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HYPERCHAOTIC ATTRACTORS OF UNIDIRECTIONALLY-COUPLED CHUA'S CIRCUITS

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In this letter we discuss properties of hyperchaotic attractors of unidirectionally-coupled Chua's circuits. Results from chaos synchronization theory allowed us to observe chaos-hyperchaos transition on 3D projections of the attractor.

1. Introduction

In the last 20 years it has been shown that chaotic behaviour is typical for three dimensional systems (see reprint collections: Cvitanovic [1984]; Hao [1986]; Kapitaniak [1992] for a large number of examples). Chaotic attractors of three-dimensional dissipative systems are characterized by one positive Lyapunov exponent which indicates a sensitive dependence on initial conditions (exponential spreading within the attractor in a direction transverse to the flow). The other two Lyapunov exponents can be either zero or negative, but the sum of all three exponents must be negative. In higher (at least four) dimensional systems, besides chaotic attractors, it is possible to find hyperchaotic attractors with two (or more) positive Lyapunov exponents [Rossler, 1979; Rossler, 1983; Kaneko, 1983; Kapitaniak and Steeb, 1991; Kapitaniak, 1991; Kapitaniak, 1993]. Such attractors involve at least two directions of spreading within the attractor in directions transverse to the flow. The evolution of the phase space volume under the action of a hyperchaotic flow was schematically described in Kapitaniak and Steeb [1991].

In what follows we investigate the hyperchaotic attractors in a pair of unidirectionally-coupled identical Chua's circuits whose combined equations of motion are

$$\dot{x}=\alpha(y-x-f(x)) \tag{1a}$$
$$\dot{y}=x-y+z+K(v-y) \tag{1b}$$
$$\dot{z}=-\beta y \tag{1c}$$
$$\dot{u}=\alpha(v-u-f(u)) \tag{1d}$$
$$\dot{v}=u-v+w \tag{1e}$$
$$\dot{w}=-\beta v \tag{1f}$$

where

$$\begin{aligned} f(x)&=bx+\frac{1}{2}(a-b)[|x+1|-|x-1|] \\ f(u)&=bu+\frac{1}{2}(a-b)[|u+1|-|u-1|] \end{aligned} \tag{2}$$

α, β, a and b are constants. The first Chua's circuit (eqs 1(a-c)) is coupled to the second one (eqs 1(d-f) in such a way that the difference between the signals y and v

$$d(t)=K(y-v) \tag{3}$$

is introduced into the first circuit as a negative feedback. $K > 0$ is the stiffness of the perturbation which we consider as a control parameter.

The outline of this letter is as follows. Section 2 describes the transition from chaos to hyperchaos in connection with the synchronization phenomenon. In Sec. 3 we present examples of hyperchaotic attractors with more than two positive Lyapunov exponents and discuss the robustness of hyperchaotic attractors in coupled systems. Finally, we summarize our results in Sec.4.

2. Transition to Hyperchaos

To describe the transition from chaotic to hyperchaotic regimes we exploit some results from chaos synchronization theory [Afraimovich, 1986; Anishchenko et al. 1991; Pecora and Carroll 1990, 1991; Endo and Chua, 1991]. When both Chua's circuits are operating in a chaotic regime, it is possible to achieve synchronization [Kocarev et al., 1993; Kapitaniak, 1994] using the above coupling. It was shown by de Sousa et al. [1992] that the boundary of the possible synchronization (according to definition by Pecora and Carroll [1990, 1991]), and nonsynchronization is strictly connected to the transition from chaotic to hyperchaotic behaviour. In this section we exploit this property to describe this transition.

In our numerical investigation we considered the following parameter values: α=10.0, β=14.87, a=-1.27 and b=-0.68, i.e. in the case of K=0 (no coupling) the dynamics of both Chua's circuits evolve along the double-scroll Chua's attractor [Chua et al.,1986, Chua, 1993). We chose slightly different initial conditions for both circuits x(0)=0.010, u(0)=0.011 y(0)=z(0)=v(0)=w(0)=0. Numerical computations have been performed using the software INSITE [Chua and Parker, 1989].

First, we calculate the Lyapunov exponents of the attractor and its associated Lyapunov dimension

$$d_L = j + \frac{\sum_{i=1}^{j} \lambda_i}{|\lambda_{i+1}|} \tag{4}$$

where j is determined by $\Sigma^{j}_{i=1}\lambda \geq 0$ but $\Sigma^{N+1}_{i=1}\lambda < 0$ [Kaplan and Yorke, 1979].

The trajectories of Eqs (1d-f) are located on a 3D manifold. If the trajectories of the whole system (1a-f) are located in this 3D manifold as well (attractor is embedded in a three-dimensional subspace of the six-dimensional phase space of Eq.(1)), then the first circuit simply reproduces the chaotic oscillations of the second circuit. In this case, all trajectories converge to the attractor of Eqs (1d-f), $d(t)\rightarrow 0$ and both circuits synchronize. The described 3D manifold exists for any value of the coupling stiffness K. This enables us to investigate the stability of the chaotic limit set located in this manifold as a function of K. The spectrum of the Lyapunov exponents of the coupled system (1) can be divided into two subsets $\lambda^{(1)}$ and $\lambda^{(2)}$, respectively, along and othogonal to the manifold. The first subset of Lyapunov exponents is associated with the second circuit (1d-f) and consists of three exponents describing the evolution of perturbations tangent to the manifold. The Lyapunov exponents of the second subset describes the evolution of the perturbations transverse to the manifold. For smaller values of K at least one $\lambda^{(2)}$-Lyapunov exponent is positive and the resulting limit set is no longer restricted to the manifold of the second circuit (1d-f) and we observe a hyperchaos regime. As shown by de Sousa et al. [1992] the $\lambda^{(2)}$-Lyapunov exponents are equivalent to the conditional or sub-Lyapunov exponents of Pecora and Carroll [1990] and this is why the chaos-hyperchaos transition in our system is strictly connected with the synchronization problem.

For values of K in the interval (0,1.17) the spectrum of the Lyapunov exponents is characterised by two positive exponents and we have a **hyperchaotic** evolution of the system. For higher values of K only one exponent is positive and the evolution takes place on a smaller 3-dimensional manifold. In Figure 1 we showed the plot of the Lyapunov dimension versus the coupling stiffness K. Figure 1 shows that the relation between the Lyapunov dimension and the coupling stiffness parameter K is not a continuous function at the transition point from chaos to hyperchaos, and at transitions to higher levels of hyperchaos (when new Lyapunov exponent becomes positive) as in the case of the unidirectionally-coupled Duffing's equations [Kapitaniak, 1993]. This result shows the possibility of a new route to hyperchaos where the attractor dimension is increased by a **jump** at the chaos-hyperchaos transition point.

For smaller values of K (K<1.17) the chaotic trajectories of of system (5) are characterized by two positive Lyapunov exponents; one in the $\lambda^{(1)}$ -subset and the other in the $\lambda^{(2)}$-subset so that in this case the two Chua's circuits cannot synchronize. For higher values of K (K>1.17) there is no positive Lyapunov exponent in the $\lambda^{(2)}$ -subset and the circuits can synchronize. In Figure 2(a,b) we showed the x-u projections of system trajectories on the x-u plane. In Figure 2(a) we observe a single-line characteristic in the synchronization regime for K=1.15 while in Figure 2(b) we observe a more complicated plot showing no synchonization between the two Chua's circuits. The simplicity of the x-u projection of the attractors in this case allows us to see the qualitative difference between chaotic and hyperchaotic attractors from these projections. Generally, this distinction is not straightforward [Kapitaniak, 1991].

In Figure 3(a,b) we show the x-u-z projections of chaotic and hyperchaotic attractors. The evolution of the projection of the chaotic attractor of Figure 3(a) takes place on a two-dimensional (x=u) plane, while the evolution of the projection of the hyperchaotic attractor of Figure 3(b) is strictly three-dimensional. The attractor of Fig. 3(a) is a classical double-scroll Chua's attractor, while the attractor of Fig. 3(b) suggests a double-double scroll.

The same x-u-z projections allow us to observe the transition from chaos to hyperchaos. The mechanism of the transition recalls classical intermittency [Pomeau and Manneville, 1980; Kohyama and Aizawa, 1984]. The trajectory evolves on a three-dimensional manifold for a long time and only occasionally does it burst to higher dimensions. This process can be observed in Figure 4(a,b). In Figure 4(a) we observe how after a relatively long evolution on a 3-dimensional manifold the trajectory leaves this manifold and tends towards one of the unstable fixed points. Figure 4(b) shows the double-double scroll attractor shortly after its birth.

3. Higher-Dimensional Hyperchaotic Attractors

To show the robustness of the hyperchaotic attractors in the coupled-Chua's circuits we investigate also a chain of three unidirectionally-coupled Chua's circuits described by

$$\begin{gathered}
\dot{x}=\alpha(y-x-f(x)) \\
\dot{y}=x-y-z+K(v-y) \\
\dot{z}=-\beta y \\
\dot{u}=\alpha(v-u-f(u)) \\
\dot{v}=u-v+w+M(s_i-v) \\
\dot{w}=\beta v \\
\cdots\cdots \\
\cdots\cdots \\
\cdots\cdots \\
\dot{r}_i=\alpha(s_i-r_i-f(r_i)) \\
\dot{s}_i=r_i-s_i+t_i+L_i(s_i-s_{i-1}) \\
\dot{t}_i=-\beta s_i
\end{gathered} \tag{5}$$

where M and L_i are constants, i=1,2,..,N-2 and each additional Chua's circuit is coupled to the previous one in the same way described earlier for coupling between the second and the first circuits. In the 3N-dimensional system described by Eq. (5), where N is the number of Chua's circuits, besides the hyperchaotic attractors with two positive Lyapunov exponents it is possible to have attractors with more than two positive exponents. Detailed investigation of the dynamics of system (5) will be given elsewhere. Here we only present some examples of hyperchaotic attractors of the 3N-dimensional system (5) shown in Figure 5(a-c). In Figure 5(a) we presented a hyperchaotic attractor with three positive Lyapunov exponents. Unfortunately in this case due to the high-dimensionality we are unable to investigate the transition to such an attractor in the way we observed the creation of the double-double scroll attractor.

Hyperchaotic attractors are robust in unidirectionally-coupled Chua's circuits and we can state the following conjecture: If in a 3N-dimensional chain of Chua's circuits (5) no two circuits synchronize with each other, then the system (5) has a hyperchaotic attractor with N positive Lyapunov exponents.

Some examples of x-u-z projections of higher-dimensional hyperchaotic attractors are shown in Figure 5(b,c). As the number of unstable fixed points increases with the number of circuits, so does the number of scrolls in the attractor. Unfortunately this can be observed only as more intense black spots on the x-u-z projection of the attractor, and for large N we will observe only a fuzzy black ellipse in this projection.

4. Conclusions

To summarize it has been demonstrated here that two coupled Chua's circuits can exhibit chaotic or hyperchaotic behaviours. Transition from chaotic to hyperchaotic regimes is characterized by a behaviour similar to the classical intermittency [Pomeau and Manneville, 1980; Kohyama and Aizawa, 1984] phenomenon with long evolutions of the hyperchaotic trajectory on a lower-dimensional chaotic attractor with occasional bursts to higher dimensions. This mechanism is similar to the mechanisms observed in the coupled generalized Van der Pol's equations [Kapitaniak and Steeb, 1991], and the unidirectionally-coupled Duffing's equations [Kapitaniak, 1993]. The special features of the coupling introduced in this system allow us to show that the Lyapunov dimension of the system during a chaos-hyperchaos transition is not a continuous function of the control parameter. This propetry is different from that found in Kapitaniak [1993] thereby pointing out the possibility of a new route to hyperchaos.

Finally we showed the way in which it may be possible to obtain hyperchaotic attractors with N positive Lyapunov exponents in a chain of N unidirectionally-coupled Chua's circuits.

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CAPTIONS

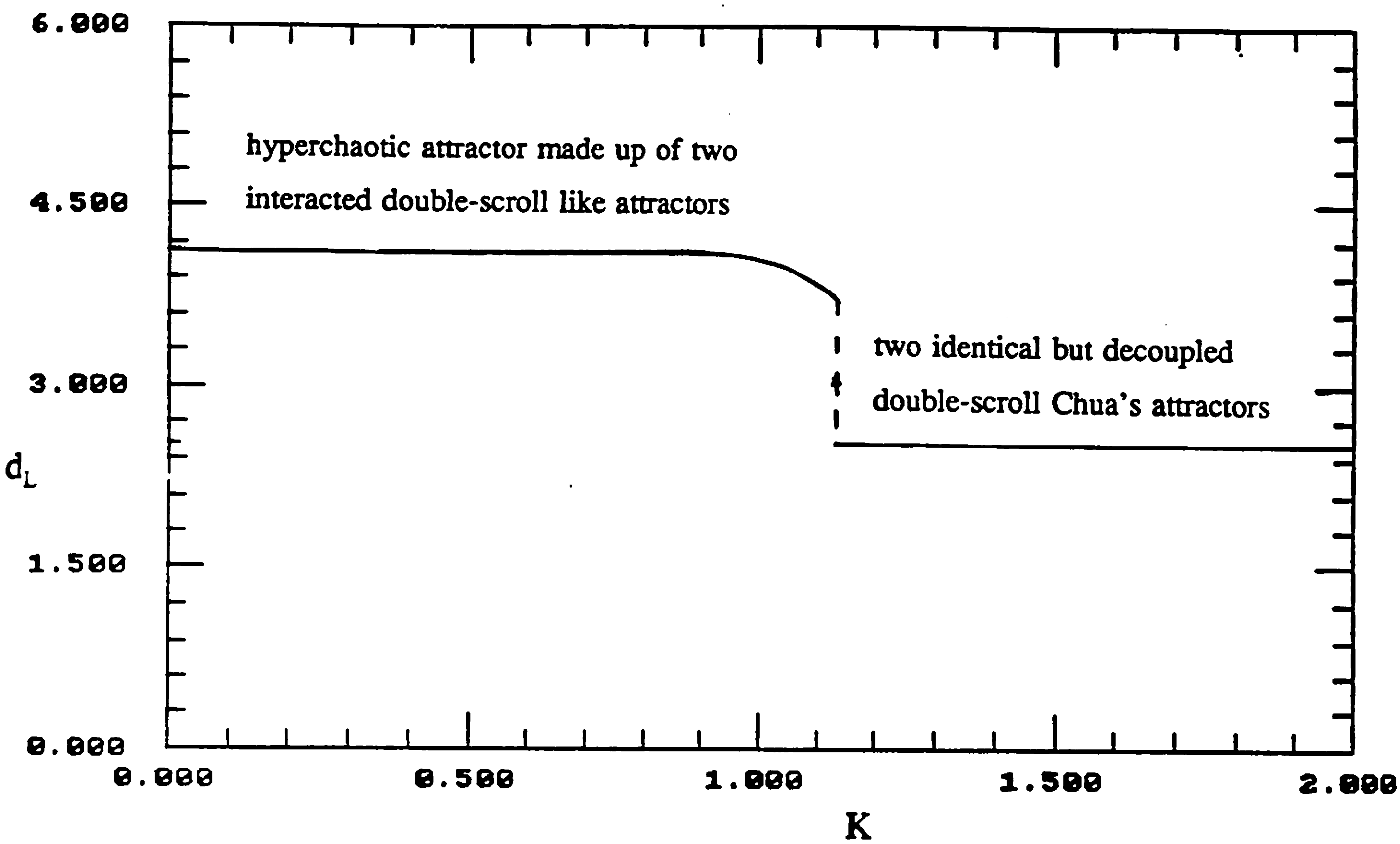
Figure 1: Plot of Lyapunov dimension versus K for Eqs (1): α=10.0, β=14.87, a=-1.27, b=-0.68.

Figure 2: x - u projection of the trajectory of Eqs (1): α=10.0, β=14.87, a=-1.27, b=-0.68; (a) K=1.15: synchronization between two circuits, (b) K=1.20: no synchronization.

Figure 3: Chaotic and hyperchaotic attractors of Eqs (1) shown in 3D x-u-z projection (a) K=1.15: chaotic attractor, (b) K=0.02: hyperchaotic attractor; λ_1=0.43, λ_2=0.41, λ_3=0, λ_4=0, λ_5=-3.74, λ_6=-3.85.

Figure 4: Details of the evolution on hyperchaotic attractor (a) escape from 3D manifold, (b) birth of the double-double scroll attractor.

Figure 5: Hyperchaotic attractors with N positive Lyapunov exponents for a chain of N unidirectionally coupled Chua's circuits; K=M=L_1=0.01 (a) N=3: λ_1=0.41, λ_2=0.49, λ_3=0.42, λ_4=0, λ_5=0, λ_6=0, λ_7=-3.27, λ_8=-3.32, λ_9=-8.47; (b) N=4, (c) N=5, λ_1=0.43, λ_2=0.42, λ_3=0.41, λ_4=0.41, λ_5=0.40, λ_6=0, λ_7=0, λ_8=0, λ_9=0, λ_{10}=0, λ_{11}=-3.80, λ_{12}=-3.82, λ_{13}=-3.82, λ_{14}=3.83, λ_{15}=-3.84.



6.000
4.500
3.000
1.500
0.000
hyperchaotic attractor made up of two
interacted double-scroll like attractors
two identical but decoupled
double-scroll Chua's attractors
d_L
0.000
0.500
1.000
1.500
2.000
K

Figure 1

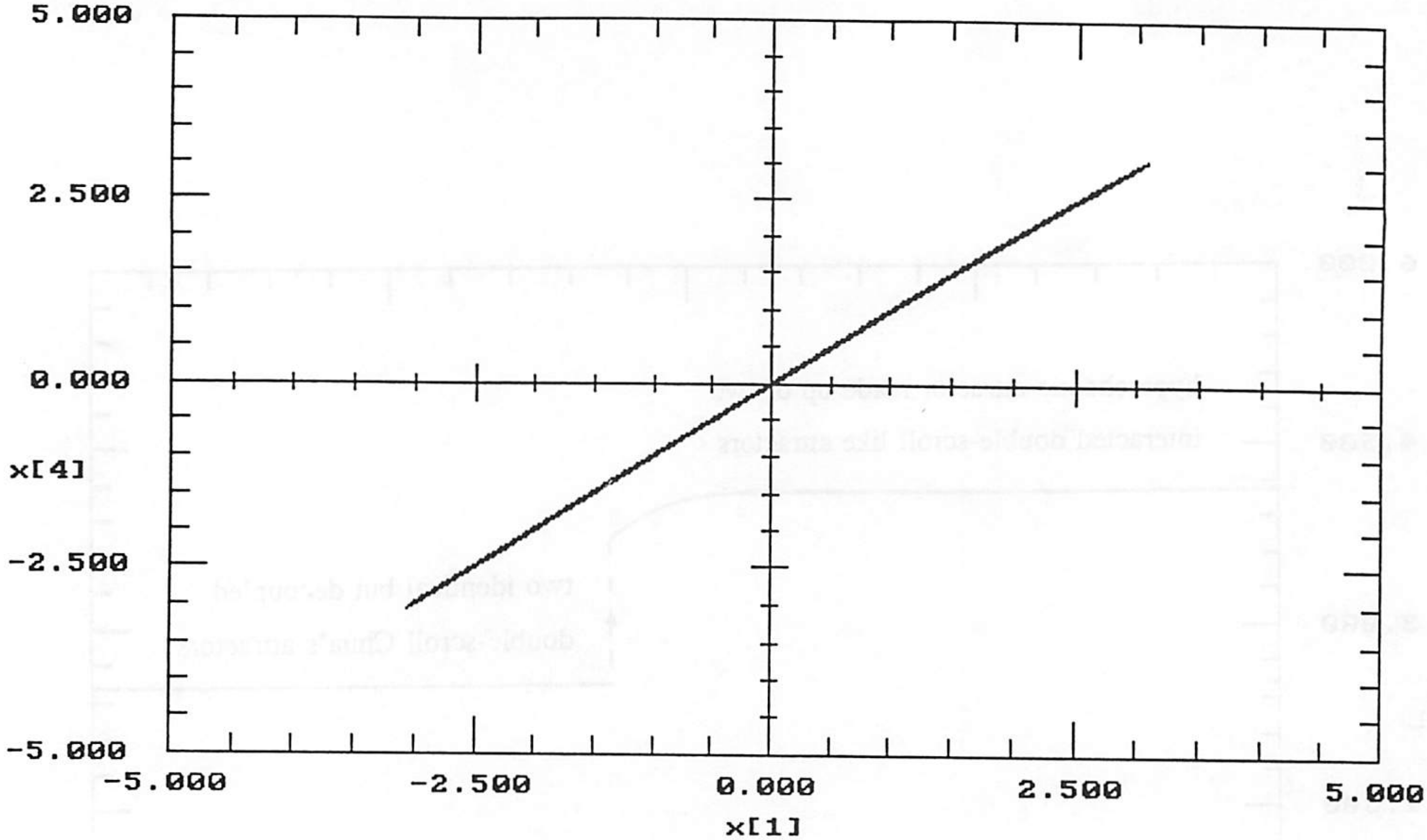


Figure 2a

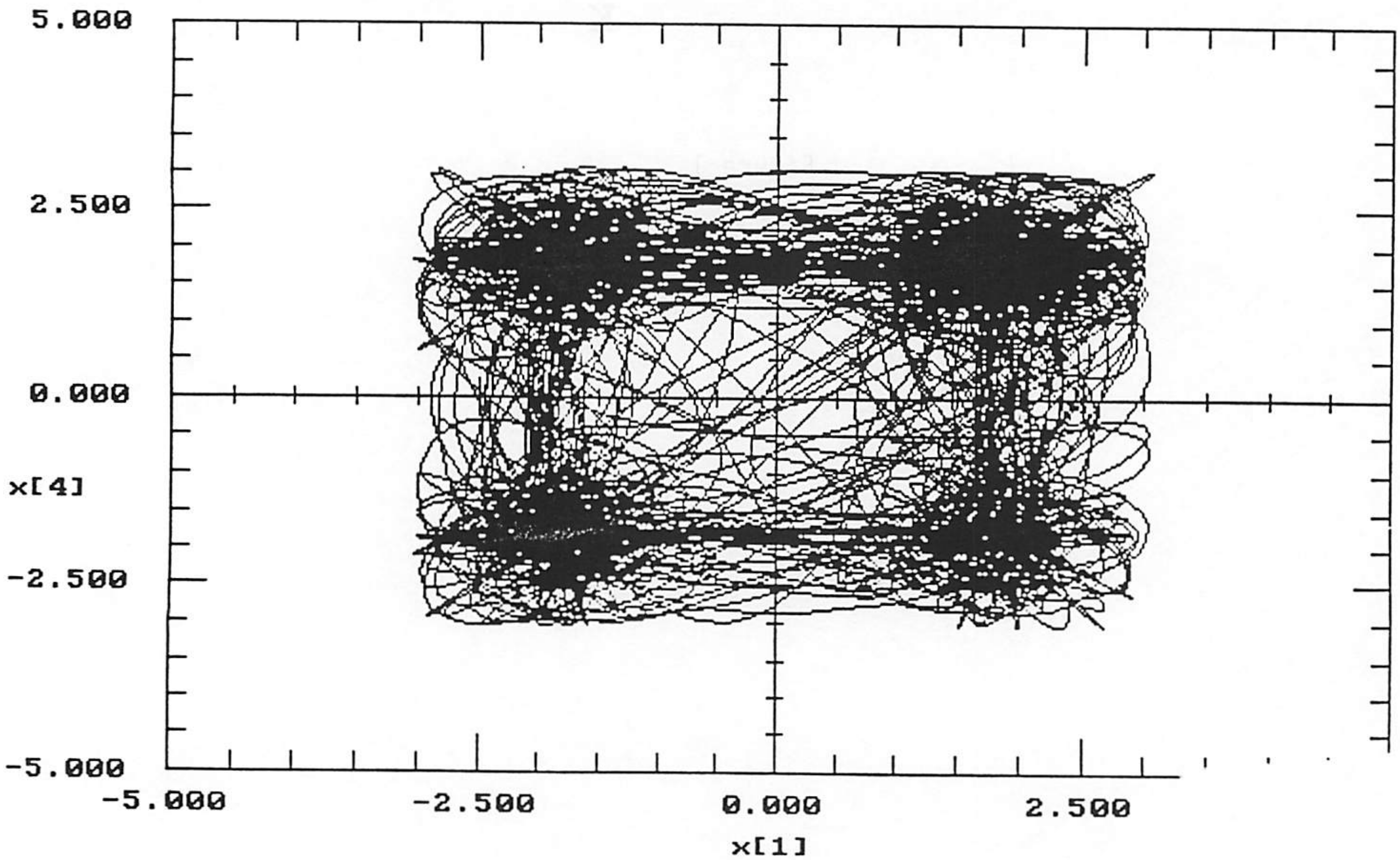


Figure 2b

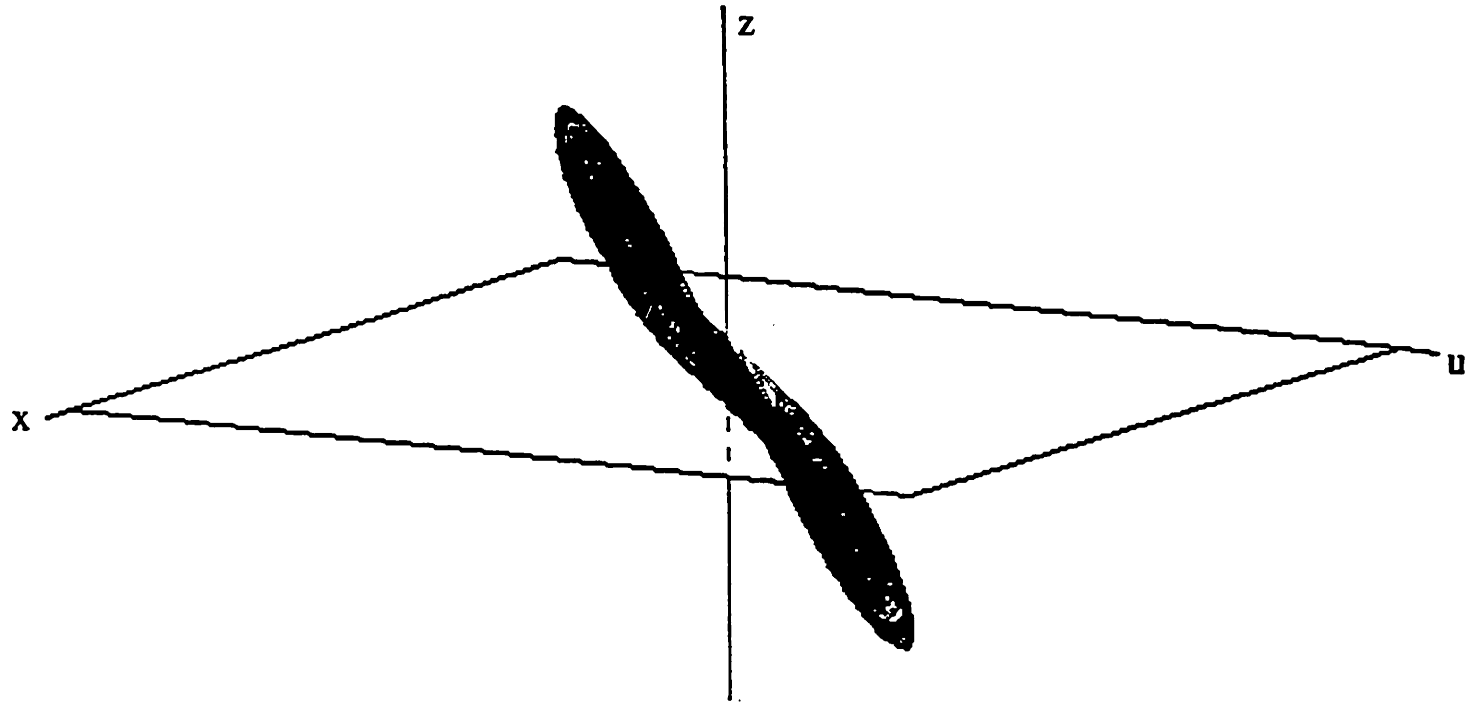


Figure 3a

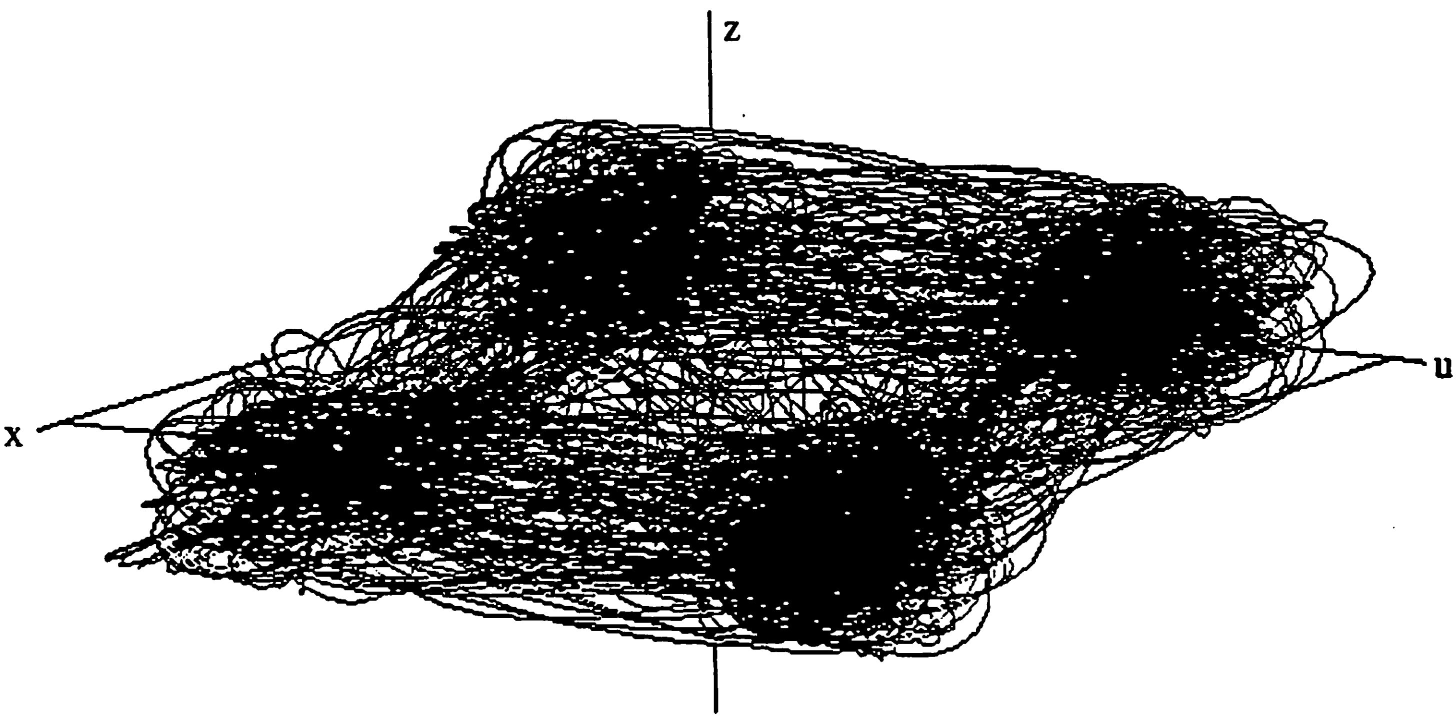


Figure 3b

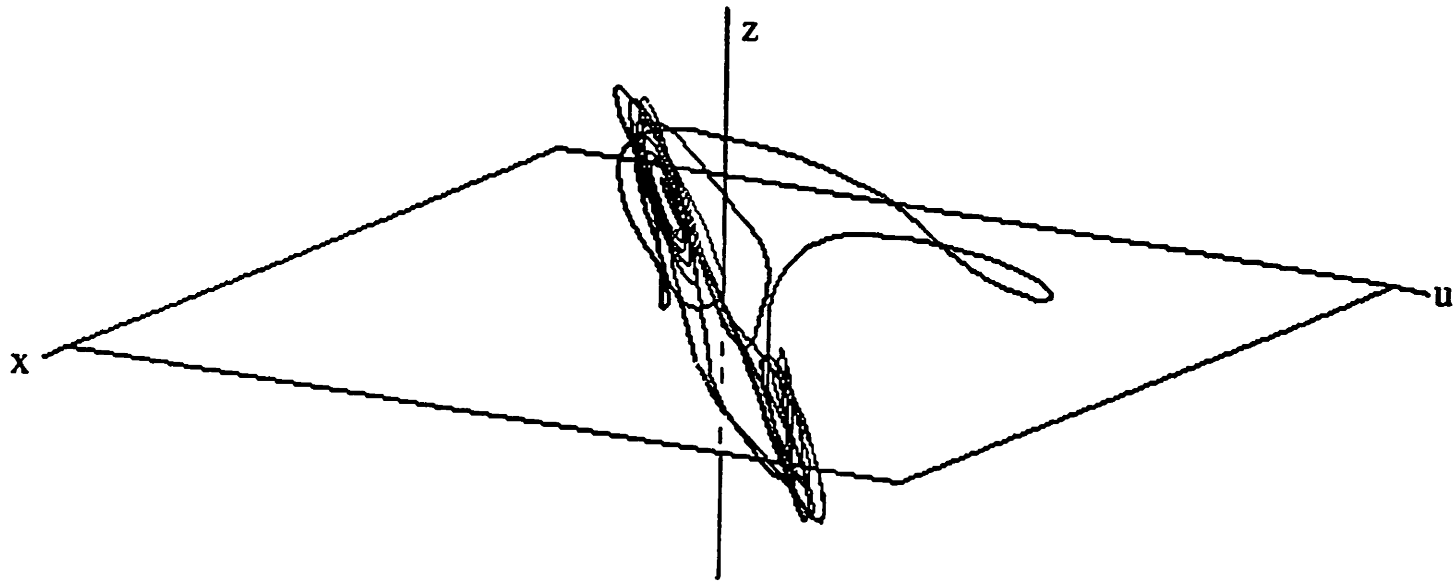


Figure 4a

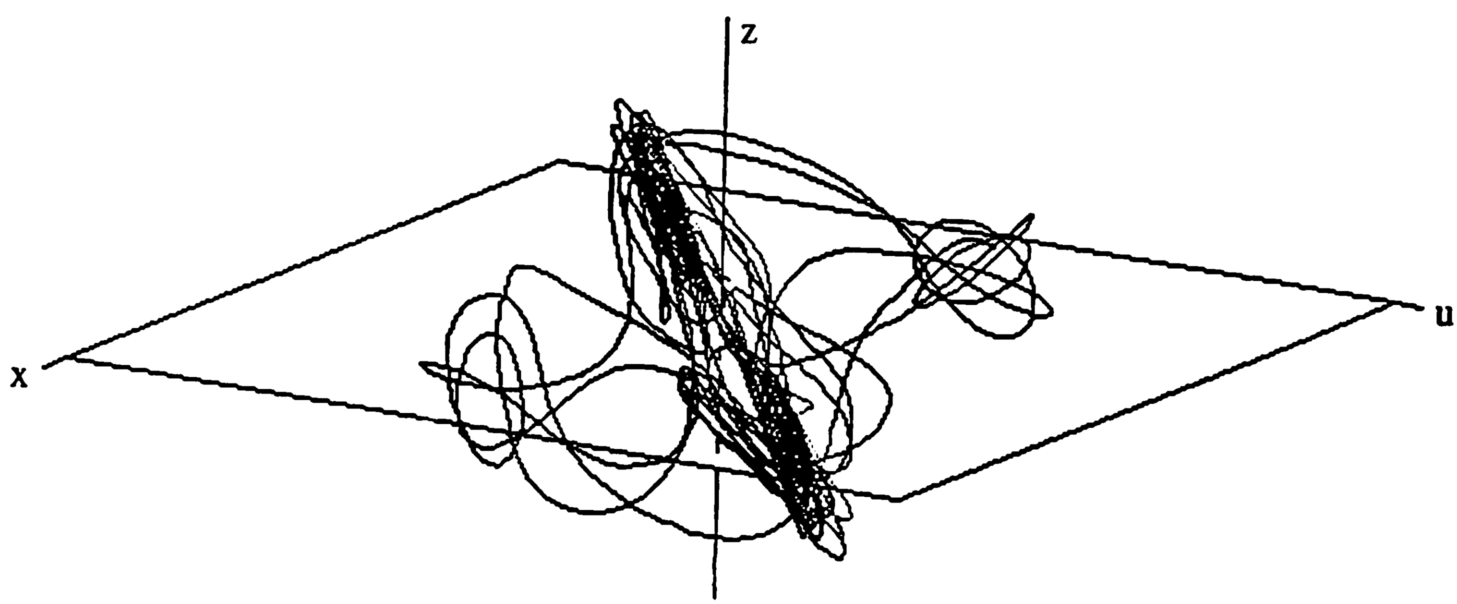


Figure 4b

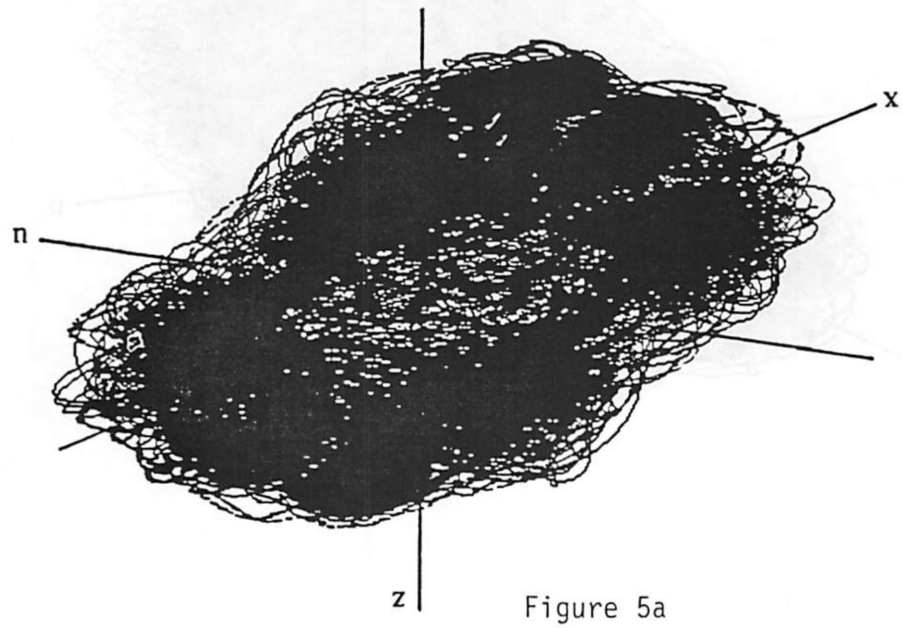


Figure 5a

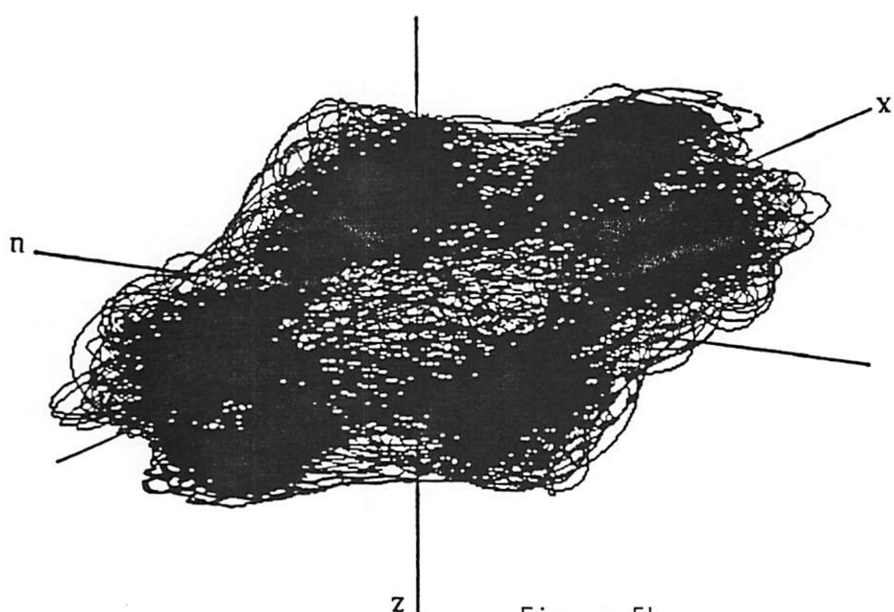


Figure 5b

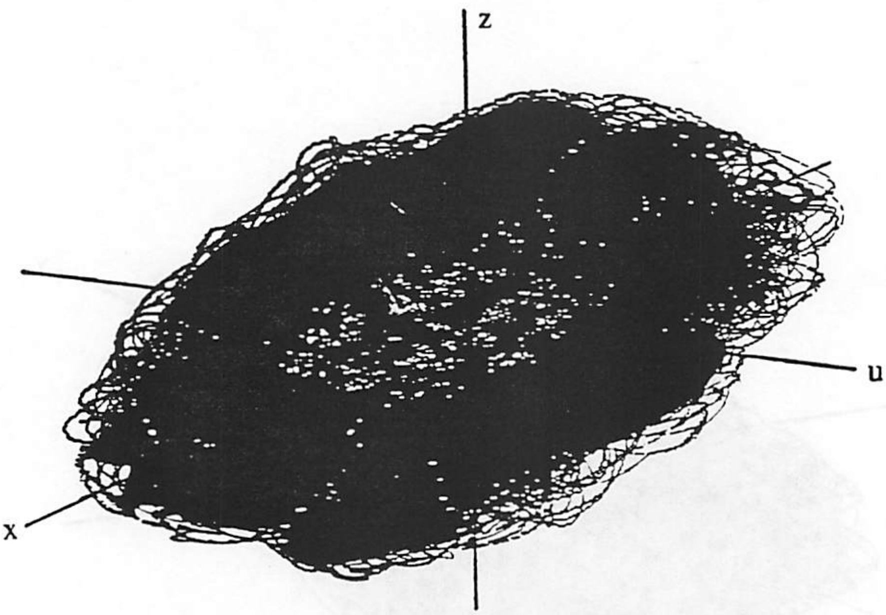


Figure 5c