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Abstract

This paper reports on the simulation, in three-dimensional CNN arrays of Chua's circuits, of basic three-dimensional scroll wave patterns observed previously from other media. Among the simulated patterns are the ***straight scroll wave, twisted scroll wave*** in both homogeneous and inhomogeneous media, as well as the ***scroll ring***. These types of waves have been obtained for only one set of circuit parameters by varying the initial conditions.

1. Introduction

Two-dimensional wave patterns have been a research subject for a long time. In particular, two-dimensional spiral structures have been observed in chemical reactions [Zhabotinsky & Zaikin,1971], the heart muscle [Allesie *et al.*,1973], and later studied in detail theoretically [Mikhailov & Krinsky,1983], [Keener & Tyson,1986], [Krinsky,1978] and in numerical experiments [Zykov,1984], [Jahnke & Winfree,1991], [Klevecz *et al.*,1992]. Three-dimensional vortex rings were first observed and described in [Winfree,1973]. Because of the complexity and high computational costs, three-dimensional analogs of planar spiral waves have begun to be studied in depth only recently, largely inspired by experimental observations of wave propagation in nerve and heart tissues [Amemori *et al.*,1987], [Chen *et al.*,1988]. In most cases, the FitzHugh–Nagumo equations and the Oregonator model were used previously to model the dynamics in excitable media [Pertsov *et* *al.*,1984], [Jahnke & Winfree,1991]. Recently it has been demonstrated that one- and two-dimensional CNN [1] arrays of Chua's circuits [Madan,1993] can support both traveling waves [V. Pérez-Muñuzuri *et al.*,1993a,1993b] and spiral waves [A. Pérez-Muñuzuri *et al.*,1993]. The CNN is an *active* medium because each discrete "cell" in the array operates in a periodic or chaotic regime when powered by a battery. The purpose of this paper is to show that the range of nonlinear wave phenomena can be further extended to three-dimensional CNN arrays. A great advantage of such arrays is that, once built in the laboratory, they will make it possible to observe the above-mentioned three-dimensional wave phenomena in real time by changing only a few circuit parameters and initial conditions. Moreover, using microelectronics technology, such an array (currently for 2 dimensions) can be fabricated in a monolithic chip the size of a fingernail.

2. Model of the Three-dimensional CNN Array and Results of Simulations

We consider three-dimensional CNN arrays of resistively coupled Chua's circuits each of which operates in a periodic (stable limit cycle) regime. The associated dynamical system is described by the following system of equations

$$\begin{aligned}
\dot{x}_{i,j,k} &= \alpha\,(y_{i,j,k} - x_{i,j,k} - f(x_{i,j,k})) + \\
&\quad D_{i,j,k}[x_{i+1,j,k} + x_{i-1,j,k} + x_{i,j+1,k} + x_{i,j-1,k} + x_{i,j,k+1} + x_{i,j,k-1} - 6x_{i,j,k}] \\
\dot{y}_{i,j,k} &= x_{i,j,k} - y_{i,j,k} + z_{i,j,k} \\
\dot{z}_{i,j,k} &= -\beta y_{i,j,k} \qquad\qquad (1 \le i \le N_i\,,\ 1 \le j \le N_j\,,\ 1 \le k \le N_k)
\end{aligned}$$

where

$$f(x) = (1/2)\,[(s_2 + s_1)x + (s_0 - s_2)(|x - B_1| - |B_1|) + (s_1 - s_0)(|x - B_2| - |B_2|)]$$

is a 3-segment piecewise-linear function with the slopes $s_0 = -0.921$ (middle segment), $s_1 = 15$ (right-hand segment), $s_2 = 0.020206$ (left-hand segment), and the breakpoints $B_1 = -1$, $B_2 = 0.0591173$. These parameters and $\alpha = 10$, $\beta = 0.334091$, as well as diffusion coefficients $D_{i,j,k} = D = 0.1$ will be fixed unless otherwise stated. Also the size of the 3-D array was the same in all simulations, namely $N_i = N_j = N_k = 50$. We used

[1] CNN is an acronym for *Cellular Neural Network* [Roska & Vandewalle,1993]. It consists of a 2- or 3-dimensional array of identical discrete electronic circuits (with possibly varying parameters), each one described by an *ordinary* differential equation.

Euler's integration routine and stepsize 0.01 to integrate the system. In all cases, zero flux (Neumann) boundary conditions were used. We consider three types of scroll waves: the straight scroll wave, twisted scroll wave, and scroll ring wave.

I. *Straight scroll waves* can be generated from the initial conditions in which the "2-D" initial conditions for spiral generation [A. Pérez-Muñuzuri *et al.*,1993] are simply stacked one on top of the other in the 3-D array in one direction. Fig.1 shows a projection of the isosurfaces of the resulting scroll wave. Note the two wavefronts corresponding to the same level of variable x for the isosurfaces.

II. *Twisted scroll waves* can be produced in two ways.

(a) In homogeneous medium, the initial conditions can be generated from those for the straight scroll wave through the rotation, by appropriate angles, of individual 2-D layers of the 3-D array. An example of this type of twisted scroll wave is given in Fig.2. According to Mikhailov [1990], twisted scroll waves in homogeneous medium are unstable, and tend to untwist for large simulation times, which was indeed the case with our particular simulation (Fig.2b).

(b) Another way of obtaining twisted scroll waves is to use an inhomogeneous medium, e.g. by introducing a uniform gradient for the diffusion coefficient, in the k–direction for instance, as follows: $D_{i,j,k} = D(1 + k/25)$, i.e. $D_{i,j,k}$ varies between 0.1 and 0.3 (Fig.3). Since larger diffusion coefficients give rise to greater wavelengths and smaller periods of rotation, the initial conditions evolve into a twisted scroll wave which, in contrast to (a), is stable [Panfilov *et al.*,1984]. Different shapes and degree of twist can be achieved by simultaneously imposing a gradient to parameters α and D (Fig.3c).

III. *Scroll ring waves* have been previously studied numerically, e.g. in [Courtemanche,1990], using the classical FitzHugh–Nagumo equations. Here we use the same parameter values as in I, II(a) and change only the initial conditions. A scroll ring can be thought of as the trace of a spiral, rotated along a circle in the three-dimensional space. In our particular case we used the mapping

$$x(i,j,k) = X(i, I(N/4) + I(\sqrt{(j-N/2-1)^2 + (k-N/2-1)^2}))$$

where N is the array size ($N = 50$ in our case), $I(r)$ is the nearest integer approximation to real number r, and $X(m,n)$ is the x–variable of the cell with coordinates (m,n) in the 2-D array of initial conditions for the spiral. The same formulae were used for the y– and z–variables. A scroll ring develops after about 1000 time units. Although circular in its early stages (Fig.4a), it developed "corners" after several thousand time units (Fig.5a), and this shape was preserved for the full length of simulation. The objects in Figs.4a,5a,6 seem to resemble tori at first sight, but their cross sections (Figs.4b,5b) reveal a scroll structure.

Finally, let us observe that the above simulations were all performed for parameter values corresponding to an active medium with periodic (limit cycle) local behavior. However, similar results can also be obtained by using excitable dynamics of the local cell behavior, e.g. by modifying the parameter values to $\beta = 0.3014987$, $D = 0.2$, $s_0 = -1.25719$, $s_2 = 0.078573$, and using the same initial conditions as above.

Some of the computations were conducted on the Silicon Graphics Iris Crimson workstation and the figures were created by using SGI EXPLORER graphics software.

Acknowledgments

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Figure captions

Fig.1. Isosurfaces for a straight scroll wave, generated from a $50 \times 50 \times 50$ array of Chua's circuits. Here, as well as in all subsequent figures, the isosurfaces correspond to the level -1.503 of variable x.

Fig.2.(a) Twisted scroll wave generated in a homogeneous discrete active medium, at time $t = 80$. Initial conditions were obtained through gradual rotation of a developed spiral by angle 4π from top to bottom of array. (b) The scroll wave has untwisted by almost 3π by the time $t = 3000$.

Fig.3.(a) Inhomogeneous discrete active media give rise to twisted scroll waves. The snapshot is at time $t = 300$. (b) Snapshot of the wave at $t = 3000$. (c) Twisted wave at $t = 1200$, obtained by imposing a gradient $\alpha_{i,j,k} = \alpha(1 + k/50)$ in addition to that of parameter D, $D_{i,j,k} = D(1 + k/50)$.

Fig.4.(a) Scroll ring at $t = 700$. (b) Cross section of the scroll ring at level $j = 25$.

Fig.5.(a) Scroll ring in its later stages of development at $t = 3000$. (b) Cross section of the scroll ring at level $j = 25$.

Fig.6. Scroll ring created through the rotation of a developed spiral ($t = 0$).

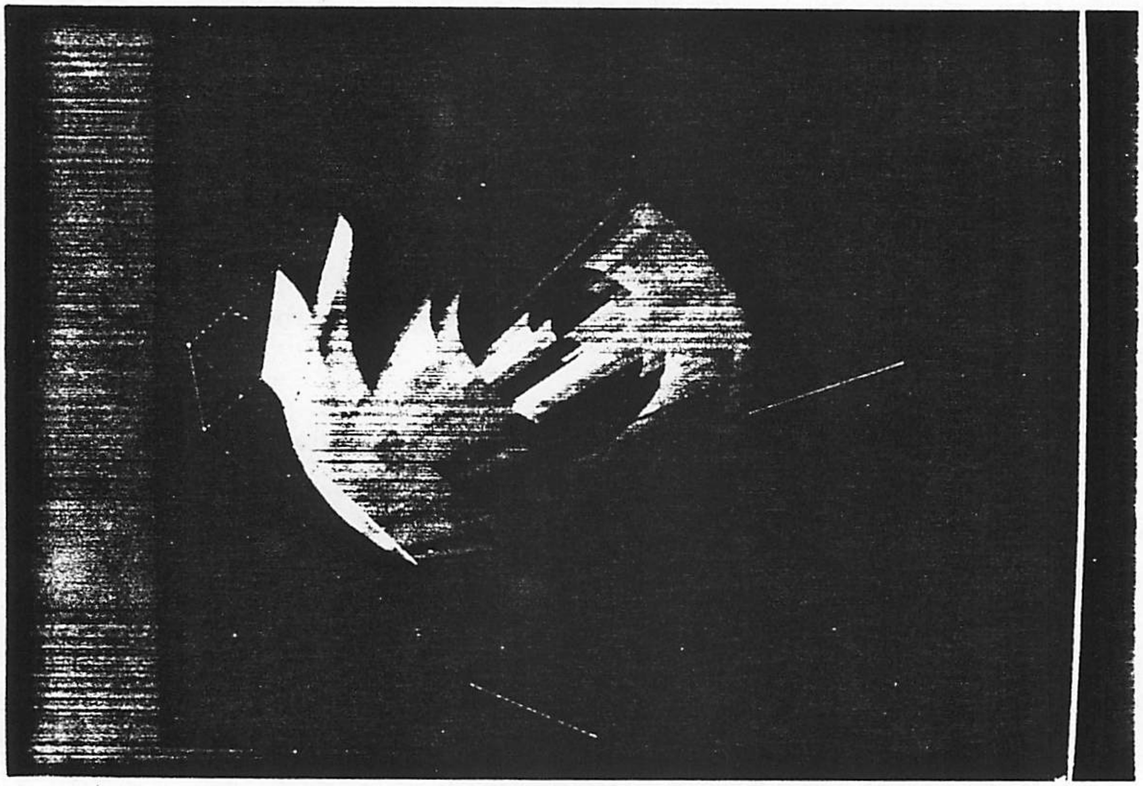

Figure 1

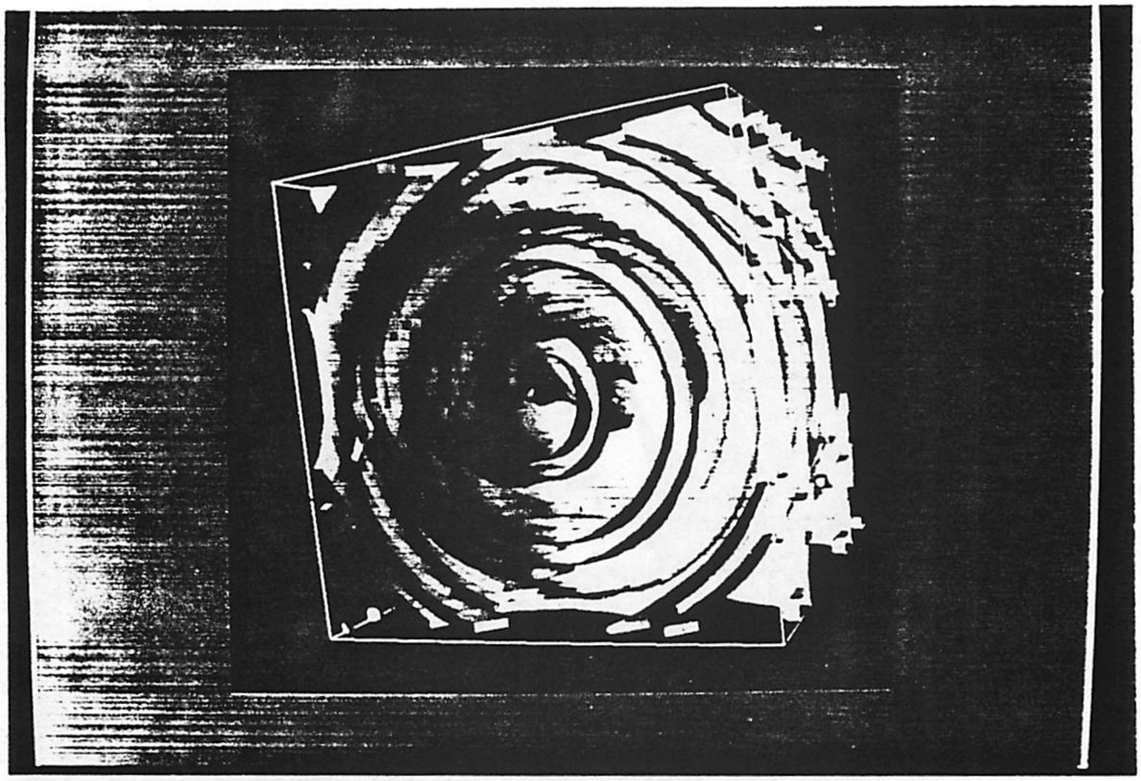

Figure 2a

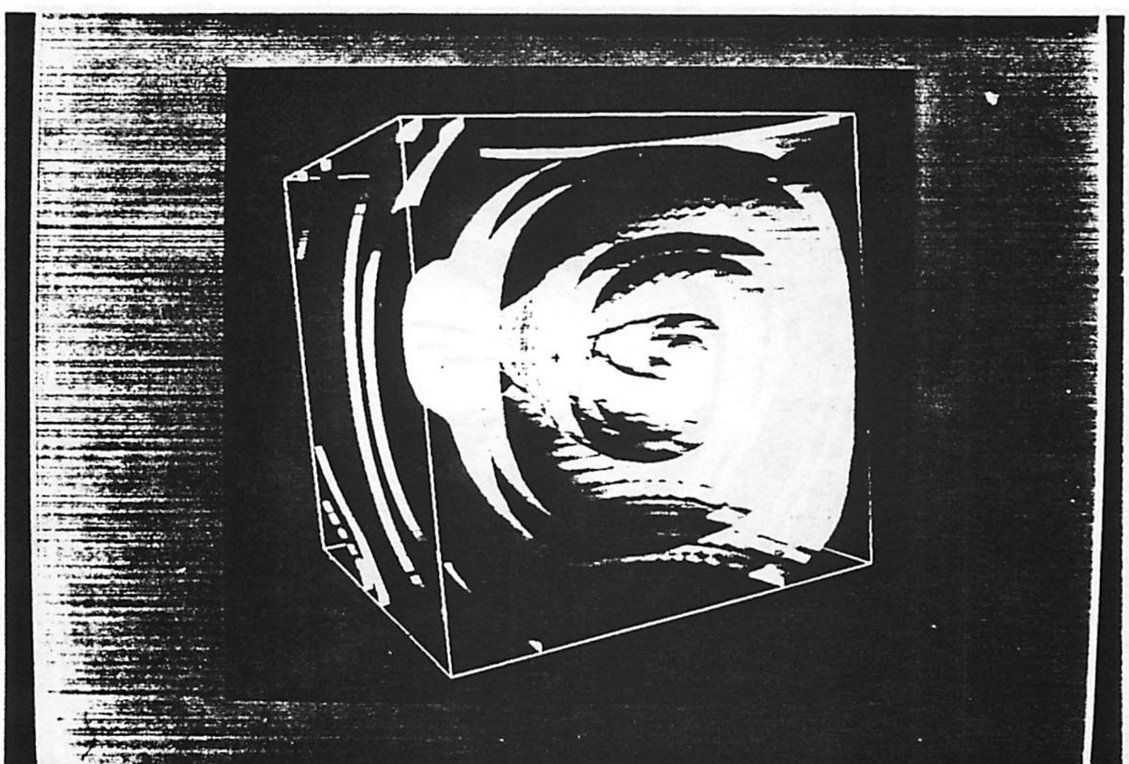

Figure 2b

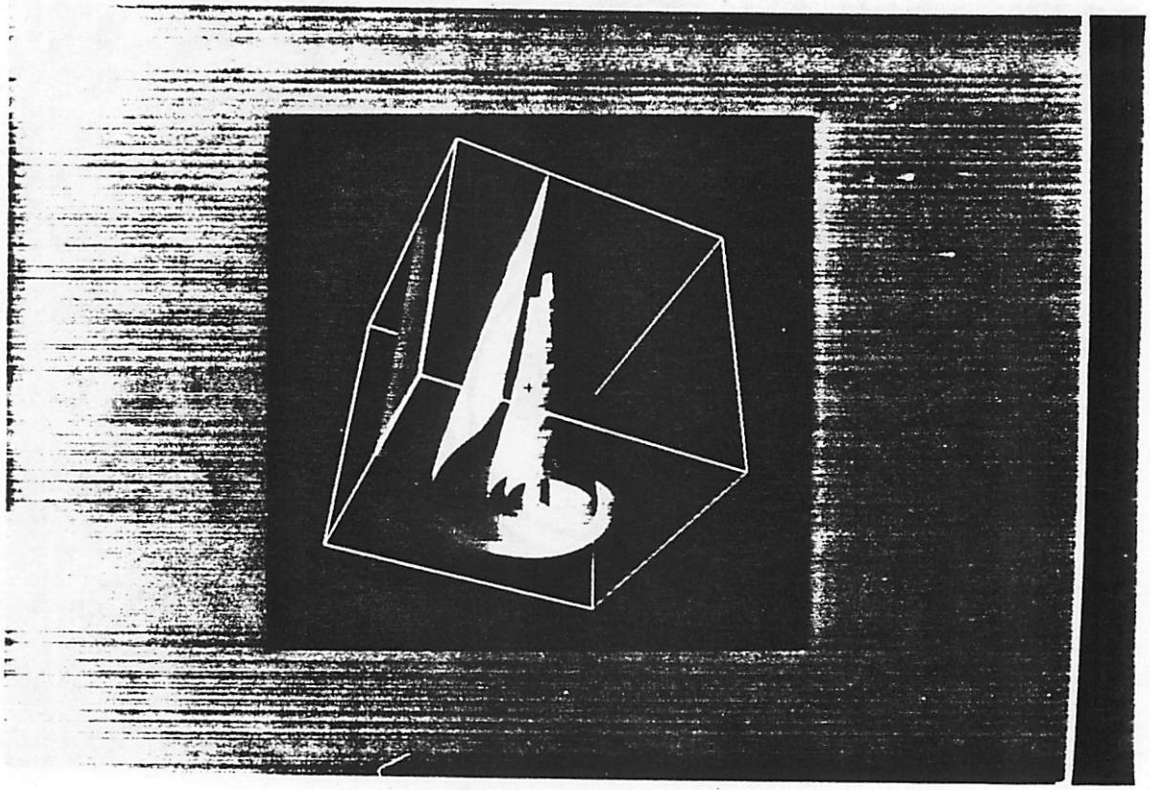

Figure 3a

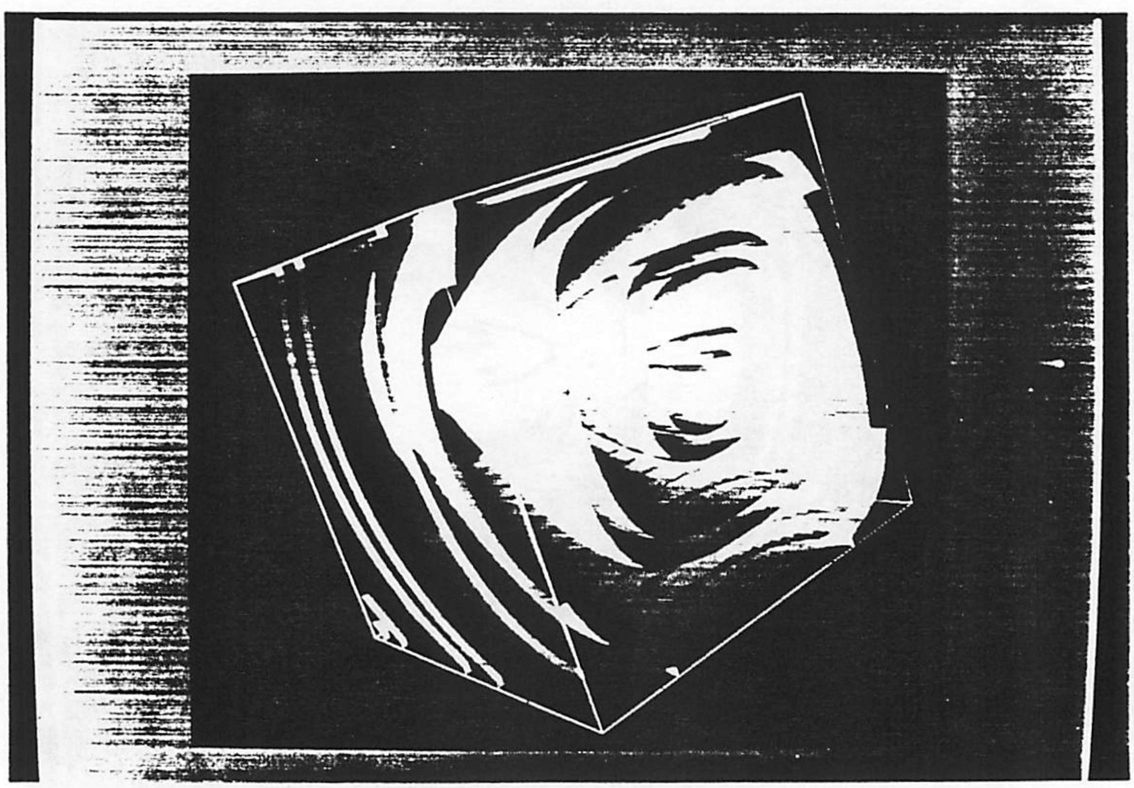

Figure 3b

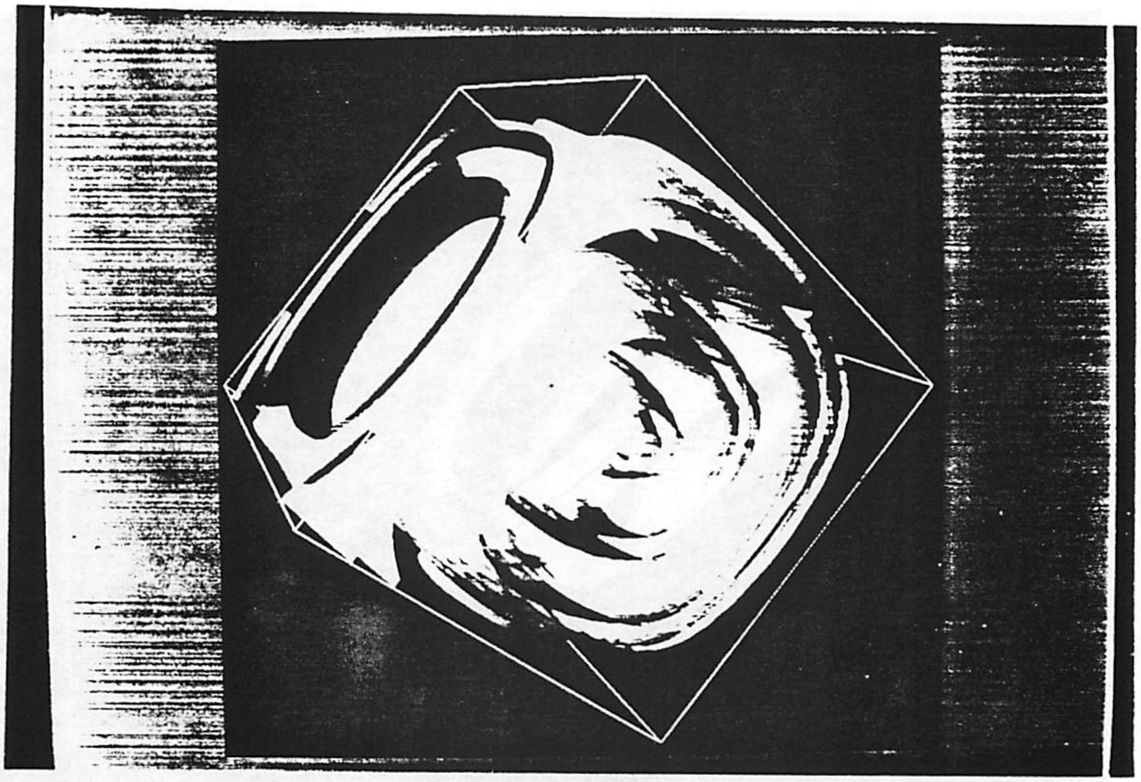

Figure 3c

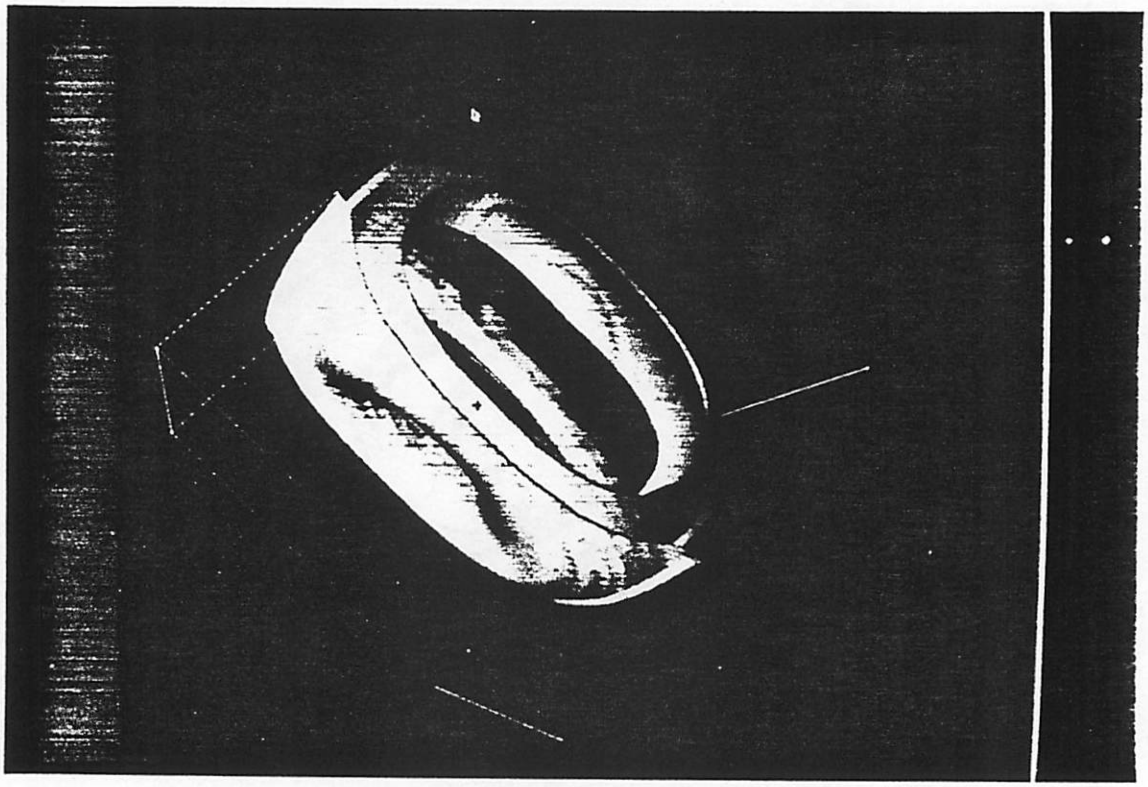

Figure 4a

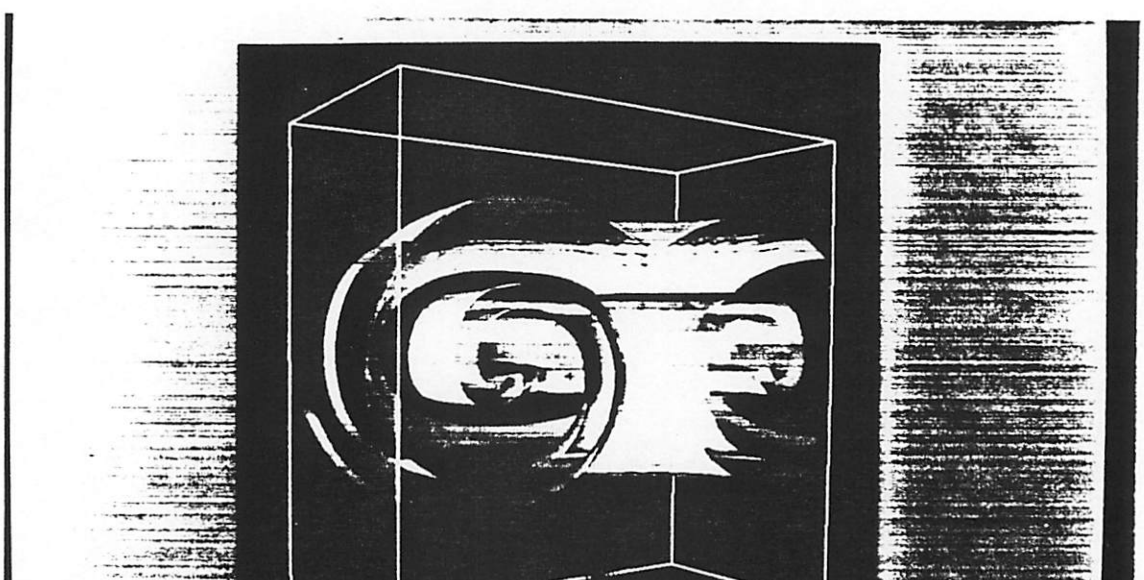

Figure 4b

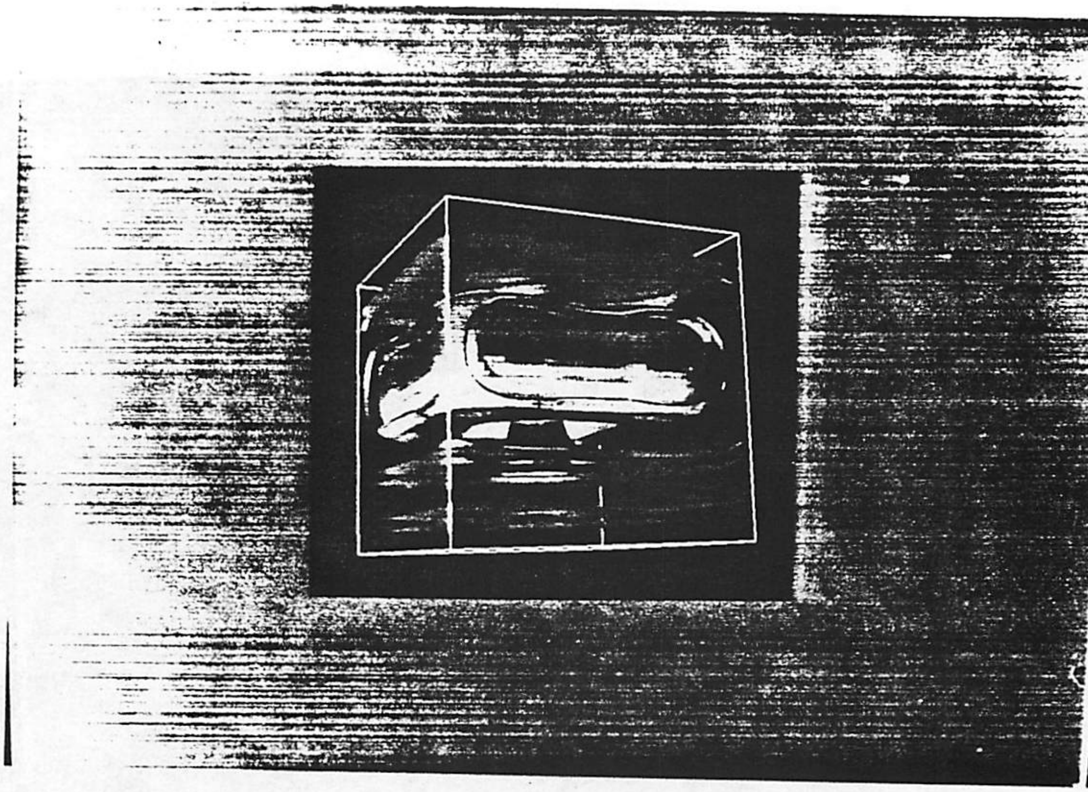

Figure 5a

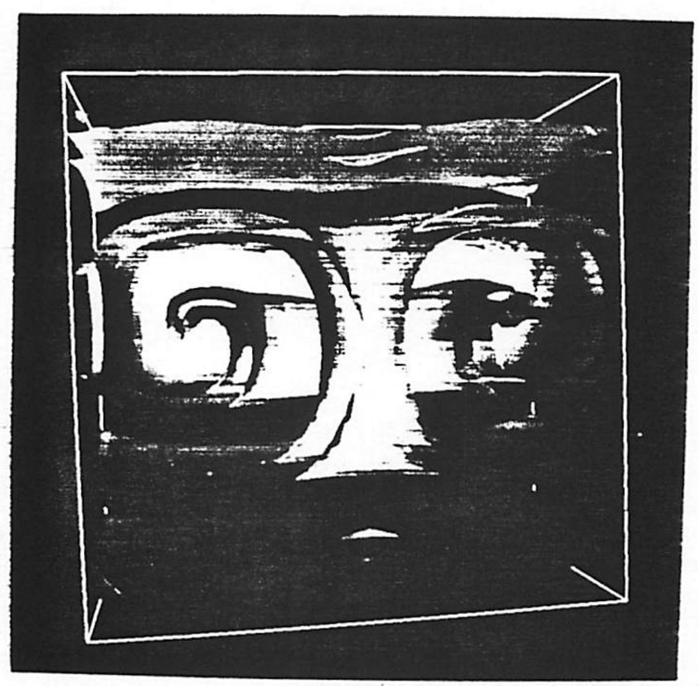

Figure 5b

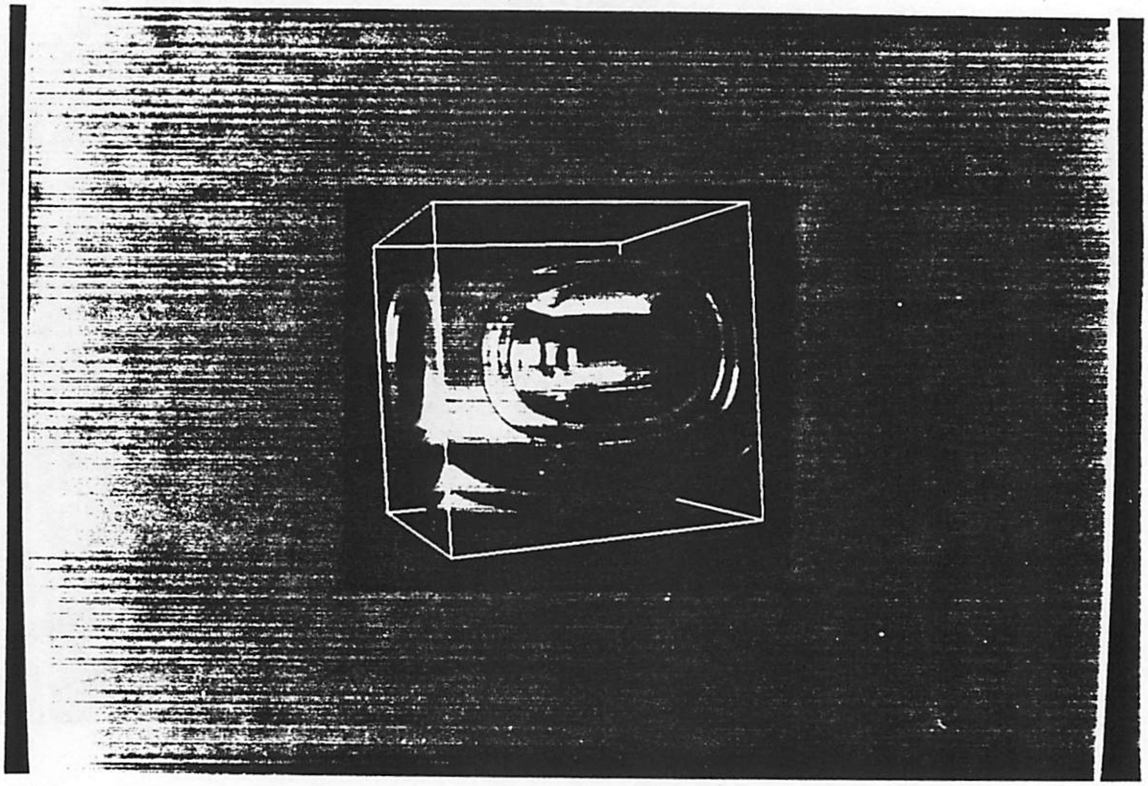

Figure 6