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Abstract

We present some further stability results for cellular neural networks with nonlinear delay-type templates. In particular, we show that there exists a globally asymptotically stable equilibrium point in CNN's with dominant nonlinear delay-type templates.

1 Introduction

Cellular Neural Networks (CNN) is a novel structure for nonlinear analog signal processing [1, 2]. Its applications for various practical problems have been demonstrated [3]. The nucleus of a specific CNN functionality is defined by the analog cloning template which is a geometric and analog code of the weights of local interactions of each cell (uniform analog processing unit).

Nonlinear and delay-type CNN's were introduced in [4]. Delay-type CNN templates are very useful in motion related applications. In a more general setting, CNN appears as an appropriate framework for sensory information processing organs [5].

In this paper we present a set of stability results of dominant nonlinear CNN's with delay.

2 General framework and earlier results

An $M \times N$ delay-type CNN with nonlinear templates is described by the state equations

$$\begin{aligned} C_x \frac{dv_{x_{ij}}(t)}{dt} &= -\frac{1}{R_x} v_{x_{ij}}(t) + \sum_{k,l} \hat{A}(i,j;k,l)(v_{y_{kl}}(t), v_{y_{ij}}(t)) \\ &+ \sum_{k,l} A^{\tau}(i,j;k,l) v_{y_{kl}}(t - \tau^A_{i,j;k,l}) + \sum_{k,l} \hat{B}(i,j;k,l)(v_{u_{kl}}(t), v_{u_{ij}}(t)) \\ &+ \sum_{k,l} B^{\tau}(i,j;k,l) v_{u_{kl}}(t - \tau^B_{i,j;k,l}) + I \end{aligned} \tag{1}$$

where $\tau^A_{ij;kl}, \tau^B_{ij;kl} \geq 0$ for all i,j,k,l. We generally assume that the A-template is space-invariant; i.e., for all i,j,k,l,m and n such that $1 \leq i,k,i+m,k+m \leq M$ and $1 \leq j,l,j+n,l+n \leq N$, $A(i,j;k,l) = A(i+m,j+n;k+m,l+n)$. Generally $\tau^A_{ij;kl}, \tau^B_{ij;kl}$ are space-invariant as well. Without loss of generality, we will assume that $C_x = 1$, $R_x = 1$. The state voltage, input voltage, output voltage of a cell, is respectively, $v_{x_{ij}}$, $v_{u_{ij}}$ and $v_{y_{ij}}$. We assume that the input is continuous and has magnitude less than 1. The output voltage is $v_{y_{ij}}(t) = \hat{f}(v_{x_{ij}}(t))$, $\hat{f}(x) = \frac{1}{2}[|x+1| - |x-1|]$. For linear templates, $\hat{A}(i,j;k,l)(v_{y_{kl}}, v_{y_{ij}}) = A(i,j;k,l)v_{y_{kl}}$.

Consider equation (1). We relabel the state variables $v_{x_{ij}}$ into a vector $\tilde{x}$ of size $n = MN$. Similarly, the input and output variables $v_{u_{ij}}$ and $v_{y_{ij}}$ are relabeled into $\tilde{u}$ and $\tilde{y}$ using the same labeling order. The invertible ordering will be called σ, i.e. $\tilde{x}_{\sigma(i,j)} = v_{x_{ij}}$. Furthermore, $\tau^A_{ij;kl}$ is ordered into a matrix $\tilde{\tau}^A$ such that $\tilde{\tau}^A_{\sigma(i,j),\sigma(k,l)} = \tau^A_{i,j;k,l}$. The same ordering is used on $\tau^B_{ij;kl}$, A, A^{τ}, $\hat{A}$, B^{τ}, and $\hat{B}$ to obtain $\tilde{\tau}^B$, $\tilde{A}$, $\tilde{A}^{\tau}$, $\tilde{A}_{nl}$, $\tilde{B}^{\tau}$, and $\tilde{B}_{nl}$ respectively. After relabeling, the state equations in (1) assume the following form of a system of functional differential equations (FDE):

$$\dot{\tilde{x}} = F(t, \tilde{x}_t) = -\tilde{x}(t) + \tilde{A}_{nl}(f_1(\tilde{x}(t))) + \tilde{A}^{\tau} f_2(\tilde{x}(t-\tau))$$

$$+\tilde{B}_{nl}(\tilde{u}(t)) + \tilde{B}^{\tau}\tilde{u}(t-\tau) + \tilde{I} \tag{2}$$

where $f_1(\tilde{x}(t))$ is defined as:

$$(f_1(\tilde{x}))_i = \hat{f}(\tilde{x}_i) \tag{3}$$

and where $\tilde{A}^{\tau} f_2(\tilde{x}(t-\tau))$ is defined as:

$$\tilde{A}^{\tau} f_2(\tilde{x}(t-\tau))_i = \sum_{k=1}^{n} (\tilde{A}^{\tau})_{ik} \cdot \hat{f}(\tilde{x}_k(t - \tilde{\tau}_{ik}^{A})) \tag{4}$$

The term $\tilde{B}^{\tau}\tilde{u}(t-\tau)$ is defined as:

$$\tilde{B}^{\tau}\tilde{u}(t-\tau)_i = \sum_{k=1}^{n} (\tilde{B}^{\tau})_{ik} \cdot \tilde{u}_k(t - \tilde{\tau}_{ik}^{B}) \tag{5}$$

We will assume that $\hat{A}$ and $\hat{B}$ are continuous, and define

$$\hat{A}_{max}(i,j;k,l) = \sup_{(x,y)\in[-1,1]^2} \hat{A}(i,j;k,l)(x,y) < \infty \tag{6}$$

$$\hat{B}_{max}(i,j;k,l) = \sup_{(x,y)\in[-1,1]^2} \hat{B}(i,j;k,l)(x,y) < \infty \tag{7}$$

Define $\bar{\tau} = \max_{i,j} \tilde{\tau}_{ij}^{A}$ to be the maximal delay and define $C_{\bar{\tau}} = C([-\bar{\tau},0],\mathbb{R}^n)$ to be the set of continuous functions into $\mathbb{R}^n$ defined on the interval $[-\bar{\tau},0]$. The function $\tilde{x}_t \in C_{\bar{\tau}}$ is defined as

$$\tilde{x}_t(s) = \tilde{x}(t+s) \qquad s \in [-\bar{\tau},0] \tag{8}$$

We regard $\xi \in \mathbb{R}^n$ as a function in $C_{\bar{\tau}}$ by setting $\xi(t) = \xi$ for $t \in [-\bar{\tau},0]$. The zero vector in $\mathbb{R}^n$ will be denoted θ. We say $\xi \leq \eta$ for $\xi,\eta \in \mathbb{R}^n$ if $\xi_i \leq \eta_i$ for all $i \in \{1,\ldots,n\}$. Similarly, $\phi \leq \psi$ for $\phi,\psi \in C_{\bar{\tau}}$ if $\phi(t) \leq \psi(t)$ for all $t \in [-\bar{\tau},0]$. We define the interval $[\xi,\eta] = \{x \in \mathbb{R}^n | \xi \leq x \leq \eta\}$. We define $C_{\bar{\tau}}^{[\xi,\eta]}$ as the set of $\phi \in C_{\bar{\tau}}$ such that $\phi(t) \in [\xi,\eta]$ for all $t \in [-\bar{\tau},0]$. The norm used on matrices will be the norm induced by the norm in $\mathbb{R}^n$. We define $\mathbb{R}_+^n = \{x \in \mathbb{R}^n | x \geq \theta\}$. For $\phi \in C_{\bar{\tau}}$, $|\phi| = \sup\{|\phi(t)| : t \in [-\bar{\tau},0]\}$

The initial conditions for the delay-type CNN is given by:

$$v_{x_{ij}}(t) = v_{0_{ij}}(t) \qquad t \in [-\bar{\tau},0] \tag{9}$$

We will assume that $v_{0_{ij}}(t)$ is a continuous function ($v_0(t) \in C_{\tilde{\tau}}$).

Symmetric, non-symmetric positive cell-linking and other types of A-templates have been shown to have stable dynamics [1, 6, 7].

In [8] it has been shown that general and CNN-type neural networks with linear templates and delay are stable under well-defined conditions. In addition, these conditions are also valid for monotone nonlinear templates [9]:

Proposition 2.1 *If a CNN with nonlinear and delay-type A-templates satisfies*

(i) $D\tilde{A}_{nl}(\tilde{x})$ is an off-diagonally nonnegative matrix for all $\tilde{x} \in [-1,1]^n$,

(ii) $\tilde{A}^\tau$ is a nonnegative matrix (the template A^τ has only nonnegative elements),

(iii) $D\tilde{A}_{nl}(\tilde{x}) + \tilde{A}^\tau$ is an irreducible matrix for all $\tilde{x} \in [-1,1]^n$,

(iv) the set of equilibria is finite,

then the union of the basins of attraction of all stable equilibrium points will be a dense open set in $\prod_{i=1}^{n} C([-\tilde{\tau}_i, 0], \mathbb{R})$, where $\tilde{\tau}_i = \max_j \tilde{\tau}_{ji}^A$.

The conclusion of the proposition implies that there are no stable limit cycles or strange attractors. For each set of initial conditions, there exists an arbitrarily small perturbation such that the trajectory will converge towards an equilibrium point.

On the other hand, in [10] it was shown that symmetric A-templates in delay-type CNN's do not necessarily imply stability and that stability is guaranteed for symmetric A-templates if the delay is small enough.

3 Stability results for dominant nonlinear and delay-type templates

The results in [8] can be extended to show that the CNN in equation 1 has an unique solution and the range of dynamics can be calculated.

Proposition 3.1 *Given the initial condition:*

$$\tilde{x}_0(t) = \phi(t), \qquad \phi(t) \in C_{\tilde{\tau}} \tag{10}$$

then the nonlinear delay-type CNN has a unique continuous solution for $t \in [0, \infty)$.

Proposition 3.2 *If the initial condition is bounded by K, then all states $v_{x_{ij}}$ of a nonlinear delay-type cellular neural network are bounded for all time in absolute value by the sum:*

$$\begin{aligned} v_{max} = & K + |I| \\ & + \max_{i,j} \left(\sum_{k,l} \left(|A^{\tau}(i,j;k,l)| + \left|\hat{A}_{max}(i,j;k,l)\right| + |B^{\tau}(i,j;k,l)| + \left|\hat{B}_{max}(i,j;k,l)\right| \right) \right) \end{aligned} \quad (11)$$

and the ω-limit points of $v_{x_{ij}}(t)$ are bounded in absolute value by $v_{max} - K$. Thus for all $\epsilon > 0$,$v_{x_{ij}}(t)$ will eventually be bounded in absolute value by $\epsilon + v_{max} - K$.

In the rest of the paper, we will assume that the input is constant ($\tilde{u}(t) = \tilde{u}$ for all t).

We consider templates which are "dominant" in the sense that the center element is much larger than the other elements in the template. This notion will be defined more precisely in the following two theorems.

Using results in [11] we can show that if the norm $\left|\tilde{A}\right|$ is less than 1, then the linear CNN becomes contractive and has a globally asymptotically stable equilibrium point. We can extend this to nonlinear delay-type CNN's as follows:

Theorem 3.1 *Assume that a finite Lipschitz constant L exists such that $|\tilde{A}_{nl}(\tilde{x}) - \tilde{A}_{nl}(\tilde{y})| \leq L|\tilde{x} - \tilde{y}|$ for all $\tilde{x}$, $\tilde{y} \in [-1,1]^n$. If a nonlinear delay-type CNN satisfy $L + |\tilde{A}^{\tau}| < 1$, then the delay-type and nonlinear CNN in equation (1) contains a globally asymptotically stable equilibrium point.*

Proof: we prove this by using a theorem of Razumikhin-type. A equilibrium point $x^*(t) = x^*$ must satisfy:

$$x^* = \tilde{A}_{nl}(f_1(x^*)) + \tilde{A}^{\tau}(f_2(x^*)) + g = T(x^*) \quad (12)$$

where $g = \tilde{B}_{nl}(\tilde{u}) + \tilde{B}^{\tau}(\tilde{u}) + \tilde{I}$. By the assumptions, x^* is a fixed point of the contraction T, therefore x^* is the unique equilibrium of the system and it exists. We use the change of variables $u = x - x^*$, and define the following Liapunov function:

$$V(u) = \frac{1}{2} < u, u > \quad (13)$$

Taking the derivative along solutions, we get

$$
\begin{aligned}
\dot{V}(u) &= <\dot{u}(t), u(t)> \;=\; <\dot{x}(t), u(t)> \\
&= <-u(t) - x^* + \tilde{A}_{nl}(f_1(u(t)+x^*)) + \tilde{A}^{\tau}(f_2(u(t-\tau)+x^*)) + g,\ u(t)> \\
&= <-u(t) + \tilde{A}_{nl}(f_1(u(t)+x^*)) - \tilde{A}_{nl}(f_1(x^*)) + \tilde{A}^{\tau}(f_2(u(t-\tau)+x^*) - f_2(x^*)),\ u(t)>
\end{aligned} \tag{14}
$$

By the choice of the nonlinearity $\hat{f}$

$$|f_1(u(t)+x^*) - f_1(x^*)| \leq |u(t)| \tag{15}$$

$$|\tilde{A}^{\tau} f_2(u(t-\tau)+x^*) - \tilde{A}^{\tau} f_2(x^*)| \leq |\tilde{A}^{\tau}||u_t| \tag{16}$$

So we have

$$\dot{V}(u) \leq -|u(t)|^2 + L < u(t), u(t)> + |\tilde{A}^{\tau}||u(t)||u_t| \leq (-1 + L + \rho|\tilde{A}^{\tau}|)|u(t)|^2 \leq 0 \tag{17}$$

for all $|u_t| < \rho|u(t)|$, where $\rho > 1$ is a real number such that $-1 + L + \rho|\tilde{A}^{\tau}| < 0$.

Then by Theorem 4.2 in [12, page 127] the equilibrium point $u = 0$ is a global attractor. Therefore $x = x^*$ is a global attractor of equation (2). ■

In other words, if the elements of the templates $\hat{A}$ and A^{τ} are small enough, then the CNN will always converge to the unique equilibrium point. Note that because of the cellular and space-invariant structure of the template, $|\tilde{A}|, L$ and $|\tilde{A}^{\tau}|$ do not depend on the size of the cell array.

The second type of dominant templates we consider are positive templates. In [4] it was shown that if $\tilde{A}$ is off-diagonally non-negative and irreducible, then the linear CNN is stable almost everywhere. In [13] it was shown that if $\tilde{A}$ is off-diagonally non-negative and $-\tilde{A}$ is row sum dominant, i.e., the sum of the absolute values of the off-diagonal elements in each row of $-\tilde{A}$ is smaller than the corresponding diagonal element, then the linear CNN has a globally asymptotically stable equilibrium point. Such A-templates have the following sign structure:

$$\begin{bmatrix} + & + & + & + & + \\ + & + & + & + & + \\ + & + & - & + & + \\ + & + & + & + & + \\ + & + & + & + & + \end{bmatrix}$$

This can be useful in analyzing templates which have a off-center on-surround character [5, 14]. This result can also be extended to linear delay-type templates:

Theorem 3.2 *If a linear delay-type CNN is such that (i) $\tilde{A}$ has nonnegative off-diagonal elements, (ii) $\tilde{A}^\tau$ has nonnegative elements and (iii) $-(\tilde{A}+\tilde{A}^\tau)$ is row sum dominant, then the CNN has a globally asymptotically stable equilibrium point.*

Proof: we prove this by applying the results in [15]. Let k be a strict bound for the ω-limit points [8], i.e. for each solution $x(t)$ of the FDE, there exist a T such that $|x(t)| < k$ for all $t > T$. In fact we will choose $k > 1$ such that

$$\begin{aligned} k &= 1 + \epsilon + |I| \\ &\quad + \max_{i,j}\left(\sum_{k,l}\left(|A^\tau(i,j;k,l)| + \left|\hat{A}_{max}(i,j;k,l)\right| + |B^\tau(i,j;k,l)| + \left|\hat{B}_{max}(i,j;k,l)\right|\right)\right) \end{aligned} \tag{18}$$

for some $\epsilon > 0$. Let $\kappa \in \mathbb{R}^n$ be defined as $\kappa_i = k$ for all $i \in \{1,\ldots,n\}$. Define the following change of variables:

$$w(t) = \tilde{x}(t) + \kappa \tag{19}$$

Then w satisfies

$$\dot{w} = F(t, w_t - \kappa) = G(t, w_t) = G(w_t) \tag{20}$$

We show that the FDE in equation (20) satisfy assumptions $H-1$,$H-2$, $H-3$ and $H-5$ in [15]. It is easy to verify that $G(2\kappa) \leq \theta$ and $G(\theta) \geq \theta$. Therefore assumption $H-3$ is satisfied. Assumption $H-1$ is satisfied because G is globally Lipschitzian [15, Remark 4]. $H-2$ is satisfied because $\tilde{A}$ has nonnegative off-diagonal elements and $\tilde{A}^\tau$ has nonnegative elements. To show that $H-5$ is satisfied, let $\xi, \eta \in \mathbb{R}^n_+$ be such that $\xi \leq \eta$ and $\xi \neq \eta$. Let i be an index such that

$\hat{f}(\eta_i - \kappa) - \hat{f}(\xi_i - \kappa)$ has the largest value. It is clear that we can choose i such that $\eta_i - \xi_i > 0$. Then

$$\begin{aligned} G_i(\xi) - G_i(\eta) &= \eta_i - \xi_i + \textstyle\sum_{j=1}^{n}(\tilde{A} + \tilde{A}^{\tau})_{ij}(\hat{f}(\xi_j - \kappa) - \hat{f}(\eta_j - \kappa)) \\ &\geq \eta_i - \xi_i + (\hat{f}(\eta_i - \kappa) - \hat{f}(\xi_i - \kappa)) \textstyle\sum_{j=1}^{n} -(\tilde{A} + \tilde{A}^{\tau})_{ij} \\ &\geq \eta_i - \xi_i > 0 \end{aligned} \tag{21}$$

Where we have used the assumption that $\sum_{j=1}^{n} -(\tilde{A} + \tilde{A}^{\tau})_{ij} \geq 0$ for all i. Therefore $H-5$ is satisfied.

By [15, Lemma 3], for all initial conditions in $C_{\overline{\tau}}^{[\theta,2\kappa]}$ the trajectory of equation (20) will converge to one equilibrium point. This means that for all initial conditions in $C_{\overline{\tau}}^{[-\kappa,\kappa]}$ all trajectory of equation (2) will converge to an unique equilibrium point. By definition of k, all trajectories will eventually enter $[-\kappa, \kappa]$. Therefore for any trajectory $\tilde{x}(t)$, there exists $T > 0$ such that $\tilde{x}_T(\cdot) \in C_{\overline{\tau}}^{[-\kappa,\kappa]}$ and the unique equilibrium point is globally asymptotically stable. ■

The property of having a globally asymptotically stable equilibrium point allows such a CNN to be used as a pattern classifier or encoder [11] in the sense that there is a nonlinear map which relates the steady state output to the input independent of the initial conditions. One could also view these CNN's as solving nonlinear equations in the sense that given any initial condition, the CNN will converge to the unique solution of a set of nonlinear algebraic equations with the set of equations being solved depending on the input.

4 Conclusions

Simple and useful stability conditions have been presented in this paper for the case when the templates are dominant nonlinear and delay-type.

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