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Abstract

Because of the periodically-time-varying nature of some circuit blocks of a communication system, namely the mixers, the noise which is generated and processed by the system has periodically-time-varying statistics. An accurate evaluation of the system output noise is considerably more complicated than in the case where all the circuit blocks are linear-time-invariant and the noise that they generate is time independent. We examine here conditions under which we can treat the noise at the output of every circuit block of a practical communication system as if it were time-invariant, in order to simplify the noise analysis without introducing significant inaccuracy in the noise characterization of the overall communication system.

1. Introduction

The concept of noise figure defined as the ratio of the signal-to-noise ratio (SNR) at the input to the SNR at the output, was introduced to describe the noise performance of circuits and receivers [4][5]. It is a convenient characterization because the noise figure of a system of cascaded blocks can be found easily from the noise figure of the individual blocks. However, the simple formulas for the noise figure of a system of cascaded blocks are based on the assumption that the noise at the input and the output of every block is a wide-sense-stationary (WSS) process. There are two reasons why the mixer output noise is in fact not WSS but has periodically time-varying statistics [6]. First, the operating points of the devices may vary with time, and second the transfer function of the noise signal from the point at which it is generated to the output can have time-varying characteristics. The mixer output noise is a cyclostationary process and its complete description requires a periodically time-varying power-spectral-density (PSD) S(f,t) [1]-[3]. An accurate evaluation of the output noise when cyclostationary noise is processed by a periodically linear-time-varying (LTV) system is considerably more complicated than the evaluation of the output noise of a linear-time-invariant (LTI) system processing WSS noise. The corresponding analysis and methodology is given in [1], and a related circuit simulator has been presented in [3].

Despite the fact that the mixer output noise is cyclostationary, the noise figure calculated using the time-average output noise PSD has been traditionally used to characterize mixers, and the simple formulas for the noise figure of a system of cascaded blocks have been used to find the noise figure of a communication system. We shall show here that this treatment provides the correct noise characterization of a receiver in most practical cases, but we will examine cases in which it could lead to an inaccurate prediction.

2. Cyclostationary noise and its time-average

Noise measuring equipment measures the PSD at a frequency f by measuring the power of the signal at the output of a very narrow-band filter around f and provides the time-average PSD [1]. Therefore, when the noise performance of a communication system is measured, the quantity that characterizes the overall system, even if this includes LTV circuit blocks, is the time-average output PSD.

The complete description of a cyclostationary signal with its time varying PSD S(f,t), as opposed to its description with S(f), the time average of S(f,t), becomes significant only when the block following the system under characterization is synchronized to the PSD variation with time. This statement will be explained on an intuitive basis, and it also gains support from the following theorem [2]: If a uniformly distributed random variable from zero to one cycle period is added to the time variable t of a cyclostationary process with PSD S(f,t), (that is, the information about the phase of the periodically varying PSD is lost) the resulting process is stationary and its statistics are the time-average of the statistics of the cyclostationary process. If the system or sensor to which the output cyclostationary noise is input does not track the PSD variation with time, the phase of S(f,t) for this system is unknown. In the absence of information about the phase of S(f,t) the process becomes stationary, with PSD equal to the time-average of S(f,t).

When a cyclostationary signal passes through a LTI filter and the time-average PSD is measured at the output, the same result would be obtained if time-averaging occurred at the input of the filter, and filtering took place on a WSS process with PSD equal to this time-average input PSD [1]. However, when a cyclostationary noise signal is fed to a time-varying system, consideration of only the time-average PSD of the input noise can lead in the general case to wrong results, as can be seen from the analysis of [1] and has been demonstrated with an example in [3]. For instance, if the time-varying-gain and the power of the input noise obtain their peak values simultaneously, considering only the time-average input noise will underestimate the output noise. The following example will help clarify the situation.

Consider that a WSS signal n(t) with PSD $S_n(f)$ is fed to a mixer A, and the output of this $n_a(t)$ is fed to a mixer B, as shown in Fig. 1(a). The random signal n(t) can represent noise present at the input of mixer A, or noise generated by its devices[1]. The mixing operation is modeled by multiplication of the input signal n(t) with a periodic waveform generated by a local oscillator, a(t) with frequency f_{oa} and b(t) with frequency f_{ob}, for mixers A and B respectively[2]. The output of mixer A is a cyclostationary process whose time-average PSD consists of copies of $S_n(f)$ shifted in frequency integer multiple of f_{oa}, and weighted by different coefficients. It is easy to see that frequency components of $n_a(t)$ in distance integer multiple of f_{oa} are correlated, since they contain the same frequency component of n(t). Correlated frequency components means that their phases are correlated. Therefore if the two components are modulated to the same frequency at the output of mixer B, the power of the resulting component is not the quadratic sum of the two individual components, as it would be in the absence of any information about their phases (or if the two components were 90^o out of phase). A random process can be cyclostationary with cycle frequency f_{oa} only if there exists correlation between two different frequency components in distance f_{oa}. The spectral correlation can be expressed in terms of the cyclic spectra, the Fourier components of the time-varying PSD, and in fact the k-th cyclic spectrum for positive k is the correlation between frequency components in distance kf_{oa}, while the 0-th order cyclic spectrum is the time-average PSD. A random process can be WSS only if any two different frequency components are uncorrelated [1]. The output of mixer B is a cyclostationary process with two cyclic frequencies f_{oa} and f_{ob}. If f_{oa} and f_{ob} are commensurate (their ratio is a rational number), $n_b(t)$ can be viewed as cyclostationary with one cycle frequency equal to the maximum common divider frequency of f_{oa} and f_{ob}.

2.1 Effect of LO frequency relation

Let us examine now the spectral content of the output of mixer B $n_b(t)$ at a frequency f_{out}. Frequency components of $n_a(t)$ at frequencies $f_{out}+kf_{ob}$, k being an integer, are folded on f_{out} as shown in Fig. 1(b). If $nf_{oa}=mf_{ob}$ for some integers n and m, there exists correlation among these components, and it is incorrect to add their power, as we would do if $n_a(t)$ were WSS, since a valid addition would require correlation terms. However if the ratio of f_{oa} and f_{ob} is not a rational number, such integers n and m do not exist and simply adding the different frequency components of the time-average PSD $S_{na}(f)$ gives the correct result, since the added terms are uncorrelated.

1. In the case of noise generated by devices with time varying operating point, this noise is cyclostationary and white and its time variation can be incorporated to the system. Therefore in any case the input noise n(t) can be considered WSS. For every noise source inside the mixer the time-varying gain is a different function.

2. At high frequencies where reactive effects are not negligible, the mixing operation is better modeled with a periodically-time-varying transfer function A(f,t) [6], instead of a periodically-time-varying gain a(t). Frequency translation is described with the Fourier components of A(f,t), the conversion transfer functions instead of the conversion gains. This difference would affect the way the amplitude of an output frequency component is calculated, but not the frequencies to which an input frequency is translated. For this reason the arguments presented here also apply at high frequencies.

In practice, the ratio of two LO frequencies generated by different free running LOs can always be considered an irrational number, since because of the random phase error they cannot track each other. The situation is different however if the two LOs are locked to a common reference frequency. In a superheterodyne receiver which employs two mixers, it is a common practice to generate the two LO signals from two PLLs with a common reference frequency, which means that f_{oa}/f_{ob} is a rational number m/n (we will assume below that m and n are such that a common integer divider of m and n greater than one does not exist). Despite this, a rational frequency ratio $f_{oa}/f_{ob}=m/n$ with m or n very large numbers is expected to have the same practical effect as an irrational frequency ratio. In fact, the LO frequencies in a receiver chain are usually chosen such that they do not have simple relation in order to avoid the spurious responses.

Assuming a smooth b(t) with low frequency content, we can see that the conversion gain of mixer B drops rapidly with the order of the sideband, and only the first few (for example up to 3 or 4) contribute significantly. Therefore, considering again the integers m and n that satisfy $f_{oa}/f_{ob}=m/n$, if m is a relatively large integer, in every set of correlated frequency components of $n_a(t)$ in distance integer multiple of $mf_{ob}=nf_{oa}$ that contribute to f_{out}, only one term contributes significantly and only a minor error is introduced by adding the power of all the components. If n is large, assuming a smooth a(t), the effect of noise correlation is also attenuated for a similar reason: the copy of n(t) around nf_{oa} has low power. Concluding, the effect of spectral correlation is insignificant if a(t) is smooth and n is large, or if b(t) is smooth and m is large, or both. Very often in practice, especially at high frequencies a(t) and b(t) are smooth functions, and unless the ratio of the two LO frequencies is a simple rational number m/n with m,n small integers, calculating the time-average at the output of the first mixer and treating it as if it were the PSD of WSS noise, cannot introduce a significant error in the estimation of noise at the output of the second mixer. An example of a mixer with non-smooth time varying gain is the sampling or subsampling mixer, since the Fourier transform of an impulse train is an other impulse train which has a high-frequency content.

The above argument can be easily visualized in the time domain with the example of Fig. 2. Consider that the time-varying power $\sigma_a(t)$ of the cyclostationary noise $n_a(t)$ - the integral of the time-varying PSD over all frequencies - at the output of the first mixer is the periodic function of time shown in Fig. 2(b). Assume that b(t) is an impulse train, so that mixer B is essentially a sampling mixer as shown in Fig. 2(a) and that we desire to estimate the time-average power of the samples at the output of the sampler. If $f_{oa}=f_{ob}$, or $f_{oa}=mf_{ob}$, we always sample $n_a(t)$ when $\sigma_a(t)$ is at the same point of the period as shown in Fig. 2(b), and if instead the time-average of $\sigma_a(t)$ is considered at the input of the sampler, we probably significantly overestimate or underestimate the output noise. In this case, since b(t) is not a smooth function of time and its spectral content does not die out at high frequencies, the effect of spectral correlation is not diminished if m is large. If f_{oa}/f_{ob}=m/n is a rational number and n is a small integer, we sample repeatedly only a few points in the period and it is possible that considering the time-average of $\sigma_a(t)$ at the input of the sampler will result in an erroneous noise estimation. However, if n is a large number, the same points of the period are repeatedly sampled, but they are many and uniformly distributed across a period, as shown in Fig. 2(c), so considering the time-average at the input of the sampler would give a practically correct result. When f_{oa}/f_{ob} is not a rational number, after long enough time the whole period is uniformly sampled and in fact the same point is never sampled twice. In this case time-averaging at the input of the sampler provides exactly the correct result.

We will now express the above in a more quantitate manner. Referring to Fig. 1, we can see that $n_b(t)$ consists of scaled copies of n(t) shifted in frequencies $k_af_{oa}+k_bf_{ob}$, where k_a and k_b are the sidebands at which the conversion gain of mixers A and B respectively is significant, determined by the spectral content of the waveforms a(t) and b(t) and possibly as we will see below by filtering the mixer outputs. If two of those frequencies coincide, the spectral correlation affects the output noise estimation. If k_a' and k_b' is a second set of mixer sidebands, the relation

$$k_af_{oa} + k_bf_{ob} = k_a'f_{oa} + k_b'f_{ob} \tag{1}$$

or

$$\frac{k_b - k_b'}{k_a - k_a'} = -\frac{f_{oa}}{f_{ob}} \tag{2}$$

can only hold if f_{oa}/f_{ob} is a rational number, as we also concluded before. Furthermore, if f_{oa}/f_{ob}=m/n, spectral correlation has an effect only if there are integers k_a, k_a', k_b, and k_b' that represent sidebands of the mixers with significant conversion gain such that

$$\frac{k_b - k_b'}{k_a - k_a'} = -\frac{m}{n} . \tag{3}$$

If for example, a(t) and b(t) are sinousoidal with frequencies mf_o and nf_o, f_o being some reference frequency, k_a, k_a', k_b, and k_b' can only be +1 and -1, and spectral correlation can have an effect only if n=m. This case corresponds to the example for the significance of the spectral correlation given in [3].

In a similar manner, one can examine the effect of spectral correlation when a third mixer C follows the chain of A and B. Denoting the frequency of C by f_{oc} and the sidebands of C with some significant conversion gain by k_c and k_c', spectral correlation affects the noise estimation only when there are sidebands of the mixers with significant conversion gain, such that

$$k_a f_{oa} + k_b f_{ob} + k_c f_{oc} = k_a' f_{oa} + k_b' f_{ob} + k_c' f_{oc}. \qquad (4)$$

If the LO frequencies are related, i.e. $f_{oa}=mf_o$, $f_{ob}=nf_o$, $f_{oc}=pf_o$, where f_o is some reference frequency and m,n, and p integers with no common divider greater than 1, (4) becomes

$$(k_a - k_a')m + (k_b - k_b')n + (k_c - k_c')p = 0 \quad . \qquad (5)$$

In this case, it is possible that relations (4) and (5) hold for low order sidebands, even if the relation of the LO frequencies is not simple. For example if f_{oa}=2000MHz, f_{ob}=660MHz and f_{oc}=10MHz the above relations are satisfied for $k_a-k_a'=1$, $k_b-k_b'=-3$, and $k_c-k_c'=-2$.

2.2 Filtering a cyclostationary noise process

If filtering takes place at the output of a mixer, as in Fig. 3(a) it is possible that the noise at the output of the filter is stationary, and no cyclostationary noise considerations need to be made, or that the characteristics of the cyclostationary noise change. Some relevant theorems have been presented in [3], but they were derived in a non intuitive way. Similar results can be found in [6][7]. These results, become rather straightforward by examining filtering of a set of correlated frequency components. Let us consider a cyclostationary noise process with cycle frequency f_o and a set of correlated frequency components in distance integer multiple of f_o. The results of [3] can be observed:

Result 1: Consider a low-pass filter with cut-off frequency $f_o/2$ or lower, as in Fig. 3(b). One can see that only one component of the set of correlated components can fall in the window $[-f_o/2, f_o/2]$ that the filter allows to pass. Therefore any frequency components at the output of the filter are uncorrelated and the output noise is stationary.

Result 2: Consider a single-sided bandpass filter, either upper band or lower band with respect to f_o, and bandwidth $f_o/2$ or less (given inaccurately in [3]), as in Fig. 3(c). After filtering, only one frequency component of the correlated set remains, and the resulting noise is stationary.

Result 3: Consider a bandpass filter with center frequency f_o and bandwidth f_o or less, (given inaccurately in [3]) as in Fig. 3(d). One can easily see that after filtering, the remaining correlated frequency components can only be in distance $2f_o$, and therefore only the stationary and the second order cyclic spectra can exist.

Many other similar results can be visualized in a similar manner. For example if the filter is a low-pass filter with a cut off frequency f_o, the resulting process can contain only the stationary and first-order cyclic spectrum. A possible application of such a result as well as of result 3 above is the following: If it is known that the random signal at the output of mixer A in Fig. 1 does not contain the n-th order cyclic spectrum, k_a-k_a' in (2) cannot be equal to n.

In a receiver chain the first mixer is typically followed by a bandpass IF filter. In this case one can apply the following theorem, which can also be verified easily by inspection: If cyclostationary noise with cycle frequency f_0 passes through a bandpass filter with bandwidth $f_0/2$ or less, and the frequencies $k(f_0/2)$ where k an integer do not fall into the passband, the output noise is stationary. This theorem has been stated in [6] but without defining clearly the necessary properties of the passband of the filter. Results 1 and 2 above can be seen as individual cases of this last theorem.

2.3 Mixing a band-limited cyclostationary noise process

In the previous section the passband characteristics of a filter following a mixer were related to the frequency of the LO waveform driving the mixer in order for the output noise signal to have certain properties. Here we will examine the case of Fig. 4(a) in which a general cyclostationary signal for which we have no information about the location of the correlated frequency components, passes through a filter and the output of the filter is fed to a mixer (or more generally a time-varying circuit). We will relate the filter characteristics with the frequency f_0 of the LO signal driving the mixer, in order for the time-average noise at the output of the mixer to be unaffected by the spectral correlation.

If the filter is low-pass with cut-off frequency $f_0/2$ or lower as shown in Fig. 4(b), no overlap will take place during mixing, and the average noise at the output will not be affected by spectral correlation. This situation appears at the back-end of a receiver where sampling (such as in a switched capacitor filter) is proceeded by an anti-alias filter.

If the filter is bandpass with center frequency f_c and bandwidth w, as in Fig. 4(c), one can see that overlap will not happen if

$$|(k-k')f_0+2f_c| > w \tag{6}$$

for all sidebands k and k' of the mixer with some significant conversion gain. This results from the observation that the positive passband will be transferred to frequency bands with center kf_0+f_c and width w, the negative passband will be transferred to frequency bands with center $k'f_0-f_c$ and width w, and to avoid overlap the centers of the two frequency bands must be in distance greater than w.

3. Two cases where spectral correlation is significant

A practical situation that deserves attention is when an interfering signal or blocker is present at the input of a receiver. If this signal is strong it can change the operating point of the devices and affect the circuit noise performance. The noise generated by the circuit will acquire cyclostationary characteristics with cycle equal to the blocker period, and if the blocker is not filtered or modulated to a different frequency, it acts as a common LO for successive cascaded blocks. In this case, although a block still can be characterized with the noise figure under the presence of a blocker, use of the formulas for cascaded blocks to estimate the noise figure of the whole receiver can possibly lead to an inaccurate prediction. This situation could arise for example when an in-band blocker is processed together with the weak desirable signal by the LNA and the RF mixer of a receiver.

Let us consider now noise introduced to a mixer from the LO port. The LO is a periodically time-varying circuit and it is possible that the noise at its output contains some cyclostationary component. The time-varying processing of this signal by the mixer tracks exactly the time variation of the noise statistics since the operating point of the mixer is determined by the LO drive and it is not correct to time-average the noise PSD at the LO output and use it as if it were a WSS process.

4. Conclusions

We examined qualitatively the significance of the cyclostationary nature of the noise generated in a communication system. We saw that cyclostationarity is equivalent to the presence of correlated components in the frequency spectrum. From the above discussion it results that in the majority of the practical cases, use of the concept of noise figure and considering only the time average component of the cyclostationary noise at the input and the output of every block does not introduce significant inaccuracy in the noise characterization of the overall system because: a) The local oscillator frequencies used usually do not have a simple relation and the situation resembles the case at which the two frequencies are noncommensurate. b) Usually filtering takes place in several places in the receiver chain which converts the cyclostationary noise to stationary noise. However, we examined practical cases where cyclostationarity cannot be ignored, namely when the subsequent stage is time-varying synchronously with the cyclostationarity, as for example when the subsequent stage is driven nonlinearly by the stage generating cyclostationary noise.

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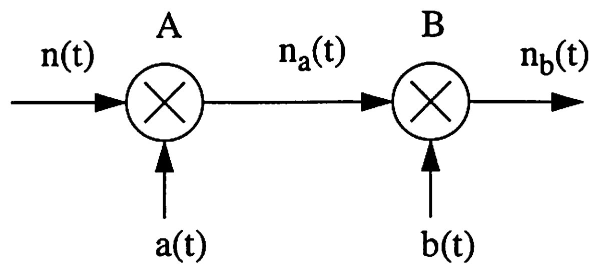
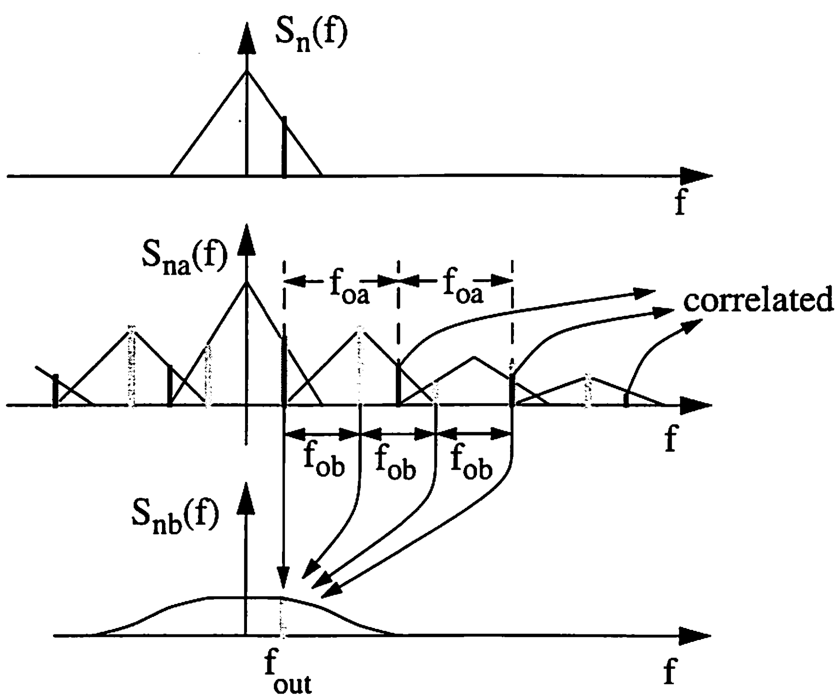
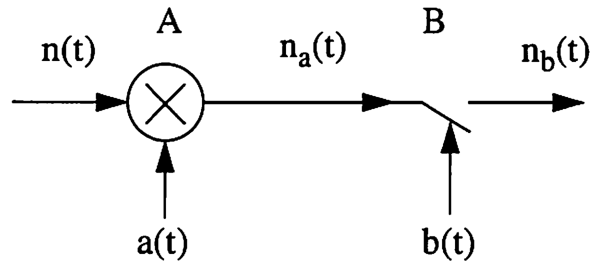


(a)

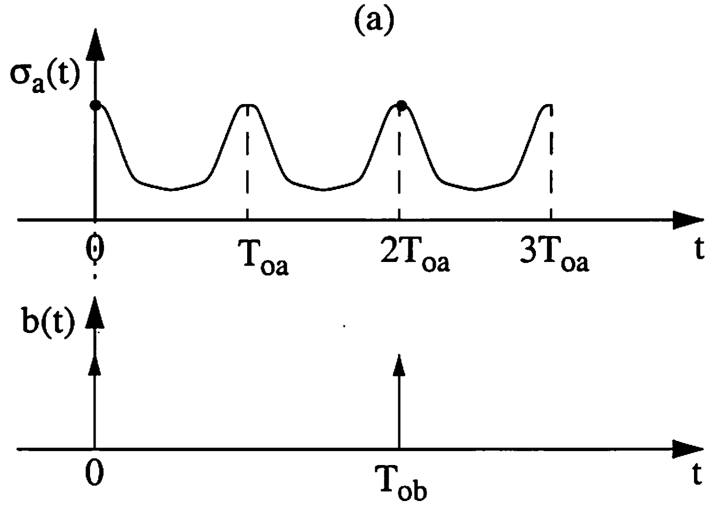


(b)

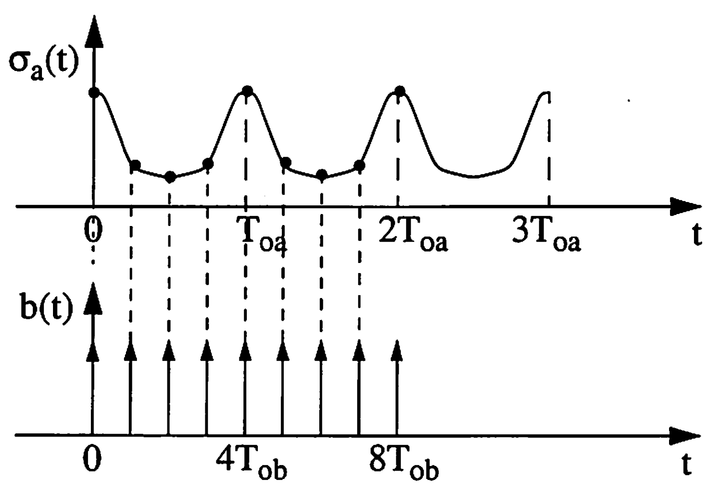
Fig. 1. a) A cascade of two mixers. (b) Time-average PSD of noise at the input, after the first mixer and the output.



(a)



(b) $f_{oa}=2f_{ob}$



(c) $f_{ob}=4f_{oa}$

Fig. 2. Sampling cyclostationary noise.

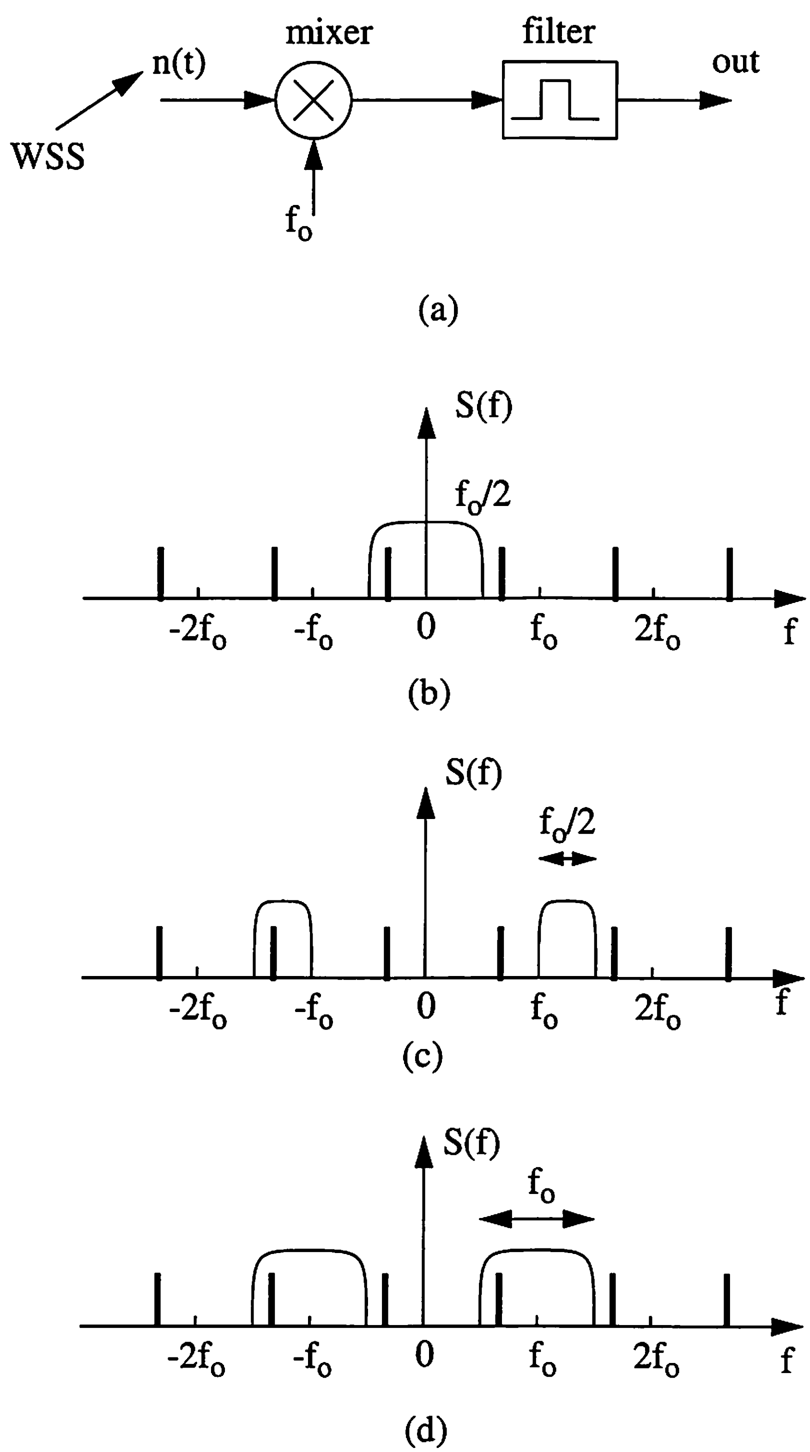


Fig. 3. Filtering cyclostationary noise.

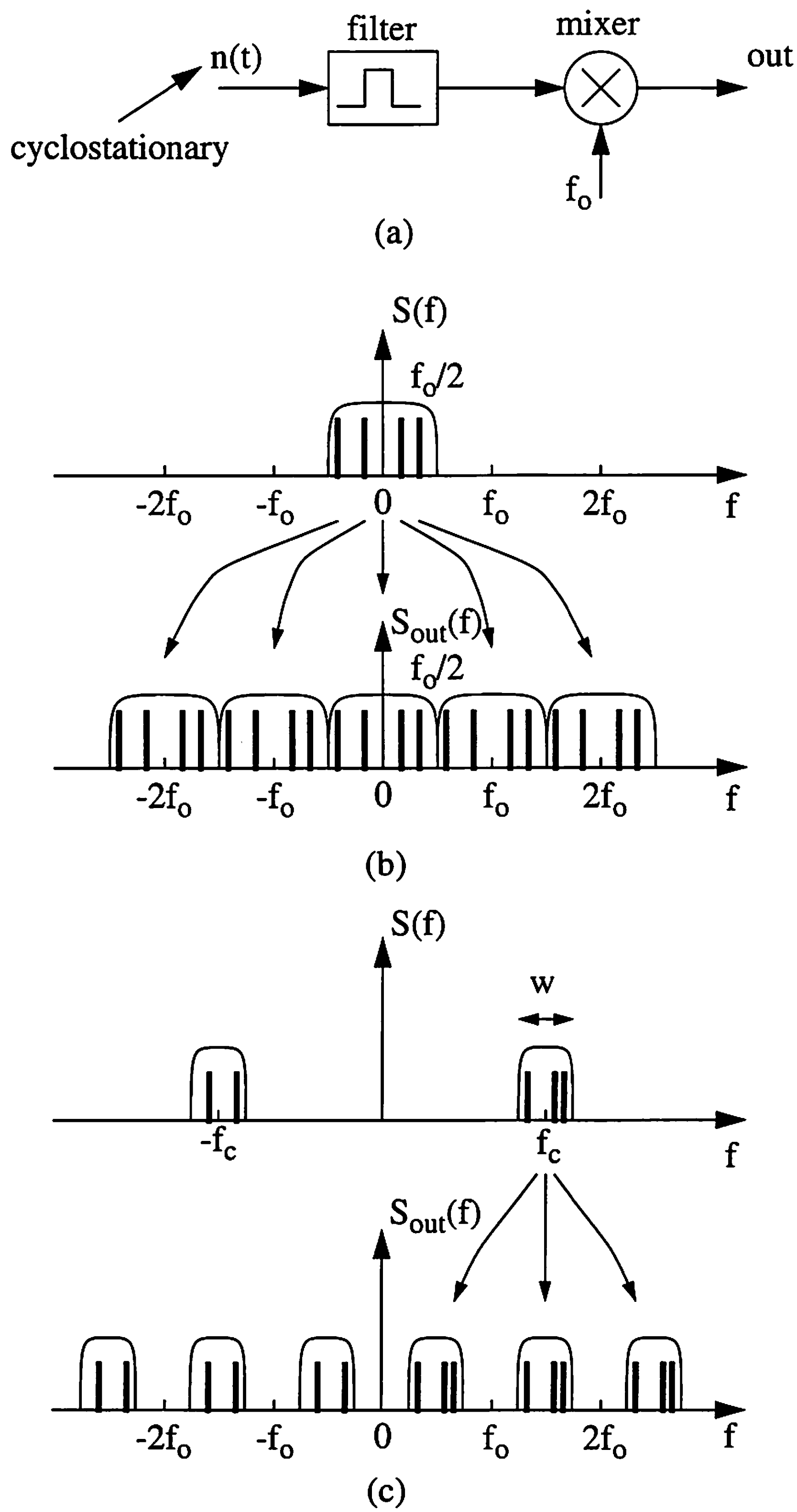


Fig. 4. Mixing band-limited cyclostationary noise