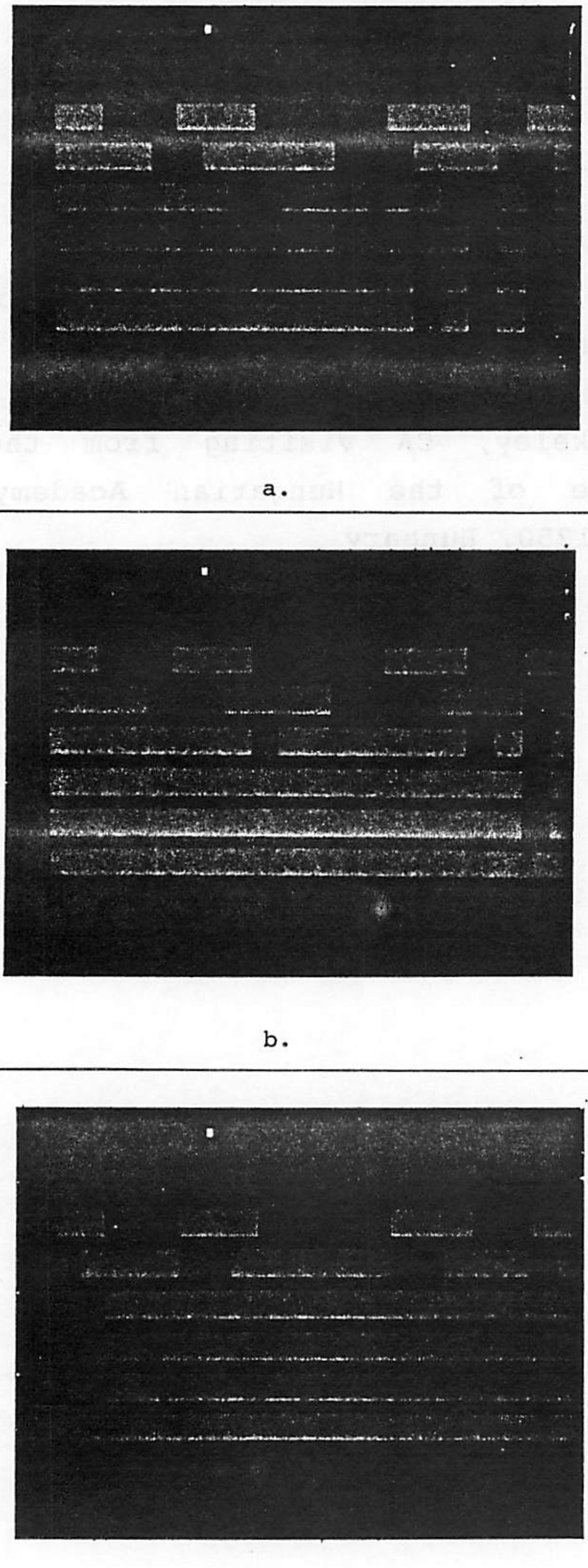

a.

b.

c.

FIGURE 3.