INDIRECT TECHNIQUES FOR ADAPTIVE INPUT OUTPUT LINEARIZATION OF NONLINEAR SYSTEMS

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Andrew Teel, Raja Kadiyala, Petar Kokotovic, and Shankar Sastry

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Indirect Techniques for Adaptive Input Output Linearization of Nonlinear Systems *

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Abstract

A technique of indirect adaptive control based on certainty equivalence for input output linearization of nonlinear systems is proven convergent. It does not suffer from the overparameterization drawbacks of the direct adaptive control techniques on the same plant. This paper also contains a semi-indirect adaptive controller which has several attractive features of both the direct and indirect schemes.

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1 Introduction

There has been much recent research in the use of adaptive control techniques for improving the input output linearization by state feedback of nonlinear systems with parametric uncertainty. Techniques of direct adaptive control (with no explicit identification) were proposed and developed in Taylor *et al.* (1989), Kanellakopoulos *et al.* (1989), Georgiou and Normand-Cyrot (1989), Sastry and Isidori (1987), (see also Sastry and Bodson (1989)). Nonlinear indirect adaptive control was initiated in Bastin and Campion (1989), Campion and Bastin (1989), Pomet and Praly (1988). It is motivated by the fact that, with exact knowledge of the plant parameters, a nonlinear state feedback law and a suitable set of coordinates can be chosen to produce linear input-output behavior. In the case of parameter uncertainty, intuition suggests that parameter estimates which are converging to their true values can be used to asymptotically linearize the system. This heuristic is known as the *certainty equivalence principle.* Indirect adaptive control differs from direct adaptive control in that it relies on an observation error to update the plant parameters rather than relying on an output error. Indirect adaptive control can be broken down into two parts. First, a parameter identifier is attached to the plant and adjusts the parameter estimates on line. These estimated parameters are then used in the linearizing control law.

In this paper we continue a program of investigating indirect adaptive control of nonlinear systems. We extend the results of Campion and Bastin (1989) and Pomet and Praly (1988) to input-output linearizable systems and adaptive tracking.

In section 2, we review two identifier structures for nonlinear systems, (they have appeared in Bastin and Campion (1989), Kreisselmeier (1977), Kudva and Narendra (1973), Luders and Narendra (1973)). Simulation results for these are given in section 3. Section 4 gives an outline of an indirect adaptive controller based on certainty equivalence along with a proof of convergence. We also present a semi-indirect adaptive controller which contains attractive features of the direct and indirect schemes. Section 5 contains a simulation comparison of a direct, indirect adaptive, semi-indirect adaptive, and non-adaptive controller methodology. Section 6 gives some conclusions.

2 Identifier Structures

Consider the system

$$\dot{x} = f(x, \theta^*) + g(x, \theta^*)u \tag{1}$$

with $x \in \mathbb{R}^n$,$u \in \mathbb{R}$,$\theta^* \in \mathbb{R}^p$ and f, g are assumed to be smooth vector fields on $\mathbb{R}^n$. Further let $f(x, \theta^*)$ and $g(x, \theta^*)$ have the form

$$\begin{aligned} f(x,\theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* f_i(x) \\ g(x,\theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* g_i(x) \end{aligned} \tag{2}$$

Here θ_i^*, $i = 1, \ldots, p$, are unknown parameters, which appear linearly, and the smooth vector fields $f_i(x)$, $g_i(x)$ are known. If we formulate the regressor

$$w^T(x, u) = [f_1(x) + g_1(x)u, \ldots, f_p(x) + g_p(x)u] \tag{3}$$

so that $w^T(x, u) \in \mathbb{R}^{n \times p}$ contains all of the nonlinearities of the system, then (1) can be written as

$$\dot{x} = w^T(x, u)\theta^* \tag{4}$$

For a system with multiple inputs, the regressor is formed in an analogous manner and (4) holds except that the notation used to define w is more involved.

2.1 Observer-based Identifier

To estimate the unknown parameters, we will use the identifier system

$$\begin{aligned} \dot{\hat{x}} &= A(\hat{x} - x) + w^T(x, u)\hat{\theta} \\ \dot{\hat{\theta}} &= -w(x, u)P(\hat{x} - x) \end{aligned} \tag{5}$$

Here $A \in \mathbb{R}^{n \times n}$ is a Hurwitz matrix and $P \in \mathbb{R}^{n \times n} > 0$ is a solution to the Lyapunov equation

$$A^T P + PA = -Q, \quad Q > 0 \tag{6}$$

This identifier is reminiscent of one proposed in Kudva and Narendra (1973), Kreisselmeier (1977). Note that $A = -\sigma I$ is a special case of the identifier. If we define $e_1 = \hat{x} - x$, the observer state error, and $\phi = \hat{\theta} - \theta^*$, the parameter error, and assume θ^* to be constant but unknown then we have the error system

$$\begin{aligned} \dot{e}_1 &= Ae_1 + w^T(x, u)\phi \\ \dot{\phi} &= -w(x, u)Pe_1 \end{aligned} \tag{7}$$

One should note the similarity of the error equation above with that of the error equation of a full order observer, although all the states are available by assumption.

Theorem 2.1 Stability of Observer-based Identifier
Consider the observer-based identifier of equation (7),

then
1. $\phi \in L_\infty$,
2. $e_1 \in L_\infty \cap L_2$,
3. If $w(x,u)$ *is bounded,*
   then $\dot{e}_1 \in L_\infty$ *and* $\lim_{t\to\infty} e_1(t) = 0$.

Remarks:

1. The proof is a standard Lyapunov argument on the function

$$V(e_1, \phi) = e_1^T P e_1 + \phi^T \phi \tag{8}$$

2. The condition on the boundedness of w is a stability condition. In particular, if the system is bounded-input bounded-state (bibs) stable with bounded input, then w is bounded. (see Sastry and Bodson (1989))

3. Theorem 2.1 makes no statement about parameter convergence. As is standard in the literature one can conclude from (7) that e_1 and ϕ both converge exponentially to zero if w is sufficiently rich, ie., $\exists \alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_1 I \geq \int_s^{s+\delta} w w^T dt \geq \alpha_2 I \tag{9}$$

   This condition is impossible to verify explicitly ahead of time since w is a function of x. If we assume that the regressor is bounded it is clearly not necessary to have the upper bound in (9). Henceforth, when we use this result we will assume that the regressor is bounded.

2.2 Filtered Regressor Identifier

Consider filtered forms of w,x given by W,W_0 and defined by

$$\begin{aligned} \epsilon \dot{W} &= -W + \epsilon w^T(x,u) \\ \epsilon \dot{W}_0 &= -W_0 + x \end{aligned} \tag{10}$$

The state can be reconstructed from the filtered regressor and filtered state as

$$x = W\theta^* + W_0 + [x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}} \tag{11}$$

The equivalence is shown by observing that

$$\begin{aligned}
\dot{x} &= \dot{W}\theta^* + \dot{W}_0 - \tfrac{1}{\epsilon}[x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}} \\
&= \tfrac{1}{\epsilon}[-W + \epsilon w^T(x,u)]\theta^* + \tfrac{1}{\epsilon}[-W_0 + x] - \tfrac{1}{\epsilon}[x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}} \\
&= \tfrac{1}{\epsilon}[-W + \epsilon w^T(x,u)]\theta^* + \tfrac{1}{\epsilon}\{-W_0 + W\theta^* + W_0 + [x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}}\} \\
&\quad -\tfrac{1}{\epsilon}[x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}} \\
&= w^T(x,u)\theta^*
\end{aligned} \tag{12}$$

We can form the estimated state as

$$\hat{x} = W\hat{\theta} + W_0 \tag{13}$$

and then, if we define $e_2 = \hat{x} - x$, we have

$$e_2 = W\phi - [x(0) - W(0)\theta^* - W_0(0)]e^{-\frac{t}{\epsilon}}. \tag{14}$$

This form of the identifier was proposed in Pomet and Praly (1988), Bastin and Campion (1989).

2.2.1 Gradient Algorithm

To estimate the unknown parameters, we can use the *gradient algorithm* which yields the following error system:

$$\dot{\phi} = -gW^T e_2 \qquad g > 0 \tag{15}$$

Theorem 2.2 Stability of Filtered Regressor Identifier Using the Gradient Method
Consider the filtered regressor identifier and the gradient algorithm of equation (15),

then
1. $\phi \in L_\infty$,
2. $e_2 \in L_2$,
3. If $w(x,u)$ *is bounded,* then $e_2, \dot{e}_2 \in L_\infty$ *and* $\lim_{t\to\infty} e_2(t) = 0$.

Remarks:

1. The proof is a standard Lyapunov argument on the function

$$V(\phi) = \frac{1}{2}\phi^T\phi \tag{16}$$

2. The condition on the boundedness of w is a stability condition. In particular, if the system is bounded-input bounded-state (bibs) stable with bounded input, then w is bounded. (see Sastry and Bodson (1989))

3. Theorem 2.2 makes no statement about parameter convergence. Parameter convergence is implied by w being sufficiently rich (cf. equation (9).)

2.2.2 Least-Squares Identifier

Another approach for estimating the parameters is the *least-squares algorithm* which can be used with the filtered regressor identifier but not with the observer-based identifier. This algorithm produces the following error system:

$$\begin{aligned} \dot{\phi} &= -\gamma\Gamma W^T e_2 \\ \dot{\Gamma} &= -\gamma\Gamma W^T W\Gamma \qquad \gamma > 0 \quad \Gamma(0) > 0 \end{aligned} \tag{17}$$

Theorem 2.3 Stability of Filtered Regressor Identifier Using the Least-Squares Method
Consider the filtered regressor identifier and the least-squares algorithm of equation (17),

then *1.* $\phi \in L_\infty$,

2. $e_2 \in L_2$,

3. If $w(x,u)$ *is bounded,*
then $e_2, \dot{e}_2 \in L_\infty$ *and* $\lim_{t\to\infty} e_2(t) = 0$.

Remarks:

1. The proof is a standard Lyapunov argument on the function

$$V(\phi) = \phi^T\Gamma^{-1}\phi \tag{18}$$

2. The same remarks as those after Theorem 2.2 concerning parameter convergence hold.

3 Identifier Examples

3.1 The Induction Motor

This model for the induction motor was presented in Luca and Ulivi (1987) for demonstrating a non-adaptive linearizing control scheme. This particular model for the motor was selected because it lends itself nicely to linearization through static state feedback thus keeping the nonlinear theory relatively simple and allowing us to focus on the implementation of the adaptive portion of the controller. This example was used for a direct adaptive controller application in Georgiou and Normand-Cyrot (1989).

The state equations for this system may be written in the familiar form of:

$$\dot{x} = f(x) + \sum_{i=1}^{3} g_i(x)u_i \quad y_i = h_i(x) \tag{19}$$

where the states are chosen to be the components of the stator current and stator flux, $x = [i_{ds}\ i_{qs}\ \phi_{ds}\ \phi_{qs}]^T$. The inputs are the projections of the supply voltage onto the stator direct and quadrature axes and the slip frequency, $u = [v_{ds}\ v_{qs}\ \omega_s]^T$.

With these choices made, we have:

$$\begin{aligned}
f(x) &= Ax = \begin{bmatrix} -(\alpha+\beta) & 0 & \frac{\beta}{L_s} & \frac{\omega}{\sigma L_s} \\ 0 & -(\alpha+\beta) & -\frac{\omega}{\sigma L_s} & \frac{\beta}{L_s} \\ -\alpha\sigma L_s & 0 & 0 & \omega \\ 0 & -\alpha\sigma L_s & -\omega & 0 \end{bmatrix} x \\
g(x) &= \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & x_2 \\ 0 & \frac{1}{\sigma L_s} & -x_1 \\ 1 & 0 & x_4 \\ 0 & 1 & -x_3 \end{bmatrix} = [g_1(x)\ g_2(x)\ g_3(x)]
\end{aligned} \tag{20}$$

where $\alpha = \frac{R_s}{\sigma L_s}$, $\beta = \frac{R_r}{\sigma L_r}$, $\sigma = 1 - \frac{M^2}{L_s L_r}$. The stator and rotor resistances are R_s and R_r, respectively. L_s and L_r represent the stator and rotor self-inductances and M is the mutual inductance.

We further partitioned the function $f(x)$ to break apart the dependence on the parameters to be identified, $\theta = [\alpha\ \beta]^T$, so that

$$\dot{x} = f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2 + f_3(x) + \sum_{i=1}^{3} g_i(x)u_i \tag{21}$$

where

$$f_1(x) = \begin{bmatrix} -x_1 & -x_1 + \frac{1}{L_s}x_3 \\ -x_2 & -x_2 + \frac{1}{L_s}x_4 \\ -\sigma L_s x_1 & 0 \\ -\sigma L_s x_2 & 0 \end{bmatrix}, \quad f_2(x) = A_2 x = \begin{bmatrix} 0 & 0 & 0 & \frac{\omega}{\sigma L_s} \\ 0 & 0 & -\frac{\omega}{\sigma L_s} & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix} x \tag{22}$$

We chose α and β as our adaptation parameters since they depend on the stator and rotor resistances, which tend to vary during operation.

3.2 Filtered Regressor Identifier

In the following there will be a slight abuse of notation as we mix time and frequency domain functions with the understanding that a time domain function preceded by a frequency domain function represents the filtering of the time domain signal by the frequency domain function. Thus, the equations of the induction motor cast into the filtered regressor identifier scheme (10) are:

$$\begin{aligned} \dot{x} &= f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2 + f_3(x) + g(x)u \\ \frac{sx}{\epsilon s+1} &= \frac{1}{\epsilon s+1}\left[f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2 + f_3(x) + g(x)u\right] \\ \frac{sx}{\epsilon s+1} - \frac{1}{\epsilon s+1}\left[f_3(x) + g(x)u\right] &= \frac{1}{\epsilon s+1}\left[f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2\right] \\ e &= \frac{1}{\epsilon s+1}\left[f_1(x)\hat{\theta}_1 + f_2(x)\hat{\theta}_2\right] \\ &- \frac{1}{\epsilon s+1}\left[f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2\right] \\ \dot{\hat{\theta}} &= -g\left[f_1(x)\ f_2(x)\right]e \end{aligned} \tag{23}$$

The constant ϵ determines the cut-off frequency of the filters. The smaller ϵ is the higher the cut-off frequency. It is clear that as ϵ tends to zero, $\frac{sx}{\epsilon s+1}$ approaches sx. In the Laplace domain sx is equivalent to the differentiation of x in the time domain. The above filter is sometimes called a *dirty derivative* since it approximates the derivative for small ϵ.

3.3 Observer Based Identifier

The observer based equations for the induction motor may be written as (see (4) - (7)):

$$\dot{x} = f_1(x)\theta^*{}_1 + f_2(x)\theta^*{}_2 + f_3(x)g(x)u$$

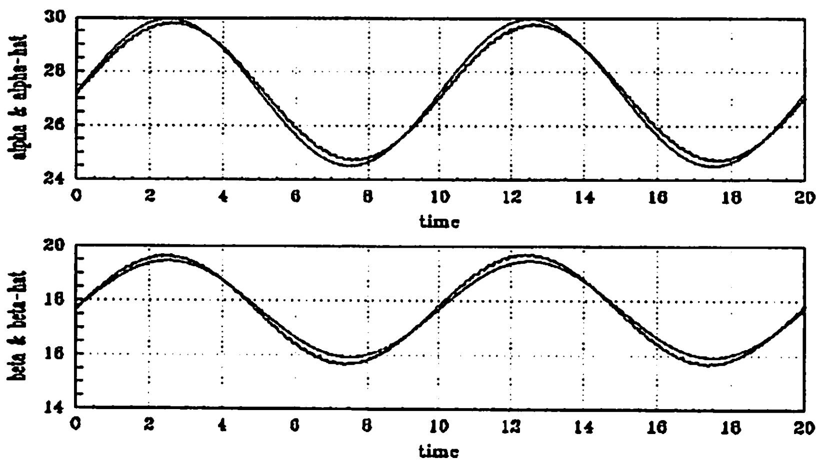

Figure 1: Open Loop Identification - Observer Based

$$
\begin{aligned}
\dot{\hat{x}} &= -\sigma\hat{x} + \sigma x + f_1(x)\hat{\theta}_1 + f_2(x)\hat{\theta}_2 + f_3(x) + g(x)u \\
\dot{e} &= -\sigma e + f_1(x)(\hat{\theta}_1 - \theta^*{}_1) + f_2(x)(\hat{\theta}_2 - \theta^*{}_2) \\
\dot{\phi} &= -w(x,u)e
\end{aligned}
\tag{24}
$$

3.4 Simulation Results

The two identifiers for the motor were simulated with $\sigma = 0.064$, $L_s = 0.179$ H, and nominal values of $\alpha = 27.232$ sec. and $\beta = 17.697$ sec. The inputs were chosen as a series of step inputs. More specifically, magnitude two (peak to peak) square waves of 1 Hz, 3 Hz, and 5 Hz were used for v_{ds}, v_{qs}, and ω_s, respectively. The parameters, α and β, varied sinusoidally according to:

$$
\begin{aligned}
\alpha &= 27.232(1 + 0.1\sin(0.2\pi t)) \\
\beta &= 17.697(1 + 0.1\sin(0.2\pi t))
\end{aligned}
\tag{25}
$$

The update gain g in the gradient algorithm (15) was chosen by looking at the error terms for a modest gain of 0.5 and then scaling g to achieve a good response, but not to make it so large to cause the parameter to move excessively for small errors or noise. In general, the maximum error multiplied by the gain should be less than one. With this in mind we picked $g = 5$.

Both the observer based identifier scheme and filtered regressor identifier scheme were quite tolerant of the choice of filter gains, σ and ϵ, respectively.

(Note that the bandwidth of the filtered regressor filter is inversely proportional to ϵ). However, too small of a gain for the observer based and too large for the filtered regressor would cause excessive lag in the tracking of the parameters. One should make sure ϵ is small enough to avoid filtering out the excitation from the input.

We found that $\sigma = 10$ and $\epsilon = 0.1$ provided good response without any ill effects from noise. These gains both result in a time constant of 0.1, which may seem large in comparison with the time constant of the motor, about $\frac{1}{17}$. This seems to be in conflict with the choice of gains for a state estimator, where a general rule of thumb is to choose the gains so that the estimator is twice as fast as the plant, but we are assuming full knowledge of the states. The identifier gains determine how fast our parameters converge.

The results were quite similar for the two identifiers. They both, however, exhibited a slight lag from the true estimates (see figures 1 and 2). The inputs could have been made richer to improve this, but overall the observer based identifier had a better response than the filtered regressor – note the difference in time scales for the two plots. The observer based scheme's response was faster (given the same transient behavior). The filtered regressor scheme could be made to be as quick as the observer, but the resulting transient response suffered (larger overshoot and ringing).

The observer based method also has the advantage of using fewer states than the filtered regressor. The latter method must filter each component of the regressor (in this case there are two zero entries), then it must filter x to create a *dirty derivative*, and finally filter the system using the parameter estimate. The observer based adds only n integrators, where n is the order of the system, to the standard p, number of parameters, integrators for the update law. For the induction motor example the filtered regressor has ten more states.

4 Indirect Adaptive Control

Nonlinear *indirect* adaptive control is motivated by the fact that, with exact knowledge of the plant parameters, a nonlinear state feedback law and a suitable set of coordinates can be chosen to produce linear input-output behavior. Linear system theory can then be applied to control the linearized portion of the system. In the case of parameter uncertainty, intuition suggests that parameter estimates which are converging to their true values can be used to asymptotically linearize the system. This heuristic is known as

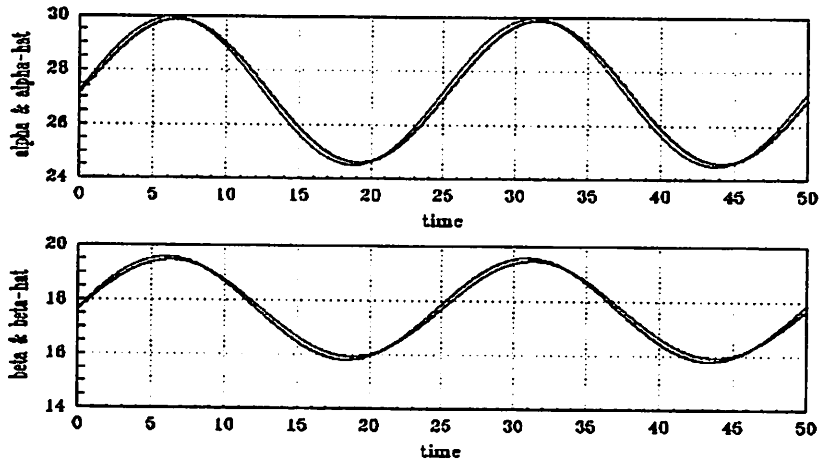

Figure 2: Open Loop Identification - Filtered Regressor

the *certainty equivalence principle.*

To fix notation, we review, following Isidori (1989), the basic linearizing theory. Consider a single-input single-output system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{26}$$

with $x \in \mathbb{R}^n$,$u \in \mathbb{R}$ and f, g, h smooth. Differentiating y with respect to time, one obtains

$$\dot{y} = L_f h + L_g h u \tag{27}$$

Here $L_f h$, $L_g h$ stand for the Lie derivatives of h with respect to f, g respectively. If $L_g h(x) \neq 0$ $\forall x \in \mathbb{R}^n$ then the control law

$$u = \frac{1}{L_g h}(-L_f h + v) \tag{28}$$

yields the linear system

$$\dot{y} = v. \tag{29}$$

If $L_g h(x) \equiv 0$, one continues to differentiate obtaining

$$y^{(i)} = L_f^i h + L_g L_f^{i-1} h u \quad i = 1, 2, \ldots \tag{30}$$

If there is a fixed integer γ such that $\forall x \in \mathbb{R}^n$ $L_g L_f^i h \equiv 0$ for $i = 0, \ldots, \gamma - 2$ and $L_g L_f^{\gamma-1} h(x) \neq 0$ then the control law

$$u = \frac{1}{L_g L_f^{\gamma-1} h(x)}(-L_f^{\gamma} h(x) + v) \tag{31}$$

yields

$$y^{(\gamma)} = v. \tag{32}$$

We stress that the linearization conditions hold in all of $\mathbb{R}^n$. Some completeness conditions on vector fields involving f,g are sufficient for this (for details see Isidori (1989) chapter 2).

The integer γ is called the *strong relative degree* of system (26). We will not consider the case where the relative degree is not defined; namely, where $L_g L_f^{\gamma-1} h(x) = 0$ for some values of x.

For a system with a strong relative degree γ, it is easy to verify that at each $x^0 \in \mathbb{R}^n$ there exists a neighborhood U^0 of x^0 such that the mapping

$$\Phi : U^0 \longrightarrow \mathbb{R}^n$$

defined as

$$\begin{array}{rcccl} \Phi_1(x) & = & \xi_1 & = & h(x) \\ \Phi_2(x) & = & \xi_2 & = & L_f h(x) \\ & & \vdots & & \\ \Phi_\gamma(x) & = & \xi_\gamma & = & L_f^{\gamma-1} h(x) \end{array} \tag{33}$$

with

$$d\Phi_i(x) g(x) = 0 \quad for \quad i = \gamma + 1, \ldots, n$$

is a diffeomorphism onto its image.

If we set $\eta = (\Phi_{\gamma+1}, \ldots, \Phi_n)^T$ it follows that the system may be written in the *normal form* (Isidori (1989)) as

$$\begin{array}{rcl} \dot{\xi}_1 & = & \xi_2 \\ & \vdots & \\ \dot{\xi}_{\gamma-1} & = & \xi_\gamma \\ \dot{\xi}_\gamma & = & b(\xi, \eta) + a(\xi, \eta) u \\ \dot{\eta} & = & q(\xi, \eta) \\ y & = & \xi_1. \end{array} \tag{34}$$

In equation (34), $b(\xi, \eta)$ represents the quantity $L_f^\gamma h(x)$ and $a(\xi, \eta)$ represents $L_g L_f^{\gamma-1} h(x)$. We assume that $x = 0$ is an equilibrium point of the system (ie. $f(0) = 0$) and we assume that $h(0) = 0$. Then the dynamics

$$\dot{\eta} = q(0, \eta) \tag{35}$$

are referred to as the *zero-dynamics* (see Isidori (1989) section 4.3 for details). The nonlinear system (26) is said to be minimum phase if the zero-dynamics are asymptotically stable.

4.1 Non-Adaptive Tracking

We now apply the normal form and the minimum phase property to the tracking problem. We desire to have $y(t)$ track a given $y_M(t)$. We start by choosing v in (31) as

$$v = y_M^{(\gamma)} + \alpha_1(y_M^{(\gamma-1)} - y^{(\gamma-1)}) + \ldots + \alpha_\gamma(y_M - y) \tag{36}$$

with $\alpha_1, \ldots, \alpha_\gamma$ chosen so that

$$s^\gamma + \alpha_1 s^{\gamma-1} + \ldots + \alpha_\gamma \tag{37}$$

is a Hurwitz polynomial. Note that $y^{(i-1)} = \xi_i$. If we define $e_i = y^{(i-1)} - y_M^{(i-1)}$ then we have

$$\begin{aligned} \dot{e} &= Ae \\ \dot{\eta} &= q(\xi, \eta) \\ \xi_i &= e_i + y_M^{(i-1)} \end{aligned} \tag{38}$$

where A is the companion matrix associated with (37), and hence is a Hurwitz matrix.

It is easy to see that this control results in asymptotic tracking and bounded states ξ provided $y_M, \dot{y}_M, \ldots, y_M^{(\gamma-1)}$ are bounded.

It can be also be shown that η remains bounded as well, assuming exponentially stable zero-dynamics and $q(\xi, \eta)$ is Lipschitz in ξ, η, by using a converse Lyapunov approach. Thus, this control yields bounded tracking. (see Sastry and Isidori (1987)).

4.2 Indirect Adaptive Tracking

In the case of parameter uncertainty, we have the system

$$\begin{aligned} \dot{x} &= f(x, \theta^*) + g(x, \theta^*)u \\ y &= h(x, \theta^*) \end{aligned} \tag{39}$$

with $\theta^* \in \mathbb{R}^p$ the vector of unknown parameters. We will make the following assumptions:

Assumption 1 Linear Parameter Dependence

The vector fields $f(x, \theta^*)$, $g(x, \theta^*)$ and the output function $h(x, \theta^*)$ in the system (39) depend linearly on the unknown parameters as

$$\begin{aligned} f(x, \theta^*) &= \textstyle\sum_{i=1}^p \theta_i^* f_i(x) \\ g(x, \theta^*) &= \textstyle\sum_{i=1}^p \theta_i^* g_i(x) \\ h(x, \theta^*) &= \textstyle\sum_{i=1}^p \theta_i^* h_i(x) \end{aligned}$$

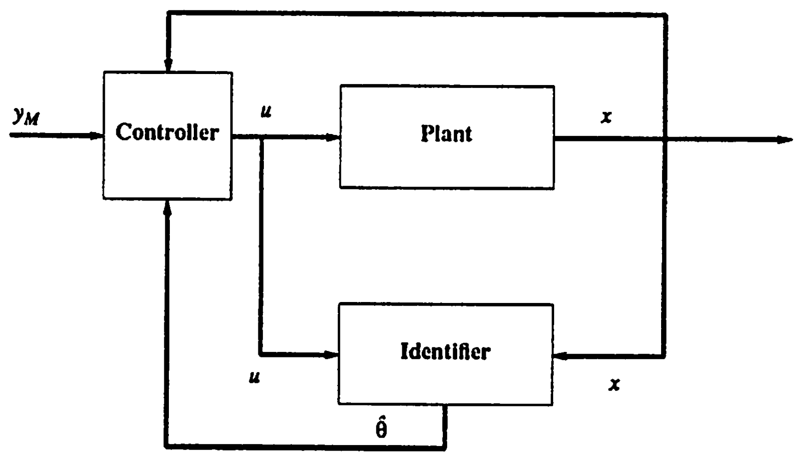

Figure 3: Block Diagram of an Indirect Adaptive Controller

where $f_i(x), g_i(x)$ are known smooth vector fields on $\mathbb{R}^n$ and $h_i(x)$ are known smooth scalar functions.

Assumption 2 Relative Degree

The relative degree of the true system (39) is γ, and for all $\hat{\theta}$ in a ball around θ^* and all x in a neighborhood of x^0

$$L_{g(x,\hat{\theta})} L^{\gamma-1}_{f(x,\hat{\theta})} h(x,\hat{\theta})$$

is bounded away from zero.

In the discussion that follows we will be using the implicit summation notation (ie. there is a summation over repeated indices) to keep the expressions manageable. For example, we will write $f(x,\theta^*)$ as $\theta_j^* f_j(x)$. Now if we pick the following diffeomorphism

$$
\Phi(x,\theta^*) = \begin{bmatrix} h(x,\theta^*) \\ L_{f(x,\theta^*)}h(x,\theta^*) \\ \vdots \\ L^{\gamma-1}_{f(x,\theta^*)}h(x,\theta^*) \\ \Phi_{\gamma+1}(x,\theta^*) \\ \vdots \\ \Phi_n(x,\theta^*) \end{bmatrix} = \begin{bmatrix} \theta^*_{j0} h_{j_0}(x) \\ \theta^*_{j_1}\theta^*_{j0} L_{f_{j_1}} h_{j_0}(x) \\ \vdots \\ \theta^*_{j_{\gamma-1}} \cdots \theta^*_{j0} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x) \\ \Phi_{\gamma+1}(x,\theta^*) \\ \vdots \\ \Phi_n(x,\theta^*) \end{bmatrix}
$$

$$
= \begin{bmatrix} \Phi_\xi(x,\theta^*) \\ -------- \\ \Phi_\eta(x,\theta^*) \end{bmatrix} = \begin{bmatrix} \xi \\ -- \\ \eta \end{bmatrix} \tag{40}
$$

with

$$
L_g\Phi_i = d\Phi_i g(x,\theta^*) = 0 \quad i = \gamma+1,\ldots,n
$$

and $\Phi_{\gamma+1}(x,\theta^*),\ldots,\Phi_n(x,\theta^*)$ chosen so that $\Phi(x,\theta^*)$ has a nonsingular jacobian matrix at x^0, then we have, in the normal form,

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
&\vdots \\
\dot{\xi}_\gamma &= \theta^*_{j_\gamma} \cdots \theta^*_{j0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \theta^*_{j_\gamma} \cdots \theta^*_{j0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x) u \\
\dot{\eta} &= q_{\theta^*}(\xi,\eta) \\
y &= \xi_1
\end{aligned} \tag{41}
$$

where

$$
q_{i\theta^*}(\xi,\eta) = L_{f(x,\theta^*)}\Phi_i(x,\theta^*) \quad \gamma+1 \le i \le n. \tag{42}
$$

We assume that $x = 0$ is an equilibrium point of the system (39) (ie. $f(0,\theta^*) = 0$) and we assume $h(0,\theta^*) = 0$. Then the dynamics

$$
\dot{\eta} = q_{\theta^*}(0,\eta) \tag{43}
$$

are referred to as the zero-dynamics. The nonlinear system (39) is said to be minimum phase if the zero-dynamics are asymptotically stable. We will now impose the following assumption:

Assumption 3 Exponentially Stable Zero Dynamics
The equilibrium point $\eta = 0$ of the zero-dynamics of the true system (39) is exponentially stable.

Now let us consider our choice for the control law. The certainty equivalence principle suggests that we pick the appropriate linearizing control law but with the unknown parameters replaced by their estimates. We choose

$$u = \frac{1}{\hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x)} [\hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \hat{v}] \tag{44}$$

To achieve tracking we pick $\hat{v}$ in the form of (36). However, we do not have exact expressions for the derivatives of y which involve unknown parameters. Instead we will use estimates of the derivatives of y obtained from the parameter estimates:

$$\hat{v} = y_M^{(\gamma)} + \alpha_1(y_M^{(\gamma-1)} - \hat{y}^{(\gamma-1)}) + \ldots + \alpha_\gamma(y_M - \hat{y}) \tag{45}$$

where

$$\hat{y}^{(i)} = \hat{\theta}_{j_i} \cdots \hat{\theta}_{j_0} L_{f_{j_i}} \cdots L_{f_{j_1}} h_{j_0}(x) \tag{46}$$

Now let us return to the normal form. Observe that $\dot{\xi}_\gamma$ can be written as

$$\begin{aligned} \dot{\xi}_\gamma \;=\;& \theta^*_{j_\gamma} \cdots \theta^*_{j_0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \theta^*_{j_\gamma} \cdots \theta^*_{j_0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x)u \\ -\;& [\hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x)u] \\ +\;& [\hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x)u] \end{aligned} \tag{47}$$

If we define the (large dimensional) vector of all multilinear parameter product errors,

$$\chi = (\hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0}) - (\theta^*_{j_\gamma} \cdots \theta^*_{j_0}) \tag{48}$$

then

$$\begin{aligned} \dot{\xi}_\gamma \;=\;& \hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{f_{j_\gamma}} \cdots L_{f_{j_1}} h_{j_0}(x) + \hat{\theta}_{j_\gamma} \cdots \hat{\theta}_{j_0} L_{g_{j_\gamma}} L_{f_{j_{\gamma-1}}} \cdots L_{f_{j_1}} h_{j_0}(x)u \\ & + \bar{z}^T(x,u)\chi \end{aligned} \tag{49}$$

Note that if $\hat{\theta} - \theta^* \equiv \phi \to 0$ as $t \to \infty$ then $\chi \to 0$ as $t \to \infty$.

Substituting the certainty equivalence control law, we have

$$\dot{\xi}_\gamma = \hat{v} + \bar{z}^T(x,u)\chi \tag{50}$$

Now notice that $\hat{v}$ can be written as

$$\begin{aligned} \hat{v} \;=\;& y_M^{(\gamma)} + \alpha_1(y_M^{(\gamma-1)}) - y^{(\gamma-1)}) + \alpha_1(y^{(\gamma-1)} - \hat{y}^{(\gamma-1)}) \\ & + \ldots + \alpha_\gamma(y_M - y) + \alpha_\gamma(y - \hat{y}) \end{aligned} \tag{51}$$

which can be seen as the exact tracking law plus an offset which is a function of the parameter error. Therefore, in the closed loop we have

$$
\begin{aligned}
\dot{e} &= Ae + z^T(x,u)\chi \\
\dot{\eta} &= q(\xi,\eta) \\
\xi_i &= e_i + y_M^{(i-1)}
\end{aligned}
\tag{52}
$$

where A is a Hurwitz matrix.

We will now state the following bounded tracking result under parameter uncertainty:

Theorem 4.1 Convergence of Indirect Adaptive Controller When Identifier Input Is Sufficiently Rich

Consider the plant of equation (39) and the control objective of tracking the trajectory y_M.

If (A1) *Assumption 1 holds (Linear Parameter Dependence),*

(A2) *Assumption 2 holds (Relative Degree),*

(A3) *Assumption 3 holds (Exponentially Stable Zero Dynamics),*

(A4) $|\chi| \to 0$ *as* $t \to \infty$,

(A5) $z^T(x,u)$ *is "cone bounded" in* x *and uniform in* u, *ie.* $|z^T(x,u)| \leq \ell_z |x| \; \forall u \in \mathbb{R}$,

(A6) A *is a Hurwitz matrix,*

(A7) $q(\xi,\eta)$ *is globally Lipschitz in* ξ,η,

(A8) $y_M, \dot{y}_M, \ldots, y_M^{(\gamma-1)}$ *are bounded*

then *the control law given by (44) and (45) results in bounded tracking for the system (39). (ie.,* $x \in \mathbb{R}^n$ *is bounded and* $y(t) \to y_M(t)$.*)*

Remarks:

1. The drawback with this result is that it needs the convergence of the identifier for its proof of asymptotic tracking. In turn, this requires the presence of sufficient richness which is not explicit in terms of conditions on the input. This is in contrast to the direct adaptive controller (Sastry and Isidori (1987)) where parameter convergence is not needed for stability and asymptotic tracking.

2. We reiterate that we are assuming the boundedness of the regressor, w, as stated in section 2. *Thus we explicitly disallow the possibility of finite escape time.*

Proof: Define b_d to be a bound on y_M and its derivatives. Then from (A8) and the definition of e,

$$|\xi| \leq |e| + b_d \tag{53}$$

From (A6) $\exists P > 0$ such that

$$A^T P + PA = -I \tag{54}$$

Because x is a local diffeomorphism of (ξ, η)

$$|x| \leq \ell_x(|\xi| + |\eta|) \tag{55}$$

From (A5) and P defined in (54)

$$|2Pz^T(x, u)| \leq \ell_z |x| \tag{56}$$

From (A3) $\exists v_2(\eta)$ and positive constants $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ satisfying

$$\begin{array}{rcl} \sigma_1|\eta|^2 & \leq & v_2(\eta) \leq \sigma_2|\eta|^2 \\ \frac{\partial v_2}{\partial \eta} q(0,\eta) & \leq & -\sigma_3|\eta|^2 \\ |\frac{\partial v_2(\eta)}{\partial \eta}| & \leq & \sigma_4|\eta|. \end{array} \tag{57}$$

From (A7)

$$|q(\xi, \eta) - q(0, \eta)| \leq \ell_q|\xi| \tag{58}$$

From these bounds

$$\begin{array}{rcl} \frac{\partial v_2}{\partial \eta} q(\xi,\eta) & = & \frac{\partial v_2}{\partial \eta} q(0,\eta) + \frac{\partial v_2}{\partial \eta}(q(\xi,\eta) - q(0,\eta)) \\ & \leq & -\sigma_3|\eta|^2 + \sigma_4\ell_q|\eta|(|e| + b_d) \end{array} \tag{59}$$

With these preliminaries, we will show that e and η are bounded. Consider a Lyapunov function for the system (52)

$$V(e, \eta) = e^T Pe + \mu v_2(\eta) \quad \mu > 0. \tag{60}$$

Taking the derivative of $V(\cdot, \cdot)$ along the trajectories of (52) yields

$$\begin{array}{rcl} \dot{V} & = & e^T(A^T P + PA)e + 2e^T P z^T(x, u)\chi + \mu \frac{\partial v_2}{\partial \eta} q(\xi, \eta) \\ & \leq & -|e|^2 + \ell_z\ell_x|e|(|e| + b_d + |\eta|)|\chi| + \mu(-\sigma_3|\eta|^2 + \sigma_4\ell_q|\eta|(|e| + b_d)) \\ & \leq & -(\frac{|e|}{2} - \ell_z\ell_x b_d|\chi|)^2 + (\ell_z\ell_x b_d|\chi|)^2 \\ & & -(\frac{|e|}{2} - (\ell_z\ell_x|\chi| + \mu\sigma_4\ell_q)|\eta|)^2 + (\ell_z\ell_x|\chi| + \mu\sigma_4\ell_q)^2|\eta|^2 \\ & & -\mu\sigma_3(\frac{|\eta|}{2} - \frac{\sigma_4\ell_q b_d}{\sigma_3}^2 + \frac{\mu(\sigma_4\ell_q b_d)^2}{\sigma_3} \\ & & -(\frac{1}{2} - |\chi|\ell_z\ell_x)|e|^2 - \frac{3}{4}\mu\sigma_3|\eta|^2 \\ & \leq & -(\frac{1}{2} - |\chi|\ell_z\ell_x)|e|^2 - [\frac{3}{4}\mu\sigma_3 - (\ell_z\ell_x|\chi| + \mu\sigma_4\ell_q)^2]|\eta|^2 \\ & & +(\ell_z\ell_x b_d|\chi|)^2 + \frac{\mu(\sigma_4\ell_q b_d)^2}{\sigma_3} \end{array} \tag{61}$$

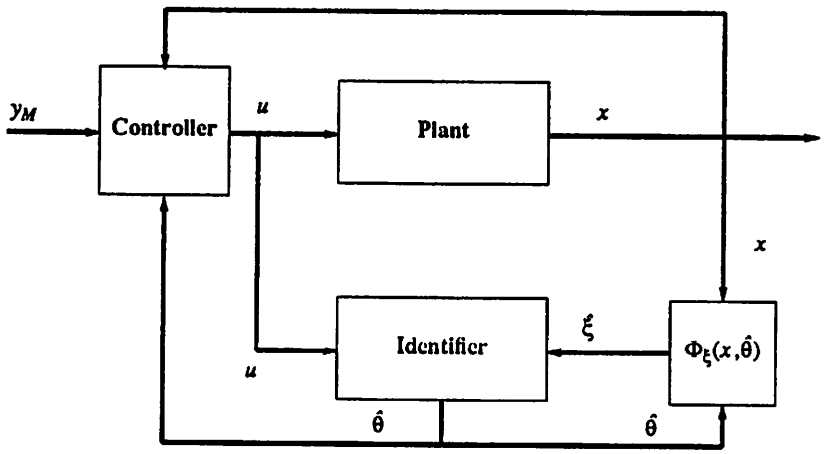

Figure 4: Block Diagram of a Semi-Indirect Adaptive Controller

Define

$$\mu_0 = \frac{\sigma_3}{4(\ell_z\ell_x + \sigma_4\ell_q)^2} \tag{62}$$

Then, for $\mu \leq \mu_0$ and $|\chi| \leq min(\mu, \frac{1}{4\ell_z\ell_x})$, we have

$$\dot{V} \leq -\frac{|e|^2}{4} - \frac{\mu\sigma_3|\eta|^2}{2} + \frac{\mu(\sigma_4\ell_q b_d)^2}{\sigma_3} + (\ell_z\ell_x b_d|\chi|)^2 \tag{63}$$

We can assume that $|\chi| \leq min(\mu, \frac{1}{4\ell_z\ell_x})$ for all $t \geq T$ from (A4). Also, the only previous restriction on μ was $\mu > 0$. Thus, for all $t \geq T, \dot{V} < 0$ when $|\eta|$ or $|e|$ is large which implies that $|\eta|$ and $|e|$, and hence $|\xi|$ and $|x|$ are bounded. If $q(\xi, \eta)$ is locally Lipschitz in (ξ, η) only on a set U and not all of $\mathbb{R}^n$ then the preceding analysis would hold so long as $|\chi(0)|$ is chosen small enough to guarantee that (ξ, η) lies in U. Consequently,

$$\dot{e} = Ae + z^T(x, u)\chi \tag{64}$$

is an exponentially stable linear system driven by an input that approaches zero asymptotically. Thus, we conclude that the tracking error converges asymptotically to zero. □

4.3 Semi-Indirect Adaptive Tracking

In this section we give a modified scheme which combines attractive features of the direct and indirect schemes; as in direct adaptive control, parameter convergence is not necessary to achieve asymptotic tracking; as in indirect adaptive control, it is not necessary to overparameterize the system. The scheme uses an observer-based identifier that is similar to the one described in section 2.1 but here the states are estimated in the coordinates of the diffeomorphism. Consequently, exact knowledge of the diffeomorphism is necessary. This is made possible by using an estimated diffeomorphism that is a function of the time-varying parameter estimate (see figure 4). These results are an extension of those found in Campion and Bastin (1989).

Consider the system (39) and allow assumption 3 to hold. We will modify assumption 1 so that $h(x)$ is no longer permitted to be a function of the parameters.

Assumption 1 A Linear Parameter Dependence in f and g

The vector fields $f(x,\theta^*)$ and $g(x,\theta^*)$ in the system (39) depend linearly on the unknown parameters as

$$\begin{aligned} f(x,\theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* f_i(x) \\ g(x,\theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* g_i(x) \end{aligned}$$

where $f_i(x), g_i(x)$ are known smooth vector fields on $\mathbf{R}^n$. The output function $h(x)$ is not permitted to be a function of the parameters.

We will also modify assumption 2 as follows:

Assumption 2 A Constant Relative Degree

For all $\hat{\theta}$ in a ball around θ^* and for all x in a neighborhood of x^0,

$$L_{g(x,\hat{\theta})}h(x) = L_{g(x,\hat{\theta})}L_{f(x,\hat{\theta})}h(x) = \ldots = L_{g(x,\hat{\theta})}L^{\gamma-2}_{f(x,\hat{\theta})}h(x) = 0$$

and

$$L_{g(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x)$$

is bounded away from zero.

This assumption is reasonable in the adaptive case because the structure of the system is known. The relative degree will drop only in very special cases. This assumption can be relaxed if parameter convergence is assumed. This trade-off will be discussed in more detail later.

For the development that follows, also consider the parametrized model

$$\begin{aligned} \dot{x} &= f(x,\theta) + g(x,\theta)u \\ y &= h(x) \end{aligned} \tag{65}$$

where $\theta \in \mathbb{R}^p$ is *fixed* and *known*. From linearization theory, if we pick the following diffeomorphism for the system (65)

$$\Phi(x,\theta) = \begin{bmatrix} h(x) \\ L_{f(x,\theta)}h(x) \\ \vdots \\ L^{\gamma-1}_{f(x,\theta)}h(x) \\ \Phi_{\gamma+1}(x,\theta) \\ \vdots \\ \Phi_n(x,\theta) \end{bmatrix} = \begin{bmatrix} \Phi_\xi(x,\theta) \\ -------- \\ \Phi_\eta(x,\theta) \end{bmatrix} = \begin{bmatrix} \xi \\ -- \\ \eta \end{bmatrix} \tag{66}$$

with

$$L_g\Phi_i = d\Phi_i g(x,\theta) = 0 \quad i = \gamma+1,\ldots,n$$

and $\Phi_{\gamma+1}(x,\theta),\ldots,\Phi_n(x,\theta)$ chosen so that $\Phi(x,\theta)$ has a nonsingular jacobian matrix at x^0, and if we choose the following control law

$$u = \frac{1}{L_{g(x,\theta)}L^{\gamma-1}_{f(x,\theta)}h(x)}[-L^{\gamma}_{f(x,\theta)}h(x)+v] \tag{67}$$

then we have the resulting closed loop system

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ &\vdots \\ \dot{\xi}_{\gamma-1} &= \xi_\gamma \\ \dot{\xi}_\gamma &= v \\ \dot{\eta} &= q_\theta(\xi,\eta) \\ y &= \xi_1 \end{aligned} \tag{68}$$

where

$$q_{i\theta}(\xi,\eta) = L_{f(x,\theta)}\Phi_i(x,\theta) \quad \gamma+1 \le i \le n$$

We can achieve bounded tracking ($y(t) \to y_M(t)$) for the system (65) in the same way as described in section 4.1.

Now consider the actual plant given in (39). We will choose, for this system, the diffeomorphism given in (66) but now x is the actual state of the plant. We will replace θ by θ^* in each of the $\Phi_i(x,\theta)$ $\gamma+1 \le i \le n$. For $\Phi_i(x,\theta)$ $1 \le i \le \gamma$, θ will be replaced by $\hat{\theta}$, the time varying parameter estimate. Observe that, under these conditions, the ξ states are no longer related simply by a chain of integrators. The chain of integrators structure is perturbed by the time varying nature of $\hat{\theta}$ and the fact that the time derivatives of ξ are taken along the trajectories of the plant states which are a function of θ^*. Consider the following two functions of x:

$$\begin{aligned} \hat{\xi} &= \Phi_\xi(x, \hat{\theta}) \\ \eta &= \Phi_\eta(x, \theta^*). \end{aligned} \tag{69}$$

This transformation is the same functional form as (66) but different in that $\hat{\xi}$ is evaluated along the estimates of θ. Taking the time derivative along the trajectories of (39) we have

$$\begin{aligned} \dot{\hat{\xi}} &= \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial x}\dot{x} + \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} \\ &= \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial x}[f(x,\theta^*) + g(x,\theta^*)u] + \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} \\ &= \begin{bmatrix} L_{f(x,\theta^*)}h(x) \\ \vdots \\ L_{f(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} \\ &\quad + \begin{bmatrix} L_{g(x,\theta^*)}h(x) \\ \vdots \\ L_{g(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} u + \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} \\ \dot{\eta} &= \frac{\partial \Phi_\eta(x,\theta^*)}{\partial x}\dot{x} \\ &= \begin{bmatrix} L_{f(x,\theta^*)}\Phi_{\gamma+1}(x,\theta^*) \\ \vdots \\ L_{f(x,\theta^*)}\Phi_n(x,\theta^*) \end{bmatrix} \\ &= q_{\theta^*}(\xi^*, \eta) \end{aligned} \tag{70}$$

where

$$\xi^* = \Phi_\xi(x, \theta^*).$$

The vector of tracking errors is defined as

$$e_i = \hat{\xi}_i - y_M^{(i-1)} \quad 1 \le i \le \gamma \tag{71}$$

and thus, the derivative of the tracking error is

$$\dot{e} = \begin{bmatrix} L_{f(x,\theta^*)}h(x) \\ \vdots \\ L_{f(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} + \begin{bmatrix} L_{g(x,\theta^*)}h(x) \\ \vdots \\ L_{g(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} u$$

$$
+\frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} - \begin{bmatrix} \dot{y}_M \\ \vdots \\ y_M^{(\gamma)} \end{bmatrix}
$$

$$
= \begin{bmatrix} L_{f(x,\theta^*)}h(x) \\ \vdots \\ L_{f(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} + \begin{bmatrix} L_{g(x,\theta^*)}h(x) \\ \vdots \\ L_{g(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} u \tag{72}
$$

$$
- \left(\begin{bmatrix} L_{f(x,\hat{\theta})}h(x) \\ \vdots \\ L_{f(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} + \begin{bmatrix} L_{g(x,\hat{\theta})}h(x) \\ \vdots \\ L_{g(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} u \right)
$$

$$
+ \left(\begin{bmatrix} L_{f(x,\hat{\theta})}h(x) \\ \vdots \\ L_{f(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} + \begin{bmatrix} L_{g(x,\hat{\theta})}h(x) \\ \vdots \\ L_{g(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \end{bmatrix} u \right)
$$

$$
+\frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} - \begin{bmatrix} \dot{y}_M \\ \vdots \\ y_M^{(\gamma)} \end{bmatrix}
$$

Observe that, from the structure of $f(x,\cdot)$,$g(x,\cdot)$,

$$
\begin{aligned}
L_{f(x,\theta^*)}L^{i}_{f(x,\hat{\theta})}h(x) &= \textstyle\sum_{j=1}^{p} \theta_j^* L_{f_j(x)}L^{i}_{f(x,\hat{\theta})}h(x) \\
L_{f(x,\hat{\theta})}L^{i}_{f(x,\hat{\theta})}h(x) &= \textstyle\sum_{j=1}^{p} \hat{\theta}_j L_{f_j(x)}L^{i}_{f(x,\hat{\theta})}h(x) \\
L_{g(x,\theta^*)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) &= \textstyle\sum_{j=1}^{p} \theta_j^* L_{g_j(x)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) \\
L_{g(x,\hat{\theta})}L^{\gamma-1}_{f(x,\hat{\theta})}h(x) &= \textstyle\sum_{j=1}^{p} \hat{\theta}_j L_{g_j(x)}L^{\gamma-1}_{f(x,\hat{\theta})}h(x)
\end{aligned} \tag{73}
$$

Also recall that u defined in equation (67) produces an exponentially stable tracking system for $\hat{\theta}$ fixed. Therefore, we pick u according to (67) with the expressions for the derivatives of y in the tracking law (36) determined assuming $\hat{\theta}$ fixed. (ie. $y^{(i-1)} = \xi_i$). Then, using assumption 2A and simplifying we have

$$
\begin{aligned}
\dot{e} &= \begin{bmatrix} \sum_{j=1}^{p}(\theta_j^* - \hat{\theta}_j) L_{f_j(x)} h(x) \\ \vdots \\ \sum_{j=1}^{p}(\theta_j^* - \hat{\theta}_j) L_{f_j(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) \end{bmatrix} \\
&\quad + \begin{bmatrix} \sum_{j=1}^{p}(\theta_j^* - \hat{\theta}_j) L_{g_j(x)} h(x) \\ \vdots \\ \sum_{j=1}^{p}(\theta_j^* - \hat{\theta}_j) L_{g_j(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) \end{bmatrix} u + Ae + \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta} \dot{\hat{\theta}}
\end{aligned}
\tag{74}
$$

Define a new variable $\bar{\xi} \in \mathbb{R}^\gamma$ with dynamics given by

$$
\begin{aligned}
\dot{\bar{\xi}} &= \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial x}[f(x,\hat{\theta}) + g(x,\hat{\theta})u] + \frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta}\dot{\hat{\theta}} + \Omega(\bar{\xi} - \hat{\xi}) \\
\bar{\xi}(0) &= \hat{\xi}(0)
\end{aligned}
\tag{75}
$$

where Ω is a Hurwitz matrix. This equation resembles (70) with two differences: (1) $\dot{x}$ is replaced by $f(x,\hat{\theta}) + g(x,\hat{\theta})u$ and (2) the additional term $\Omega(\bar{\xi} - \hat{\xi})$ appears. Define

$$
\varepsilon = \bar{\xi} - \hat{\xi}. \tag{76}
$$

Then

$$
\begin{aligned}
\dot{\varepsilon} &= \Omega\varepsilon + \begin{bmatrix} \sum_{j=1}^{p}(\hat{\theta}_j - \theta_j^*) L_{f_j(x)} h(x) \\ \vdots \\ \sum_{j=1}^{p}(\hat{\theta}_j - \theta_j^*) L_{f_j(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) \end{bmatrix} \\
&\quad + \begin{bmatrix} \sum_{j=1}^{p}(\hat{\theta}_j - \theta_j^*) L_{g_j(x)} h(x) \\ \vdots \\ \sum_{j=1}^{p}(\hat{\theta}_j - \theta_j^*) L_{g_j(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) \end{bmatrix} u
\end{aligned}
\tag{77}
$$

We now specify the parameter update law

$$
\dot{\hat{\theta}} = g(x,u,\hat{\theta},t). \tag{78}
$$

Define

$$
\begin{aligned}
M(x,\hat{\theta},u) &= \begin{bmatrix} L_{f_1(x)}h(x) & \dots & L_{f_p(x)}h(x) \\ \vdots & & \vdots \\ L_{f_1(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) & \dots & L_{f_p(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x) \end{bmatrix} \\
&\quad + \begin{bmatrix} L_{g_1(x)}h(x)u & \dots & L_{g_p(x)}h(x)u \\ \vdots & & \vdots \\ L_{g_1(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x)u & \dots & L_{g_p(x)} L_{f(x,\hat{\theta})}^{\gamma-1} h(x)u \end{bmatrix}
\end{aligned}
\tag{79}
$$

and

$$\phi = \hat{\theta} - \theta^*. \tag{80}$$

Then we have

$$\begin{aligned} \dot{\varepsilon} &= \Omega\varepsilon + M\phi \\ \dot{\phi} &= g(x, u, \hat{\theta}, t). \end{aligned} \tag{81}$$

Using the Lyapunov function candidate

$$V(\varepsilon, \phi) = \varepsilon^T P_0 \varepsilon + \phi^T \phi \qquad \Omega^T P_0 + P_0 \Omega = -I \tag{82}$$

and taking the time derivative along the trajectories of (81) leads to choosing

$$g(x, u, \hat{\theta}, t) = -M^T P_0 \varepsilon \tag{83}$$

for the parameter update law. In this case, since

$$\dot{V} = -\varepsilon^T \varepsilon \tag{84}$$

we can conclude that, $\forall t \geq 0$,

$$\begin{aligned} |\varepsilon(t)| &\leq \rho|\phi(0)| \quad \rho = \sqrt{\lambda_{min}^{-1}(P_0)} \\ |\phi(t)| &\leq |\phi(0)| \end{aligned} \tag{85}$$

and hence ε is a bounded L_2 function.

To study the stability of the tracking error system (74) we will define

$$\zeta = e + \varepsilon \tag{86}$$

Then the tracking error e can be seen as the output of a linear, time-varying filter given by

$$\begin{aligned} \dot{\zeta} &= A\zeta + [(\Omega - A) - \tfrac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta} M^T P_0]\varepsilon \\ e &= \zeta - \varepsilon \end{aligned} \tag{87}$$

We will now state the following bounded tracking result under parametric uncertainty:

Theorem 4.2 Convergence of Semi-Indirect Adaptive Controller *Consider the plant of equation (39) and the control objective of tracking the trajectory y_M.*

If (A1) *Assumption 1A holds (Linear parameter dependence in f,g),* (A2) *Assumption 2A holds (Constant relative degree),*

(A3) *Assumption 3 holds (Exponentially stable zero dynamics),*

(A4) $q_{\theta^*}(\xi^*, \eta)$ *is globally Lipschitz in* ξ^*, η,

(A5) $\Phi_\xi(x, \theta)$ *is globally Lipschitz in* θ *and uniform in* x,
ie. $|\Phi_\xi(x, \theta^*) - \Phi_\xi(x, \hat{\theta})| \leq \ell_\phi |\phi| \; \forall x \in \mathbf{R}^n$,

(A6) A *is a Hurwitz matrix,*

(A7) $\frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta} M^T$ *is "cone bounded" in* x *and uniform in* $u, \hat{\theta}$,
ie. $|\frac{\partial \Phi_\xi(x,\hat{\theta})}{\partial \theta} M^T| \leq \ell_{\tilde{N}} |x| \; \forall u \in \mathbf{R}, \; \forall \hat{\theta} \in \mathbf{R}^p$,

(A8) $y_M, \dot{y}_M, \ldots, y_M^{(\gamma-1)}$ *are bounded,*

(A9) $|\phi(0)|$ *bounded as a function of specified Lipschitz constants*

then *the control law* u *given in (67) results in bounded tracking for the system (39). (ie.* $x \in \mathbf{R}^n$ *is bounded and* $y(t) \to y_M(t)$ *as* $t \to \infty$*).*

Proof: Define $\tilde{b}_d$ to be a bound on y_M and its derivatives. From (A8) and the definition of e,

$$|\xi| \leq |e| + \tilde{b}_d \tag{88}$$

From the definition of ζ,

$$|e| \leq |\zeta| + |\varepsilon| \tag{89}$$

From (A6) $\exists P > 0$ such that

$$A^T P + PA = -I \tag{90}$$

Because x is a local diffeomorphism of (ξ, η),

$$|x| \leq \ell_x(|\xi| + |\eta|) \tag{91}$$

From (A7) and P defined above

$$|2P[(\Omega - A) - \frac{\partial \Phi_\xi(x, \hat{\theta})}{\partial \theta} M^T P_0]| \leq \ell_N |x| + c \tag{92}$$

For simplification purposes, define a new constant $b_d = \tilde{b}_d + c$ which will be used later. From (A3) $\exists v_2(\eta)$ and positive constants $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ satisfying

$$\begin{array}{rcl} \sigma_1 |\eta|^2 & \leq & v_2(\eta) \leq \sigma_2 |\eta|^2 \\ \frac{\partial v_2}{\partial \eta} q_{\theta^*}(0, \eta) & \leq & -\sigma_3 |\eta|^2 \\ |\frac{\partial v_2(\eta)}{\partial \eta}| & \leq & \sigma_4 |\eta|. \end{array} \tag{93}$$

From (A4)

$$|q_{\theta^*}(\xi^*,\eta) - q_{\theta^*}(0,\eta)| \le \ell_q|\xi^*| \tag{94}$$

From (A5),

$$\begin{aligned} |\xi^*| &\le |\xi| + |\Phi_\xi(x,\theta^*) - \Phi_\xi(x,\hat\theta)| \\ &\le |\xi| + \ell_\phi|\phi| \end{aligned} \tag{95}$$

From these bounds

$$\begin{aligned} \tfrac{\partial v_2}{\partial \eta} q_{\theta^*}(\xi,\eta) &= \tfrac{\partial v_2}{\partial \eta} q_{\theta^*}(0,\eta) + \tfrac{\partial v_2}{\partial \eta}(q_{\theta^*}(\xi^*,\eta) - q_{\theta^*}(0,\eta)) \\ &\le -\sigma_3|\eta|^2 + \sigma_4\ell_q|\eta|(|e| + b_d + \ell_\phi|\phi|) \end{aligned} \tag{96}$$

With these preliminaries we can show that ζ and η are bounded. Consider a Lyapunov function candidate for the system (87) and the η dynamics which are driven by (87):

$$V(\zeta,\eta) = \zeta^T P\zeta + \mu v_2(\eta) \quad \mu > 0. \tag{97}$$

Taking the time derivative along (87) and the η dynamics yields

$$\begin{aligned} \dot V &= \zeta^T(A^TP + PA)\zeta + 2\zeta^T P[(\Omega - A) - \tfrac{\partial \Phi_\xi(x,\hat\theta)}{\partial \theta} M^T P_0]\varepsilon + \mu\tfrac{\partial v_2}{\partial \eta} q(\xi,\eta) \\ &\le -|\zeta|^2 + \ell_N\ell_x|\zeta|(|\zeta| + |\varepsilon| + \tilde b_d + c + |\eta|)|\varepsilon| \\ &\quad +\mu(-\sigma_3|\eta|^2 + \sigma_4\ell_q|\eta|(|\zeta| + |\varepsilon| + \tilde b_d + c + \ell_\phi|\phi|)) \\ &\le -(\tfrac{|\zeta|}{2} - \ell_N\ell_x(|\varepsilon| + b_d)|\varepsilon|)^2 + (\ell_N\ell_x(|\varepsilon| + b_d)|\varepsilon|)^2 \\ &\quad -(\tfrac{|\zeta|}{2} - (\ell_N\ell_x|\varepsilon| + \mu\sigma_4\ell_q)|\eta|)^2 + (\ell_N\ell_x|\varepsilon| + \mu\sigma_4\ell_q)^2|\eta|^2 \\ &\quad -\mu\sigma_3(\tfrac{|\eta|}{2} - \tfrac{\sigma_4\ell_q(|\varepsilon|+b_d+\ell_\phi|\phi|)}{\sigma_3})^2 + \tfrac{\mu(\sigma_4\ell_q(|\varepsilon|+b_d\ell_\phi|\phi|))^2}{\sigma_3} \\ &\quad -(\tfrac{1}{2} - \ell_N\ell_x|\varepsilon|)|\zeta|^2 - \tfrac{3}{4}\mu\sigma_3|\eta|^2 \\ &\le -(\tfrac{1}{2} - \ell_N\ell_x\rho|\phi(0)|)|\zeta|^2 - [\tfrac{3}{4}\mu\sigma_3 - (\ell_N\ell_x\rho|\phi(0)| + \mu\sigma_4\ell_q)^2]|\eta|^2 \\ &\quad +(\ell_N\ell_x(\rho|\phi(0)| + b_d)\rho|\phi(0)|)^2 + \tfrac{\mu(\sigma_4\ell_q(\rho|\phi(0)|+b_d\ell_\phi|\phi|))^2}{\sigma_3} \end{aligned} \tag{98}$$

Define

$$\mu_0 = \frac{\sigma_3}{4(\ell_N\ell_x\rho + \sigma_4\ell_q)^2} \tag{99}$$

Then, for $\mu \le \mu_0$ and $|\phi(0)| < min(\mu, \frac{1}{4\ell_N\ell_x\rho})$, we have

$$\begin{aligned} \dot V \;\le\; & -\tfrac{|\zeta|^2}{4} - \tfrac{\mu\sigma_3|\eta|^2}{2} + \tfrac{\mu(\sigma_4\ell_q(\rho|\phi(0)|+b_d\ell_\phi|\phi(0)|))^2}{\sigma_3} \\ & +(\ell_N\ell_x(\rho|\phi(0)| + b_d)\rho|\phi(0)|)^2 \end{aligned} \tag{100}$$

Therefore $\dot V < 0$ when $|\zeta|$ or $|\eta|$ is large which implies that $|\zeta|$ and $|\eta|$ are bounded. This implies $|e|$ is bounded, which implies $|\xi|$ is bounded.

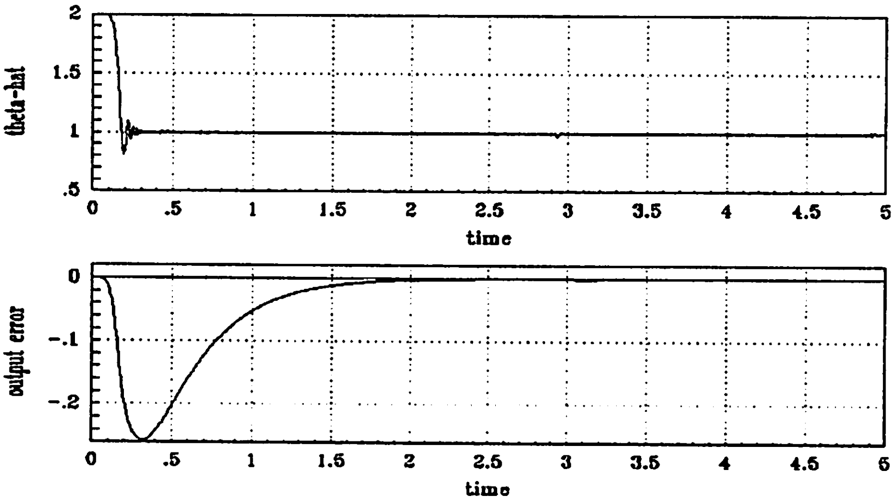

Figure 5: Indirect Adaptive Controller - Observer Based

This, together with $|\eta|$ bounded implies $|x|$ is bounded. This implies $|M|$ is bounded and consequently $\dot{\varepsilon}$ is bounded. Therefore, since $\varepsilon \in L_2$, $\varepsilon \to 0$ as $t \to \infty$. We have then that (87) is an exponentially stable linear filter driven by an input that approaches zero asymptotically. Thus we can conclude that the tracking error, which is the output of this filter, converges asymptotically to zero. □

5 Closed Loop Simulations

5.1 Comparison of Methods

We will qualitatively compare five nonlinear control schemes, namely direct, indirect, and semi-indirect adaptive control, non-adaptive nonlinear control and sliding mode control. The system we choose to simulate is:

$$\begin{aligned} \dot{x}_1 &= x_2 + \theta\psi(x_1, x_2) \\ \dot{x}_2 &= u \\ y &= x_1 \\ \psi(x_1, x_2) &= x_1[10 + \sin(x_1)] \end{aligned} \tag{101}$$

This plant is easily linearized with

$$u = -\frac{\partial \psi}{\partial x_1}[\theta x_2 + \theta^2 \psi(x_1, x_2)] + v \tag{102}$$

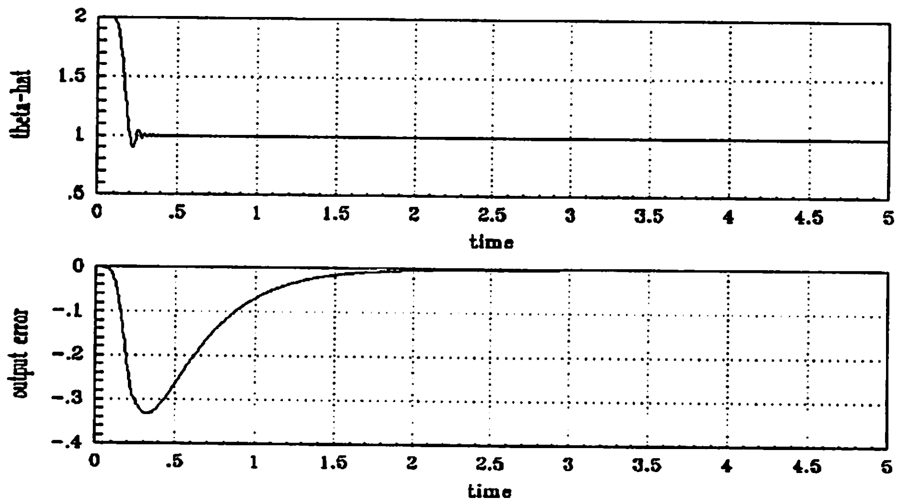

Figure 6: Semi-indirect Adaptive Controller

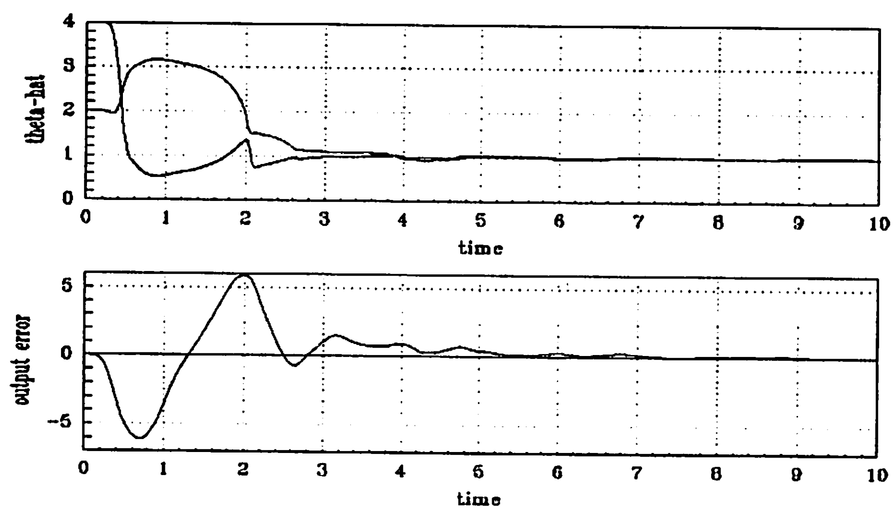

Figure 7: Direct Adaptive Controller

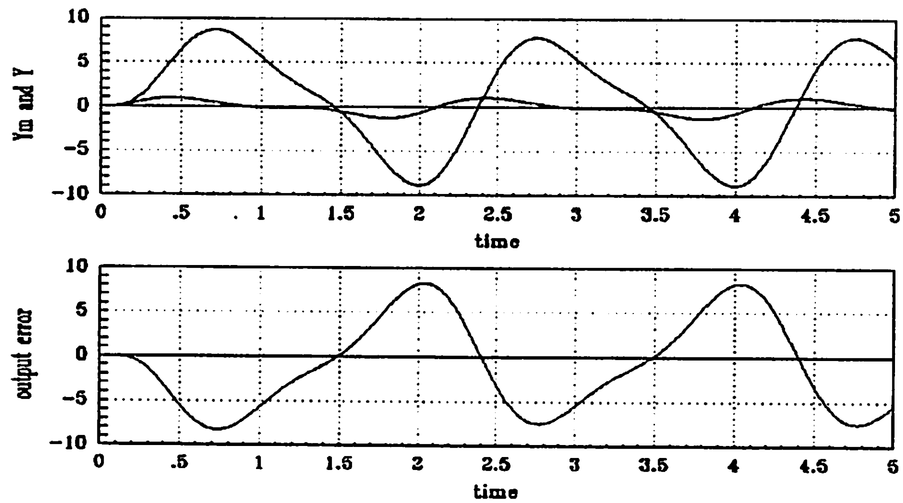

Figure 8: Non-Adaptive Controller

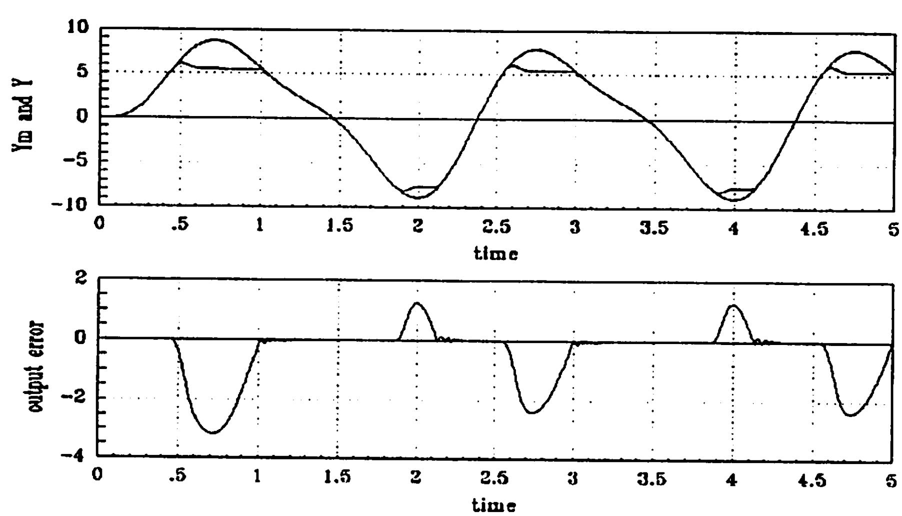

Figure 9: Sliding Mode Controller

and output tracking is achieved by

$$v = \ddot{y}_M + \alpha_1(\dot{y}_M - \dot{y}) + \alpha_2(y_M - y) \tag{103}$$

except in the case of sliding mode, where the $\alpha_2(y_M - y)$ term is replaced by $k\text{sgn}(y_M - y)$. We picked $\alpha_1 = 30$, $\alpha_2 = 200$, and $k = 2000$ to provide good nominal tracking.

The equation for the semi-indirect parameter update is:

$$\dot{\hat{\theta}} = \psi(x_1, x_2)g_{11}(\hat{\xi}_1 - \bar{\xi}_1) + \psi(x_1, x_2)\frac{\partial \psi}{\partial x_1}g_{12}(\hat{\xi}_2 - \bar{\xi}_2) \tag{104}$$

with

$$\begin{aligned} \dot{\bar{\xi}}_1 &= x_2 + \psi(x_1, x_2)\hat{\theta} - g_{21}(\bar{\xi}_1 - \hat{\xi}_1) \\ \dot{\bar{\xi}}_2 &= \frac{\partial \psi}{\partial x_1}\hat{\theta}(x_2 + \psi(x_1, x_2)\hat{\theta}) + \psi(x_1, x_2)\dot{\hat{\theta}} - g_{22}(\bar{\xi}_2 - \hat{\xi}_2) + u \end{aligned} \tag{105}$$

where the constants g_{ij} are gains and from equation (69)

$$\begin{aligned} \hat{\xi}_1 &= x_1 \\ \hat{\xi}_2 &= x_2 + \hat{\theta}\psi(x_1, x_2) \end{aligned} \tag{106}$$

The true value was $\theta^* = 1$ and $\hat{\theta}$ was initially at 2. For the direct adaptive controller a second parameter had to be added, $\theta_2 = \theta^2$, with an initial value of 4. The reference signal was picked to be $10sin(\pi t) + 5sin(2\pi t)$ to provide adequate excitation.

Using the same criterion for the observer based identifier from 3 we picked $g = 500$ and $\sigma = 50$. We similarly set $g_{ij} = 50$, but the update gain was scaled back to 1 since our error in the transformed space was smaller for the semi-indirect scheme. The update gains for the direct controller were again determined by looking at the errors and scaling them accordingly. The gains were set to 1000 and 2000 for the first and second components of the regressor.

5.2 Simulation Results

The indirect scheme, with the observer based identifier, and the semi-indirect controller performed quite well compared with the other methods, as can be seen in figure 5 and figure 6. The parameter θ converged to the correct value in less the one second and the output error, $y - y_M$ was driven to zero. The identifier was quite robust to choices of update gains and estimator gains, σ. Virtually all reasonable values yielded convergence in less than one second. The indirect scheme was also able to handle larger perturbations in $\dot{x}_1$, such as $\psi(x_1, x_2) = {x_1}^2$ for large values of x_1. With this $\psi(x_1, x_2)$, the non-adaptive controller became unstable, but for the indirect scheme the identifier was able to converge quickly enough to stabilize the system. It does seem that in most cases the excitation provided by system instability drives the parameters to their true values, thus allowing the controller to stabilize the plant. In fact $\psi(x_1, x_2)$ is the regressor for this system, thus

$$\frac{d\hat{\theta}}{dt} = -g\psi(x_1, x_2)(x_1 - \hat{x}_1) \tag{107}$$

so the estimation error, $x_1 - \hat{x}_1$, is driven to zero.

The direct scheme did not converge nearly as fast as the indirect, as shown in figure 7 – note the different time scale. The parameters were approaching their true values around six seconds, and the output error was driven to zero, which is what the direct method guarantees without any claims on excitation. Any hopes of speeding up the convergence would be by increasing the update gain or by increasing the amplitude of the reference signal. This would increase the elements in the regressor, and would cause the identifier to be ill-conditioned. In fact the update gain had to be reduced by a factor proportional to the square of the increase in the reference signal amplitude. The identifier in the direct scheme also has more states than the observer based identifier for the indirect. These extra states, six in all, come from filtering the regressor for the generation of the augmented output error used to drive the parameter updates and also from the additional parameter, θ^2, which needs to be identified. The adaptive schemes are compared in figure 10.

The non-adaptive scheme, figure 8, performed as well as could be expected. The tracking gains could have been increased in hopes of swamping out the perturbation caused by $\psi(x_1, x_2)$, but in anything other than a noiseless environment this would be ill advised.

The sliding mode method steered the output error to zero, but when the perturbation was large, $\psi(x_1, x_2)$ was at its maximum, the system could not swamp it out as quickly. The gain k was set at 2000. Larger gains caused considerable chattering in the regions were $\psi(x_1, x_2)$ was not at its maximum and would also send the numerical integrator into fits. The results are shown in figure 9.

Comparison of Adaptive Methods			
Criterion	Scheme		
	Indirect	Semi Indirect	Direct
Parameter Convergence	Fastest	Fast	Slow
Sensitivity to Adaptation Gains	Not Very	Slightly	Very
Ease of Implementation	Easy	Moderate	Difficult
Needs Overparametrization	No	No	Yes
Needs Constant Relative Degree to Prove Tracking	No	Yes	No
Needs Parameter Convergence to Prove Tracking	Yes	No	No

Figure 10: Method Comparison

5.3 Non-Constant Relative Degree

We investigated the semi-indirect control scheme further by simulating a system which does not have a constant relative degree. Clearly the constant relative degree assumption is sufficient for asymptotic tracking, but, as will be seen, it is not a necessary condition.

The system we picked was a simple third order plant described by

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta x_3 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u \\ y &= x_1\end{aligned} \tag{108}$$

We let the initial $\hat{\theta}$ be 0.1, and had $\theta^* = 0$. Hence, the relative degree would decrease for non-zero θ, and the actual relative degree would be different from the initial relative degree of the estimated system. The linearizing control law was applied to the system with the same type of tracking law to close the loop as above, namely

$$v = y_M{}^{(3)} + \alpha_1(\ddot{y}_M - \ddot{y}) + \alpha_2(\dot{y}_M - \dot{y}) + \alpha_3(y_M - y) \tag{109}$$

where $\alpha_1 = 9, \alpha_2 = 26, \alpha_3 = 24$, and the input was picked to be $6[sin(2\pi t) + sin(0.25\pi t)]$. The results, shown in figure 11, reveals that the closed loop

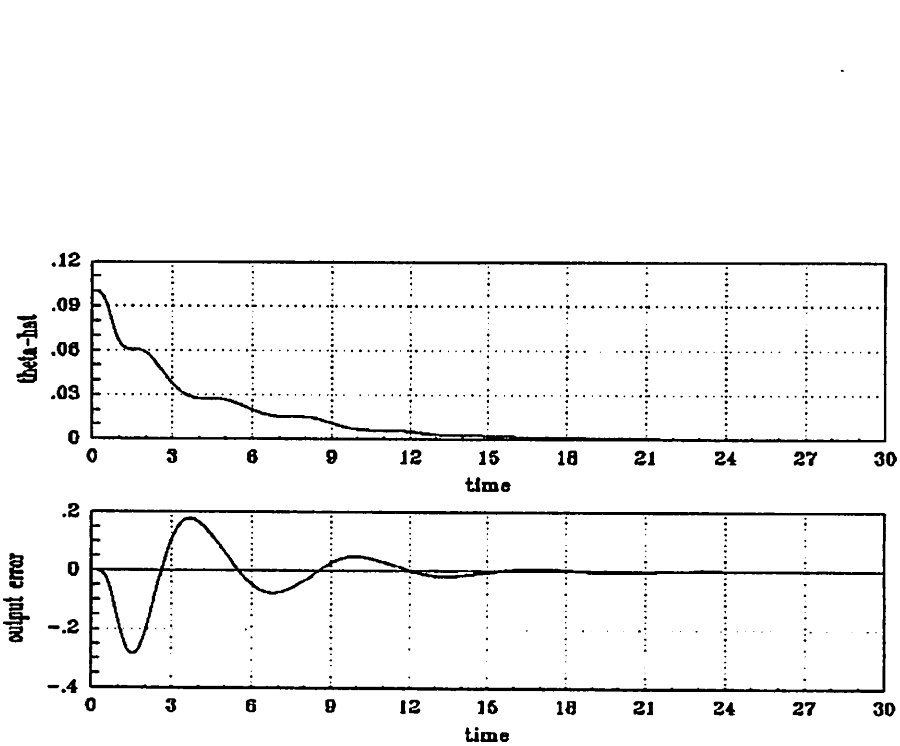

Figure 11: Semi-indirect Adaptive Controller with a Non-Constant Relative Degree Plant

system was able to track the input, thus showing it is not necessary to have the relative degree fixed. In fact, it turns out, as previously stated, that if the relative degree were changing then we must have parameter convergence for the semi-indirect method to asymptotically track an input. Thus, if we assume constant relative degree for the semi-indirect method, then we do not need to have parameter convergence to achieve asymptotic tracking, but if we did not want to assume constant relative degree then we would need to have the parameters converge. The later is, interestingly enough, the same assumption necessary to show tracking in the indirect case. The semi-indirect scheme thus allows us two scenarios. If we are certain of the structure of our plant and can guarantee that the relative degree will not change in the neighborhood of interest, then we do not need to have strict requirements on the richness of the input. On the other hand, if we are not sure of the structure of our plant or have parasitic effects which may easily change the relative degree, then we must have a rich input to assure parameter convergence,thus giving us asymptotic tracking. It should be noted that in all the simulations that have been run (numerous but certainly not exhaustive) a system has yet to be seen where the parameters do not converge in the closed loop with just about any non-zero input.

5.4 Induction Motor

The induction motor from section 3 was linearized and had its loop closed by a simple proportional-derivative controller used to servo the rotor flux ($k_p = 235$, $k_v = 22$) and proportional controllers for the motor torque ($k_m = 10$) and q-component of the stator flux ($k_q = 180$), see Luca and Ulivi (1987). The commanded torque was a step input of 1000 Nm while the rotor flux was a step input of 6.88 V/sec.

The indirect scheme was employed with the observer based identifier with the same gains as used previously. The system worked extremely well, as seen in figure 12. The initial estimates for the sinusoidally varying parameters (25) were about half of their true values ($\alpha_0 = 10$, $\beta_0 = 8$). After the initial transients died out, the identifier was able to converge within one second and track the parameters. In fact if we look at the conditioning of ww^T over an integration window, as in (9), then we can see the inputs to the identifier are quite rich. This seems surprising since the reference signals were simple step inputs. This would imply that the parameter variation alone was enough to cause adequate excitation as shown in the lower plot of figure 12. The linearizing control law was also decoupling. This was verified by changing the individual gains for the input channels separately and noticing their effects solely on the corresponding outputs.

6 Conclusion

In this paper, we have presented convergence results for two nonlinear adaptive control schemes. We presented an output tracking result using indirect adaptive control. This approach was based on certainty equivalence for input output linearization of nonlinear systems. Examples of identification schemes based on observation errors were also presented. The form of the identifier did not need to be specified for the convergence result and over-parameterization was not necessary. However, the result was based on an assumption of identifier convergence. Simulation results were presented for this indirect adaptive control scheme using a familiar induction motor model. Simulation results were also presented on another system to compare this scheme with a direct adaptive scheme, semi-indirect adaptive controller, a non-adaptive control scheme and a sliding mode scheme.

We also presented an output tracking result using a semi-indirect adaptive control scheme.

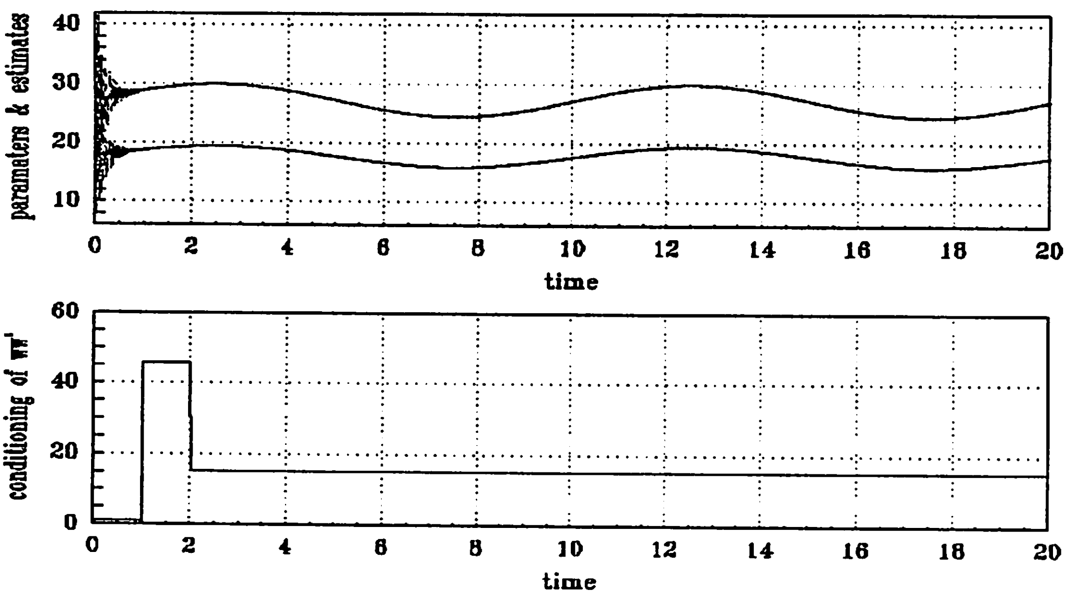

Figure 12: Closed Loop Induction Motor

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