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Transmission of Digital Signals by Chaotic Synchronization

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Abstract

The transmission of digital signals by means of chaotic synchronization is demonstrated numerically as well as experimentally via Chua's circuit.

Recently Pecora and Carroll [1990], [1991a], [1991b] demonstrated the possibility to synchronize chaotic (sub)systems with a common driving signal. In this letter we explore the possible applications of this *chaotic synchronization* approach. In view of their typical broadband spectra, chaotic signals are ideal candidates for *masking* and *spread spectrum* communication applications. The main idea is to use the synchronization mechanism to transmit digital signals where, for example, a 1 corresponds to a synchronized state and a 0 to desynchronization. To keep the presentation as simple as possible we will consider only three dimensional continuous systems. Generalizations for high dimensional systems or iterated maps are straight forward.

Following the approach of Pecora and Carroll we decompose the given (chaotic) dynamical system

$$\begin{aligned} \dot{x} &= f(x,y,z) \\ \dot{y} &= g(x,y,z) \\ \dot{z} &= h(x,y,z) \end{aligned} \tag{1}$$

into two subsystems. Without loss of generality we shall divide the state variables (x,y,z) from system (1) into (x) and (y,z). The first subsystem (x) is considered as the *drive*, and the second subsystem (y,z) as the *response*. A copy (y_A, z_A) of the response system is then subjected to the drive signal $x(t)$. If the Lyapunov exponents of the response system (y,z) driven by x are all negative it synchronizes with the driving signal, i.e. the differences $y - y_A$ and $z - z_A$ converge to zero. In the case of a communication system, the receiver of signal x in general does not have access to the signals y and z to compare against and thus we cannot determine whether the subsystem (y_A, z_A) is synchronized or not. To determine whether there indeed is synchronization, reference signals are needed. They can be generated from the known variables x, y_A and z_A by means of a second subsystem B. The second subsystem can in principle be any of the following types:

$$\begin{array}{ccc} (y_A) & \rightarrow & (x_{B1}, z_{B1}) \\ (z_A) & \rightarrow & (x_{B2}, y_{B2}) \\ (x, y_A) & \rightarrow & (z_{B3}) \\ (x, z_A) & \rightarrow & (y_{B4}) \\ (y_A, z_A) & \rightarrow & (x_{B5}) \end{array} \tag{2}$$

Using one or several of these subsystems provides the receiver with reference signals x_{B1}, x_{B2}, x_{B5}, y_{B2}, y_{B4}, z_{B1} or z_{B3} that may be compared with each other and with x, y_A or z_A in order to verify the presence of synchronization. Of course, again the subsystems have to be stable.

To illustrate this idea we shall consider in the following chaotic synchronization of Chua's circuit [Chua, 1992]. This piecewise-linear system shows a family of chaotic attractors and can easily be implemented by hardware [Kennedy, 1992],[Cruz and Chua,1992]. Two members of this family are the Rössler and the Double Scroll attractor. In this paper we shall use the Rössler attractor to illustrate numerically the transmission of digital signals via Chua's circuit. But we will use the Double Scroll attractor to demonstrate the method's experimental feasibility.

The state equations for Chua's circuit are given by

$$\begin{aligned}\frac{dv_1}{dt} &= \frac{1}{C_1}\left[\frac{v_2 - v_1}{R} - f(v_1)\right]\\ \frac{dv_2}{dt} &= \frac{1}{C_2}\left[\frac{v_1 - v_2}{R} + i_3\right]\\ \frac{di_3}{dt} &= \frac{1}{L}\left[-v_2 - R_0 i_3\right]\end{aligned} \tag{3}$$

where v_1, v_2 and i_3 are the voltage across C_1, the voltage across C_2 and the current through L, respectively.

The nonlinear characteristic of the circuit is given by

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)\left[|v_1 + B_p| - |v_1 - B_p|\right]. \tag{4}$$

For our numerical experiments we used two Rössler chaotic attractors. The parameters for the first one are $R = 1001\Omega$, $R_0 = 20\Omega$, $G_a = \frac{-1}{878\Omega}$, $G_b = \frac{1}{2339\Omega} - \frac{1}{878\Omega}$, $B_p = 1\text{V}$, $L = 12\text{mH}$, $C_1 = 17\text{nF}$ and $C_2 = 178\text{nF}$. To generate the second chaotic attractor we used $R = 1001\Omega$, $R_0 = 20\Omega$, $G_a = \frac{-1}{878\Omega}$, $G_b = \frac{1}{2339\Omega} - \frac{1}{878\Omega}$, $B_p = 1\text{V}$, $L = 13.3\text{mH}$, $C_1 = 18.8\text{nF}$ and $C_2 = 197\text{nF}$. With these values equation (3) possesses two different but qualitatively similar chaotic attractors.

For the following discussion we use the abbreviations $x = v_1$, $y = v_2$ and $z = i_3$. Since Chua's circuit is a piecewise-linear system the stability analysis can partly be done analytically and some of the resulting (conditional) Lyapunov exponents do not depend on the driving signal. The only unstable subsystem is (x, y), i.e. the z-variable can not be used to transmit the signal. With x−driving; i.e. $(x) \rightarrow (y_A, z_A)$, all subsystems listed in (2) except for (x_{B2}, y_{B2}) can be used to generate a reference signal. Therefore the following quantities may serve in principle to monitor the state of synchronization, because they all have to converge to zero.

$$\begin{aligned}d_1 &= x - x_{B1} & d_5 &= z_A - z_{B1}\\ d_2 &= x - x_{B5} & d_6 &= z_A - z_{B3}\\ d_3 &= x_{B1} - x_{B5} & d_7 &= z_{B1} - z_{B3}\\ d_4 &= y_A - y_{B4} & & \end{aligned} \tag{5}$$

For the vector field (3) under investigation it turned out that most of the stable subsystems are also stable when driven with the "wrong" signal (i.e.

with a time series from the other attractor). This means that the subsystems (x_{B1}, z_{B1}) and (x_{B5}) will converge to the same (response) trajectory. Therefore d_1 equals d_2 and d_3 always converges to zero (i.e. even without synchronization). The differences d_4 to d_7 also converge to zero for the same reason. For x-driving the only quantity that can be used to monitor the state of synchronization is thus $\Delta_0 = d_1 = d_2$ for the first chaotic source, and analogously Δ_1 for driving with the signal from the second chaotic attractor. For other dynamical systems, or different choices of the driving signals, other combinations may be more suitable.

The convergence properties of the differences Δ_0 and Δ_1 are shown in Fig.1c and Fig.1d. Figure 1a shows the binary signal b_{in} to be transmitted and Fig. 1b gives the driving signal $s(t)$ derived from two different chaotic attractors of Chua's circuit (3). Whenever a message 0 occurs in the binary signal we use the voltage v_1 of the first attractor to drive the subsystems of the receiver and the difference Δ_0 converges to zero. For each message 1, the v_1-coordinate of the second chaotic attractor is transmitted and Δ_1 tends to zero.

As can be seen in the figure the differences Δ_0 and Δ_1 are strongly fluctuating. In order to recover the digital signal we smoothed these fluctuactions with a moving average filter of length equal to 40 time steps. The results are shown in Fig.1e and Fig.1f. From the smoothed differences $a_0 = a_0(t)$ and $a_1 = a_1(t)$ we derived the binary output $b_{out} = b_{out}(t)$ with the following rule:

$$\begin{array}{ll} a_0 < \varepsilon \ \wedge \ a_1 > \varepsilon \ \Rightarrow & b_{out} = 0 \\ a_0 > \varepsilon \ \wedge \ a_1 < \varepsilon \ \Rightarrow & b_{out} = 1 \\ \quad \text{otherwise} & b_{out}(t_+) = b_{out}(t_-). \end{array} \tag{6}$$

where t_+ denotes the right-hand limit at time t, and t_- denotes the left-hand limit at time t. The threshold was chosen to be $\varepsilon = 0.1$. The resulting digital signal b_{out} is shown in Fig.1g and agrees up to a time shift with the original input given in Fig.1a.

The experimental set-up is shown in Fig.2. The transmitter consists of one Chua's circuit. Note that C_1 in the transmitter is the voltage-controlled capacitance. For our experiment, we used two Double Scroll attractors. The parameters of the first attractor are: $G_a = -0.409mS$, $G_b = -0.756mS$, $B_p = 1.08V$, $C_1 = 10.04nF$, $C_2 = 102.2nF$, $R = 1.747k\Omega$, $R_0 = 20\Omega$ and $L = 18.77mH$. The parameters for the second attractor are the same except for $C_1 = 9.79nF$. To recover the transmitted signal $s(t)$, it is sufficient to monitor the state of syncronization of one attractor. In our experiment, the receiver consists of two partial Chua's circuits: the (y_A, z_A)- subsystem driven by the transmitted signal x and the (x_{B1}, z_{B1})-subsystem (in our case it is only the (x_{B1})-subsystem) driven by the signal y_A [Kocarev et al, 1992]. Two such circuits, each monitoring the synchronization state of one (of the two) transmitted chaotic attractor, may have been used in conjunction with rule (6). But while such a scheme improves the reliability of the communication, it is not necessary for the demonstration of our basic idea. To receive the synchronized state the receiver's parameters are matched with the transmitter's parameters of the first Double Scroll attractor. Figure 3 shows the quality of synchronization between the transmitter and the receiver. The horizontal axis corresponds to v_1 of the transmitter and the vertical axis to v_1 of the receiver.

To transmit a desynchronized state, all the parameters are still matched except for the voltage-controlled capacitance C_1 of the transmitter which is switched with a stepwise voltage source (see Fig.2) from the matched value of 10.04nF to 9.79nF. For this latter value of C_1, the transmitter still operates in a double scroll mode but its x-component is completely out of synchronization with x' of the receiver.

An alternating sequence of '1' and '0' shown at the top of Fig. 4 is applied to the voltage-controlled capacitance. At the bottom of this figure, the transmitted "chaotic" signal, $s(t)$ is shown. The information signal is again shown at the top of Fig. 5 and the recovered signal $b_{out}(t)$ is displayed at the bottom of Fig. 5 which clearly shows that the original information is recovered at the receiver.

In this letter we have demonstrated numerically as well as experimentally the application of chaotic synchronization to transmit digital signals (binary sequences). Two strange attractors of Chua's circuit are used as sources for the chaotic driving signal. For each 0 in the binary sequence, we transmit the first coordinate (voltage v_1) of the first attractor, and for all 1's, we transmit the first coordinate of the second attractor. The transmitted signal is thus a chaotic (broadband) signal. We have chosen two different chaotic signals for two reasons. In order to mask the information, another chaotic or random signal has to be sent in any case during those periods of time when the chaotic driving signal is switched off. The use of two chaotic signals with mutually exclusive synchronization properties improves the reliability of the communication and allows us to decide whether there has been sent a message or not. Instead of using two different chaotic attractors one may also use different coordinates from the same attractor. We have succesfully tested this approach, for example, by alternating the x and y driving signals to switch between two synchronized states. For some systems it may be possible just to reverse the sign of the transmitted variable in order to synchronize different subsystems. The proper choice of the driving signals depends of course very much on the additional constraints of the communication process and is an important field for future research. Another research topic is the design of high-dimensional dynamical systems that provide as many useful reference signals as possible. In any case our results show that secure communication based on chaotic synchronization is possible and may lead (perhaps in combination with other standard techniques) to important innovations.

Acknowledgement

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Figure captions

Fig.1: Transmission of digital signals: numerical results

(a) Binary input signal $b_{in}(t)$

(b) Transmitted signal $s(t)$

(c) Response Δ_0

(d) Response Δ_1

(e) 40-points moving average of Δ_0

(f) 40-points moving average of Δ_1

(g) Recovered binary signal b_{out} computed with rule (6) and $\varepsilon = 0.1$

Fig.2: Transmission of digital signals: experimental set-up

(a) Block diagram

(b) Practical realization of the transmitter. Note that C_1 is the voltage-controlled capacitance.

(c) Practical realization of the receiver

Fig.3: Chaotic synchronization between the transmitter and the receiver: v_1 of the receiver versus v_1 of the transmitter.

Fig.4: Top of figure: Information to be transmitted.
Bottom of figure: Actual signal transmitted.

Fig.5: Top of figure: Information to be transmitted.
Bottom of figure: Recovered signal at the receiver.

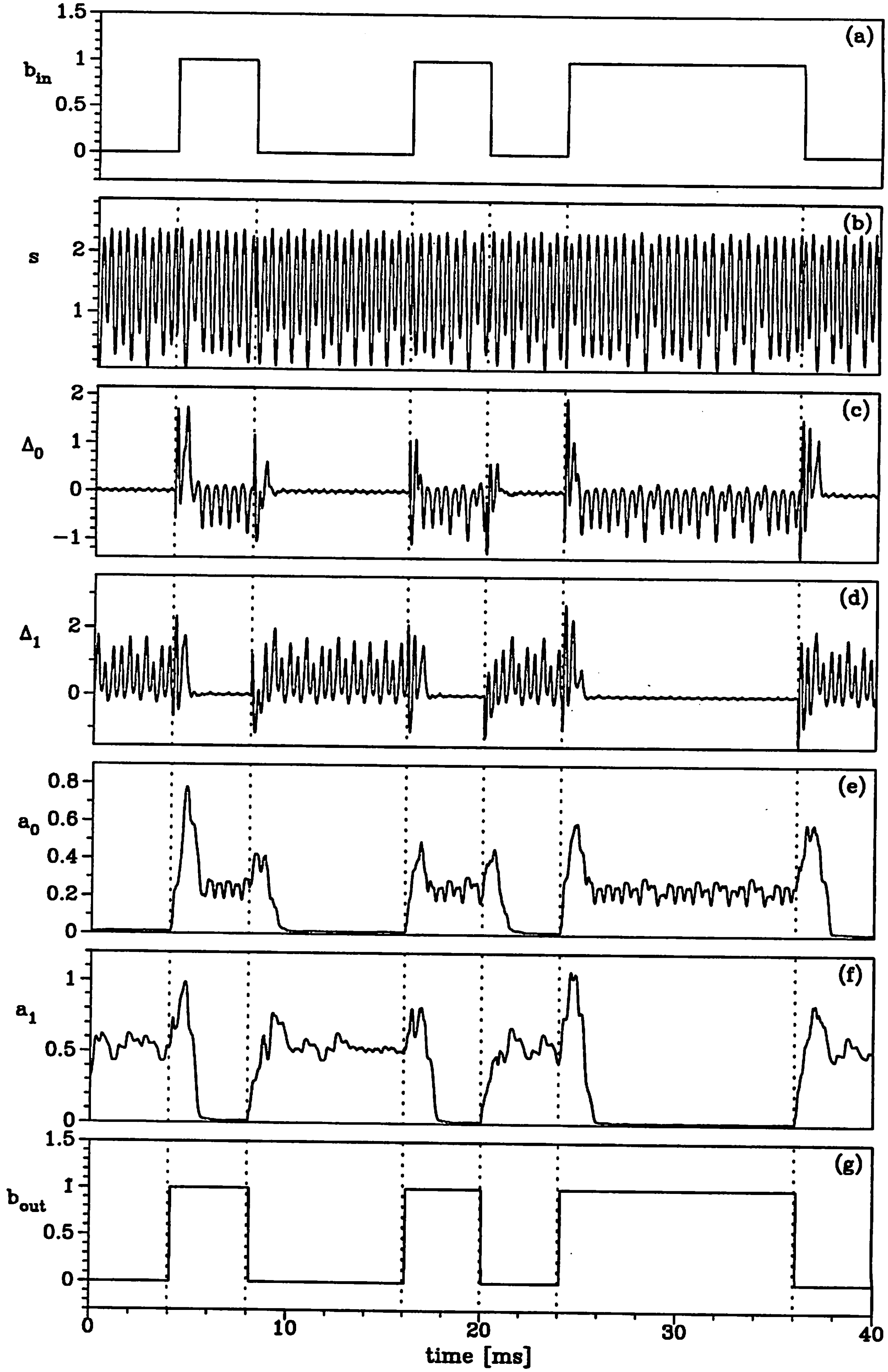

Fig. 1

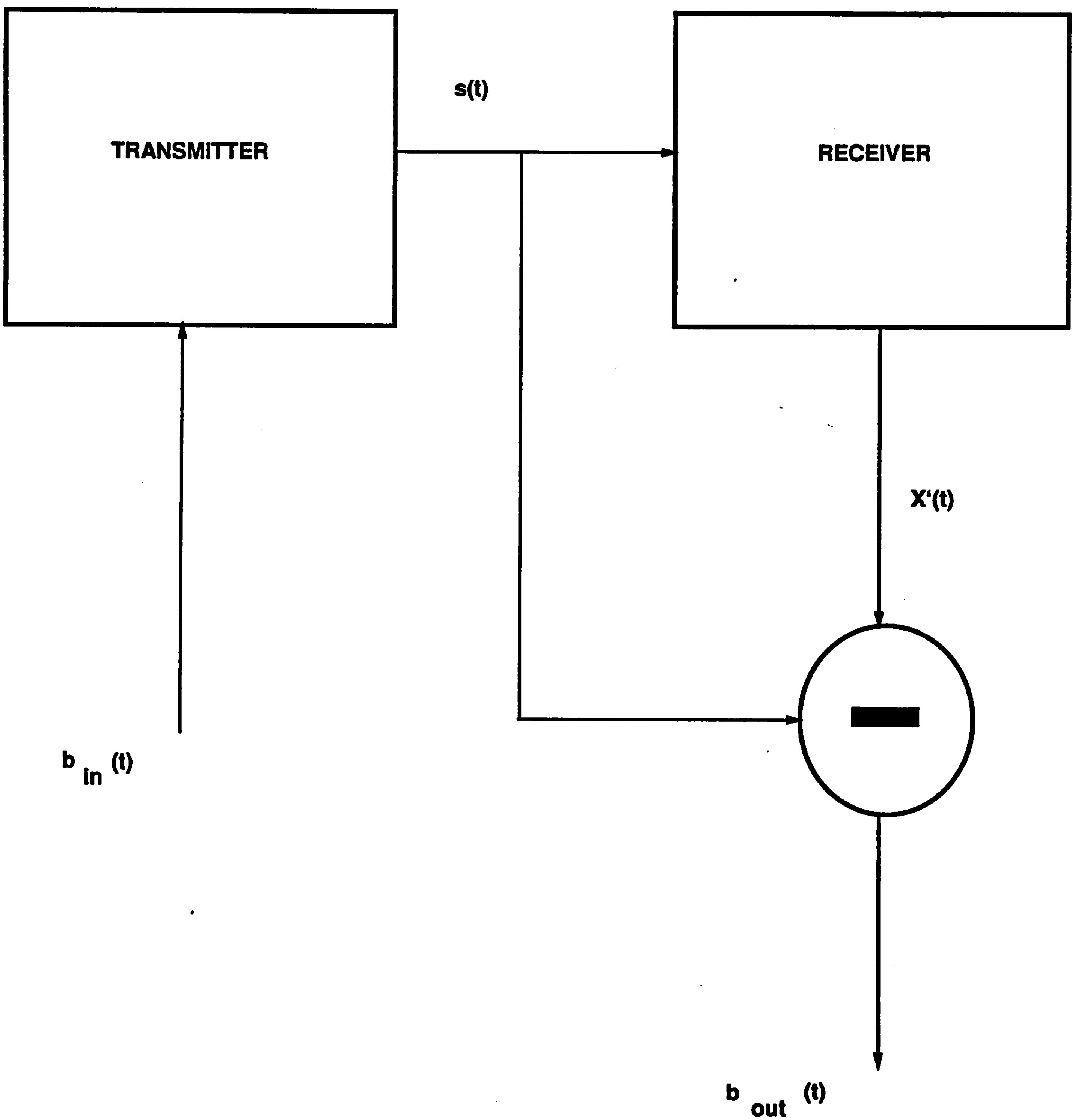

Fig. 2a)

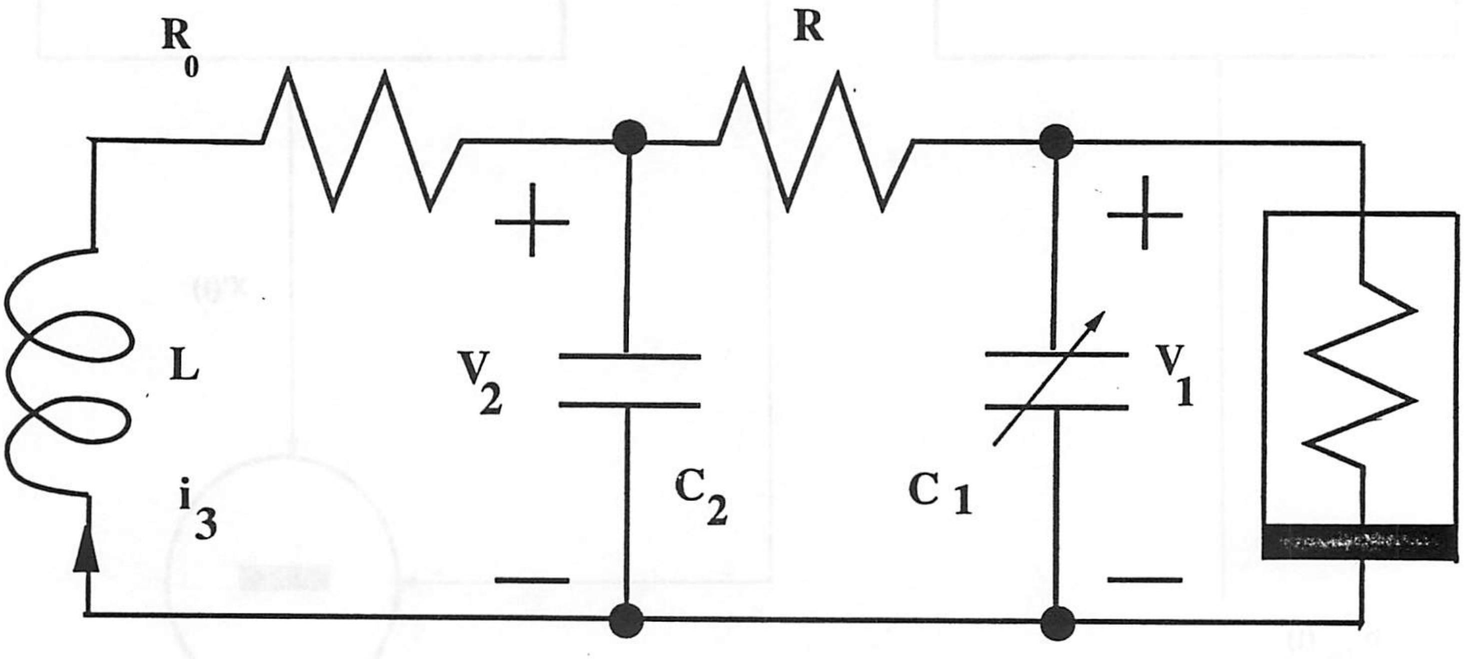

Fig. 2b)

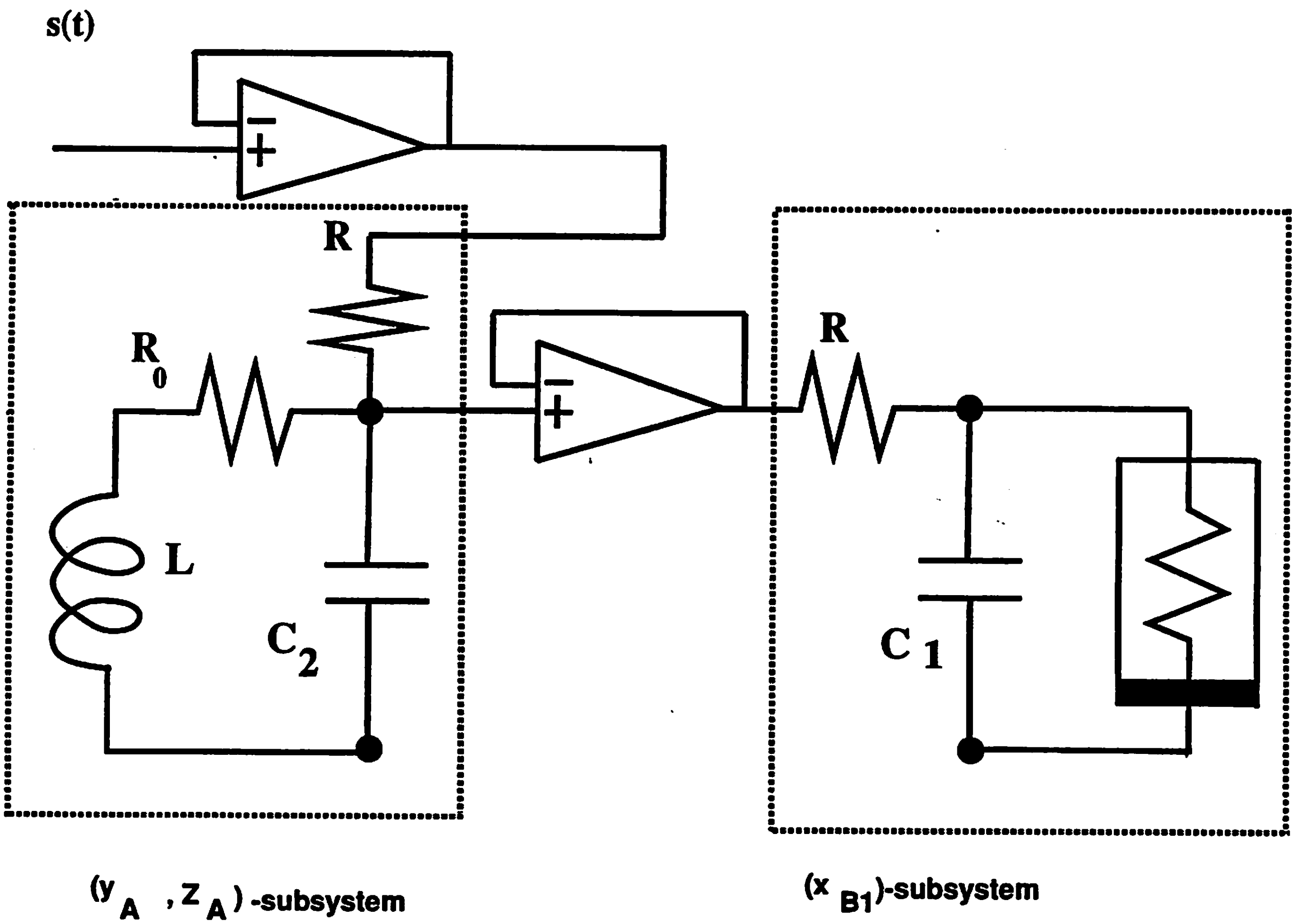

Fig. 2 c)

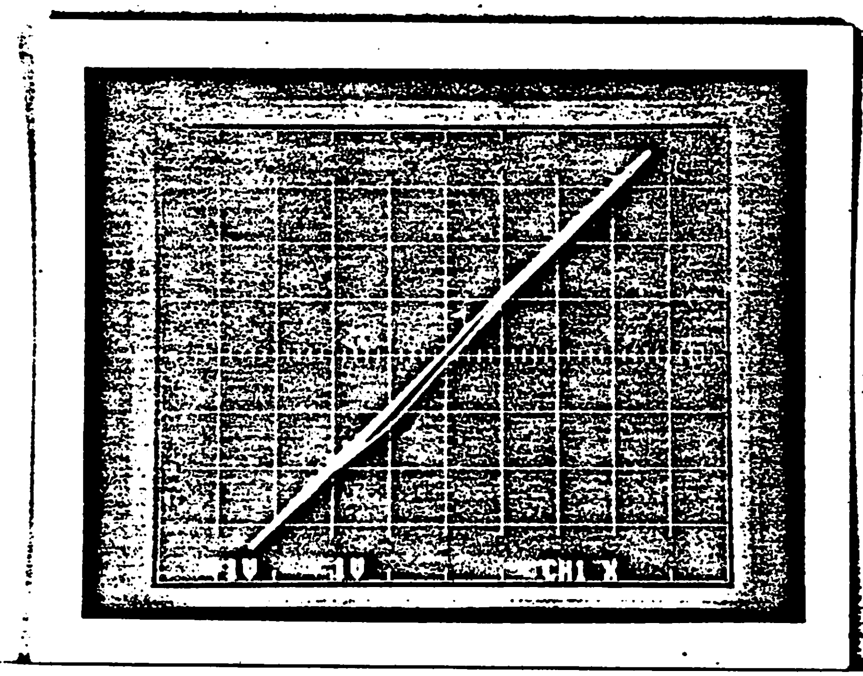

Fig. 3

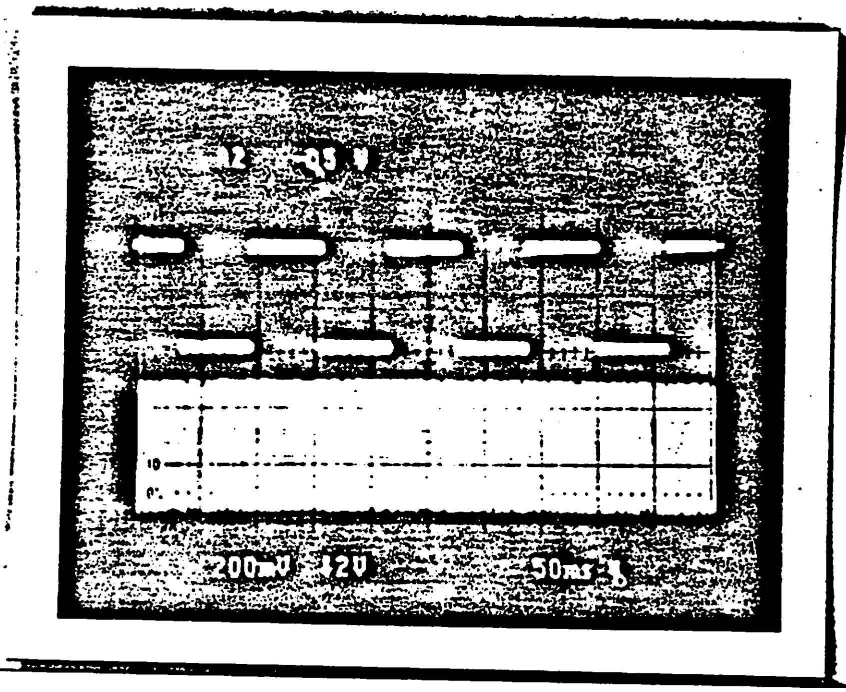

Fig.4

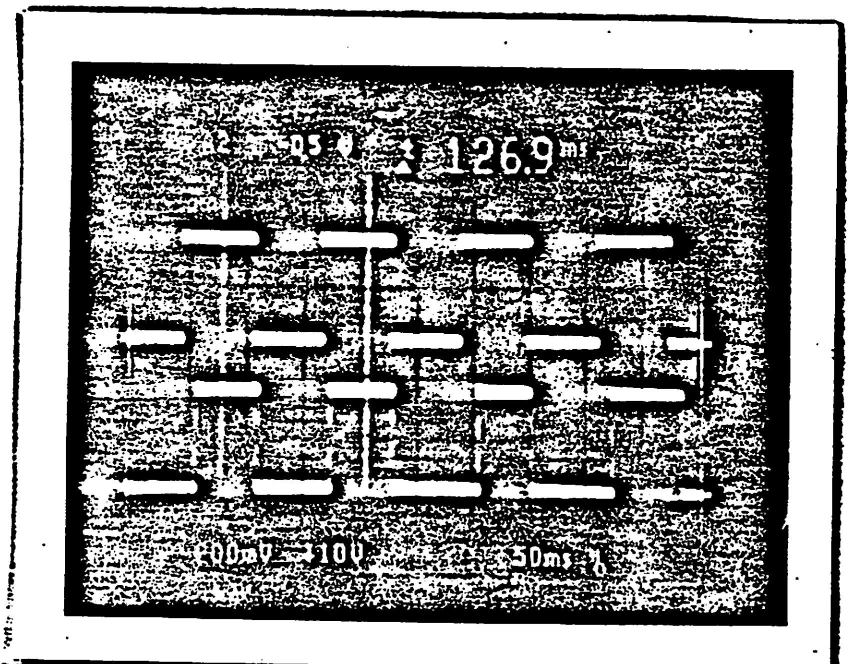

Fig. 5