SYNCHRONIZING NONAUTONOMOUS CHAOTIC SYSTEMS WITHOUT PHASE-LOCKING

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Synchronizing Nonautonomous Chaotic Systems without Phase-locking

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Abstract

Pecora and Carroll [1] have shown how two nonautonomous chaotic circuits driven by periodic forcing can be synchronized using the master-slave driving principle. However, in their scheme, the periodic forcing in both circuits needs to be phase-locked through some additional circuitry for the system to synchronize. In this paper, we show two ways in which this can be avoided.

In the first scheme, the two circuits are connected in a master-slave driving configuration and the periodic forcing is included in the driving signal such that it eliminates the need for the slave circuit to have an external periodic forcing signal. In addition, we can recover the periodic forcing signal at the slave circuit.

In the second scheme, the two circuits are connected in a mutual coupling configuration. The two circuits will synchronize regardless of what the periodic forcing signal of the two circuits are. In particular, the two periodic forcing signals could have different phase, different frequency, or different shape.

We discuss two interpretations of these synchronization schemes. First, we consider them as communication systems when the periodic forcing signal is replaced by a properly encoded information signal. Second, we consider them as synchronization schemes for nonidentical systems by considering the external forcing signal as an error signal due to the difference between the two systems.

1 Introduction

Pecora and Carroll [1] have shown how two nonautonomous chaotic circuits driven by periodic forcing can be synchronized. Since the two circuits need to be identical to synchronize, the periodic forcing in the two circuits must have the same phase. In their implementation, the two periodic forcing signals need to be phase-locked through some additional circuitry for the two circuits to synchronize. In [2], computer simulations were performed on two nonautonomous chaotic circuits where the periodic forcing signals in both circuits also have the same phase. We show here two cases where such phase-locking is not necessary for synchronization.

First, we show a master-slave driving scheme where the periodic forcing is included in the driving signal, and which eliminates the need for the slave system to have an external periodic forcing signal. We can then recover the periodic forcing signal from the driving signal.

Second, we show a mutual coupling scheme, where the two systems are synchronized regardless of whether the periodic forcing signals in the two systems are identical or not.

One of the key properties of these synchronization schemes for nonautonomous systems is that the external inputs to the two systems do not need to have the same phase. In fact, they can be arbitrary and completely different. This allows us to use them as communication systems or synchronization schemes for two systems which are not identical. These schemes can be considered as communication systems when the periodic driving signal is replaced by a properly encoded information signal. These schemes are considered as synchronization schemes for two systems which are not identical when the external forcing signal is considered as an error signal due to mismatch between the two systems.

In this paper, we assume that we can write state equations for all the nonlinear circuits that we consider and that for each initial condition, there exists a unique solution for all time.

The organization of this paper is follows. In Sec. 2 we will discuss the master-slave synchronization scheme. In Sec. 3 we discuss the possibility of synchronization through linear mutual coupling such that the periodic forcing in both systems can be different. In Sec. 4 we discuss the possibility of using these schemes as communication systems by replacing the periodic driving signals by properly encoded information signals. In Sec. 5 we discuss how we can synchronize two systems which are not identical.

2 Master-slave Synchronization Scheme

In the communication systems proposed in [3, 4], the slave system is synchronized to the master system, even though the master system has an information signal injected into it, while the slave system does not. As the information signal is an external source, this can be considered as synchronization of nonautonomous systems. We will use the same principles for our first synchronization scheme.

The chaotic system we use (Fig. 1) is a second order nonautonomous circuit, a modification of the circuit proposed in [5]. In [5], the periodic forcing is in series with the linear resistor, while in Fig. 1, the periodic forcing is in series with the nonlinear resistor. For the circuit to be chaotic, we choose R, C and L to be passive and R_1 to be active and have a monotone v-i characteristic.

The state equations for this circuit are given by:

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C}\left(i_2 - f_c(v_1 + s_c(t))\right) \\ \frac{di_2}{dt} &= -\frac{1}{L}\left(v_1 + i_2 R\right) \end{aligned} \tag{1}$$

where $s_c(t) = A_c \sin(\Omega t)$ is the periodic forcing function and the v-i characteristic of the voltage-controlled Chua's diode $f_c(v)$ is a 3-segment piecewise-linear function given by

$$f_c(v) = G_b v + \frac{1}{2}(G_a - G_b)\left(|v+E| - |v-E|\right) \tag{2}$$

where $E > 0$.

After normalization using $G = \frac{1}{R}$, $x = \frac{v_1}{E}$, $y = \frac{i_2}{GE}$, $\tau = \frac{t}{|C/G|}$, $a = \frac{G_a}{G}$, $b = \frac{G_b}{G}$, $\omega = \Omega|C/G|$, $\beta = \frac{C}{LG^2}$, $s(t) = \frac{s_c(|C/G|t)}{E}$, $A = \frac{A_c}{E}$, and redefining τ as t, we obtain the following dimensionless equations:

$$\begin{aligned} \frac{dx}{dt} &= k(y - f(x + s(t))) \\ \frac{dy}{dt} &= k\beta(-x - y) \end{aligned} \tag{3}$$

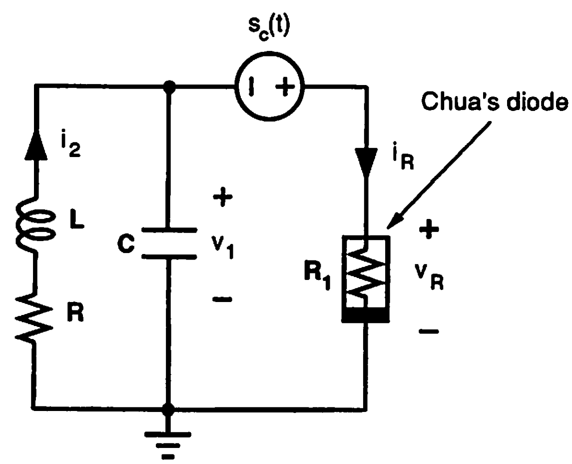

Figure 1: Nonautonomous chaotic circuit 1. For the circuit to be chaotic, we choose R, C and L to be passive and R_1 to be active and have a monotone v-i characteristic.

where $k = 1$ if $\frac{C}{G} > 0$ and $k = -1$ if $\frac{C}{G} < 0$, $s(t) = A\sin(\omega t)$ and

$$f(x) = bx + \frac{1}{2}(a-b)\left(|x+1| - |x-1|\right) \tag{4}$$

We choose the following set of parameters: $a = -1.37$, $b = -0.84$, $\omega = 0.4$, $A = 0.5$, $\beta = 0.895$, and $k = 1$. A chaotic attractor for these parameters is shown in the x-y plane in Fig. 2.

We couple two identical chaotic circuits in the following scheme (Fig. 3) which is similar to the scheme proposed for the autonomous Chua's circuit in [3]. It is based on the idea proposed in [6] that to synchronize two chaotic systems, the parts of the system that are responsible for the instability of the system are used as driving. The corresponding normalized state equations are given by:

$$\begin{aligned} \frac{dx}{dt} &= k(y - f(x + A\sin(\omega t))) \\ \frac{dy}{dt} &= k\beta(-x - y) \\ \frac{d\tilde{x}}{dt} &= k(\tilde{y} - f(x + A\sin(\omega t))) \\ \frac{d\tilde{y}}{dt} &= k\beta(-\tilde{x} - \tilde{y}) \end{aligned} \tag{5}$$

When k, $\beta > 0$ (or R,L,$C > 0$) this setup will synchronize, i.e. $x(t) \to \tilde{x}(t)$ as $t \to \infty$. This implies that $\tilde{s}_c(t)$ in Fig. 3 approaches $s_c(t)$ as $t \to \infty$ ($\tilde{s}(t)$ approaches $s(t)$ in the normalized equations, where $\tilde{s}(t) = x + s(t) - \tilde{x}$). The proof that this setup will synchronize is similar to that in [3].

An alternative circuit implementation of Eqs. (5) is to transmit the current i_R to the receiver circuit as shown in Fig. 4. This implementation will also synchronize the two circuits. If we assume that the nonlinear resistor R_1 is both current and voltage controlled (i.e., $f_c(v)$ is one-to-one), then $\tilde{s}_c(t)$ will approach $s_c(t)$ as $t \to \infty$.

3 Mutual Coupling Synchronization Scheme

The nonautonomous chaotic system that we use is the same circuit as Fig. 1, except that we interchange the linear resistor and the nonlinear resistor, as shown in Fig. 5. Note that this circuit is the dual circuit of the circuit in [5] except that we replace the current source in the dual circuit by a Thévenin equivalent voltage source. However, for our synchronization scheme we will use an active linear resistor and a passive

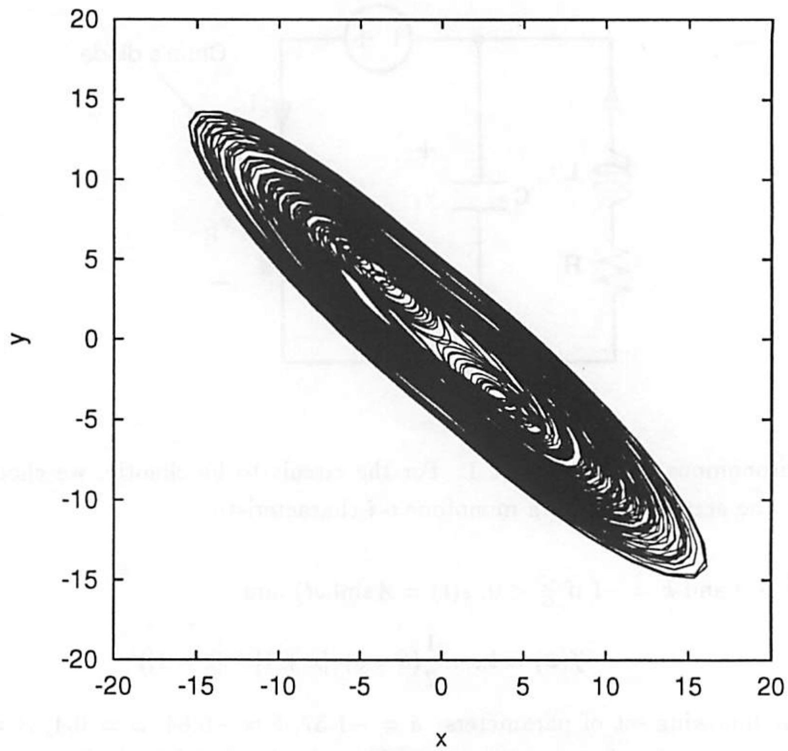

Figure 2: Chaotic attractor for system (3) in the x-y plane. The parameters are $a = -1.37$, $b = -0.84$, $\omega = 0.4$, $A = 0.5$, $\beta = 0.895$, and $k = 1$.

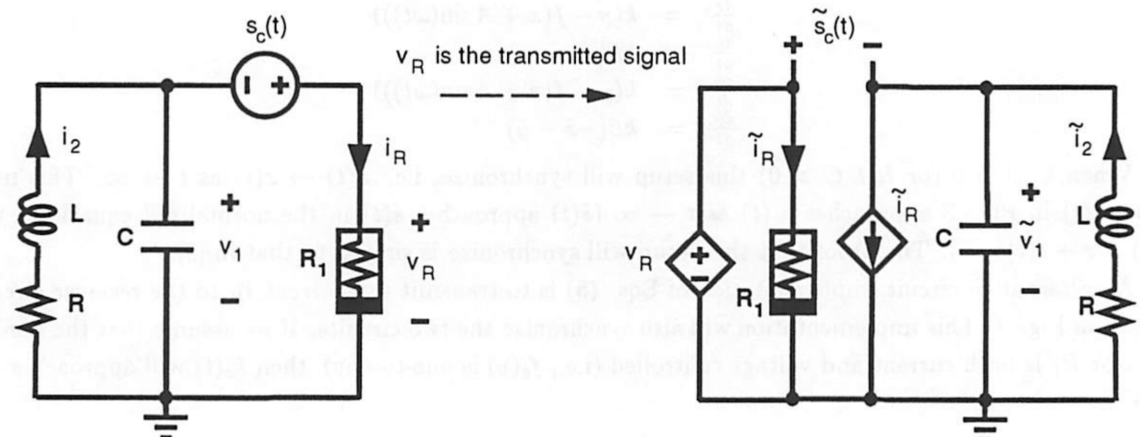

Figure 3: Two nonautonomous chaotic circuits coupled through unidirectional coupling. This can be viewed as a communication system when the coupling is considered as the tranmission of the signal v_R. The voltage accross the controlled voltage source is v_R and the current through the controlled current source is $\tilde{\imath}_R$. The recovered signal $\tilde{s}_c(t)$ will asymptotically approach $s_c(t)$ as $t \to \infty$.

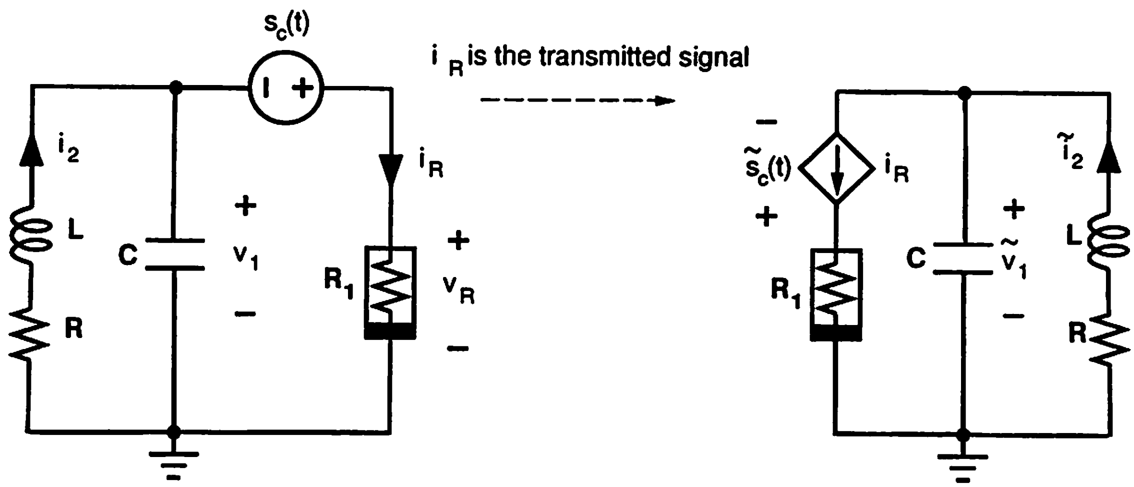

Figure 4: Alternative way of connecting two nonautonomous chaotic circuits through unidirectional coupling. This can be viewed as a communication system when the coupling is considered as the tranmission of the signal i_R. The current through the controlled current source is i_R. The recovered signal $\tilde{s}_c(t)$ will asymptotically approach $s_c(t)$ as $t \to \infty$ assuming that R_1 is both voltage and current controlled.

nonlinear resistor with a monotone v-i characteristic. Note that since the nonlinear resistor has a passive and monotone v-i characteristic, the linear resistor must be active for the system to become chaotic (and exhibit sensitive dependence on initial conditions) as otherwise the system will have a unique steady state solution [7].

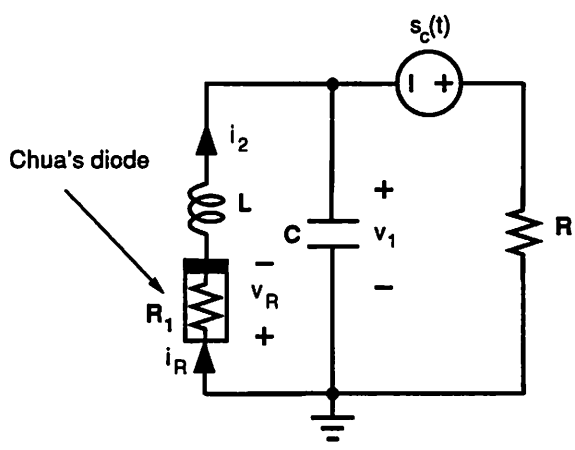

Figure 5: Nonautonomous chaotic circuit 2. For the system to become chaotic, we use an active linear resistor and a Chua's diode with a passive monotone v-i characteristic.

The state equations for this circuit are given by:

$$\begin{aligned} \frac{dv_1}{dt} &= \frac{1}{C}\left(i_2 - \frac{1}{R}(v_1 + s_c(t))\right) \\ \frac{di_2}{dt} &= -\frac{1}{L}\left(v_1 + g_c(i_2)\right) \end{aligned} \tag{6}$$

where $s_c(t) = A_c \sin(\Omega t)$ is the periodic forcing function and the v-i characteristic of the current-controlled

Chua's diode $g_c(i)$ is a 3-segment piecewise-linear function given by

$$g_c(i) = R_b i + \frac{1}{2}(R_a - R_b)(|i + I| - |i - I|) \tag{7}$$

where $I > 0$.

After normalization using $G = \frac{1}{R}$, $x = \frac{v_1 G}{I}$, $y = \frac{i_2}{I}$, $\tau = \frac{t}{|C/G|}$, $a = GR_a$, $b = GR_b$, $\omega = \Omega|C/G|$, $\beta = \frac{C}{LG^2}$, $s(t) = \frac{s_c(|C/G|t)G}{I}$, $A = \frac{A_c G}{I}$, and redefining τ as t, we obtain the following dimensionless equations:

$$\begin{aligned} \frac{dx}{dt} &= k(y - x - s(t)) \\ \frac{dy}{dt} &= k\beta(-x - f(y)) \end{aligned} \tag{8}$$

where $k = 1$ if $\frac{C}{G} > 0$ and $k = -1$ if $\frac{C}{G} < 0$, $s(t) = A\sin(\omega t)$ and f is as defined in Eq. (4).

We choose the following set of parameters: $a = -1.27$, $b = -0.68$, $\omega = 0.5$, $A = 0.2$, $\beta = 1.4$, and $k = -1$. There exists a set of corresponding circuit parameter values such that the inductor and the capacitor are passive, the nonlinear resistor has a passive monotone v-i characteristic and the linear resistor is active, i.e. C, L, R_a, $R_b > 0$, $R < 0$. A chaotic attractor for these parameters is shown in the x-y plane in Fig. 6.

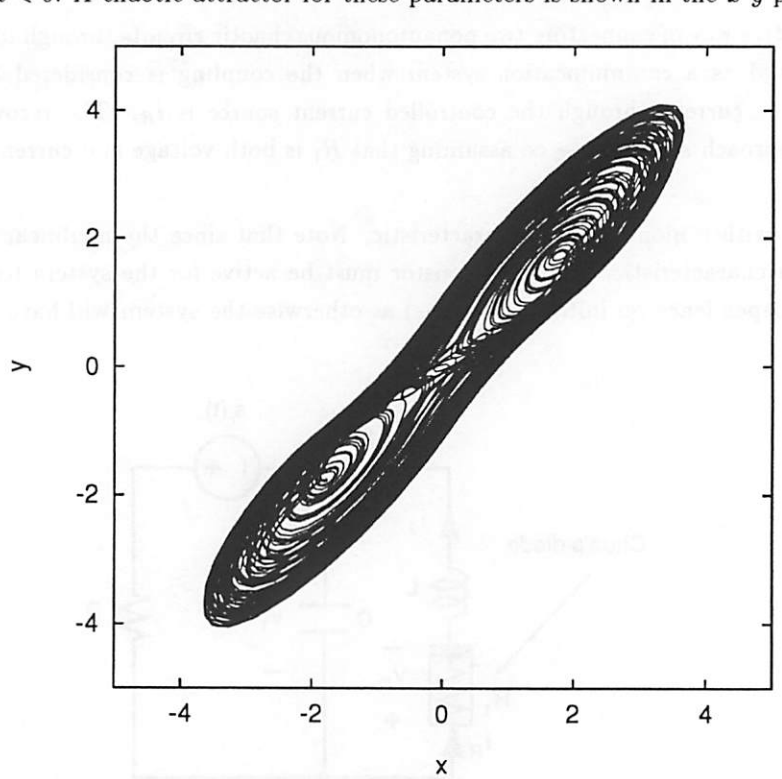

Figure 6: Chaotic attractor for system (8) in the x-y plane. The parameters are $a = -1.27$, $b = -0.68$, $\omega = 0.5$, $A = 0.2$, $\beta = 1.4$, and $k = -1$.

We will sychronize two such circuits by connecting a linear resistor of resistance R_c accross the two linear resistors, as shown in Fig. 7.

The normalized state equations of the system in Fig. 7 are:

$$\begin{aligned} \frac{dx}{dt} &= k(y - x - s(t) + \gamma(\tilde{x} + \tilde{s}(t) - x - s(t))) \\ \frac{dy}{dt} &= k\beta(-x - f(y)) \\ \frac{d\tilde{x}}{dt} &= k(\tilde{y} - \tilde{x} - \tilde{s}(t) + \gamma(x + s(t) - \tilde{x} - \tilde{s}(t))) \\ \frac{d\tilde{y}}{dt} &= k\beta(-\tilde{x} - f(\tilde{y})) \end{aligned} \tag{9}$$

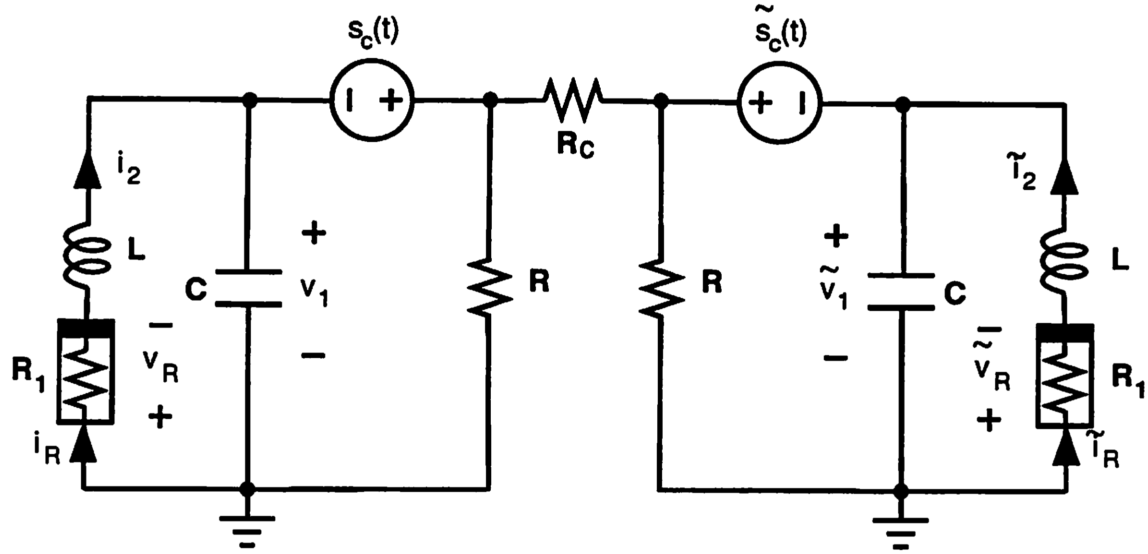

Figure 7: Two nonautonomous chaotic circuits coupled through a linear resistor. The signal $\tilde{v}_1(t)$ approaches $v_1(t)$ as the system is synchronized.

where $\gamma = \frac{1}{GR_c}$.

We choose $\gamma = -\frac{1}{2}$. This corresponds to $R_c = -2R$, so that if $R < 0$ is an active resistor, R_c will be a passive resistor. Eqs. (9) can then be rewritten as

$$\begin{aligned} \frac{dx}{dt} &= k(y - [\tfrac{1}{2}(x+\tilde{x}) + \tfrac{1}{2}(s(t)+\tilde{s}(t))]) \\ \frac{dy}{dt} &= k\beta(-x - f(y)) \\ \frac{d\tilde{x}}{dt} &= k(\tilde{y} - [\tfrac{1}{2}(x+\tilde{x}) + \tfrac{1}{2}(s(t)+\tilde{s}(t))]) \\ \frac{d\tilde{y}}{dt} &= k\beta(-\tilde{x} - f(\tilde{y})) \end{aligned} \tag{10}$$

If we set $\eta(t) = -\frac{1}{2}(x+\tilde{x}) - \frac{1}{2}(s(t)+\tilde{s}(t))$, then by [6, Corollary 1], this system will asymptotically synchronize (i.e. $\tilde{x} \to x$ and $\tilde{y} \to y$ as $t \to \infty$) if the following system is uniformly asymptotically stable for all $\eta(t)$.

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \beta \end{pmatrix} \begin{pmatrix} ky \\ k(-x - f(y)) \end{pmatrix} \tag{11}$$

The function $h(x,y) = (-ky, -k(-x-f(y)))^T$ is uniformly increasing for $k=-1, a<0, b<0$ since

$$\begin{aligned} (x-x', y-y')(h(x,y) - h(x',y')) &= -k(x-x')(y-y') + k(x-x')(y-y') + k(y-y')(f(y)-f(y')) \\ &= -(y-y')(f(y)-f(y')) = -s(y-y')^2 \end{aligned}$$

where $s = \frac{f(y)-f(y')}{y-y'}$ which depends on y and y' satisfies $s \le \max(a,b) < 0$ for all $y \neq y'$. Thus system (11) is uniformly asymptotically stable for all $\eta(t)$ by Theorem 6 in [6] and so system (10) will asymptotically synchronize for $a<0, b<0$, $\beta > 0$ and $k = -1$.

Note that when the two systems are synchronized ($x = \tilde{x}$ and $y = \tilde{y}$), the two systems behave both as a single system (8) with the external source replaced by the average of the two sources.

4 Communication Systems Based on Chaotic Synchronization

There has been many approaches to implementing communication systems based on chaotic synchronization [3, 4, 8, 9, 10, 11, 12, 13]. In these systems, the input signal is scrambled or converted to a chaotic signal in the transmitter and this chaotic signal is transmitted to the receiver. Nearly all of them utilize an autonomous chaotic system and the information signal does not play a significant role in generating the chaos. We have discussed earlier the relationship between communication systems using chaotic synchronization and synchronizing nonautonomous chaotic systems by considering the information signal as external input.

Let us now consider the above synchronization schemes in this light. The synchronization schemes in the previous sections can be considered as communication systems if the periodic signal $s_c(t)$ in Fig. 3 (resp. signals $s_c(t)$ or $\tilde{s}_c(t)$ in Fig. 7) is replaced by an encoded information signal that oscillates at the proper rate. Some examples of encoding of binary information signals that could be used are coded PCM (Manchester pulses), FSK or PSK. For FSK and PSK, the frequencies of the keys should be chosen such that system (1) (resp. system (6)) is chaotic.

Consider Fig. 3. The information signal is $s_c(t)$, and this is scrambled by the circuit, the scrambled signal v_R is transmitted, and in the receiver the signal $\tilde{s}_c(t)$ is recovered which approaches $s_c(t)$. The signal $\tilde{s}_c(t)$ is now an information-bearing signal at the slave circuit (receiver). Some differences between this scheme and the other communication schemes using chaos are:

- The information signal plays a crucial role in generating the chaotic signal to be transmitted which potentially can lead to a secure communication system that is harder to break.
- The minimum number of dimensions needed to generate chaos is less (2 versus 3).

The scheme in Fig. 7 considered this way can be redrawn as a bidirectional communication system, as shown in Fig. 8. Both circuits transmit and receive to each other at the same time. $s_c(t)$ and $\tilde{s}_c(t)$ are both information signals. The signal $s_c(t)$ is recovered in the second system as $r_c(t)$ ($r_c(t) \to s_c(t)$ as $t \to \infty$) and $\tilde{s}_c(t)$ is recovered in the first system as $\tilde{r}_c(t)$ ($\tilde{r}_c(t) \to \tilde{s}_c(t)$ as $t \to \infty$).

Although it cannot be considered as secure communication system as in [3, 8], since both transmitted signals can be intercepted, it can nevertheless serve as a system to modulate the information signal which can be demodulated at the receiver.

5 Synchronization of Nonidentical Systems

In this section we use the two synchronization schemes to synchronize two systems which are not identical. Since the external forcing voltage source can be arbitrary, we can add a nonlinear (resistive or dynamic) one-port in series with it and the system will still synchronize. Since the two systems are not identical and can have different dimensions, synchronization here means that the state variables in one system which have a corresponding counterpart in the other system will approach each other as $t \to \infty$ (see Definition 8 in [6]).

For the master-slave configuration (Fig. 3) this leads to Fig. 9, where the one-port is shown as N_2.

As an example, assume the one-port N_2 consists of a linear capacitor in series with a nonlinear resistor. The resulting system is shown in Fig. 10. We assume that R_1 is voltage controlled and the driving point characteristic of R_1 in series with R_2 is also voltage controlled. Then $\tilde{v}_1 \to v_1$ and $\tilde{i}_2 \to i_2$ as $t \to \infty$.

When the one-port N_2 is dynamic, this synchronization scheme is similar to the homogeneous driving scheme of Pecora and Carroll [14] where the driven system is a smaller dimensional system than the driving system (see [6, Section 4.1]).

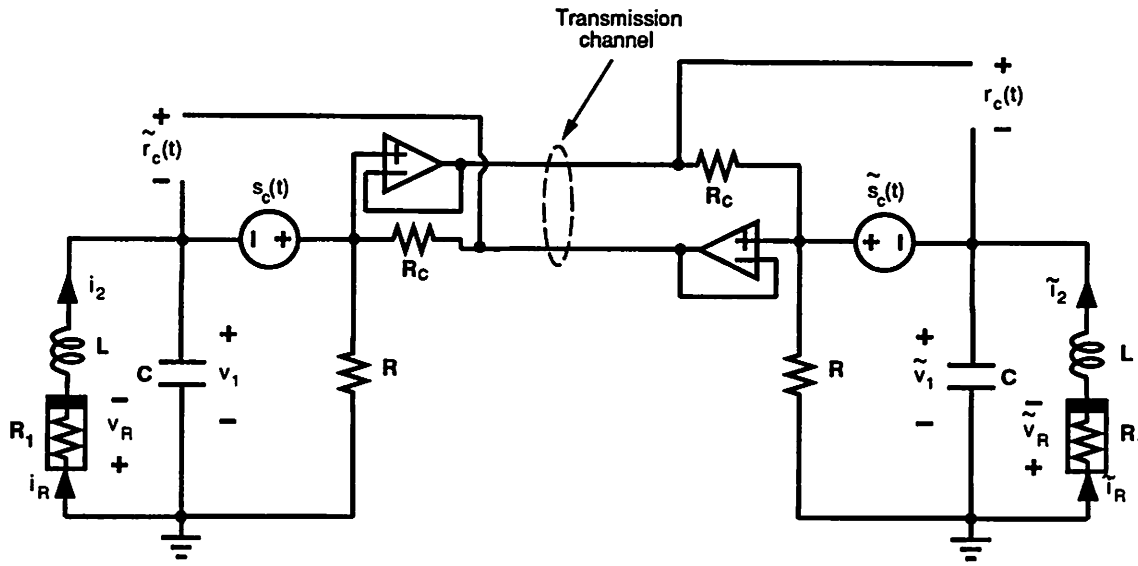

Figure 8: The system in Fig. 7 redrawn as a bidirectional communication system. The signal $s_c(t)$ is recovered in the second system as $r_c(t)$ ($r_c(t) \to s_c(t)$ as $t \to \infty$) and $\tilde{s}_c(t)$ is recovered in the first system as $\tilde{r}_c(t)$ ($\tilde{r}_c(t) \to \tilde{s}_c(t)$ as $t \to \infty$).

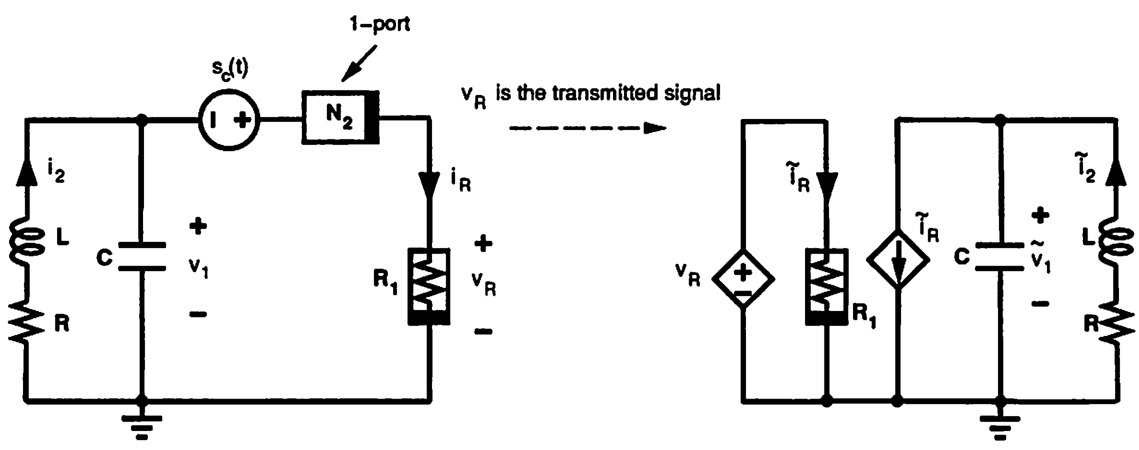

Figure 9: Synchronization of nonidentical systems. The one-port N_2 can be dynamic or resistive.

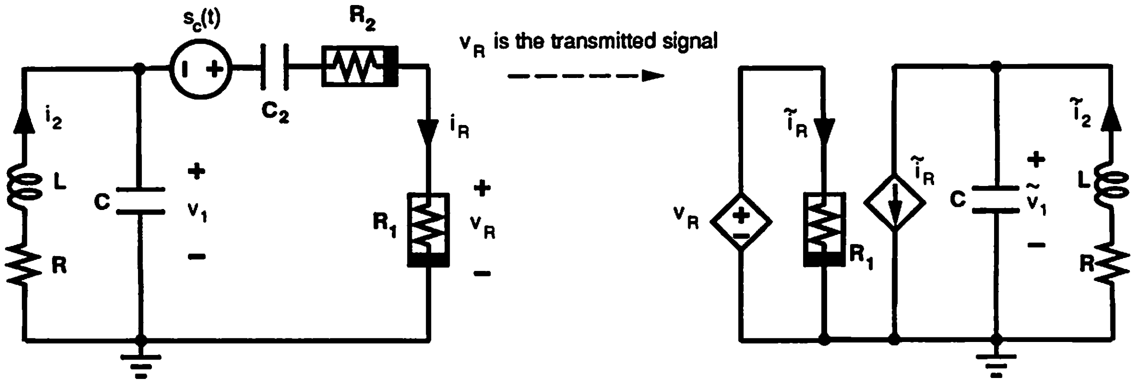

Figure 10: Figure 9 redrawn when the one-port N_2 is a linear capacitor in series with a nonlinear resistor. $\tilde{v}_1 \to v_1$ and $\tilde{i}_2 \to i_2$ as $t \to \infty$.

Similarly, two one-ports can be connected in series to the independent sources in Fig. 7 and the system will still synchronize[1] in the sense that $\tilde{v}_1 \to v_1$ and $\tilde{i}_2 \to i_2$ as $t \to \infty$ (Fig. 11).

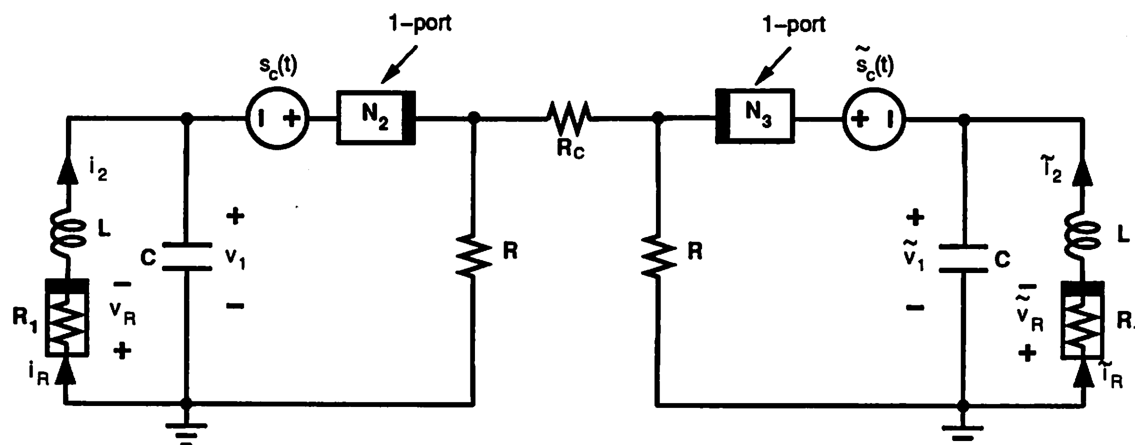

Figure 11: Figure 7 with two one-ports N_2 and N_3 inserted in series with the external sources. The system synchronizes in the sense that $\tilde{v}_1 \to v_1$ and $\tilde{i}_2 \to i_2$ as $t \to \infty$.

These exact same approaches can be used to synchronize two nonidentical *autonomous* systems with a similar topology. Examples of such systems are Chua's circuit and Chua's oscillator [15, 16].

6 Conclusions

We have shown how two nonautonomous chaotic systems can be synchronized without the need to explicitly phase-lock the periodic forcing in the two systems. The key feature of these synchronization schemes is that the external forcing of the two systems can be arbitrary and do not need to be identical. This allows us to consider them as communication systems and synchronization schemes for nonidentical systems.

In particular, in the master-slave configuration the slave system does not need an external periodic forcing and can recover the periodic forcing of the master system. This suggests the possibility of using this as a

[1] As long as we can write state equations for the entire system.

communication system by replacing the external periodic forcing signal in the master circuit by a properly encoded information signal, which can then be recovered in the slave circuit.

In the mutual coupling scheme, the two periodic forcing signal can be completely different. This suggests the possibility to use this as a bidirectional communication system with the two systems both receiving and transmitting at the same time.

Both schemes can also be considered as synchronization schemes for two systems which are not identical.

The reason these synchronization schemes work is due to the special topology of the circuits being synchronized. It would be interesting to see if synchronization without phase-locking is possible for other circuits.

Acknowledgements

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