15 Years of Scherk-Collins Saddle Chains

Carlo H. Séquin

Electrical Engineering and Computer Sciences
University of California at Berkeley

Technical Report No. UCB/EECS-2010-41
http://www.eecs.berkeley.edu/Pubs/TechRpts/2010/EECS-2010-41.html

April 11, 2010

15 Years of Scherk-Collins Saddle Chains

Carlo H. Séquin

University of California, Berkeley

My collaboration with Brent Collins started 15 years ago. When I saw a picture of *Hyperbolic Hexagon* (Fig.1), I wanted to find out who was the artist behind this elegant form, which looked simultaneously mathematical and quite natural. So I picked up the phone and contacted the artist Brent Collins, a wood sculptor living in Gower, MO. We talked for about 45 minutes, and in this first phone conversation we established that this piece indeed has a mathematical underpinning, even though Brent was not aware of it when he first carved it; the connection was made by Prof. George Francis at the University of Illinois. The shape shown in Figure 1 can be interpreted as a section from Scherk's *Second Minimal Surface* (Fig.2a) bent into a closed loop. This geometrical form also looks quite "natural," because minimal surfaces are the shapes assumed by a soap film suspended in an arbitrary wire frame. Mathematically they are defined as having zero mean curvature everywhere, thus forming perfectly balanced saddle shapes at every point of the surface.

Figure1: *Hyperbolic Hexagon*

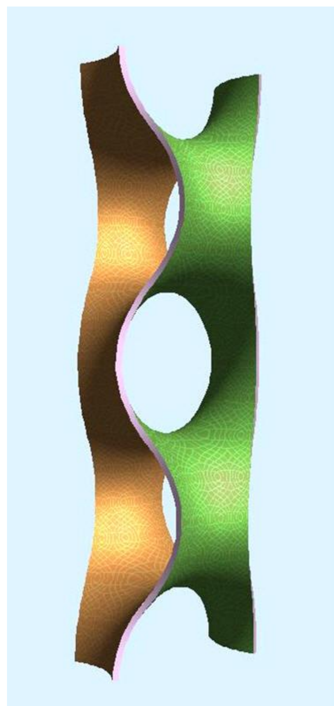

Figure 2a: *Scherk tower*

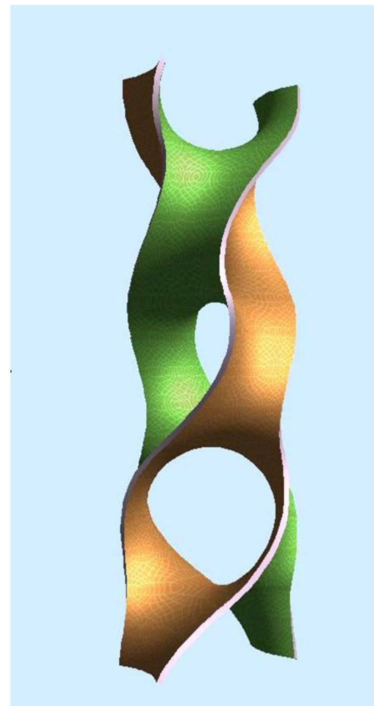

Figure 2b: *Twisted Scherk tower*

Once we understood this shape as a chain of six hole-saddle pairs bent into a closed toroid, it was unavoidable to ask what would happen, if we had a different number of holes and saddles in that loop. After some analysis, it becomes clear that when there is an odd number of such "stages" or "stories," the ends do not join naturally, since every other stage is turned by 90° with respect to its neighbors above and below. We can still make them join smoothly, if we twist the whole "Scherk Tower" longitudinally (Fig.2b) by any odd multiple of 90°. Next, in our phone discussion, we realized that under those circumstances the two different sides of this surface would be merged, thus forming a single-sided Möbius configuration. Adding any twist to the Scherk tower also interlinks the edges of this sculpture in interesting ways. In the un-twisted *Hyperbolic Hexagon* all four edges form their own separate closed loops. But with a moderate amount of twist these edges will merge into one another after one pass around the twisted toroid; and it will then take several laps around the toroid to reach the starting point again. Analyzing the multiple intertwined torus knots that can be formed in such twisted structures is rather intriguing.

While these mathematical questions are quite fascinating, Brent, as an artist, is mostly concerned with the question whether any of these possible configurations make attractive geometrical sculptures that look good from most viewing angles. For that reason, Brent first builds smaller scale models from embroidery hoops, PVC piping, wire meshing, bees wax, etc, for any larger-scale sculpture that he plans to carve in wood. Making such a model may itself take a couple of weeks. In our first phone conversation we already had 3 or 4 novel ideas, and in our subsequent frequent phone discussions we typically added one or two more ideas each week. Thus it became clear that Brent could not possibly keep up with making models for all these concepts to judge their visual appeal. At this point I decided that a computer-based visualization tool was our only hope of keeping abreast of our own ideas. The result was *Sculpture Generator I*, a special purpose modeling program written in the C language, the geometry kernel of which comprised only about 3000 lines of code [C. H. Séquin: *Virtual Prototyping of Scherk-Collins Saddle Rings*. Leonardo, Vol 30, No 2, pp 89-96, 1997]. Within a few months I had a first prototype running that allowed me to specify the number of stories in the loop, their combined total twist, the azimuth orientation of the edges around the smaller radius of the torus, as well as the thickness and extension of the flanges, and various simulated surface properties (Fig.3). Now I could try out many different ideas and parameter combinations in a matter of minutes, and then focus on the most promising configurations and optimize them for their aesthetic appeal. Or I could investigate such questions as: What is the tightest loop into which a Scherk tower with only 3 stories could be bent. The latter question led to my version of the *Minimal Trefoil* (Fig.4).

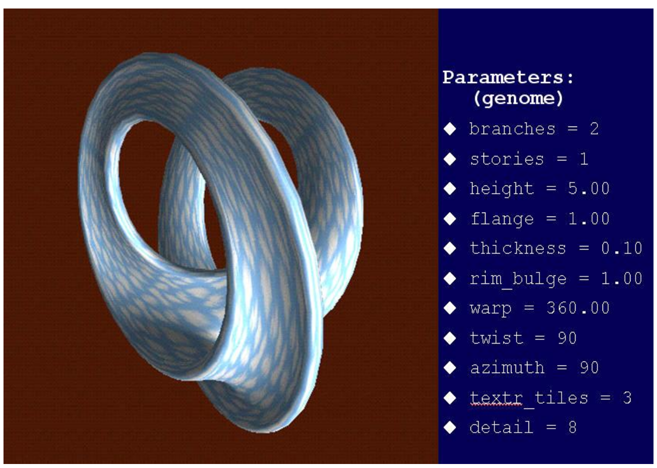

Figure 3: *Sculpture Generator I: Single-story toroid*

Figure 4: *Minimal Trefoil in bronze*

By the end of 1996, *Sculpture Generator I* had grown into a fairly robust program with many different output modalities. Besides compelling virtual renderings, the program also could output a boundary representation of these solid shapes in the .STL file format, which was the lingua franca of the many layered manufacturing machines that started to emerge around that time. I also added a program module that could output full-size blue prints of section cuts taken at regular intervals, say 7/8 of an inch apart, corresponding to the thickness of the wood-boards from which Brent Collins was planning to build such sculptures. *Hyperbolic Hexagon II* was the first large sculpture that Brent built from a set of blue prints generated by *Sculpture Generator I*. He cut the individual profiles from 7/8 inch thick walnut boards, assembled them with industrial strength glue, then fine-tuned the shape with hand tools, and honed the surface to perfection. Figure 5 shows Brent Collins holding our first truly collaborative piece.

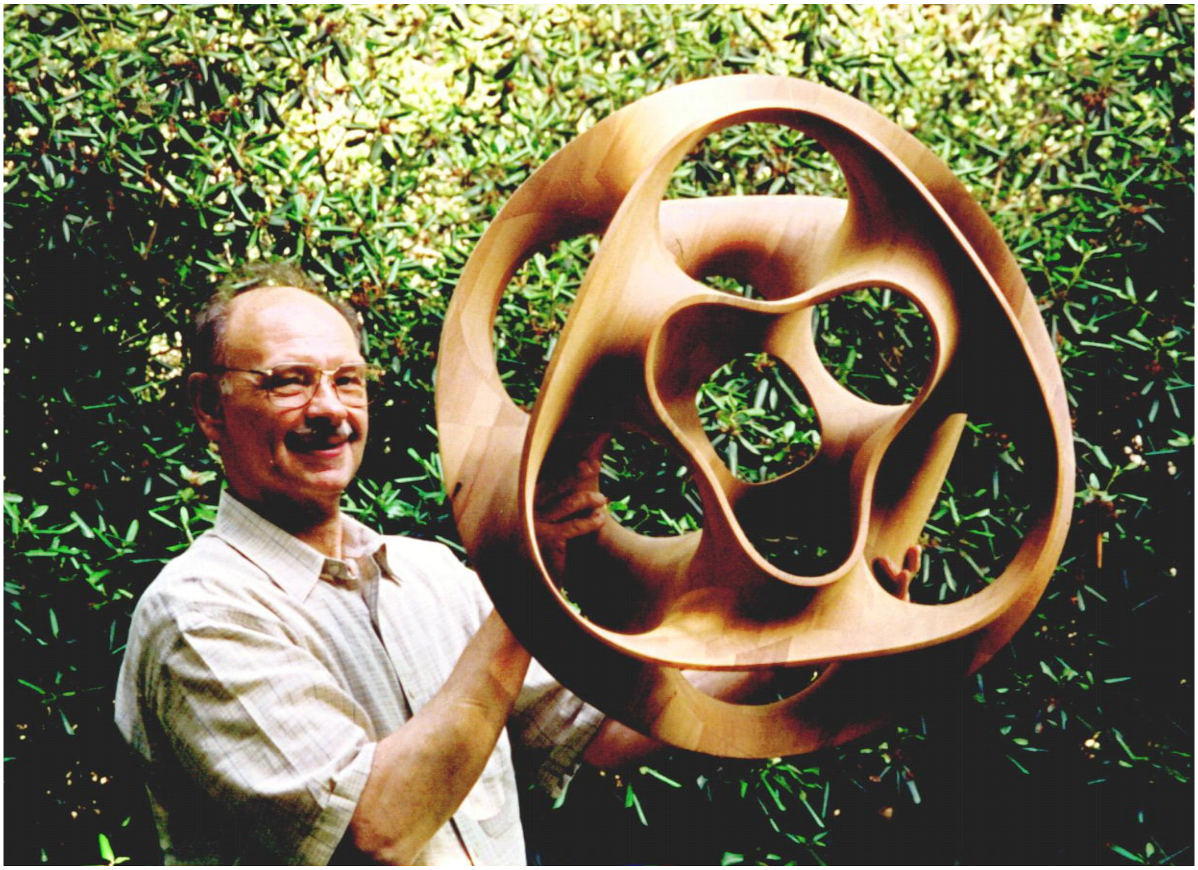

Figure 5: *Brent Collins holding the wood master of Hyperbolic Hexagon II*

A few months later we built *Heptoroid* in the same way; this was a ring with seven 4th-order saddles and a twist of 135°. And while Brent claims that he might have been able to design the shape of *Hyperbolic Hexagon II* without the help of a computer, he readily admits that this would not be possible for the *Heptoroid*. Below is a cross-eye stereo picture of this twisted shape.

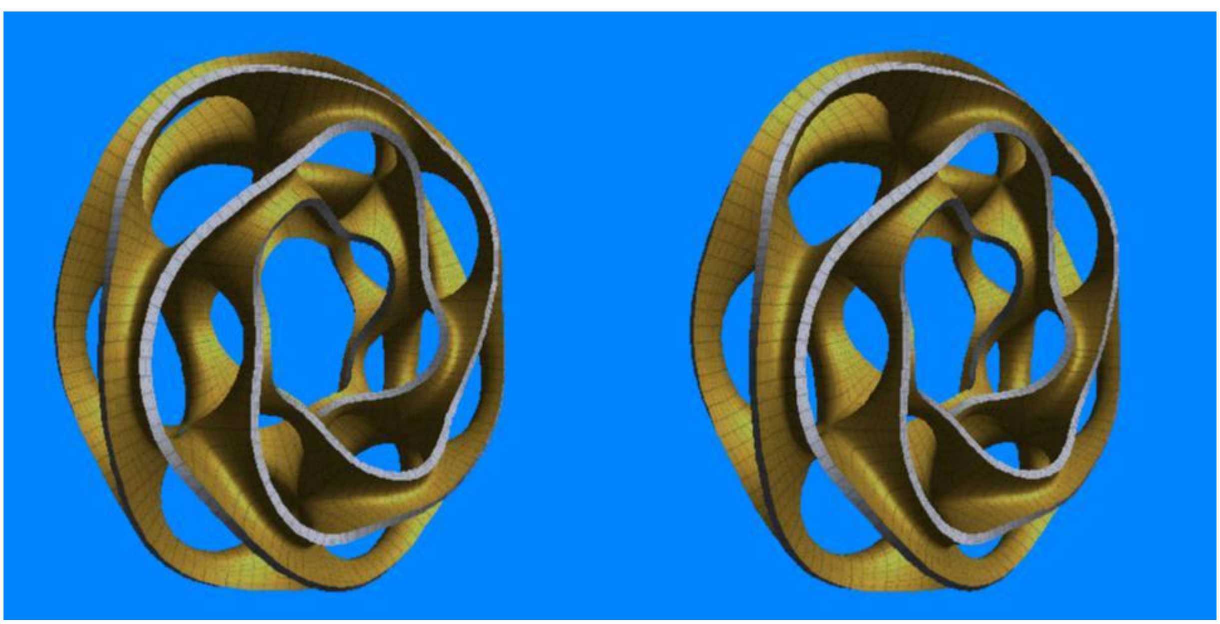

Figure 6: *Cross-eye stereo picture of the Heptoroid geometry*

Around that time, the College of Engineering at U.C. Berkeley acquired a few rapid prototyping machines on which I could now produce small-scale versions of these sculptures within a day or two. That is when I really started to experiment with my *Sculpture Generator I*, producing dozens of small prototypes, 3 to 4 inches in diameter (Fig.7), or "hyper-sculptures" consisting of several closely related individual pieces (Fig.8).

Figure 7: *Three cinque-foils made on an FDM machine*

Figure 8: *Family of Twelve Trefoils*

Some of the more attractive shapes were subsequently fabricated at a scale of about 8 inches in diameter. I then sent some of these plastic maquettes, made on a Fused Deposition Modeling (FDM) machine, to Steve Reinmuth in Eugene, OR, for bronze casting. Steve had figured out how these ABS-plastic shapes could be converted into bronze casts in a "lost-ABS" investment casting process, directly analogous to the traditional lost-wax process. Examples of that conversion process are: *Cohesion* (Fig.9) and *Scherk Tower* (Fig.10).

Figure 9: *Cohesion (bronze)*

Figure 10: *Scherk Tower (bronze)*

Cohesion consists of a Scherk-Collins saddle-ring of only two stories with 3-way monkey saddles (Fig.9), and *Scherk Tower* is a twisted straight-line cut from Scherk's *2nd Minimal Surface*. The latter piece was recently re-issued with a new green patina (Fig.10) to show what a large-scale sculpture might look like, if placed in a reflecting pond with water running down from the top over its sides and dripping from one story to the next one below.

Following the notion that "bigger is better," Brent and I are always looking for opportunities to take one of our collaborative designs and build it at a larger scale. Figure 11 shows a 12-inch diameter bronze cast of *Solar Circle* a 12-story saddle ring with 270° twist. In 2007 it was cast as a 10-foot diameter polyester and glass-fiber ring and hung in an atrium in a housing development near Kansas City (Fig.12).

Figure 11: *Solar Circle (bronze)*

Figure 12: *Millennium Arch (polyester)*

Another motif consisting of three monkey saddles in a tight loop with a 270° twist was also built at several different scales. The largest one was *Whirled White Web*, a 12-foot tall snow sculpture (Fig.14) carved in a span of five days at the snow-sculpting championships in Breckenridge, CO, in 2003. Even though it collapsed only 45 minutes after judging had ended, it was awarded the silver medal. The same motive was re-engineered in 2008 to serve as an annual award trophy for the Euro-Graphics conference (Fig.13).

Figure 13: *Euro-Graphics conference award trophies*

Figure 14: *Whriled White Web, silver-medal winner, 2003 snow sculpting championships, Breckenridge, CO*

Figure 15: *Doubly Wound Quad (bronze, gold-plated)*

As I continued to play with *Sculpture Generator I*, some of its built-in limitations started to frustrate me. Thus, over the next few years I gradually enhanced the program's capabilities and expanded its conceptual boundaries. For instance, why should the hole-saddle chain only go around the toroidal loop once? Wrapping it around the loop more than once will typically produce a complex, self-intersecting mess. But if we select an odd number of stories, so that the stories intertwine gracefully on subsequent passes, and also judiciously select the amount of twist and the width of the flanges, then we can obtain a clean, non-self-intersecting solid with a proper 2-manifold surface. One demonstration that this can be done is the *Doubly Wound Quad* (Fig.15), of which Steve Reinmuth said that this was his most difficult casting job ever!

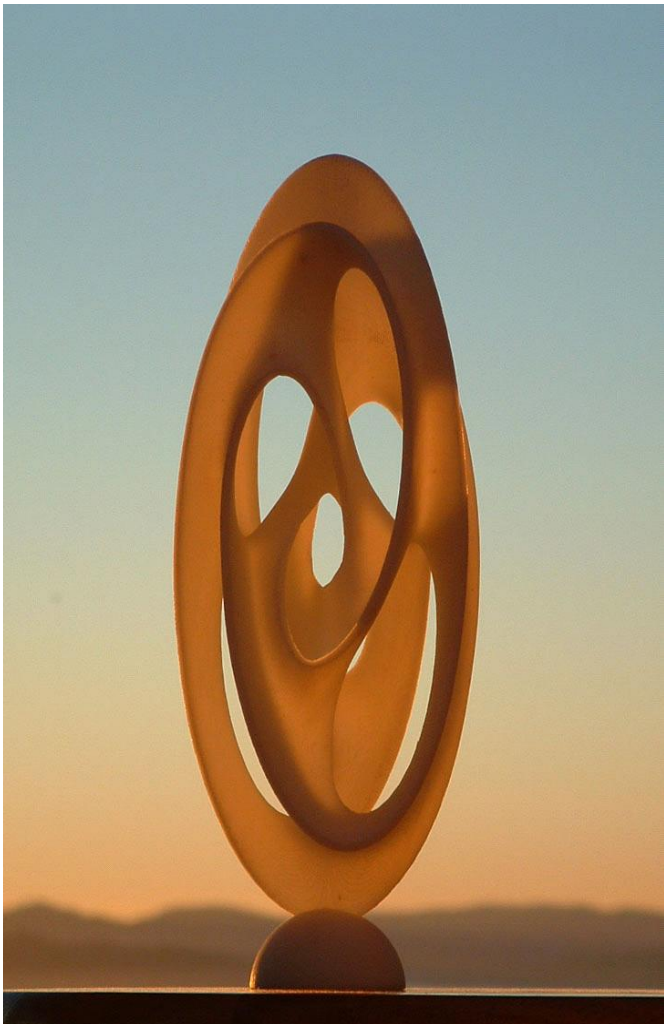

Figure 16: *Totem 2 (ABS plastic)*

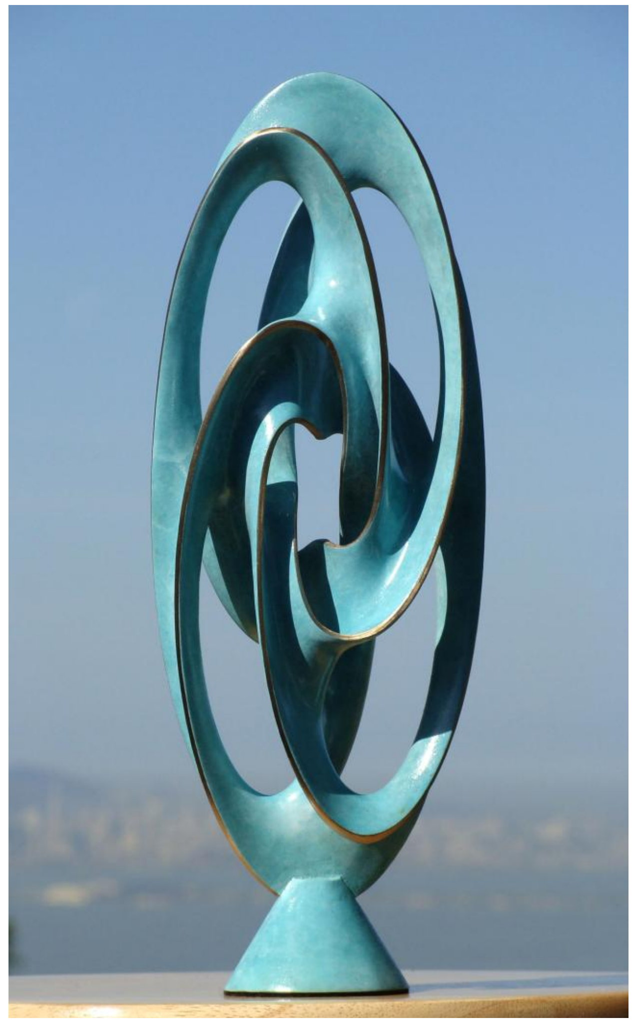

Figure 17: *Totem 4 (bronze)*

As another extension of the domain of *Sculpture Generator I* a program module allowing non-uniform, affine scaling of the toroidal geometry was added. Sculptures often can be made to look more dramatic and more interesting if their symmetry is broken in controlled ways. This extension made possible the design of several sculptures in the *Totem* family (Fig.16 and 17).

Nevertheless, at some point this *saddle-ring* paradigm runs out of steam. There are many other sculptures with quite different design paradigms. To capture their essence requires completely new programs. Over the last 10 years, several other sculpture modeling environments have been created by my students – those will be the subject of future publications.

Figure 18: *Carlo Séquin next to a bronze cast of Hyperbolic Hexagon II placed at U.C. Berkeley*

This review concludes with another picture of *Hyperbolic Hexagon II*, which recently has been cast in bronze and is now standing in the lobby of the CITRIS building on the U.C. Berkeley campus. The wood master for *Hyperbolic Hexagon II* had been standing in the Mathematics Lounge of Wesleyan College for several years. There it was discovered by the film makers of *Watchmen* and borrowed for an appearance in Dr. Manhattan’s studio. Eventually, a mold was formed from that wood master, and Steve Reinmuth's Bronze Studio in Eugene, OR, then reproduced the shape in bronze by investment casting. That studio also created the special patina, which turned a “geometric model” into a wonderful piece of art. Thus *Hyperbolic Hexagon II* is truly a collaborative effort between Brent Collins, Steve Reinmuth, and Carlo Séquin.