How much white space is there?

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***Abstract*— This paper uses data from the FCC and the 2000 USA Census to estimate the actual white space (in terms of area/population recovered) available for cognitive radios in TV bands in the continental USA. The available white space is the intersection of white spaces resulting from two different viewpoints. The "pollution" viewpoint reflects the cognitive radios' perspective: where is interference from the primary tolerable? The primary users' perspective is called the "protection" viewpoint: where can a secondary operate without generating excessive 'harmful interference' to the primary? For most practical systems that may operate in the TV bands, it turns out that the operational limit will largely come from the pollution-oriented view.**

The amount of white space resulting from the protection-oriented view is greatly effected by the amount of 'harmful interference' allowed. Harmful interference is crisply quantified by the fading margin eroded by the secondary's potential operation. Based on the FCC's November 14^{th} ruling in 2008, the median (across TV towers) erosion of the fading margin is $\sim$ 1dB. Furthermore, the available white space depends on the scale of secondary users — while the current DTV channel allocation cannot accommodate new TV stations, about five channels per person are available for 4W fixed transmitters.

We propose a principled way for regulators to choose the protection margin for primaries that can be eroded by secondary operation. This approach quantifies the political tradeoff between person-channels gained for potential white-space usage versus person-channels lost for broadcasters as we vary the protection margin. For the choice of protection margin(s) used by the FCC, the overall tradeoff is at least 30:1 while being approximately 3:1 at the margin — that is three additional people gain a channel for potential white-space use for every additional person that potentially loses reception of a channel of broadcast television.

Finally, the data validates the conservatism of fixed threshold rules for white-space detection — the -114dBm rule for ATSC signals fails to recover most available white space except in areas of low population density.

I. Introduction

In its notice and order issued on November 14, 2008, the FCC has decided to open up DTV bands to the operation of cognitive radio devices [1]. All locations where a cognitive radio can operate according to this ruling constitute a 'white space'. Technologists, regulators, business strategists and economists are interested in the amount of white space this ruling enables.

There are two viewpoints that govern the amount of 'white space' available. The 'protection' viewpoint is the main concern of the November 14^{th} ruling and stipulates that a cognitive radio can only operate in locations where it cannot generate harmful interference to TV receivers. The 'pollution' oriented viewpoint considers instead that the attractiveness of locations for secondary operation increases with increased distance from the primary transmitter due to the reduction in interference from the primary user. Both viewpoints exclude regions around a television tower and the actual white space available is the intersection of the white space resulting from these two viewpoints.

To quantify the white space available under the 'pollution' viewpoint, we can set limits on the amount of acceptable interference (above noise level) that a secondary can tolerate. For the protection viewpoint, we need a similar (and consistent) notion of 'harmful interference' and the protection that we guarantee to TV receivers. The notion advanced in this paper is idea of an 'eroded fading margin'. At every distance from the TV transmitter there exists a fading margin which *protects* the TV receiver from bad fading events. Once a secondary starts transmission, it raises the noise floor and hence erodes some of that fading margin. This protection margin dictates the number of TV receivers at the edge that are sacrificed to enable secondary operation.

The main contributions of this paper are three-fold:

- The actual white space available in the continental United States is estimated based on the 'pollution' viewpoint in Section II and the 'protection' viewpoint in Section III. We also evaluate the available white space using a variety of detection approaches and with respect to the actual population density. Furthermore, we also look at white space availability as the scale (tower/height) of the primary is varied. In doing so we negatively answer the question: "Is there enough white space for another 1MW DTV transmitter?"

- Section IV proposes a principled way for regulators to make the essentially political choice of the protection margin. This method is based on examining the tradeoff between the number of person-channels gained for potential white space use and those lost for potential broadcast use as the protection margin is varied. Based on estimates of broadcast use and projections for white-space market penetration, regulators can choose the appropriate margin.

- Section V evaluates the FCC's sensing specification of -114dBm for ATSC signals and show how a 'one size fits all' rule of this nature has to be conservative and cannot recover most white space.

There are a few limitations of this study that should be pointed out. Firstly, to calculate the available white space we assumed that the all the licensed transmitters in the FCC high power DTV transmitters database and the master low power transmitter database are all transmitting [2], [3] and neglected some of the clauses from the FCC ruling [1]. If some of the transmitters are off, then sensing rules can help reclaim the additional white space. Secondly, we assume the the ITU propagation models predict the reality on the ground to a fair degree [4]. Thirdly, we are overestimating the number of people served by broadcasters today by assuming that everyone in the noise limited contour can receive TV signal.

II. White Space using the pollution viewpoint

The pollution-oriented view of whitespace takes the perspective of a rational self-interested cognitive radio — a band is attractive when it has a low noise-floor. Implicit in this view is the idea that primary user transmissions must be considered as raising the noise floor from the point of view of the secondary user. Each television tower can thus be viewed as having a pollution radius around it in which the band is unattractive for secondary use. It must be pointed out that there exist theoretical approaches where a secondary transmitter would prefer to be closer to the primary transmitter so as to decode the primary signal and use dirty-paper-coding techniques (DPC) and simultaneously boost the primary signal in the direction of interference [5], [6]. However, it has also been shown that this approach is not robust since simple phase uncertainty can significantly lower the performance of such schemes [7]. Knowledge of the primary user's codebook is useful if the secondary receiver can actually decode the primary signal and use multiuser detection. Otherwise, it has been shown that the secondary system is forced to treat the primary transmission as noise [8].

To calculate the available white space using the pollution viewpoint, we used the FCC's transmitter databases which list the latitude, longitude, effective radiated power (ERP) and transmitter height above sea level for all licensed high power and low power transmitters [2], [3]. These databases were combined with the ITU-R recommendations on propagation (ITU-R P-1536-3) to calculate the values of the pollution radius for all towers.

Appendix II uses the example of the KCNS transmitter on TV channel 39 to illustrate the procedure used to calculate the value for the pollution radius. A location is considered available for white space operation in a given TV channel if it does not fall within the pollution radius of any transmitter of that channel. Table I quantifies the available white space as the average number of white space channels available per location for various values of the acceptable pollution level. The 15dB pollution level was inferred from current IEEE 802.11g systems; a survey of IEEE 802.11g systems shows that raising the interference level to 15dB above noise only deteriorates the data rate from 54MBps to 12MBps. The 5dB pollution level was calculated by analyzing the business case for Wireless ISPs for rural areas [2].

Table I breaks down the TV channels into four frequency bands: Low VHF (LVHF - channels 2, 5 and 6), High VHF (HVHF - channels 7 to 13), Low UHF (LUHF - channels 14 to 51) and High UHF (HUHF - channels 52 to 69). As a percentage of the total number of channels in a band, there is significant white space available in the low VHF bands which runs counter to our intuition that propagation at low frequencies is rather good. The main reason for this is the remarkably low number of DTV stations in the low VHF bands – after the DTV transition, there will be only 31 high power DTV stations in channels 2, 5 and 6. Similarly, there is significant white space in the high UHF bands since there are no high power TV stations in these bands. However, these bands cannot be used for white space devices since they have been reallocated to uses other than television. From a white space recovery standpoint, the band of interest is the low UHF band – for these bands and a tolerable interference level of 5dB, the average number of white space channels per location is ~ 16 which is substantial.

A. Adjacent channel considerations in the pollution viewpoint

Another consideration for secondary operation is interference received from TV transmitters on adjacent channels. For example, if a secondary device operates on channel 39, it would not want to be too close to TV transmitters in channels 38 and 40. The amount of interference that can be tolerated on adjacent channels depends on the characteristics of the secondary user's receiver (receiver filter etc). As an illustration, suppose that the channel-select filter can attenuate adjacent channels by 20dB or 40dB. The impact can be seen in Table I. For the lower UHF channels, with a tolerable adjacent channel pollution level of 45dB above noise and a tolerable co-channel pollution of 5dB, the average number of white space channels per location drops from 16.2 (assuming perfect channel-select filters) to 15.6.

B. White space calculations using actual population density

Another way to evaluate available white space is to consider the number of people that are potential users of white space devices. This number is more valuable for businesses to determine revenue potential and also for regulators interested

[1] We neglected the more stringent emission requirements for the 602-620MHz bands (Section 15.709 [1]). We also neglected the locations of cable headends, fixed broadcast auxiliary service (BAS) links, and PLMRS/CMRS devices (Section 15.712 [1])

[2] Estimated annual costs for each wireless ISP tower is $22000. For a 50% profit margin per tower and a monthly subscription rate of $25 per month we need $\sim$ 147 installations $((\$44000/12)/\$25)$. Average rural population density in the US is around 6.6 people per km^2 [9]. Assuming 4 people per family and a 50% penetration, we need to service a area of $178km^2$ i.e. service radius 7.5 km. At this distance, the secondary received power is -94dBm. For a minimum of 1MBps service the operational SNR for 802.11g is 7dB, hence we can tolerate -101dBm of interference which is 5dB above the noise floor.

in the public good. To calculate available white space in terms of population, we used the US Census data of 2000 which lists the population density per zip code [10]. Furthermore, the region occupied by a zip code is specified as a polygon [11]. Using this data we can determine the average number of white space channels available per person. For low UHF channels and a tolerable pollution level of 5dB, the average number of white space channels is ~ 9 which is roughly half of the number assuming a uniform population density across the country. This is because TV transmitters are constructed to serve areas of high population density and hence excluding areas around TV towers significantly reduces the population that can be served by white space devices. This observation, however, is not true for the low VHF channels. For the low VHF channels the amount of white space increases as we move from a uniform population density assumption to the actual population density. This is because the high power DTV stations (after the February transition) in channels 2, 5 and 6 are located in areas of low population density – the actual population density around these regions is almost half the uniform population density.

The impact for the UHF channels is even more severe when interference from adjacent channels is considered – for a main channel tolerable pollution level of 5dB and an adjacent channel pollution level of 45dB, the average number of white space channels per person is 6.5. This is a third (15.6 to 6.5) of the numbers obtained assuming a uniform population density.

III. White Space using the protection viewpoint

A. *Defining white space for the protection viewpoint*

In Figure 1, the noise limited (r_{nl}) coverage area is the broadcast coverage area with no interference from other transmissions. Within this area, the nominal TV SNR is greater than the target SINR (Δ), *i.e.* for all distances $r \leq r_{nl}$:

$$P_t - l_n(r) - N_0 \geq \Delta$$

where P_t is the transmit power in dBm, $l_n(r)$ is the nominal path loss at a distance r from the transmitter in dB, N_0 is the noise level in dBm and Δ is the target SINR in dB. At every distance from the TV transmitter there exists a fading margin ($\psi(r)$) which 'protects' the TV receiver from bad fading events.

$$\psi(r) = P_t - l_n(r) - N_0 - \Delta$$

Once a secondary user starts transmission it raises the noise floor (In this paper, we ignore the effect of multiple simultaneous secondary users. Multiple low power secondary users act as a single high power user [12], [13]). Regulators/system designers must decide the protection margin ψ_t that they are willing to erode to enable secondary operation. Undoubtedly, the TV receivers close to r_{nl} can loose service due to the additional interference. However, corresponding to the selection of ψ_t is a protected radius r_p where the original fading margin was equal to ψ_t. With cognitive radio transmission, this margin is reduced to zero but all TV receivers within r_p are still nominally protected – they can still receive the TV signal by positioning their antennas appropriately.

$$r_p = l_n^{-1}(P_t - \psi_t - N_0 - \Delta)$$

The protected radius together with the transmit power/height of the secondary transmitter can then be used to calculate the no-talk radius for cognitive radio operation (r_n). The cognitive radio can only transmit if it is outside the no-talk area of the primary transmitter in question. To calculate this we must first calculate the value of $r_n - r_p$ *i.e.* the distance beyond the protected radius where the secondary can safely transmit. In other words we need to determine the distance r_n such that a transmission from r_n results in a SINR of Δ at r_p. We started by calculating the allowable interference level at r_p (I_{r_p}) as:

$$I_{r_p} = 10 \log_{10}(10^{\frac{P_t - l_n(r_p) - \Delta}{10}} - 10^{\frac{N_0}{10}}) \qquad (1)$$

Next we calculate $r_n - r_p$ as:

$$r_n - r_p = l_o^{-1}(P_s - I_{r_p})$$

where P_s is the transmit power (in dBm) of the secondary user, I_{r_p} is the maximum tolerated interference power (in dBm) from (1) and $l_o(r)$ is the optimistic path loss from the secondary to the primary.

For each 6MHz TV channel, the 'white space' can be defined as all area that is not within any relevant tower's no-talk radius.

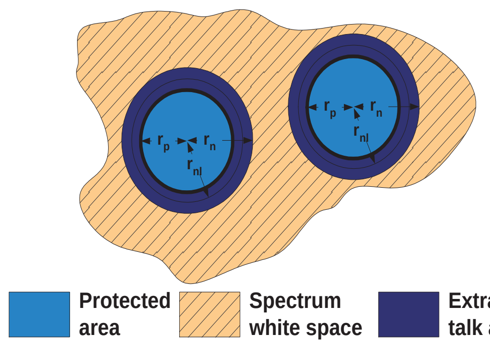


Fig. 1. Definition of spectrum 'white space' from a primary protection perspective. The 'extra no-talk area' is the extra space where there are no protected primary users and yet secondary users are not permitted to transmit.

B. *Calculating the available white space from the protection viewpoint*

As in Section II, we used the FCC's transmitter databases and the ITU propagation curves to calculate the white space [4], [14]. Appendix III uses the example of the KCNS transmitter on channel 39 to illustrate the procedure used to calculate the values for r_p. We assumed that the nominal path loss model ($l_n(r)$) is the ITU's F(50, 90) path loss *i.e.* the protection radius (r_p) is the distance where the SNR is Δ for 50% of the locations, 90% of the time. To determine the maximum possible white space available we assumed a hypothetical 'zero Watt' transmitter which only causes

Tolerable pollution level above noise	*By Area*				*By Population*			
	LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69	LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69
5dB Main channel only	1.69	1.74	16.3	15.9	1.83	1	8.71	15.5
5dB Main channel and 25dB adjacent channels	1.6	1.29	13.7	15.4	1.63	0.496	4.93	14
5dB Main channel and 45dB adjacent channels	1.67	1.62	15.6	15.8	1.73	0.714	6.52	14.8
10dB Main channel only	1.94	2.59	20	16.4	2.08	1.68	11.8	16
10dB Main channel and 30dB adjacent channels	1.8	1.69	15.5	15.7	1.83	0.67	5.87	14.2
10dB Main channel and 50dB adjacent channels	1.9	2.3	18.6	16.3	1.96	1.09	8.24	15.1
15dB Main channel only	2.16	3.43	23.3	16.8	2.29	2.42	15	16.3
15dB Main channel 35dB adjacent channels	2.04	2.5	19.1	16.3	2.07	1.07	8.18	14.7
15dB Main channel and 55dB adjacent channels	2.13	3.17	22.2	16.7	2.17	1.65	11.2	15.6

TABLE I

COMPARISON OF AVAILABLE WHITE SPACE BASED ON TOLERABLE POLLUTION LEVELS FOR VARIOUS FREQUENCY BANDS (LOW VHF (LVHF - CHANNELS 2, 5 AND 6), HIGH VHF (HVHF - CHANNELS 7 TO 13), LOW UHF (LUHF - CHANNELS 14 TO 51) AND HIGH VHF (HVHF - CHANNELS 52 TO 69)). WHITE SPACE IS MEASURED AS THE AVERAGE NUMBER OF WHITE SPACE CHANNELS AVAILABLE PER LOCATION/PERSON IN THE CONTINENTAL UNITED STATES.

harmful interference if it is right next to a TV receiver *i.e.* for this hypothetical transmitter $r_n - r_p$ is defined to be zero. Table II shows the white space available for such a transmitter for various values of the eroded fading margin ϕ_t. About $\sim$ 19 channels per person are available on average for white space use in the lower UHF bands even with a erosion of $\sim$ 1dB of protection margin. (A 1dB erosion in protection margin translates into an effective 20% reduction in the transmit power from the link-margin perspective of a primary user.)

Next, Table III calculates the white space for a 4W[3] with a height above average terrain (HAAT) of 30m: both are the maximum allowed by the FCC for fixed white space devices [1]. For this secondary user, the value of $r_n - r_p$ is calculated using the procedure outlined in Appendix IV. We assume that the optimistic path loss model ($l_o(r)$) is the ITU's F(50, 10) path loss *i.e.* the secondary has to be at a distance $(r_n - r_p)$ beyond the protection radius to ensure that the signal would be higher than I_{r_p} for 50% of the locations only 10% of the time. This ensures that secondary users beyond r_n only interfere for at most 10% of the time. With this scale of a secondary user, the average number of white space channels per person is reduced by 4 to 14.9.

C. Impact of Secondary power on white space availability

As seen earlier, the available white space is determined by the power control rule selected by the secondary – we lose 4 white space channels as we go from a hypothetical 'zero watt' transmitter to a 4W, 30m transmitter. How does white space availability scale with the transmit power of the secondary? Figure 2 helps us answer this question. If we assume that the secondaries are 1MW transmitters (90dBm), then there is not much white space available to accommodate them. On the other hand, portable (100mW) and fixed white space devices (4W) have plenty of white space available. All calculations assumed a transmitter HAAT of 30m. The white space availability is significantly less at higher transmit powers if we assume that transmitter height scales with the transmit power (a 1MW transmitter would sit on a higher tower).

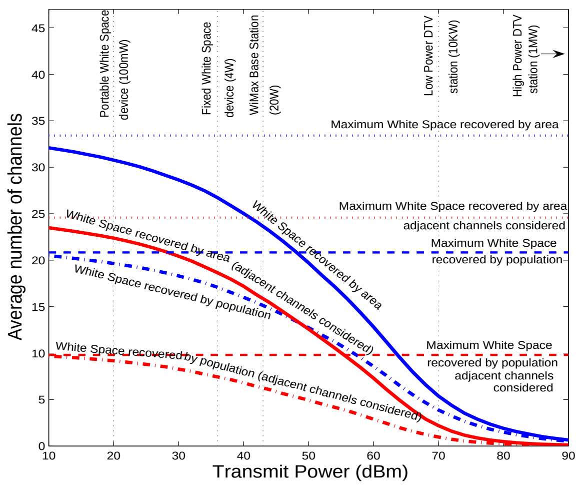


Fig. 2. Impact of the power control rule for the secondary user. If we assume secondary users are 1MW transmitters (90dBm), then there is not much white space available to accommodate them. On the other hand small scale primaries (4W) have plenty of white space available. (all calculations assumed a transmitter HAAT of 30m). The maximum number of channels is 47 (channels 2 to 51 excluding 3,4 and 37).

D. Available White Space considering adjacent channels

The FCC is concerned that the operation of cognitive radios in a given band will significantly impact TV receivers in adjacent bands. Hence they require that a cognitive radio should only transmit in a given band if TV receivers in the

[3] [1] actually specify a 1W maximum transmit power with a 6dBi antenna which translates (in a worse case scenario) into 4W transmit power towards the primary.

Margin	*Detection Scheme*	*By Area*				*By Population*			
		LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69	LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69
0.1dB	Geolocation	2.5	4.67	26.7	16.9	2.55	3.64	18.3	16.4
	Geolocation with adjacent channels	2.24	2.58	16.6	15.1	2.22	1.16	6.39	13.6
1dB	Geolocation	2.52	4.75	27	17	2.56	3.71	18.7	16.4
	Geolocation with adjacent channels	2.26	2.69	17.1	15.3	2.24	1.22	6.74	13.7
2.73dB	Geolocation	2.55	4.9	27.8	17.1	2.58	3.84	19.4	16.5
	Geolocation with adjacent channels	2.31	2.89	18.2	15.5	2.28	1.35	7.43	13.9

TABLE II

MAXIMUM POSSIBLE WHITE SPACE USING A HYPOTHETICAL "ZERO WATT" TRANSMITTER FOR VARIOUS VALUES OF THE PROTECTION MARGIN THAT IS ERODED. THE 1DB (2.73DB) PROTECTION MARGIN IS THE AVERAGE (MAXIMUM) PROTECTION MARGIN ERODED ACROSS ALL TOWERS USING THE FCC'S DEFINITION OF HARMFUL INTERFERENCE. FOR ADJACENT CHANNELS, IT IS ASSUMED THAT THE ACCEPTABLE INTERFERENCE LEVEL IS 27DB MORE THAN WHAT IS ACCEPTABLE CO-CHANNEL.

Environment	*Detection Scheme*	*By Area*				*By Population*			
		LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69	LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69
No Multipath	Geolocation	1.52	2.85	22.4	16.2	1.71	2.11	14.9	15.8
	Geolocation with adjacent channels	1.39	1.55	14.1	14.5	1.46	0.64	5.37	13.2
Multipath	Geolocation	1.56	2.73	21.9	16.1	1.75	2	14.4	15.8
	Geolocation with adjacent channels	1.42	1.47	13.8	14.4	1.48	0.609	5.2	13.2

TABLE III

COMPARISON OF THE AVERAGE NUMBER OF WHITE SPACE CHANNELS AVAILABLE PER USER IN VARIOUS ENVIRONMENTS AND FREQUENCY BANDS FOR A 4W SECONDARY TRANSMITTER WITH A HAAT OF 30M AND A 1DB EROSION OF THE FADING MARGIN OF PRIMARY RECEIVERS.

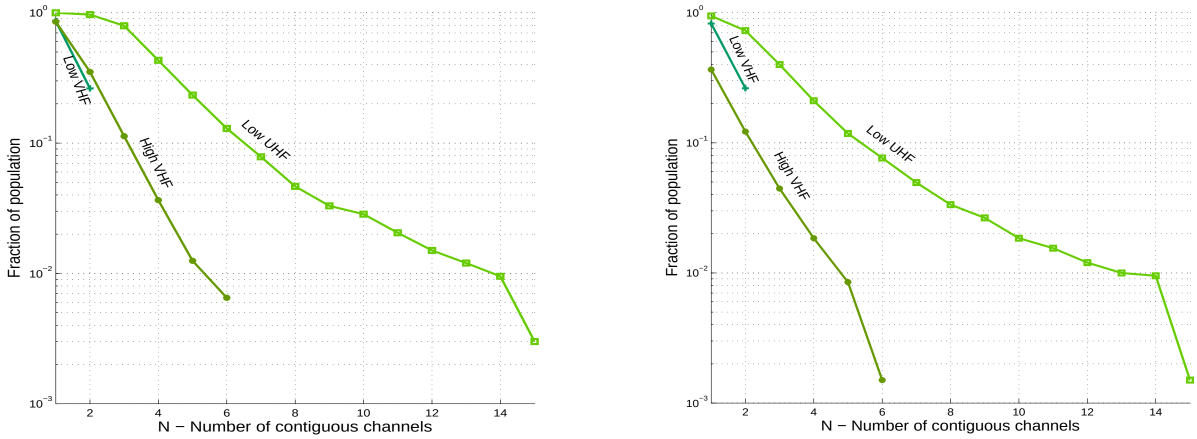
band and in the adjacent band can be protected. Consider a TV receiver on channel 39 attempting to receive the KCNS transmission from Sutro tower. Assume channel 40 is empty and being used by the secondary user. The first source of concern is that the secondary signal's 'skirts' falling in channel 39 itself will harm the TV receiver. This can be dealt with by engineering the secondary transmission 'skirts' to a acceptable level.

The second source of concern is that the secondary's transmission in channel 40 will impact the TV receiver. Each TV receiver has a requirement for the interference level in adjacent bands which depends on the characteristics (receive filter, etc.) of the TV receiver. This cannot be altered and hence must be budgeted for. To determine the impact of this rule on available white space, we first determined the no-talk radius for adjacent bands (this procedure is detailed in Appendix V). We considered a location/person to be available for white space operation in a given TV channel only if it out of the no-talk area of all transmitters in its channel as well as the adjusted no-talk area of all transmitters in adjacent channels. This rule significantly reduces the available white space as can be seen in Tables II,III and IV. For example, for the low UHF bands the maximum number of average whitespace channels per person drops from around 19 to around 7 assuming a 1dB tolerable erosion, dropping further to only about 5 for a 4W transmitter at 30m HAAT.

A way to avoid the problem of adjacent channels is to position the secondary transmission away from the channel boundary. This can be achieved by finding contiguous bands of channels and then positioning the transmission in the center of this contiguous frequency band. Figure 3 shows the fraction of the population that can harness contiguous channels of a given length. It is interesting to note that the fraction of people that can find two free contiguous channels (without concerns for adjacent channels) is higher than the fraction that finds a single free channel while considering adjacent channels (See Figure 3 – the fraction of the population that can receive 2 contiguous channels with only co-channel considerations is 0.97 while only 0.94 fraction of the population can receive one channel with adjacent channel considerations.). This suggests that it may be worthwhile to search for 2 contiguous channels and then create a waveform in the center of the 12MHz band.

E. Available White Space considering multipath fading

The ITU-R propagation model does not incorporate multipath fading. To study the impact of multipath we created new path loss tables for different multipath scenarios (See Appendix VIII). To remain conservative we neglected multipath for calculating r_p. For calculating $r_n - r_p$ we assumed Rayleigh fading. The impact of multipath is to increase $r_n - r_p$ which reduces the available white space.



(a) **(b)**

Fig. 3. Investigation of the number of contiguous channels available in different frequency bands (Lower VHF, Higher VHF, and Lower UHF) using different protection rules (a) Co-channel interference only (b) Considering adjacent channels as well.

IV. A PRINCIPLED WAY TO CHOOSE THE PROTECTION MARGIN

As we add the potential for using white-space devices for some people, we potentially sacrifice the utility for other people who have been watching over-the-air broadcast television. This tradeoff can be changed by modifying the protection margin offered to TV users (ψ), but it is essentially a political tradeoff since it involves weighing and balancing the interests across different parties. As we adjust the margin we change the protection radius r_p and the no-talk radius r_n. Figure 4(a) shows the average number of channels available for white space usage per user as the erosion margin is varied. With zero eroded margin, there is no white space. However, a zero margin also does not mean that all users can view all possible broadcast television channels – the average number of reliable broadcast channels per person is limited to 23 channels. As we increase the eroded margin, the number of broadcast channels diminishes slightly but there is a huge gain in the number of white space channels per user.

This tradeoff is better studied by examining the cumulative and marginal (instantaneous) gain-loss tradeoff curves. Let P_m^W be the average number of person-channels[4] available for white space usage at an eroded margin of m dB. Similarly let P_m^B be the number of person-channels available for broadcast television usage at a margin of m dB. Then the red and blue curves in Figure 4(a) are the values of P_m^W and P_m^B respectively for different values of m. For a given margin, $\frac{P_m^W - P_0^W}{P_m^B - P_0^B}$ represents the cumulative gain-loss tradeoff for each margin setting. The values assuming a uniform population distribution and the actual population density are shown in Figure 4(b) as the solid and dotted black curves respectively. Further, $\frac{dP_m^W/dm}{dP_m^B/dm}$ is the marginal (instantaneous) gain-loss tradeoff at an eroded margin of m dB. The values assuming a uniform population distribution and the actual population density are shown in Figure 4(b) as the solid and dotted red curves respectively. These instantaneous tradeoff curves tell us the number of person-channels gained for potential white space use versus those lost for broadcast television viewing by varying the erosion margin settings around m.

At small erosion margin ($\sim .1dB$) the tradeoff is skewed towards white space usage – we gain a lot of white space person-channels for a few broadcast person-channels lost. For instance, if we increase the erosion margin infinitesimally beyond 0.1dB, we gain 20 white space person-channels for every broadcast person-channel lost. This unequivocally provides not just the motivation for allowing white space usage, but also provides regulators with a way to choose the tradeoff point by using estimates of social utility and market penetration.

We can repeat this exercise by considering the margin of protection as a fraction of the original noise-limited broadcast area that we are giving up. Let r_{nl} represent the noise limited radius of operation and r_p be the protected radius of operation, then we can measure the area margin eroded as the fraction $1 - (\frac{r_p}{r_{nl}})^2$. With this definition of the margin we can retrace the trade-off curves as shown in Figures 4(c-d).[5]

Due to variations in tower power and height, a single erosion margin translates into a range of protected radius values. The range spread and medians (across towers) of the area lost can be seen in Figure 5. The histogram of r_p values shows a bimodal distribution whose median increases with the margin. The spread in the area lost also increases with the margin as can be seen from the divergence of the 90^{th} and 10^{th} percentiles.

[4]If the total population is P and N_p is the number of channels that can be received by person p, then $\frac{1}{P}\sum_{p=1}^{P} N_p$ is the average number of person-channels available per person.

[5]The marginal (instantaneous) person-channels gained versus those lost shown in Figure 4(d) is a polynomial fitted version of the actual. This fitting procedure is robust for $r_p \geq 0.02$.

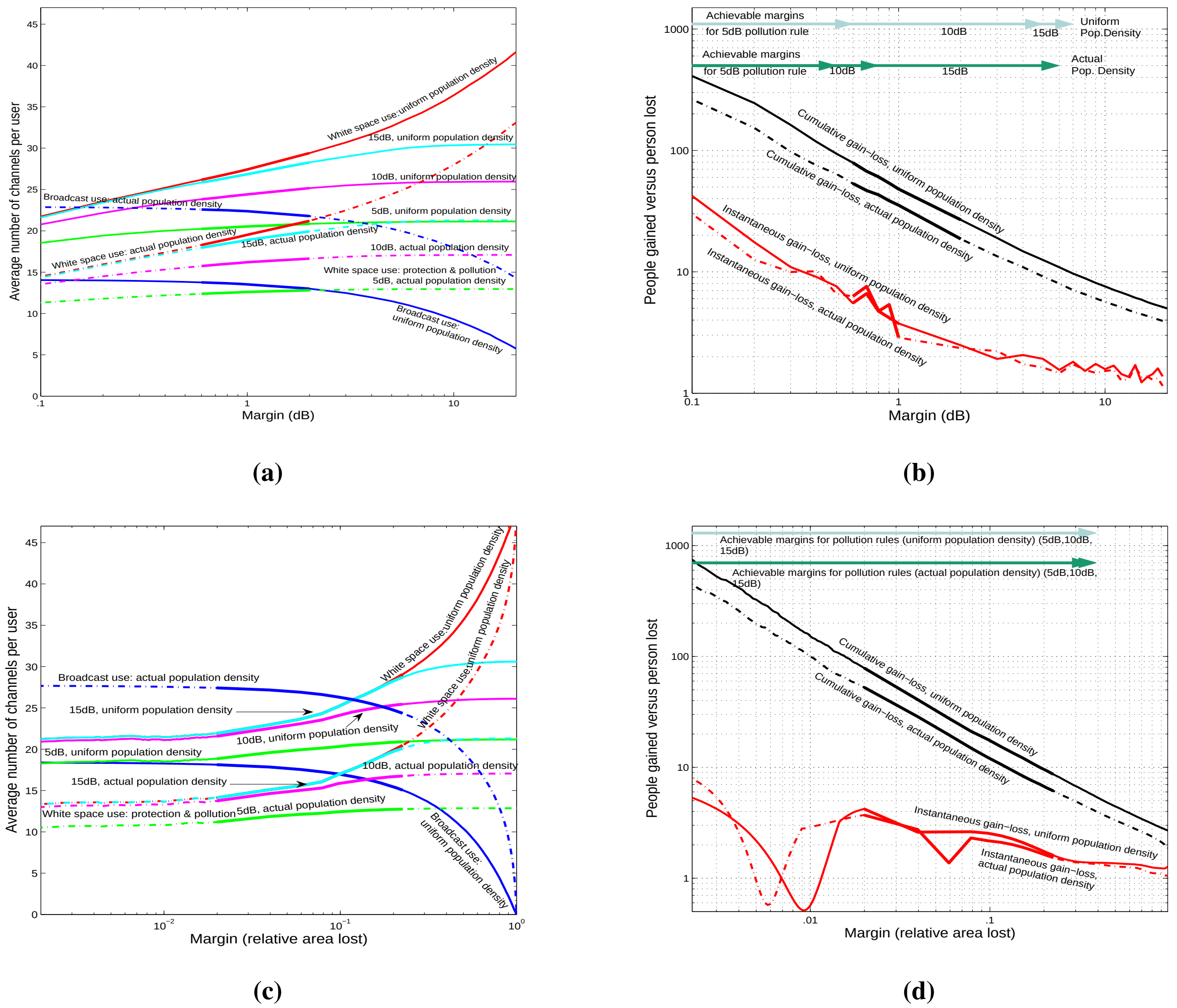


Fig. 4. (a) Average number of channels available for white space/broadcast usage per person as the protection margin (in dB) is varied. (b) The cumulative/marginal (instantaneous) gain-loss tradeoff as the protection margin is changed. (c) Average number of channels available for white space/broadcast usage per person as the margin in terms of relative area lost is varied (d) The cumulative/marginal (instantaneous) gain-loss tradeoff for white space usage as the relative area lost is varied. The light blue and dark blue lines at the top of (b) and (d) represents the range of margins achievable under a combined pollution/protection viewpoint.

Finally, the protection perspective and the pollution perspective can be combined. Depending on the chosen values for the erodable margin and the tolerable pollution level, the perspective that dominates white space availability can change. In Figure 4(a) the average number of channels per person for white space usage increases as the erosion margin is increased, but saturates after a certain point where the pollution viewpoint starts to dominate. Physically, this means that the additional white-space opened up is not considered to be worth using. The same goes when we consider the margin in terms of relative broadcast area lost (Figure 4(c)). This limit on the achievable tradeoff can also be viewed in Figures 4(b) and (d) where the tradeoff points achievable by the pollution viewpoint are marked out. Increasing the erosion/area margin beyond these points is not advisable since the additional whitespace recovered is too polluted to be worthwhile.

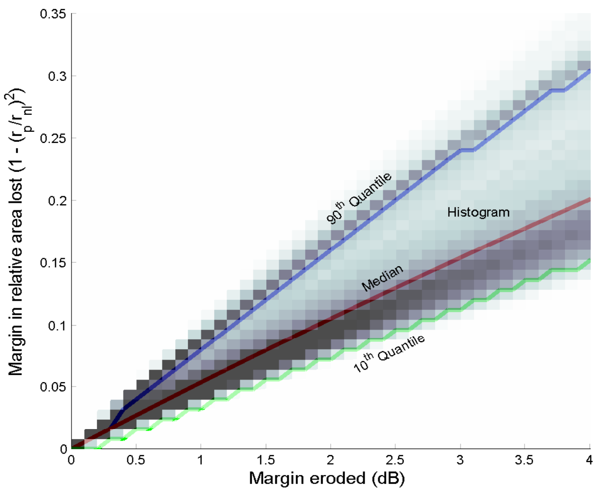


Fig. 5. Variation in the broadcast area lost as the protection margin is varied.

V. Impact of FCC rules on white space availability

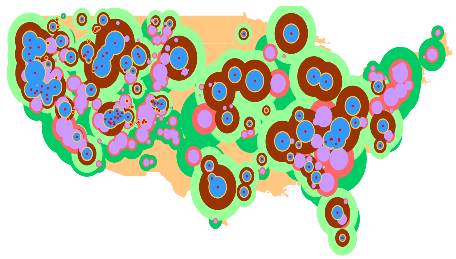
The FCC has developed a slightly different approach to determine the no-talk radius around a tower. They assume that the protected radius is the same as the Grade B contour and the interference level at the protected radius should be 23dB lower than the signal level. Furthermore, the FCC specifies the value of $r_n - r_p$ as a distance that should be used irrespective of the channel frequency (See Section 15.712 in [1]). The details of using this methodology to calculate available white space is given in Appendix VI. Table IV shows the white space available using the FCC style definition of the no talk radius. These calculations were performed for a 4W, 30m (HAAT) secondary transmitter. Comparing Table IV with Table III we can see that the FCC style rules predict very similar white space values for the lower UHF bands but not for the VHF bands. For the values of r_n and r_p calculated using the FCC method, the average margin eroded in the low VHF bands is $\sim$2.73dB while for the high VHF bands it is $\sim$2.3dB. Since white space increases with increasing margin (See Table II) the FCC method shows more white space in these bands as compared to the margin method. For the low UHF bands the margins used in both methods are comparable and hence the white space numbers are similar.

The FCC has also proposed a detection sensitivity of -114dBm for ATSC signals. Such a detection rule has to be conservative in order to protect the primary user in worst case fading scenarios. However, in the average case such a rule tends to loose a significant portion of the white space [15]. To evaluate the amount of white space recovered by the -114dBm rule we determined the distance at which the signal dropped to the -114dBm level for 50% of the locations, 50% of the time (F(50, 50) rule). Ideally a cognitive radio user can be expected to adjust his/her antenna so as to catch the worst fades (remember: if the signal decays below -114dBm the user can *use* the channel). But such manoeuvering cannot be performed for all transmitters and channels simultaneously. Hence considering the F(50, 50) propagation model is reasonable. From Table IV it is clear that the -114dBm rule reduces the available white space by a factor of three. This effect is even more dramatic when white space by population is considered. In this case the available white space is reduced by a factor of five.

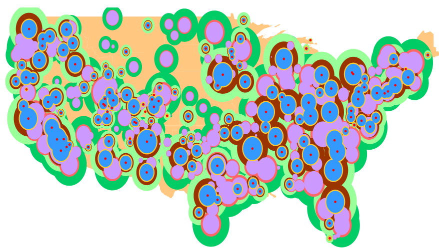
The impact of the -114dBm rule is visible in Figures 6(a) and (b) which visually depict the available white spaces for various rules for channel 5 and channel 40. For both channels, the -114dBm style rules with adjacent channel considerations knocks out a considerable amount of area.

The inability of the -114dBm rule in recovering any area in the West and East Coasts of the continental USA is shown in Figure 7 by plotting the number of channels that are available on a hypothetical trip from Berkeley, CA to Washington, D.C. along Interstate 80.

Figure 8 shows the probability of getting at least a given number of channels using various detection and inference rules. This plot enables one to determine the quality of service that can be offered to white space users. For example, if we need 5 channels available, then we can only ensure this for 26% of the population using the -114dBm rule.



(a)



(b)

(c)

Fig. 6. (a) Visual representation of the available white space for channel 5 using various usage guidelines. (b) Visual representation of the available white space for channel 40 using various usage guidelines. (c) Legend for the visual representation.

The tradeoff analysis using person-channels gained versus those lost can be extended to fixed threshold (eg. -114dBm) rules. As seen earlier, setting the threshold rule in a conservative manner results in a significant loss of area. For every setting of the threshold rule, we gain white space for people outside the region where the signal falls below the threshold for 50% of the locations, 50% of the time (the F(50, 50) point). A secondary at such a location will cause the primary receiver to see degradation in its SINR. Areas around the primary where the SINR is still above the target SINR will be able to receive a DTV signal but regions

Environment	*Detection Scheme*	*By Area*				*By Population*			
		LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69	LVHF 2,5,6	HVHF 7-13	LUHF 14-51	HUHF 52-69
No Multipath	Geolocation ([1] Section 15.712)	2.4	4.13	23.8	16.4	2.5	3.24	16.1	15.9
	Geolocation with adjacent channels ([1] Section 15.712)	2.17	2.35	14.9	14.7	2.21	1.1	5.82	13.3
	Geolocation	1.87	3.35	22.4	16.1	2.08	2.54	14.8	15.7
	Geolocation with adjacent channels	1.71	1.89	14.1	14.4	1.81	0.82	5.36	13.1
	-114dBm rule	0.985	0.409	7.7	13.8	1.13	0.167	2.57	13.6
	-114dBm rule with adjacent channels	0.631	0.0505	2.63	9.83	0.639	0.004	0.284	8.87
Multipath	Geolocation	1.86	3.2	21.9	16	2.07	2.42	14.4	15.6
	Geolocation with adjacent channels	1.7	1.8	13.8	14.3	1.8	0.775	5.18	13
	-114dBm rule	1.09	0.524	8.85	14.3	1.25	0.225	3.29	14
	-114dBm rule with adjacent channels	0.73	0.0705	3.19	10.4	0.74	0.008	0.38	9.46

TABLE IV

COMPARISON OF THE AVERAGE NUMBER OF WHITE SPACE CHANNELS AVAILABLE PER USER IN VARIOUS ENVIRONMENTS, FREQUENCY BANDS AND THE METHODOLOGY SPECIFIED IN THE FCC REPORT FOR A 4W 30M TALL TRANSMITTER. LVHF DENOTES THE LOWER VHF TV BANDS (CHANNELS 2-6), HVHF DENOTES THE HIGHER VHF TV BANDS (CHANNELS 7-13), LUHF DENOTES THE LOWER UHF TV BANDS (CHANNELS 14-51) AND HVHF DENOTES THE HIGHER VHF TV BANDS (CHANNELS 52-69) USING THE FCC'S DEFINITION OF WHITE SPACE. THE TOP TWO ROWS USE THE DEFINITION OF $r_n - r_p$ FROM SECTION 15.712 IN [1] WHILE THE NEXT TWO ROWS CALCULATE $r_n - rp$ AS PER APPENDIX VI. THE -114DBM SENSING RULE FOR CO-CHANNELS TRANSLATES INTO A -110DBM RULE FOR ADJACENT CHANNELS.

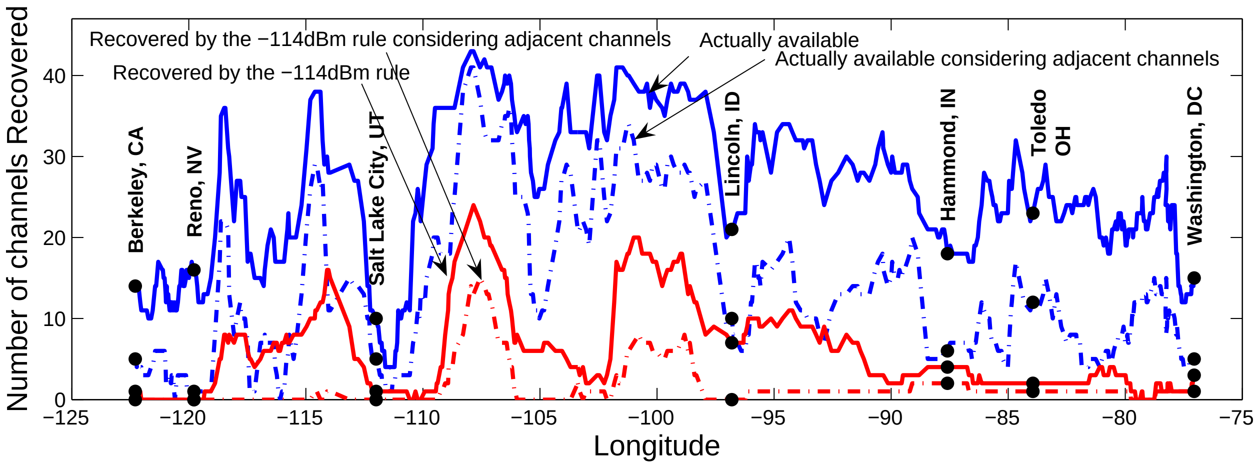


Fig. 7. Number of channels available on a trip from Berkeley, CA to Washington, D.C. The availability of white spaces is greatly reduced at the two coasts where most of the population lives.

close to the noise limited radius will not. By varying the detection threshold we can examine the tradeoff between person-channels gained for white space versus those lost for broadcast. Figure 9(a) shows the average number of channels available for broadcast television usage as the threshold is varied.

The reasonable interpretation of the -114dBm rule is that it represents the fear of the FCC – that the -114dBm rule is really a -94dBm rule (for instance) with a 10dB sensing margin for deep fading events. In other words, for mobile secondary users, the FCC cannot trust the placement and hence assumes that there will exist a secondary radio which nominally receives a signal of -94dBm but ends up seeing a 10dB fading event when trying to sense it. This is rational because the assumption is that there are many such mobile devices deployed and any one of them could transmit and harm TV receivers. On the other hand, any particular secondary user cannot assume deep fading events and hence under the -114dBm rule is unable to access vast amounts of whitespace.

The average number of broadcast channels resulting from this viewpoint can be seen in Figure 9(a) for sensing fading margins of 0dB, 10dB and 20dB. For a 0dB fading margin, the average number of broadcast users remains relatively flat for a range of detection thresholds (-120dBm to -100dBm). This is because the secondary interference from these distant

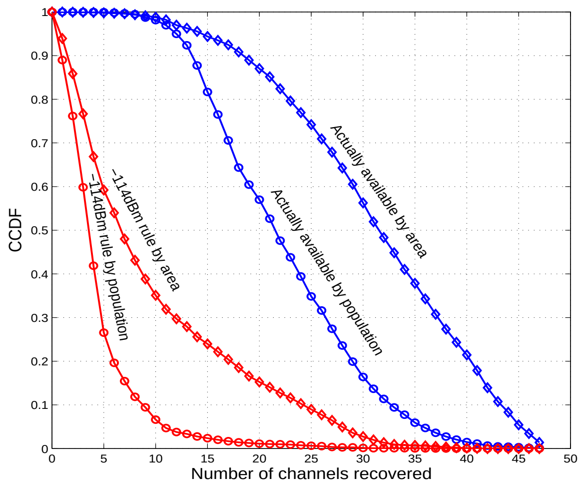


Fig. 8. Probability of getting at least a given number of channels using FCC style geolocation and fixed threshold (-114dBm) rules for uniform and actual population densities. For this plot adjacent channels are ignored.

locations is still much lower than the DTV noise floor. Since the -114dBm rule corresponds to large distances, the tradeoff point is much higher (500-1000 person-channels gained to 1 lost - see Figure 9(b)) for a fading margin of 0dB. For sensing fading margins of 10dB and 20dB, the tradeoff curves plummet considerably before they rise again. This is because low population densities in rural areas result in few white space users being added. However the broadcasters' fear of deep fading events means that they expect a large number of suburban broadcast users to lose service.

Figures 10(a) and (b) shows how 'one size fits all' style rules like the -114dBm style rule does not yield consistent area loss across towers. Figures 10(a) and (b) show the histogram and median/percentiles of the relative area lost as we change the threshold rule. The wide spread in the 10^{th} and 90^{th} rules show the problem with fixed threshold rules – they effect different towers in a vastly different manner.

VI. Conclusions and Recommendations

The objective of this paper is to quantify the amount of white space that is actually available for secondary use. The FCC's high and low power databases gave details of all licensed TV (digital and analog) stations and their latitude, longitude, effective radiated power (ERP) and transmitter heights above sea level. The ITU-R recommendations on propagation (ITU-R P-1536-3) and the US Census data of 2000 which lists the population density per zip code were used to get the white space availability per location/person. We compared the amount of white space resulting from the 'pollution' (stay away from the primary to avoid its interference) and 'protection' (stay away from the primary to avoid interference to it) viewpoints. Engineering suggests that the erosion margin should be set such that white space resulting from the two viewpoints is matched.

Quantifying white space as the average number of white space channels per location, we have shown that the main channels of relevance are the lower UHF channels (channels 14-51) in which we could get $\sim$ 15 channels per person. However this number drops significantly (to $\sim$ 5) when adjacent channels also have to be protected. Furthermore, the amount of white space is a function of the scale of the secondary. While we could get almost twenty-two channels per person (even without the upper UHF bands) for a hypothetical 'zero Watts' secondary, the number of white space channels for a 1MW secondary is almost zero.

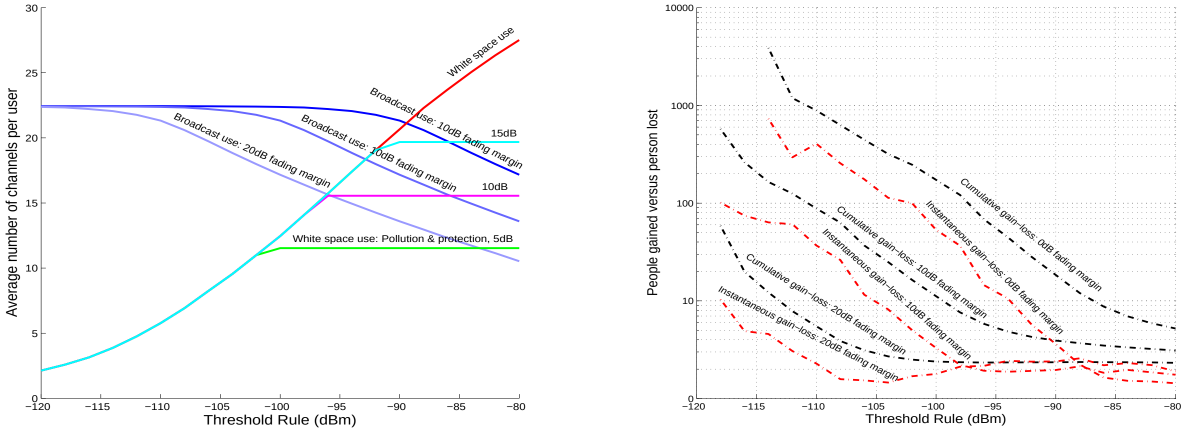
Fixed threshold rules (for example the -114dBm rule proposed by the FCC for ATSC signals) are very conservative and result in almost no channels per person (especially when adjacent channels are considered). Under these observations, while geo-location may seem to be the way to go, sensing can be used to complement geo-location to determine channels which transmitters are not actually transmitting.

Lastly, we set forth a principled way to set the erosion margin. This principled approach considers the person-channels gained for white space use versus those lost from broadcast use. Based on the relative weighing of white space person-channels and broadcast person-channels, regulators can set forth the appropriate protection margin to be sacrificed for secondary use. Using the FCC's report, we have "reverse engineered" the tolerable erosion margin as around 1dB. This will allow the community to propose new sensing strategies to follow the spirit of this margin while not being as conservative as the -114dBm rule.

Appendix I
Introduction to the Example

We shall use the running example of the KCNS transmitter in San Francisco to illustrate the calculations. The KCNS digital transmitter occupies channel 39 and is housed on Sutro tower at a height above sea level of 459m and a transmit power of 1000kW. To determine the propagation characteristics of this tower we need to calculate the effective height of the tower — called HAAT for height above average terrain. To calculate the HAAT, the elevation of 50 random points are taken at distances between 3km and 16km around the tower at 8 (or more) evenly spaced radials from the transmitter site (These calculations use the Globe Terrain database available from the National Geophysical Data Center [16]). The elevation points along each radial are averaged, then the radial averages are averaged to calculate the average height of the terrain. Subtracting this value from the height of the tower above sea level gives the HAAT of the tower. For the KCNS transmitter this value turns out to be 430m. This is mainly because San Francisco is at an average elevation of $\sim$ 16meters above sea level.

Before we continue, we should also determine the operational SINR of the KCNS transmitter. According to [1] the



(a) **(b)**

Fig. 9. (a) Average number of channels available for white space/broadcast usage per person as the detection sensitivity (in dBm) and the assumed sensing fading margin (in dB) is changed. (b) The cumulative/incremental gain-loss tradeoff as the detection sensitivity (in dBm) and sensing fading margin (in dB) is varied. These plots are for the actual population density.

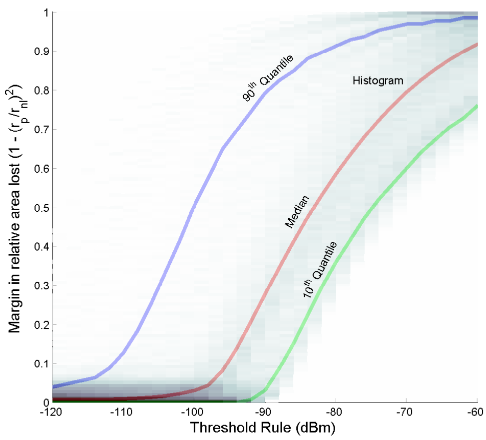


Fig. 10. Relative area lost for a range of sensing threshold rules

Desired Signal to Interference ratio (SIR) ratio at the Grade-B contour is 23dB. With noise power over a 6MHz band at -106dBm we can calculate the required SINR at the Grade-B contour. These values are shown in Table V. For channels 39, the required SINR is 15.46dB.

Appendix II
Calculating the pollution radius

The maximum interference that a secondary can tolerate (γ) is set as a value above the noise level. To calculate the pollution radius, we need to find the distance where the signal power drops to $\gamma + N_0$dBm for 50% of the locations, 50% of the time (the F(50, 50) curve from the ITU specifications). For the KCNS tower and a $\gamma = 5$dB, the pollution radius is 220km.

Appendix III
Calculating the protection radius r_p

The FCC calculations give rise to a range of erosion margin values (See Figure 4(a)) the median of which is 1dB. We shall use this as the target erosion margin ($\psi = 1$dB). Hence r_p is the distance at which the signal strength is greater than $N_0 + \Delta + \psi = -89.75dBm$) for 50% of the locations, 90% of the time. For channel 39, this translates to a electric field of 41.16dBu ($E_{r_p}(dBu)$). $E_{r_p}(dBu)$ was used to calculate the r_p using the following procedure.

- The Effective Radiated Power is converted to Effective Isotropic radiated Power ($EIRP(dBm)$ = $ERP(dBm)$ + $2.15dB$). For the KCNS transmitter the EIRP (dBm) is 92.5dBm.
- The Electric field at a distance of 1m from the transmitter is calculated as $E_{1m}(dBu) = 104.8 + EIRP(dBm)$, which is 197.3dBu for the KCNS transmitter.
- The required path loss is calculated as $RPL(dB) =$

Parameter	*Description*	*Value*
Δ	Minimal Operational SINR	LVHF: 19.69dB ULHF: 19.13dB ULHF: 17.33dB - 14.67dB
$E_{r_p}(dBu)$	Electric field at the protected radius (r_p)	
$E_{r_{nl}}(dBu)$	Electric field at the noise limited radius (r_{nl})	
$E_{r_b}(dBu)$	Electric field at the Grade B radius (r_b)	See Table VI
ψ	Margin eroded	1dB
$I_{r_p}(dBm)$	Interference at the protected radius	
$N_0(dBm)$	Noise in a 6MHz band	-106.22dBm

TABLE V
PARAMETERS TO BE USED IN WHITE SPACE CALCULATIONS.

Channels	*Required Field Strength (dBu)*
$2-6$	23
$7-13$	36
$14-69$	41

TABLE VI
ATSC (DIGITAL) FIELD STRENGTH DBU REQUIRED AT THE GRADE B CONTOUR.

Channels	*Formula to convert from dBu to dBm*
$2-6$	P(dBm) = E(dBu) -111.8
$7-13$	P(dBm) = E(dBu) -120.8
$14-69$	P(dBm) = E(dBu) $-130.8+20\log_{10}(\frac{615}{\frac{f_h+f_l}{2}})$

TABLE VII
DBM TO DBU CONVERSION VALUES FOR VARIOUS FREQUENCIES. f_l AND f_h ARE THE CHANNEL'S LOWER AND HIGHER FREQUENCY LIMITS (IN MHZ) RESPECTIVELY. [17]

$E_{1m} - E_{r_p}$ which turns out to be 156.3dB for KCNS.

- The ITU-R recommendations are used to determine the maximum distance (beyond 1m) at which the path loss is less than (or equal to) RPL for 50% of the locations, 90% of the time. (The ITU-R recommendations provide a mechanism to extrapolate the tables for different distances, heights and frequency [4]).

For KCNS, the r_p calculated using this method is 129.3km.

We further assumed that all television signals are ATSC signals *i.e.* the assumption is made that low power and Class A transmitters would switch to ATSC signals with the same power.

APPENDIX IV
CALCULATING THE NO-TALK RADIUS (r_n)

To calculate r_n we first calculated the distance beyond the protected radius where the secondary can transmit ($r_n - r_p$) *i.e.* we need to determine the distance r_n such that a transmission from r_n results in a signal level at r_p of I_{r_p} (See Equation 1). After we converted I_{r_p} to an electric field, we used the procedure outlined in Appendix III to determine the value of $r_n - r_p$ with the exception that we used the $F(50, 10)$ propagation curves for predicting the distance. This was to ensure that transmissions from a secondary just outside r_n can cause harmful interference only 10% of the time.

For the KCNS station, I_{r_p} was -112.1dBm which translated into a electric field strength of 18.83dBu. This required the secondary transmitter to be 20.57km outside KCNS's protected radius.

APPENDIX V
CALCULATING THE NO-TALK RADIUS (r_n) FOR ADJACENT CHANNELS

For adjacent channels, the additional distance beyond the protected radius that we need to budget reduces since the TV receiver can tolerate higher interference from adjacent channels. The FCC specifies that the adjacent channel interference can be $\sim$ 27dB higher than the desired signal. Using this value, $r_n - r_p$ was calculated for adjacent channels as per the procedure outlined in Appendix IV except that I_{r_p} can be as high as -62.1dBm. To create this interference level, the secondary would have to be 1.1km from r_p.

APPENDIX VI
CALCULATING r_n USING THE FCC METHOD

The FCC assumes that the protection radius is the Grade B contour (r_b). The required field strength ($E_{r_b}(dBu)$) at the Grade B contour is defined by the FCC for ATSC signals [1] as shown in Table VI. Hence for the KCNS transmitter we need to determine the distance at which the received signal is above 41dBu for 50% of the locations, 90% of the time. This distance turns out to be 129.8km.

To calculate $r_n - r_b$ we found the distance beyond r_b such that a transmission from r_n results in a signal level at the Grade-B contour of $E_{r_b} - 23dBu$. Since E_{r_b} is only a function of the frequency (and not the transmit power/height of the primary transmitter) there is a single $r_n - r_p$ value for a given channel. The procedure outlined in Appendix III can be used to determine this value of $r_n - r_b$ with the exception that we use the the $F(50, 10)$ propagation curves for predicting the distance. For the KCNS transmitter, this distance turns out to be 21.42km.

An alternative approach adopted by the FCC also specifies the value of $r_n - r_p$ to be used for all channels (See Section 15.712 [1]).

Appendix VII
Calculating the radius for single-threshold sensing rules

A sensing based approach does not distinguish between different towers. The FCC's -114dBm rule imposes a 20dB margin on the nominal signal level at r_n. To determine the area lost due to such a rule, we first converted the -114dBm power to an equivalent dBu value using the equations in Table VII. Using the procedure outlined in Appendix III we calculated the equivalent distance. As discussed in the main text we use the $F(50, 50)$ curves for predicting distances. For KCNS the -114dBm rule would wipe out all area to a distance of 255km.

Next, we derived an equivalent rule for adjacent channels. In Appendix V we had determined r_n for the adjacent channel. At this new r_n we calculated the average signal level and added a budget of 20dB to obtain a -110dBm sensing rule for adjacent channels. With such a rule the area around the KCNS station that needs to be excluded for usage in 223km. It should be noted that the FCC specifies a single -114dBm rule for all channels (co-channel and adjacent channel sensing).

Appendix VIII
Incorporating Multipath into the Path Loss Model

The effect of multipath fading is incorporated in the calculations of r_p, $r_n - r_p$ and the distance corresponding to the -114dBm rule. To remain conservative we neglected multipath for calculating r_p (with multipath, the F(50, 90) point is lower and this leads to the underestimation of r_p). For determining $r_n - r_p$ we assumed Rayleigh multipath – this increased the value of $r_n - r_p$ for the KCNS tower from 20.57km to 24km. For the -114dBm rule the opposite was true; the distance excluded shrinks from 255km to 240km.

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