Constructing an Augmented Rhombicuboctahedron (RCO)

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Abstract

Using the OCTA-TETRA kit [2] polyhedral objects are constructed based on the small rhombicuboctahedron.

1. Introduction

The small rhombicuboctahedron (RCO) is one of the Archimedean solids. It has 8 equilateral triangles and 6+12 square faces -- one each for the faces and edges of the underlying cube (Fig.1a). One can *augment* this polyhedron by placing 3- and 4-sided pyramids made from equilateral triangles onto each of the 26 faces of the RCO. This creates a "*stellated*" form (Fig.2a); but it is not a true stellation of the RCO, since the pyramid faces are not formed as extensions of the neighboring faces of the RCO.

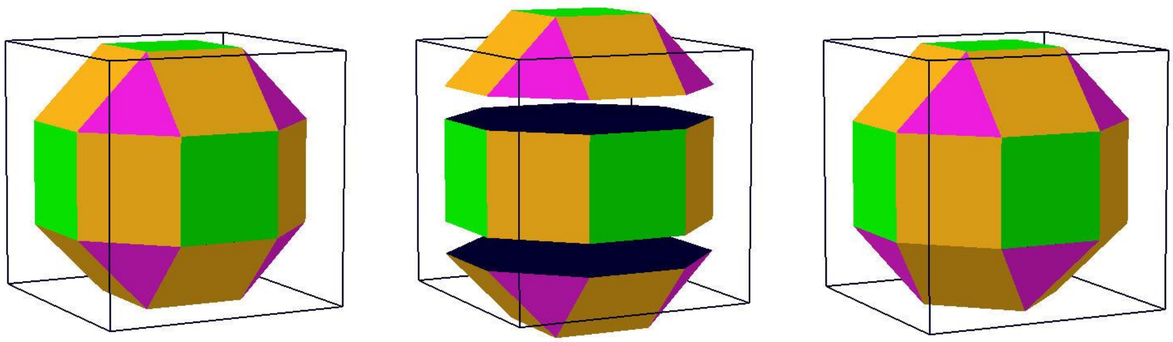

Figure 1: *Rhombicuboctahedron (RCO) and Pseudo-RCO in the frame of a reference cube; (a) RCO; (b) cut into 3 slices; (c) bottom slice rotated by 45° to yield the Pseudo-RCO.*

Recently a controversy [1] has erupted around this shape (Fig.2b) as depicted in a book by the 15-century mathematician Luca Pacioli. One guesses that this rather complicated drawing was done by Leonardo da Vinci. The issue of discussion arises, since that drawing does not really depict an augmented RCO (Fig.2a), but an augmented form of the Pseudo-RCO (Fig.2c). In this object, which is not a semi-regular solid, the lower part of the polyhedron is twisted through 45° (Fig.1b,c) with respect to the proper RCO. My analysis of this drawing can be found on-line [1].

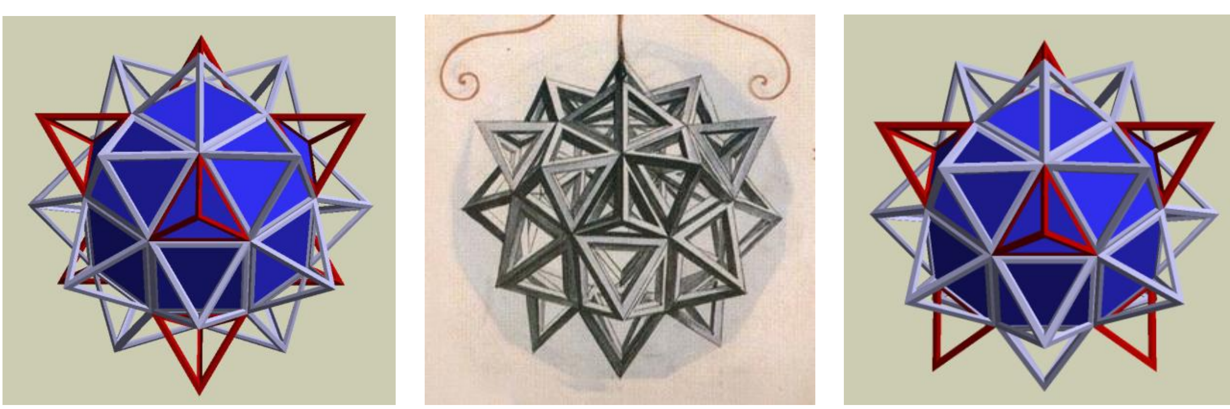

Figure 2: *Augmentations: (a) of the RCO; (b) Leonardo's(?) figure; (c) augmented Pseudo-RCO.*

2. OCTA-TETRA Construction

When I received Dan's OCTA-TETRA sample kit [2], I was immediately tempted to build an augmented RCO from these modular cardboard pieces. However, I did not just want to connect eight 3-sided pyramids and eighteen 4-sided pyramids into a flimsy, wobbly outer shell of an augmented RCO. Ideally I would have liked to fill the body of the RCO solidly with OCTAs, TETRAs or similar small and stable polyhedral modules. But I quickly realized that the internal angles of the RCO were not amenable to such a decomposition. The interior angles (e.g., 135°) just do not accommodate the dihedral angles of the OCTA or the TETRA. Thus I realized that I would have to "bend" the OCTA-TETRA assembly rules to some degree. In my approach I was encouraged by Uncle Dan's workbook #64 that describes how to put six 4-sided pyramids, or half-OCTAs, around a cube. So I decided to build complete OCTAs underneath the twelve pyramids that sit on the bevelled edges of the underlying cube frame of this structure. They are shown in orange in Figure 3.

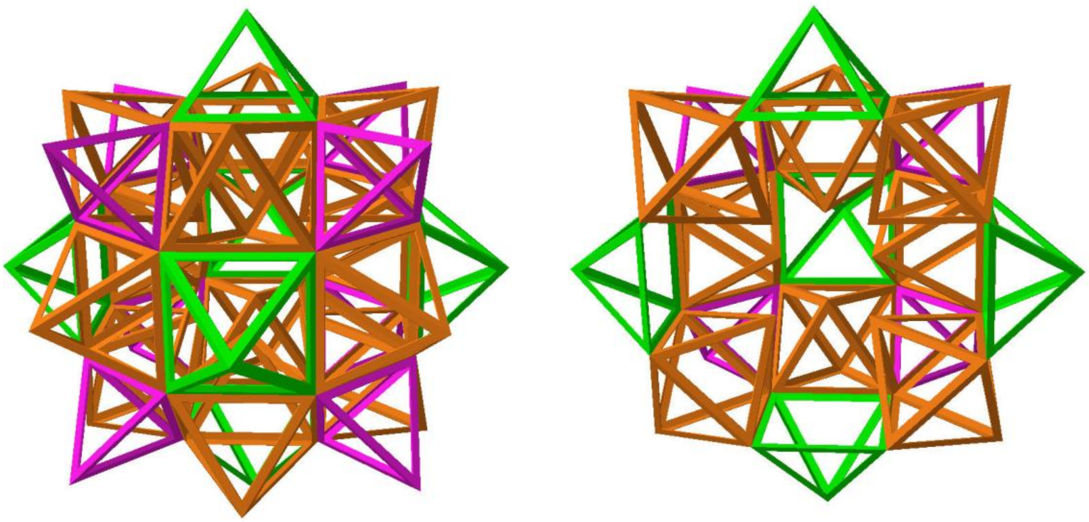

Figure 3: *The organization of (a) the augmented RCO with its 12 complete OCTAs and with another 14 pyramids that extend inwardly to hold the other pieces together; (b) a partial 2/3 assembly.*

The other six 4-sided and eight 3-sided pyramids also have an internal structure, but that structure is kept "open" and provides triangular flaps that attach to the regular OCTAs. Figure 4 shows all three modules, OCTA, OCTA*, and TETRA*, that are needed for the construction of the augmented RCO.

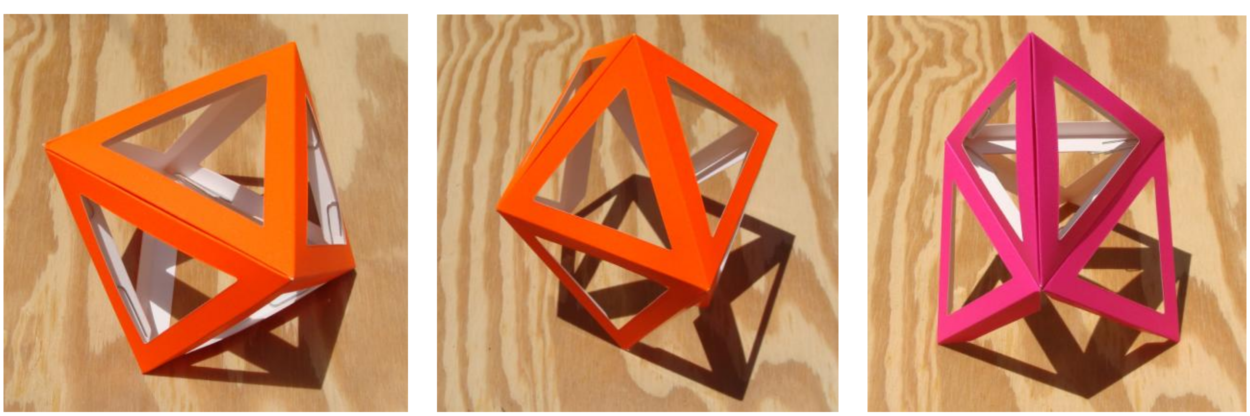

Figure 4: *The three different modules used in the construction of the augmented RCO: (a) regular OCTA; (b) extended, open half-OCTA = "OCTA*"; (c) extended TETRA = "TETRA*".*

Here is how I built the two special modules. For the case of the 3-sided pyramid, the solution is obvious: Build a complete TETRA, and attach the middle portion of another TETRA stencil to its base (Fig.5a); but leave its 3 faces open as giant flaps that will connect to the inner parts of adjacent OCTAs.

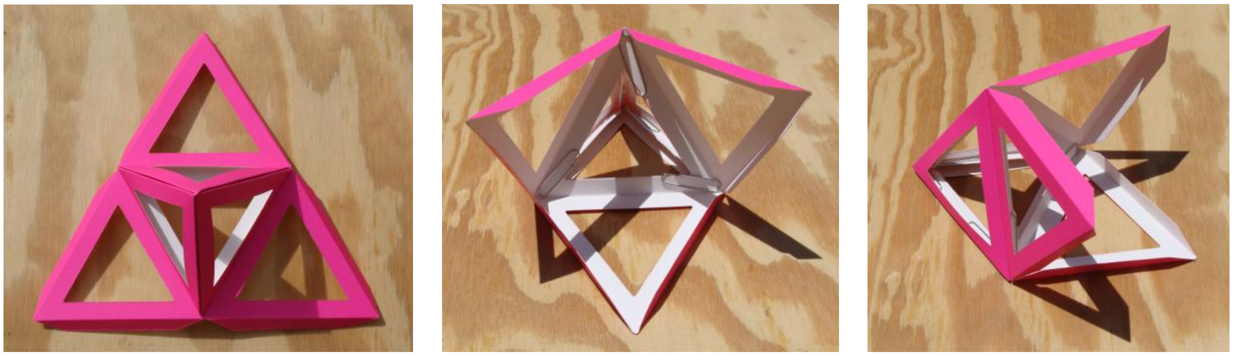

Figure 5: *The TETRA* module: (a) TETRA on top of open stencil; (b) bottom view, and (c) side view.*

It took me a little longer to see that by combining two OCTA stencils as shown in Figure 6a, I could also obtain an equivalent 4-sided pyramid structure with four dangling flaps (Fig.6b,c).

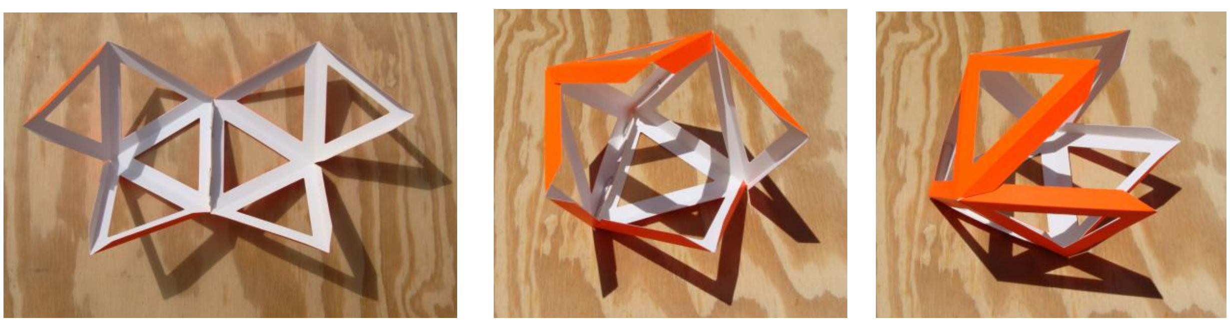

Figure 6: *The OCTA* module: (a) assembly of two stencils; (b) bottom view, and (c) side view.*

Once I had decided on these modules and how the whole thing would hold together, construction could begin in earnest! First I built 12 regular OCTA's. Next I constructed 8 TETRAs, connecting the weakest face of each (lacking any paper clips so far) to the central portion of an open TETRA stencil (Fig. 5a). I also assembled 6 open OCTA* modules following the scheme illustrated in Figure 6.

Now I was ready to assemble these modules. I first connected the loose flaps of an OCTA* structure to the "lower" (inner) triangles of four regular OCTAs. Then I added four TETRA*s into the open corners between neighboring OCTAs, connecting two of their open flaps to the adjacent OCTAS. The resulting sub-assembly, consisting of four regular OCTAs, four TETRA*s, and one central OCTA*, is shown in Figure 7. This constitutes roughly one third of the whole augmented RCO structure. Up to this point the assembly is quite easy, since everything is readily accessible. Figure 7 also gives a glimpse at the inside of this structure (Fig.7b), as well as a side view (Fig.7c).

As the next major step we have to add a belt of eight 4-sided pyramids to the bottom of this structure. This can easily be done by first adding four regular OCTAs, one each to the four unused flaps of the four TETRA* structures. Since these flaps are flexibly hinged to the TETRA*s, their insides are readily accessible. Next, an OCTA* element has to be inserted into each of the four gaps between the four OCTAs. Again, this is not difficult, since a first loose flap can always be attached to a regular OCTA in the 1/3 subassembly (Fig.7). Two more flaps are attached to the newly added regular OCTAs. This results in the 2/3 assembly, shown in Figure 8, which has a relatively flat bottom with four loose flaps (Fig.8b).

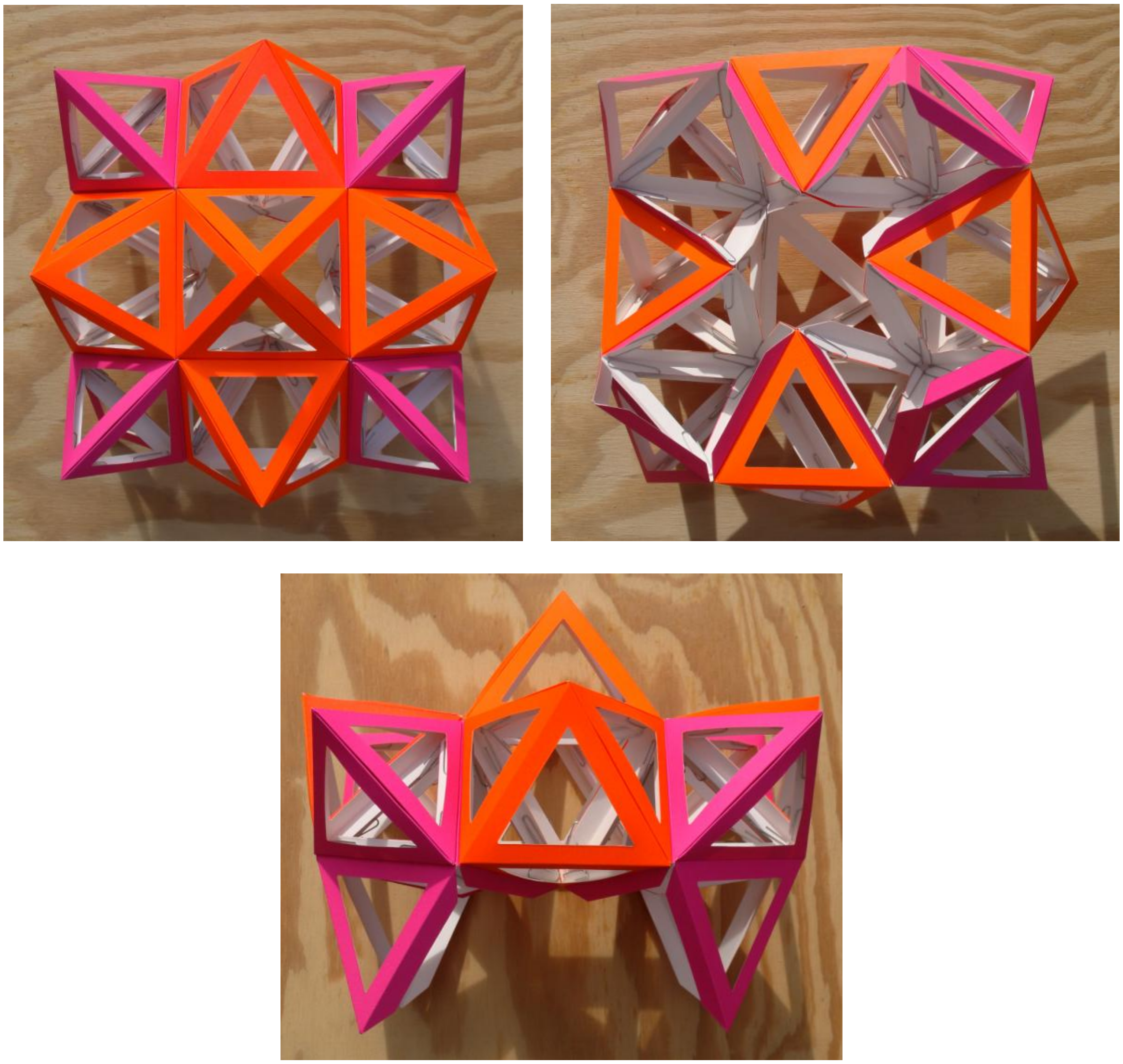

Figure 7: *A 1/3 sub-assembly: (a) seen from the top, (b) from the bottom, and (c) from its side.*

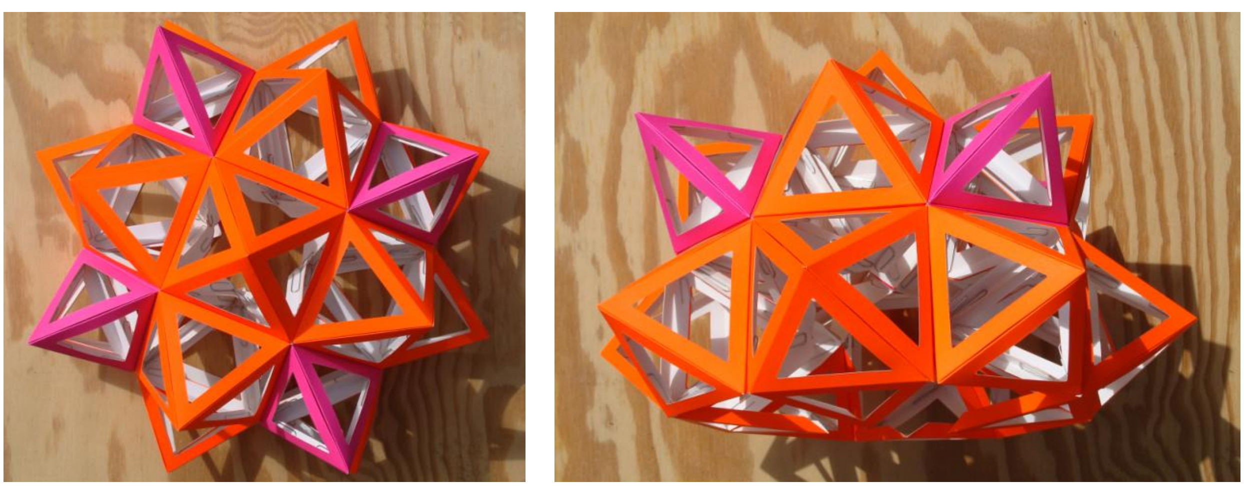

Figure 8: *The 2/3 assembly: (a) seen from the top and (b) from its side.*

Now I just had to build another 1/3 subassembly as shown in Figure 7 and attach it to the bottom of the 2/3 assembly. This is more difficult than the steps so far, since no helping hand can reach in from the inside and help guide the paper clips into their proper locations. In this context I found that the use of a pincette (tweezers) is a great help (see last section of this report). I actually found this so convenient, that I started to use tweezers even for the construction of the individual modules and subassemblies.

After connecting all eight loose flaps, four from the 2/3 assembly and four from the new 1/3 subassembly, the complete Augmented RCO emerges (Fig.9).

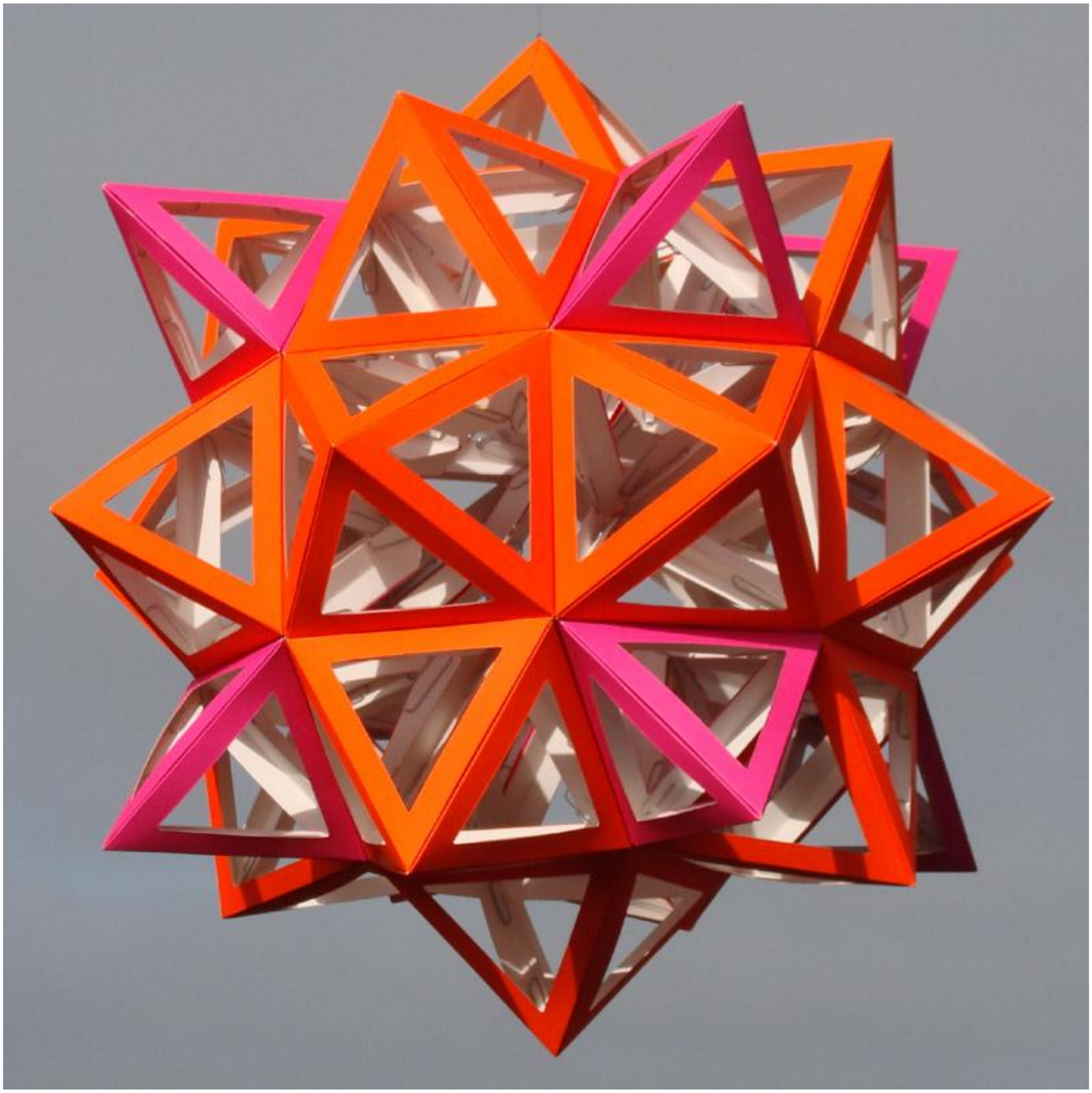

Figure 9: *The completely assembled Augmented RCO.*

I am quite happy with the finished model! Too bad, that Pacioli did not have an OCTA-TETRA kit, or he could have done the drawing of the Augmented RCO himself [1] rather than having to rely on somebody else who may have tricked him ...

3. Pseudo-RCO

The controversy mentioned above [1] now raises the question: Can we also build the augmented Pseudo-RCO depicted in Figure 2c? This poses some extra problems. The nice pattern that emerges from the RCO where every regular OCTA is surrounded by four open OCTA* or TETRA* elements is now disturbed by the 45° rotation of the lower part of this polyhedron against the upper part (Fig.1c). In order to have every loose, hinged flap meet a rigid structure for good stability, the bottom 1/3 subassembly has to be re-built in a manner that differs from the structure shown in Figure 7. In this revised bottom assembly we want the inwards-pointing parts of the 3-sided pyramids to be solid, and the four 4-sided pyramids between them to have the open flaps. Thus we need a new module: the "double-TETRA" shown in Figure 10a. The construction of the new 1/3 subassembly can follow the same conceptual sequence used for the one shown in Figure 7. However, now we start with a regular OCTA in the center and attach one flap each of four OCTA*s to its lower sides. Then we insert the four double-TETRAs into the four corner gaps. The resulting modified 1/3 subassembly is shown in Figures 10b and 10c. Attaching this to the 2/3 assembly shown in Figure 8 results in the complete Augmented Pseudo-RCO (Fig.10d).

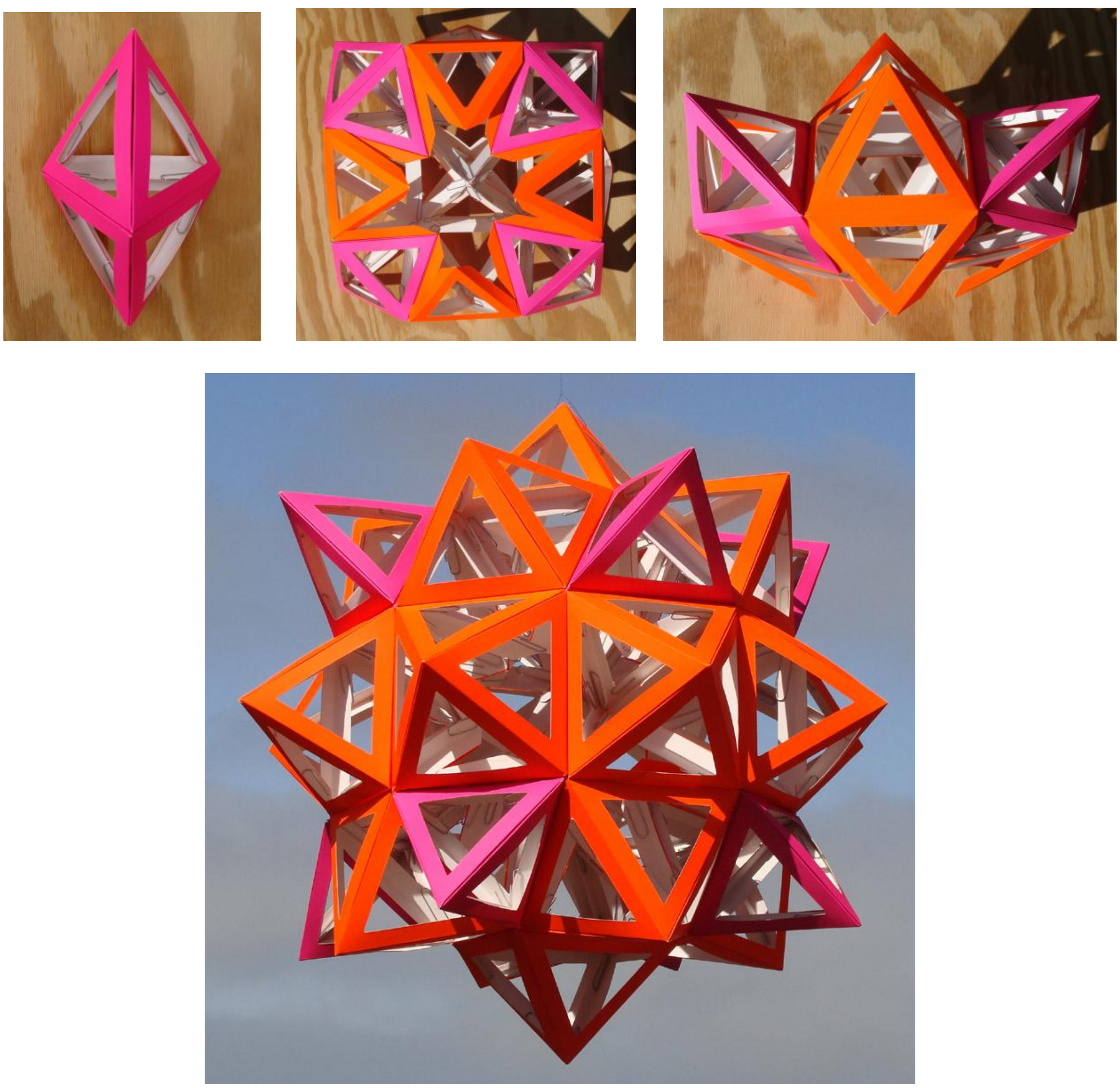

Figure 10: *Modifications needed for the Pseudo-RCO: (a) the double-TETRA module, (b) the resulting 1/3 subassembly seen from its inside, and (c) in a side view; (d) the completed Augmented Pseudo-RCO.*

4. Decremented RCO

Another variation that occurred to me is to create a "Decremented RCO" by replacing all faces of the RCO (or of the Pseudo-RCO) with inward pointing pyramid shells. The geometrical structure of this polyhedron is shown in Figure 11. With all pyramids pointing inward, there is enough tension emerging from the very sharp dihedral angles between adjacent pyramids, so that the structure will assume a nice "spherical" shape and form a rather rigid body without the needs for any additional internal bracing.

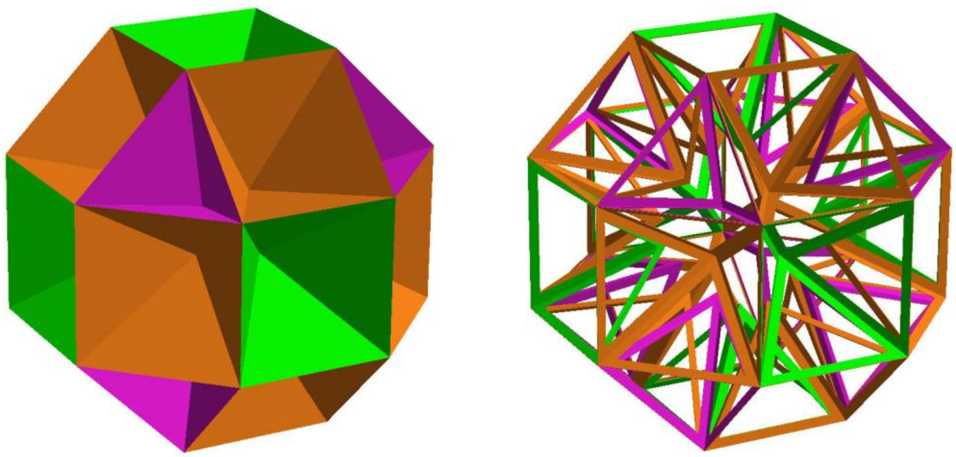

Figure 11: *Decremented RCO: (a) as a solid polyhedron, and (b) as a frame structure.*

In order to make this object look pretty, we fold all the stencils the "wrong way" so that the color shows on the concave side. We can obtain the necessary modules by turning the creases between adjacent faces of the OCTA and TETRA stencils into valley folds, but still folding the flanges in the usual direction (Fig.12). To obtain a consistent treatment of all such pyramids, we trim away one of the triangular faces of the TETRA stencil, but we do it in such a way that a standard-size trapezoidal flap is left where we remove that face; Figures 12a,b show how this is done.

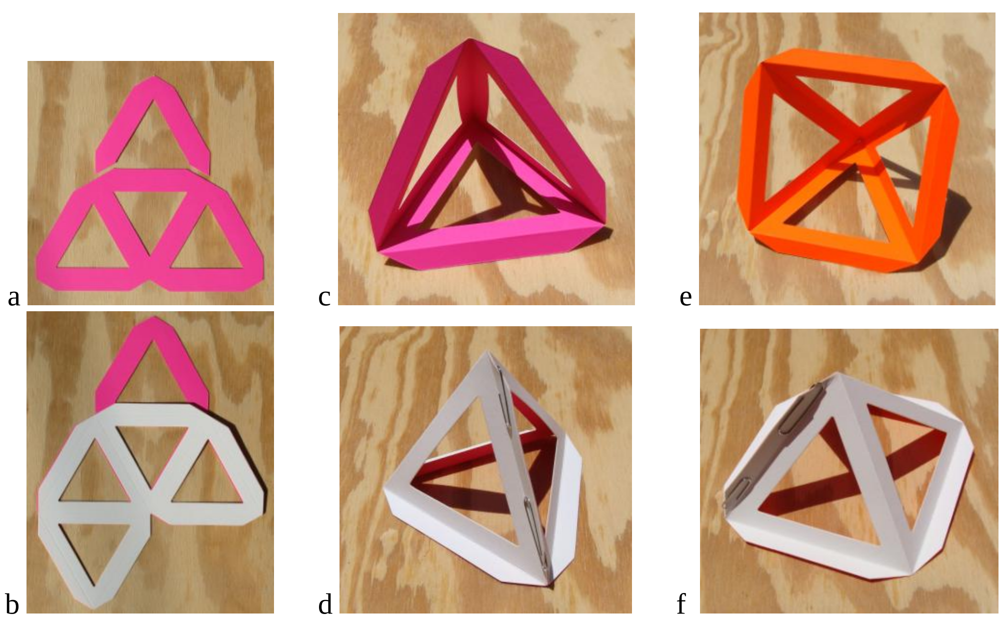

Figure 12: *Construction of the modules for the Decremented RCO: (a,b,c,d) the 3-sided pyramids, and (e,f) the 4-sided pyramids; (c,e) outside views and (d,f) inside views, respectively.*

The assembly of the whole Decremented RCO is rather straight-forward, at least in the beginning. Adjacent modules just get attached to their neighbours by tying their loose flanges together; and in these early stages of assembly, these flanges are easily accessible (Fig.13). Since I was running out of paper clips on a Sunday evening, and since I had no intentions to ever take this model apart again, I broke another assembly rule: I started to use staples for the easily accessible, outwards-protruding flanges (see last section of this report).

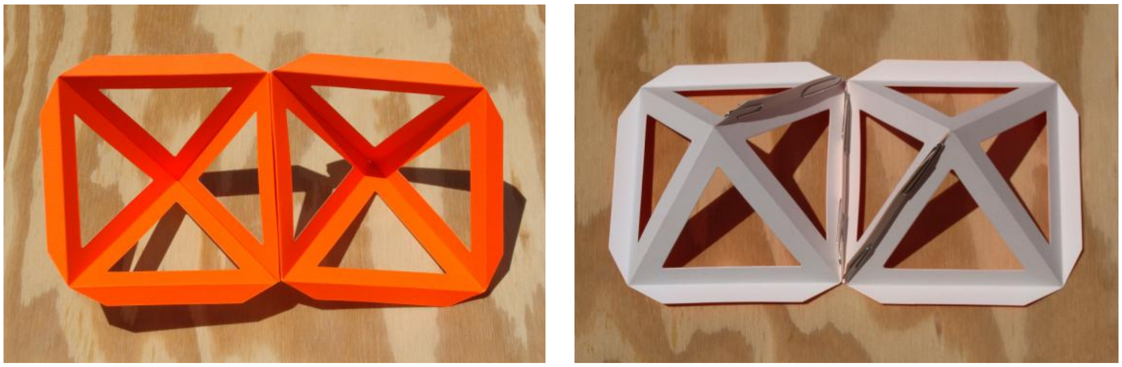

Figure 13: *Assembly of two adjacent 4-sided pyramids: (a) outside view, and (b) inside view.*

I started by forming a cluster of four 4-sided pyramids surrounding a central 4-sided pyramid. Into the four open corners I then fit four 3-sided pyramids (Fig.14). I first attached those pyramids with just one flange to keep that structure flexible and to maintain easy access to all the flanges on the inside that needed to be connected. To attach the second flanges of the 3-sided pyramids, the whole subassembly now had to be bent into a “hemispherical” shape, which pushed some of the flange-pairs that needed connecting to the inside (Fig.14). This rendered them inaccessible to a standard stapler, and I thus resorted again to paper clips, inserting them with the help of tweezers.

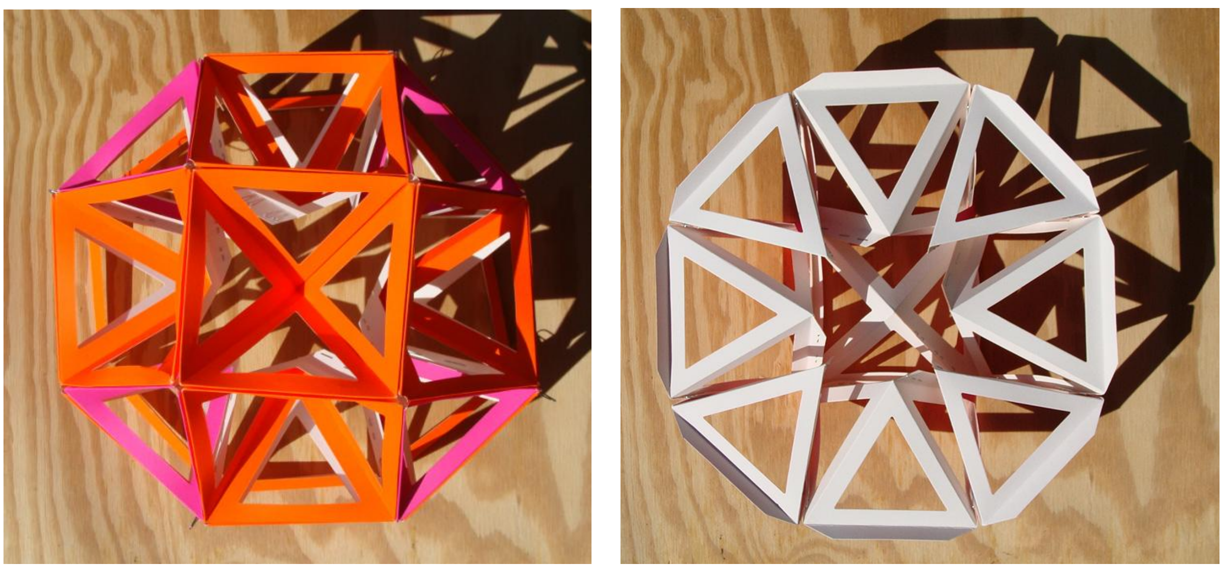

Figure 14: *1/3 subassembly of a decremented RCO.*

I then continued in the same spirit and attached four 4-sided pyramids to the four 4-sided pyramids at the perimeter of the 1/3 subassembly. To those I attached one more pyramid each to one side. All this could be done again with staples, because the new attachments were still very pliable.

Now the eight new pyramids had to be interconnected to form the central belt of the RCO. This created some tension throughout the whole structure and forced it into a "hemispherical" shape (Fig.15). To keep it roughly in this shape while connecting individual pairs of adjacent flanges, I used some external paper clips to just hold some of the struts of adjacent pyramids together. Those four pairs of paper clips can be seen sticking out in Figure 15c.

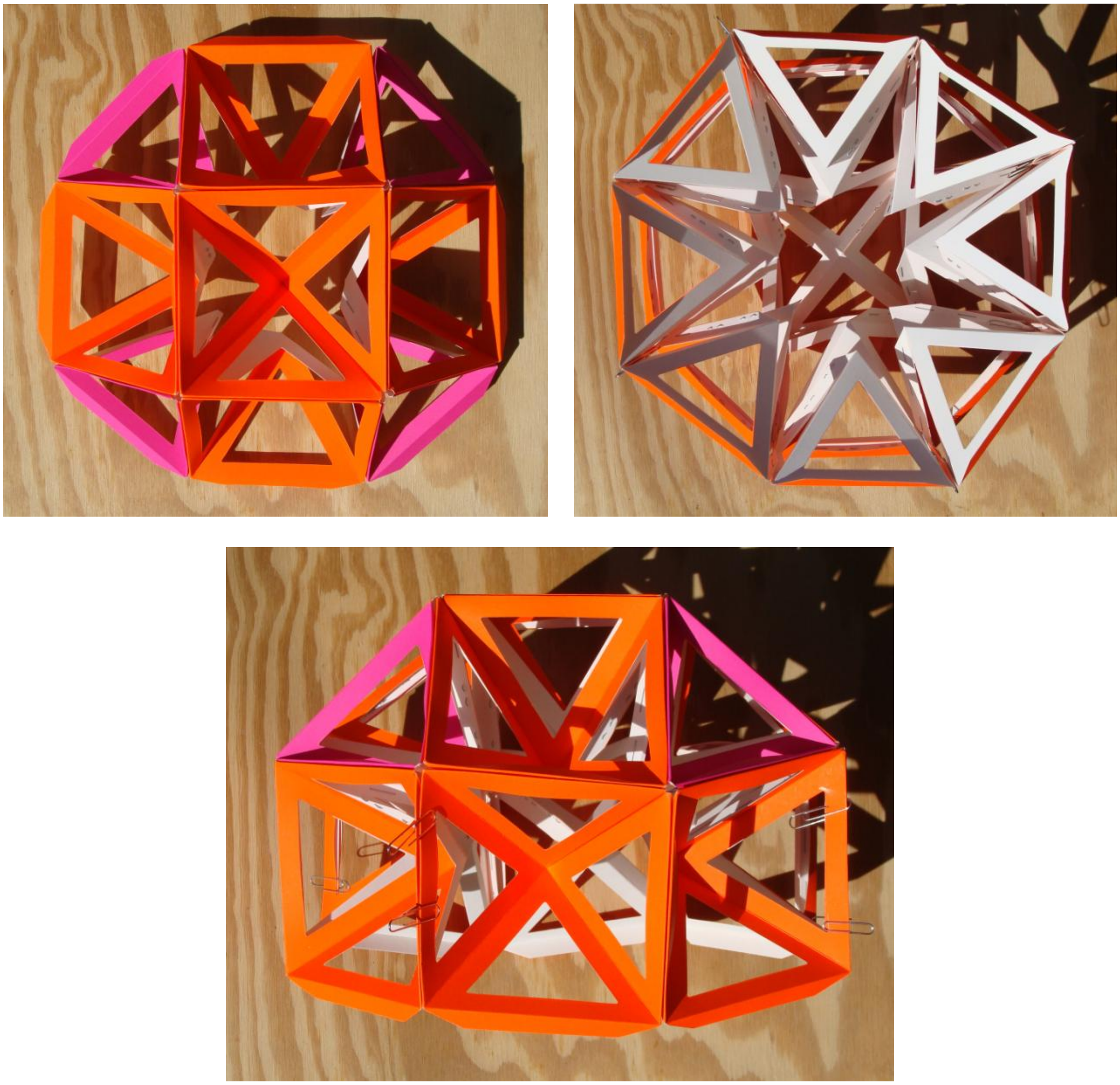

Figure 15: *2/3 subassembly of a decremented RCO.*

A second 1/3 subassembly as depicted in Figure 14 was constructed next. This was then combined with the 2/3 assembly (Fig.15), by connecting the eight pairs of flanges along their respective perimeters. This was done with paper clips only, so that this connection could be undone and the 1/3 subassembly could be rejoined after a 45° rotation, if I ever wanted to build a model of a Decremented Pseudo-RCO. Actually, jointing together these eight flange pairs was by far the most challenging assembly step of any of the structures discussed in this report; the paper clips had to be fit into very narrow grooves that were difficult to access. The completed construction of a Decremented RCO is shown in Figure 16.

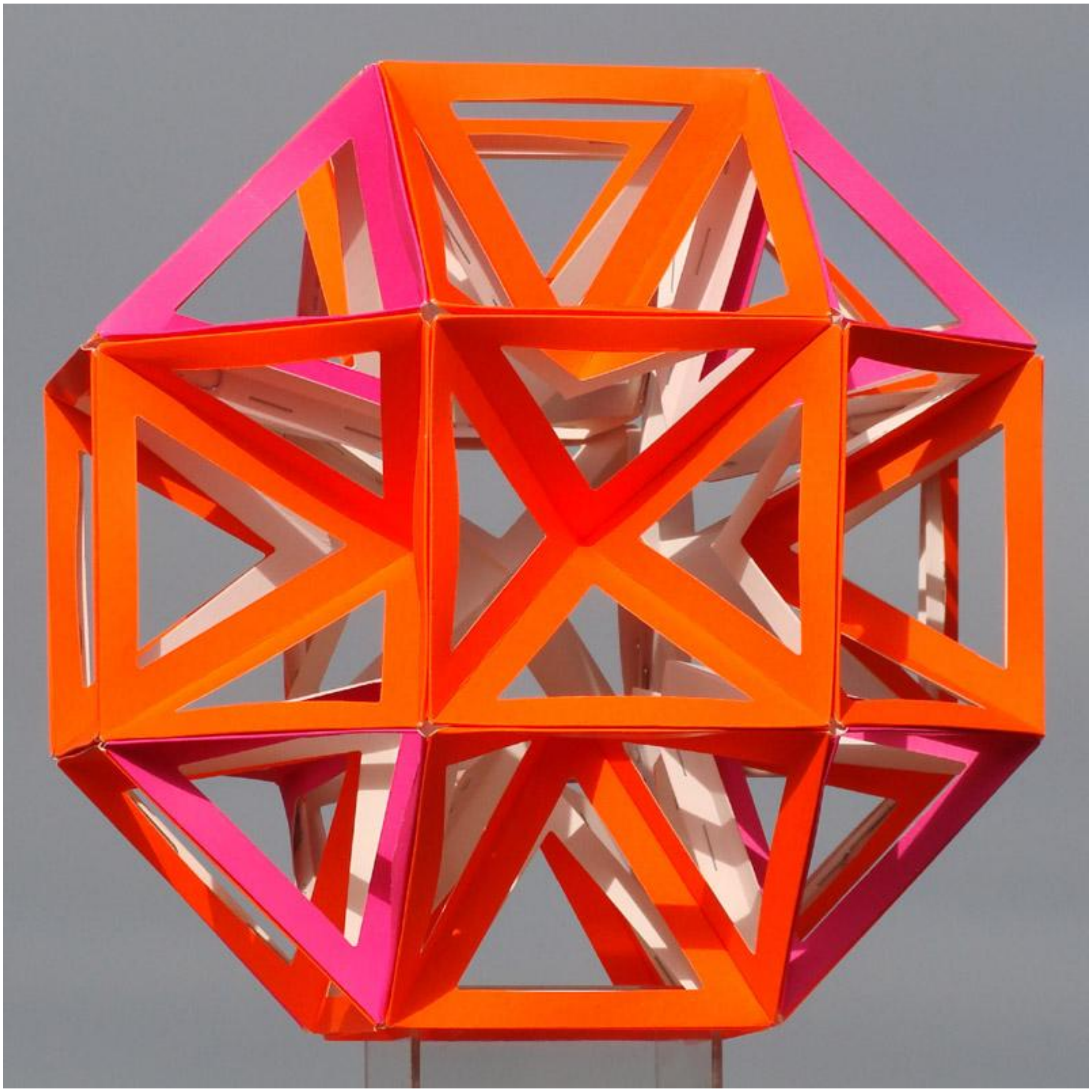

Figure 16: *The completed Decremented RCO.*

5. Assembly Techniques

Uncle Dan’s instructional videos give many good hints on how to grab and insert the paper clips into individual modules and emerging assemblies. They are very useful for the initial stages of construction. But at some point the structure might become packed too densely for medium-sized fingers to reach in there and place that paper clip into the perfect location. Tweezers come to the rescue! In less congested areas, where the fingers may still be able to put the paper clip roughly where it belongs, the tweezers still come in handy to push that clip down to the bottom of the flange or all the way into a corner. For even harder-to-reach areas, the tweezers may be used for the whole insertion process. But then we have to pay careful attention to the exact way in which the paper clip is held, because we need to be able to apply a moderate twisting force that opens the clip so as to grab a pair of flanges between its two wire loops.

Figure 17 shows two ways that have worked for me. The first one is for a side-wards insertion at a 90° angle (Fig.17a); the second one is for an in-line insertion in the direction of the tweezers (Fig.17b). Note how the clip is gripped just behind the end of the big wire loop. Then, by laying the large loop on top of a flange and pushing the tweezers downwards, the small loop opens downwards and can thus slip behind the targeted flange (Fig.17c).

Figure 17: *Use of tweezers to insert a paper clip: (a) side-wards insertion; (b) grip for in-line insertion; (c) action on a half-OCTA.*

For the inside-out modules of the Decremented RCO I have used another assembly technique. The flanges of the individual modules (Fig.12) stand out so invitingly that one cannot resist using a stapler – particularly if one has run out of paper clips on a Sunday evening. Also for attaching such modules to one another and for forming small subassemblies, the flanges are still readily accessible to a stapler while the structures are still pliable (Fig.18).

Figure 18: *Construction with staples: (a) stapling together the flanges of two adjacent 3-sided pyramids; (b) the two connected pyramids folded open towards their eventual relative position.*

Acknowledgements

Thanks to "Uncle Dan" for creating the neat OCTA-TETRA system and for sending me an evaluation kit.

References

[1] C. H. Séquin, *Lost in Triangulation ??*: – http://www.cs.berkeley.edu/~sequin/X/Leonardo/leonardo.html

[2] D. Suttin, *The OCTA-TETRA Construction Set*: – http://www.homespun4homeschoolers.com/octa-tetra.htm