ERROR-BASED ADAPTIVE NONLINEAR CONTROL AND REGIONS OF FEASIBILITY

by

Andrew R. Teel

Memorandum No. UCB/ERL M91/96

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Error-based Adaptive Nonlinear Control and Regions of Feasibility *

Andrew R. Teel
Department of Electrical Engineering
and Computer Sciences
University of California
Berkeley, CA 94720

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Abstract

We recast a recently developed adaptive stabilization algorithm for pure-feedback form nonlinear systems into an error-based algorithm. This enlarges the subset of pure-feedback form nonlinear systems that can be stabilized globally (with respect to the state of the system.)

1 Introduction

Several recent nonlinear adaptive control algorithms have focused on stabilization and tracking for systems that can be described in pure-feedback form. The development of these algorithms were initiated in [3] and have been refined in [4]. These schemes fall into the category of *direct* adaptive control in that the parameter estimates are driven by the mismatch between the plant states and the control objective (stabilization or tracking) for these states. These algorithms have not been cast into an error-based or *indirect* framework. By indirect adaptive control we mean that the parameter estimates are driven by the mismatch between the plant states and a dynamic estimate of the plant states. For recent examples of this approach, see [2], [5] and [6]. An appealing feature of the indirect approach is that parameter estimates that begin close to the actual parameter values remain close to the actual parameter values. This feature can play an important role in the feasibility of the adaptive control algorithm. For instance, consider the following academic example:

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta x_2^3 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u\end{aligned} \tag{1}$$

This system is in pure-feedback form. For this system, the feasibility region of [3] is expressed as a set $\mathcal{F} = B_x \times B_\theta$ where B_x is an open set in $\mathbf{R}^3$ and B_θ is an open set in $\mathbf{R}$ such that

$$|1 + \theta x_2^2| > 0 \quad \forall x \in B_x \quad \forall \theta \in B_\theta$$

We see that one possible feasibility region is given by $B_x = \mathbf{R}^3$ and $B_\theta = \mathbf{R}_+$ so that the global stabilization problem is possible. However, the direct algorithms of [3] and [4] cannot guarantee that θ remains in B_θ unless the initial state $x(0)$ is sufficiently small. Reformulating the algorithm of [3] as an error-based algorithm will eliminate the restriction on the size of the initial state.

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2 The Class of Systems and Feasibility Regions

For simplicity, we will consider single-input systems of the form

$$\begin{aligned} \dot{x}_1 &= \theta^T f_1(x_1, x_2) \\ \dot{x}_2 &= \theta^T f_2(x_1, x_2, x_3) \\ &\vdots \\ \dot{x}_{n-1} &= \theta^T f_{n-1}(x_1, \ldots, x_n) \\ \dot{x}_n &= \theta^T [f_n(x) + g_n(x)u] \end{aligned} \tag{2}$$

Here $\theta \in \mathbf{R}^p \times \{1\}$ is the vector of unknown parameters augmented to allow for terms that are independent of the nominal unknown parameters $\theta^* \in \mathbf{R}^p$. i.e.

$$\theta = \begin{bmatrix} \theta^* \\ 1 \end{bmatrix}$$

The vector $g_n \in \mathbf{R}^{p+1}$ is smooth and the smooth vectors $f_i \in \mathbf{R}^{p+1}$ are such that $f_i(0) = 0$. Geometric conditions for transforming a general single-input nonlinear system into this form (locally) generalize easily from the conditions in [1] and [3].

We demonstrate our algorithm by solving the adaptive stabilization problem. (As in [3], the algorithm presented here naturally extends to the tracking and multi-input problems.) Our algorithm is most powerful when the feasibility region is global in the state x but (possibly) not global in the parameter θ. Consequently, following [3], we make the following definition:

Definition 1 *A feasibility region for the system (2) is any connected set $\mathcal{F} \subset \mathbf{R}^p \times \{1\}$ such that*

$$\begin{aligned} |\theta^T \tfrac{\partial f_i}{\partial x_{i+1}}| &> 0 \quad for \quad i = 1, \ldots, n-1 \\ |\theta^T g_n(x)| &> 0 \end{aligned}$$

for all $x \in \mathbf{R}^n$ and for all $\theta \in \mathcal{F}$.

Remarks.

1. As noted in [3], the sets $\mathcal{F}$ are connected sets where the system is full-state linearizable.
2. It is important to note that feasibility regions are connected. For example, in the case that $p = 1$ it may be true that the conditions of definition 1 are satisfied for all $\theta \neq 0 \times \{1\}$. However $\mathcal{F} = (0 \times \{1\})^c$ is not a feasibility region.

We now restrict the augmented parameter vector $\theta \in \mathbf{R}^p \times \{1\}$ so that our algorithm remains feasible. To do so, let $\{S_\theta^i\}$ be the collection of sets known to contain θ and define $S_\theta = \cap S_\theta^i$. Further, let $\{\mathcal{F}^j\}$ be the collection of feasibility regions such that $S_\theta \subset \mathcal{F}^j$ and define $\mathcal{F} = \cup \mathcal{F}^j$. ($\mathcal{F}$ is connected since $\theta \in \mathcal{F}^j$.)

Assumption 1 *If $\mathcal{F} \neq \mathbf{R}^p \times \{1\}$ then we assume:*

1. *If $p = 1$ and $\mathcal{F}$ is unbounded, then $clos(S_\theta) \subset \mathcal{F}$*
2. *otherwise, $S_\theta \subset B_{(r,\theta')} \times \{1\} \subset B_{(2r,\theta')} \times \{1\} \subset \mathcal{F}$ where $B_{(r,\theta')} \subset \mathbf{R}^p$ is a ball of radius r centered at some $\theta' \in \mathbf{R}^p$.*

Remark. We see that when the entire space $\mathbf{R}^p \times \{1\}$ is not a feasibility region, we restrict the possible values of the unknown parameter vector θ. In the case of one unknown parameter, we do not necessarily restrict θ to lie in a bounded set. For example, if $\mathcal{F} = \mathbf{R}^+ \times \{1\}$ then it is sufficient to know that $\theta \in \mathbf{R}_+ \times \{1\}$. If $\mathcal{F} = (0, +\infty) \times \{1\}$ then it is sufficient to know that $\theta \in [\epsilon, +\infty) \times \{1\}$ for some $\epsilon > 0$.

If $p = 1$ and $\mathcal{F}$ is bounded or if $p > 1$, we restrict θ to lie in a bounded set. For example, if $\mathcal{F} = \mathbf{R}_+ \times \mathbf{R}_+ \times \{1\}$, then we require θ to lie in some ball such that a ball of twice the radius and centered at the same point is contained in $\mathcal{F}$. The reason for this will become clear in the stability proof.

3 The Stabilization Algorithm

We recast the basic algorithm of [3] into an error-based algorithm.

Step 0. Define $z_1 = x_1$.

Step 1. The previous step gives

$$\dot{z}_1 = \theta^T f_1(x_1, x_2) \tag{3}$$

Now define

$$z_2 = \hat{\theta}_1^T f_1(x_1, x_2) \tag{4}$$

where $\hat{\theta}_1$ is an estimate of θ. Substituting (4) into (3) yields

$$\begin{aligned} \dot{z}_1 &= z_2 + [\theta - \hat{\theta}_1]^T f_1(x_1, x_2) \\ &= z_2 + [\theta - \hat{\theta}_1]^T w_1(z_1, z_2, \hat{\theta}_1) \end{aligned} \tag{5}$$

(We will demonstrate in the stability proof that assumption 1 ensures this algorithm is feasible and hence the inverse relation between z_2 and x_2 is well-defined. We write w_1 as a function of z_1, z_2 and $\hat{\theta}_1$ for completeness. When implementing this algorithm, it will be easier to employ this function expressed in the original coordinates x_1, x_2.)

We choose the update law for $\hat{\theta}_1$ to be driven by the mismatch between the state z_1 and a dynamic estimate of this state $\hat{z}_1$:

$$\begin{aligned} \dot{\hat{z}}_1 &= -\alpha_1(\hat{z}_1 - z_1) + z_2 \\ \dot{\hat{\theta}}_1 &= (z_1 - \hat{z}_1) w_1(z_1, z_2, \hat{\theta}_1) \end{aligned} \tag{6}$$

where $\alpha_1 > 0$.

Step 2. The previous step gives

$$\begin{aligned} \dot{z}_2 &= \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_2} \theta^T f_2(x_1, x_2, x_3) + \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_1} \theta^T f_1(x_1, x_2) + (z_1 - \hat{z}_1) w_1^T(z_1, z_2, \hat{\theta}_1) f_1(x_1, x_2) \\ &= \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_2} \theta^T f_2(x_1, x_2, x_3) + \theta^T \psi_1(z_1, z_2, \hat{\theta}_1) + \chi_1(z_1, z_2, \hat{z}_1, \hat{\theta}_1) \end{aligned} \tag{7}$$

Now define

$$z_3 = \hat{\theta}_1^T \frac{\partial f_1}{\partial x_2} \hat{\theta}_2^T f_2(x_1, x_2, x_3) + \hat{\theta}_2^T \psi_1(z_1, z_2, \hat{\theta}_1) + \chi_1(z_1, z_2, \hat{z}_1, \hat{\theta}_1) \tag{8}$$

where $\hat{\theta}_2$ is an (independent) estimate of θ. Substituting (8) into (7) yields

$$\dot{z}_2 = z_3 + [\theta - \hat{\theta}_2]^T w_2(z_1, z_2, z_3, \hat{z}_1, \hat{\theta}_1, \hat{\theta}_2) \tag{9}$$

We choose the update law for $\hat{\theta}_2$ to be driven by the mismatch between the state z_2 and a dynamic estimate of this state $\hat{z}_2$:

$$\begin{aligned} \dot{\hat{z}}_2 &= -\alpha_2(\hat{z}_2 - z_2) + z_3 \\ \dot{\hat{\theta}}_2 &= (z_2 - \hat{z}_2)w_2(z_1, z_2, z_3, \hat{z}_1, \hat{\theta}_1, \hat{\theta}_2) \end{aligned} \tag{10}$$

where $\alpha_2 > 0$.

Step i: i=3,...,n-1. The previous step gives

$$\begin{aligned} \dot{z}_i &= \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_2} \cdots \hat{\theta}_{i-1}^T \tfrac{\partial f_{i-1}}{\partial x_i} \theta^T f_i(x_1, \ldots, x_{i+1}) \\ &\quad + \theta^T \psi_i(z_1, \ldots, z_i, \hat{z}_1, \ldots, \hat{z}_{i-2}, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}) + \chi_i(z_1, \ldots, z_i, \hat{z}_1, \ldots, \hat{z}_{i-1}, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}) \end{aligned} \tag{11}$$

Now define

$$\begin{aligned} z_{i+1} &= \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_2} \cdots \hat{\theta}_{i-1}^T \tfrac{\partial f_{i-1}}{\partial x_i} \hat{\theta}_i^T f_i(x_1, \ldots, x_{i+1}) \\ &\quad + \hat{\theta}_i^T \psi_i(z_1, \ldots, z_i, \hat{z}_1, \ldots, \hat{z}_{i-2}, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}) + \chi_i(z_1, \ldots, z_i, \hat{z}_1, \ldots, \hat{z}_{i-1}, \hat{\theta}_1, \ldots, \hat{\theta}_{i-1}) \end{aligned} \tag{12}$$

where $\hat{\theta}_i$ is an (independent) estimate of θ. Substituting (12) into (11) yields

$$\dot{z}_i = z_{i+1} + [\theta - \hat{\theta}_i]^T w_i(z_1, \ldots, z_{i+1}, \hat{z}_1, \ldots, \hat{z}_{i-1}, \hat{\theta}_1, \ldots, \hat{\theta}_i) \tag{13}$$

We choose the update law for $\hat{\theta}_i$ to be driven by the mismatch between the state z_i and a dynamic estimate of this state $\hat{z}_i$:

$$\begin{aligned} \dot{\hat{z}}_i &= -\alpha_i(\hat{z}_i - z_i) + z_{i+1} \\ \dot{\hat{\theta}}_i &= (z_i - \hat{z}_i)w_i(z_1, \ldots, z_{i+1}, \hat{z}_1, \ldots, \hat{z}_{i-1}, \hat{\theta}_1, \ldots, \hat{\theta}_i) \end{aligned} \tag{14}$$

where $\alpha_i > 0$.

Step n. The previous step gives

$$\begin{aligned} \dot{z}_n &= \hat{\theta}_1^T \tfrac{\partial f_1}{\partial x_2} \cdots \hat{\theta}_{n-1}^T \tfrac{\partial f_{n-1}}{\partial x_n} \theta^T g_n(x) u \\ &\quad + \theta^T \psi_n(z_1, \ldots, z_n, \hat{z}_1, \ldots, \hat{z}_{n-2}, \hat{\theta}_1, \ldots, \hat{\theta}_{n-1}) + \chi_n(z_1, \ldots, z_n, \hat{z}_1, \ldots, \hat{z}_{n-1}, \hat{\theta}_1, \ldots, \hat{\theta}_{n-1}) \end{aligned} \tag{15}$$

We choose the input

$$u = \Delta^{-1}[-\hat{\theta}_n^T \psi_n - \chi_n - k_1 z_1 - \ldots - k_n z_n] \tag{16}$$

where k_i are the coefficients of a Hurwitz polynomial and

$$\Delta = \hat{\theta}_1^T \frac{\partial f_1}{\partial x_2} \cdots \hat{\theta}_{n-1}^T \frac{\partial f_{n-1}}{\partial x_n} \hat{\theta}_n^T g_n(x) \tag{17}$$

and $\hat{\theta}_n$ is an (independent) estimate of θ. (We will demonstrate in the stability proof that assumption 1 ensures the algorithm remains feasible and, hence, Δ^{-1} is well-defined.)

Substituting (16) into (15) yields

$$\dot{z}_n = -k_1 z_1 - \ldots - k_n z_n + [\theta - \hat{\theta}_n]^T w_n(z_1, \ldots, z_n, \hat{z}_1, \ldots, \hat{z}_{n-1}, \hat{\theta}_1, \ldots, \hat{\theta}_n) \tag{18}$$

We choose the update law for $\hat{\theta}_n$ to be driven by the mismatch between the state z_n and a dynamic estimate of this state $\hat{z}_n$:

$$\begin{aligned} \dot{\hat{z}}_n &= -\alpha_n(\hat{z}_n - z_n) - k_1 z_1 - \ldots - k_n z_n \\ \dot{\hat{\theta}}_n &= (z_n - \hat{z}_n)w_n(z_1, \ldots, z_n, \hat{z}_1, \ldots, \hat{z}_{n-1}, \hat{\theta}_1, \ldots, \hat{\theta}_n) \end{aligned} \tag{19}$$

where $\alpha_n > 0$.

Step n+1. Consider the set S_θ and $\mathcal{F}$ of assumption 1. If $\mathcal{F} = \mathbf{R}^p \times \{1\}$, then $\hat{\theta}_i(0)$ can be chosen anywhere in $\mathbf{R}^p \times \{1\}$. Otherwise, if $p = 1$ and $\mathcal{F}$ is unbounded then the projection of $\mathcal{F}$ onto $\mathbf{R}$ has either a well-defined least upper bound or greatest lower bound, but not both. Denote whichever is well-defined by β. Finally, let $\hat{\theta}_i(0)$ be that point in the closure of S_θ with the shortest distance to $(\beta, 1) \in \mathbf{R} \times \{1\}$. If $\mathcal{F}$ is bounded or $p > 1$ then consider the ball $B_{(r,\theta')}$ associated with S_θ as defined in assumption 1. Choose the initial state of the parameter estimates as

$$\hat{\theta}_i(0) = \begin{bmatrix} \theta' \\ 1 \end{bmatrix} \tag{20}$$

This, together with $x(0)$ completely defines $z(0)$. Now choose the initial state of the state estimates such that $\hat{z}(0) = z(0)$.

Remarks.

1. It is clear that $\hat{\theta}_{i,p+1}(0) = \theta_{p+1} = 1$. Consequently, updating $\hat{\theta}_{i,p+1}$ is not necessary.

2. It follows from the algorithm and the above remark that the dimension of the dynamic adaptive compensator is $np + n$. The n additional states are due to estimating the states dynamically to construct an error-based identifier. These additional states are not found in the algorithm of [3].

3. Because of the error-based scheme we are able to place the poles of the certainty equivalence z dynamics arbitrarily with the Hurwitz polynomial coefficients k_i.

4. Let $f(x,\theta)$ denote the drift vector field and $g(x,\theta)$ denote the input vector field both associated with (2) and let $h(x) = x_1$. It follows from the algorithm that if

$$\hat{\theta}_i \equiv \theta, \quad \hat{z}_i \equiv z_i \quad for \quad i = 1, \ldots, n \tag{21}$$

then

$$z_i = L^{i-1}_{f(x,\theta)} h(x)$$

and

$$u = (L_{g(x,\theta)} L^{n-1}_{f(x,\theta)} h(x))^{-1} [-L^n_{f(x,\theta)} h(x) - k_1 h(x) - \ldots - k_n L^{n-1}_{f(x,\theta)} h(x)]$$

The condition (21) is an equilibrium point of the identifier, independent of the value of z. Consequently, if (21) is satisfied at $t = 0$ then the the control implemented for $t \geq 0$ is an exact linearizing control.

5. As seen in step n+1, the selection of the initial value of $\hat{\theta}$ is not arbitrary. It is selected to ensure that the algorithm remains feasible.

4 Closed-loop Stability

In this section we prove the following theorem:

Theorem 4.1 (Adaptive Regulation) *Under assumption 1, if the algorithm of section 3 is applied to the system (2), the resulting closed loop system is such that*

$$\lim_{t \to \infty} x = 0 \tag{22}$$

for all $x(0) \in \mathbf{R}^n$.

Proof. The algorithm of section 3 yields the following closed loop system:

$$\begin{aligned}
\dot{z}_1 &= z_2 + (\theta - \hat{\theta}_1)^T w_1 \\
&\vdots \\
\dot{z}_{n-1} &= z_n + (\theta - \hat{\theta}_{n-1})^T w_{n-1} \\
\dot{z}_n &= -k_1 z_1 - \ldots - k_n z_n + (\theta - \hat{\theta}_n)^T w_n \\
\dot{\hat{z}}_1 &= -\alpha_1(\hat{z}_1 - z_1) + z_2 \\
&\vdots \\
\dot{\hat{z}}_{n-1} &= -\alpha_{n-1}(\hat{z}_{n-1} - z_{n-1}) + z_n \\
\dot{\hat{z}}_n &= -\alpha_n(\hat{z}_n - z_n) - k_1 z_1 - \ldots - k_n z_n \\
\dot{\hat{\theta}}_1 &= (z_1 - \hat{z}_1) w_1 \\
&\vdots \\
\dot{\hat{\theta}}_n &= (z_n - \hat{z}_n) w_n \\
\dot{\theta} &= 0
\end{aligned} \tag{23}$$

We make the following linear coordinate change:

$$\begin{bmatrix} e \\ \hat{z} \\ \phi_i \\ \theta \end{bmatrix} = \begin{bmatrix} I & -I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & -I \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} z \\ \hat{z} \\ \hat{\theta}_i \\ \theta \end{bmatrix} \tag{24}$$

The dynamics of (23) in the new coordinates become:

$$\begin{aligned}
\dot{e}_1 &= -\alpha_1 e_1 - \phi_1^T w_1 \\
&\vdots \\
\dot{e}_n &= -\alpha_n e_n - \phi_n^T w_n \\
\dot{\phi}_1 &= e_1 w_1 \\
&\vdots \\
\dot{\phi}_n &= e_n w_n \\
\dot{\hat{z}}_1 &= \hat{z}_2 + \alpha_1 e_1 + e_2 \\
&\vdots \\
\dot{\hat{z}}_{n-1} &= \hat{z}_n + \alpha_{n-1} e_{n-1} + e_n \\
\dot{\hat{z}}_n &= -k_1 \hat{z}_1 - \ldots - k_n \hat{z}_n + \alpha_n e_n - k_1 e_1 - \ldots - k_n e_n \\
\dot{\theta} &= 0
\end{aligned} \tag{25}$$

We denote by A_k the controllable canonical form matrix corresponding to the Hurwitz polynomial

$$s^n + k_n s^{n-1} + \ldots + k_2 s + k_1$$

We then choose $P > 0$ to satisfy

$$A_k^T P + P A_k = -I \tag{26}$$

To prove stability, we choose the following Lyapunov function candidate:

$$V = \mu[\frac{1}{2}(e^T e + \sum_{i=1}^{n} \phi_i^T \phi_i)] + \frac{1}{2}\hat{z}^T P \hat{z} + \theta^T \theta \tag{27}$$

The derivative of V along the trajectories of (25) is given by

$$\dot{V} = \mu(\sum_{i=1}^{n} -\alpha_i e_i^2) - \hat{z}^T \hat{z} + \hat{z}^T M e \tag{28}$$

where M is a constant matrix independent of μ. It is obvious that $\exists \mu > 0$ such that $\dot{V} \leq 0$ for all $e, \hat{z}, \phi, \theta$. This establishes the stability (i.s.L.) of the closed loop system.

We now focus on the dynamics of the identifier itself to verify that the proposed algorithm is indeed feasible. The n identifier systems are given by

$$\begin{aligned} \dot{e}_1 &= -\alpha_1 e_1 - \phi_1^T w_1 \\ \dot{\phi}_1 &= e_1 w_1 \\ &\vdots \\ \dot{e}_n &= -\alpha_n e_n - \phi_n^T w_n \\ \dot{\phi}_n &= e_n w_n \end{aligned} \tag{29}$$

Consider the Lyapunov function candidate for the ith system of (29):

$$V_i = \frac{1}{2}(e_i^2 + \phi_i^T \phi_i) \tag{30}$$

The derivative for V_i along the trajectories of the ith system of (29) is given by

$$\dot{V}_i = -\alpha_i e_i^2 \tag{31}$$

Since $\dot{V}_i \leq 0$ for all e_i, ϕ_i we can conclude that

$$V_i(t) \leq V_i(0) \tag{32}$$

Since we have chosen $\hat{z}_i(0)$ such that $e_i(0) = 0$ we can then conclude that

$$||\phi_i(t)|| \leq ||\phi_i(0)|| \tag{33}$$

We only need to consider the case when $\mathcal{F} \neq \mathbf{R}^p \times \{1\}$. If $p = 1$ and $\mathcal{F}$ is unbounded, we have chosen $\hat{\theta}_i(0) = (s, 1) \in clos(S_\theta)$ for some $s \in \mathbf{R}$. Define $E_\ell = (-\infty, s] \times \{1\}$ and $E_r = [s, +\infty) \times \{1\}$ and let E denote the one set, E_l or E_r, that is contained in $\mathcal{F}$. (One and only one will satisfy this condition since $\mathcal{F}$ is unbounded but not $\mathbf{R} \times \{1\}$.) We then have $\theta \in S_\theta \subset E \subset \mathcal{F}$. The choice of $\hat{\theta}_i(0)$, the definition of E, the fact that $\theta \in E$ and (33) imply $\hat{\theta}_i(t) \in E$ for all $t \geq 0$. Since $E \subset \mathcal{F}$ it follows that the proposed algorithm is feasible. For $p > 1$ or $\mathcal{F}$ bounded, we have chosen $\hat{\theta}_i(0)$ such that $||\phi_i(0)|| \leq r$. Since we know that $\theta \in S_\theta \subset B_{(r,\theta')} \times \{1\}$ it follows from (33) that $\hat{\theta}_i(t) \in B_{(2r,\theta')} \times \{1\}$ for all $t \geq 0$. Finally, since $B_{(2r,\theta')} \times \{1\} \subset \mathcal{F}$ it follows that the proposed algorithm is feasible.

We now demonstrate asymptotic stability of the state x. First, from (31) it follows that

$$\int_0^\infty \sum_{i=1}^{n} \alpha_i e_i^2 < \infty \tag{34}$$

Next, from the stability of the overall system (see (28)) it follows that $\dot{e}_i$ is bounded. With this we are able to conclude that

$$\lim_{t \to \infty} e_i = 0 \tag{35}$$

Then a simple application of the Bellman-Gronwall lemma to the dynamics of $\hat{z}$ shows that

$$\lim_{t \to \infty} \hat{z} = 0 \tag{36}$$

From (35),(36) and (24) we conclude that

$$\lim_{t \to \infty} z = 0 \tag{37}$$

Finally, from the algorithm of section 3, since $f_i(0) = 0$ and from the definition of a feasibility region, z is a global diffeomorphism of x without translation. Hence,

$$\lim_{t \to \infty} x = 0 \tag{38}$$

□.

5 Conclusion

We have modified the nonlinear adaptive algorithm of [3] to produce an error-based algorithm. This allows global stabilizability for a larger subset of pure-feedback nonlinear systems. The algorithm was demonstrated on the single-input stabilization problem but easily extends to the multi-input and tracking problems.

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