Solid State Tesla Coils and Their Uses

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Research Project

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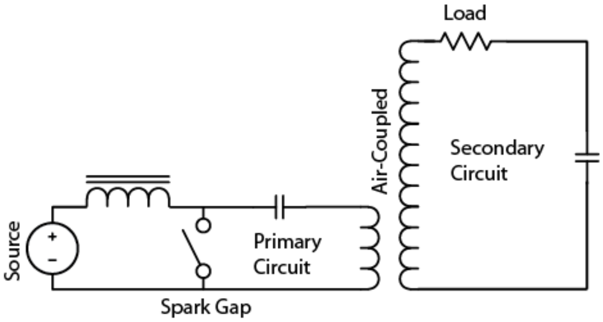
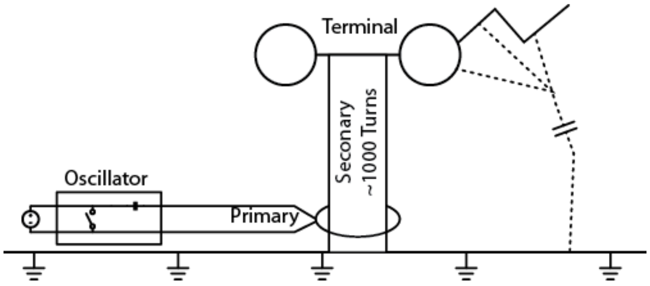
Abstract – The solid state Tesla coil is a recently-discovered high voltage power supply. It has similarities to both the traditional Tesla coil and to the modern switched-mode flyback converter. This report will document the design, operation, and construction of such a system. Possible industrial applications for the device will also be considered.

I. INTRODUCTION – THE TESLA COIL

Around 1891, Nikola Tesla designed a system consisting of two coupled resonant circuits. The primary circuit is a spark gap oscillator that can be charged from a high voltage power supply. When the voltage across the capacitor is sufficient to turn on the spark gap, the primary circuit begins to oscillate and transfer its energy to the secondary resonator. After the transfer is complete, the spark gap turns off so that the primary circuit can re-charge for the next cycle. In this manner, each cycle adds energy to the secondary circuit.



The system was originally conceived as a wireless transmitter, and was indeed used for this purpose until the 1920s. Since then, most interest in the device has centered on its ability to generate high voltages across the secondary circuit load. One physical arrangement of the circuit has proven to be especially convenient for generating electric arcs, streamers, and corona discharge. A secondary coil is wound as a single-layer solenoid and is oriented in a vertical configuration. A metal terminal with a large surface area is affixed to the top of this coil, and serves as one plate of a capacitor in the secondary resonant circuit. The base of the secondary coil is connected to the ground, which forms the other end of the capacitor. If sparks are emitted from the metal terminal, they can be modeled as a resistive load, completing the secondary RLC circuit.

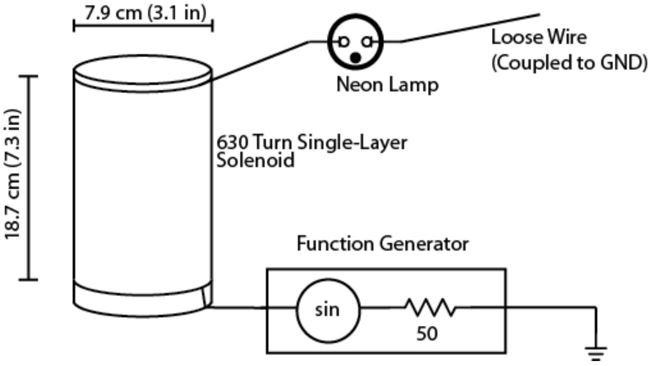


For reasons that will be discussed later, the traditional Tesla coil now has very few practical uses other than the production of sparks and special effects. Nevertheless, numerous hobbyists and professional engineers have continued to study the machine and produce improvements over the original design. As a result, modern technology has found its way into an age-old invention. Recently, it has become practical to replace the spark gap of a Tesla coil with an entirely different type of switch.

II. THE SOLID STATE TESLA COIL (SSTC)

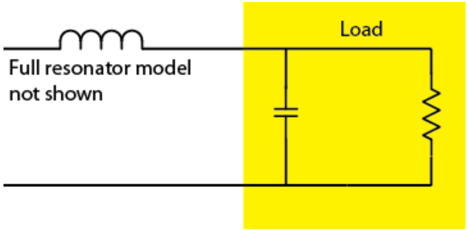
If a solid state inverter is used to feed energy into the system, the primary coil no longer needs to be driven by a resonant circuit. In fact, it can even be eliminated entirely. This is known as the base feed drive method. It has some major shortcomings, but provides a starting point for analyzing the behavior of the more practical two-coil SSTC.

The base-fed SSTC is nothing more than an air-core inductor connected to a signal generator that is set to the coil's resonant frequency. A simple low-power experiment can demonstrate the behavior of such a system. In the following circuit, a 1V RMS, 637kHz sine wave is used to power a 50V neon indicator lamp. The resonator coil is wound with a single layer of 30AWG copper magnet wire on a plastic tube.



The neon lamp circuit provides a very convenient method for measuring the resonant frequency of a solenoid if an oscilloscope is not available. It also provides additional insights that will be useful when designing circuits for high power tests.

The voltage rise that enables the neon lamp to turn on can be explained by modeling the system as a narrowband impedance matching circuit.



The neon lamp has both capacitive and resistive components. It is in series with the coil, which has a large inductive component. Since most high voltage loads consist of electrodes separated by a gas or vacuum, this voltage rise effect applies to other Tesla coil circuits as well. It is, however, difficult to predict the exact voltage rise unless detailed models of both the load and the coil are available.

As mentioned before, the resonant frequency of the coil connected to a neon lamp has been found to be 637kHz. Is this the only resonant frequency? It would be if the coil was an ideal inductor. However, it turns out that that 637kHz is actually the quarter wave resonant frequency of the system, and that the neon tube will also light up at 1570kHz if the drive voltage is increased slightly. An oscilloscope was used to find an additional resonant mode at 2984kHz. It is important to note that these higher resonant modes do not occur at exact integer multiples of the quarter resonant frequency. Therefore, if a square voltage waveform is used to drive the coil (as it will in a later section), the harmonics do not excite the resonator. This means that although the drive voltage is a square wave, the current is a perfect sinusoid, as would be expected for a simple RLC circuit.

In practice, the Tesla coil resonator is only operated at the quarter-wave resonant frequency, and is modeled as a series LC circuit. The inductance can be estimated using Wheeler's approximation for a single-layer solenoid, where r and l are in inches and n is the number of turns [1]:

$$L = \frac{r^2 * n^2}{9 * r + 10 * l}$$

By this method, the inductance of the resonator is found to be 11.0mH. This number agrees with the measured value of 12.04mH.

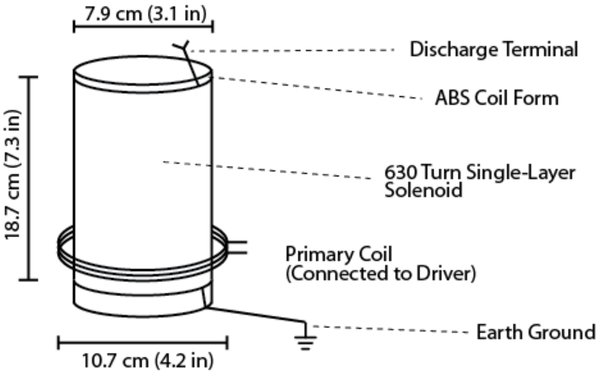
The resonant frequency of the system is more difficult to estimate because several factors contribute to the effective capacitance of the coil and the load. ETesla6 is an open-source program that uses a finite element simulation to provide a reasonable estimate [2]. More information is available from the Tesla Secondary Simulation Project [3]. The simulated resonant frequency is 669kHz, with a capacitance of 5.03nF. The measured resonant frequency of this coil is approximately 660kHz when no load is connected. It drops to around 640kHz when large streamers are emitted from the discharge terminal, and can go much lower if a load with a high capacitance is used. For example, if the free end of the neon lamp is instead connected directly to ground, the resonant frequency drops to 410kHz.

To summarize, the resonant circuit of a solid state tesla coil will be modeled as a driven RLC circuit. To characterize the RLC circuit, three parameters must be identified. The inductance can easily be found by calculation or measurement, and the capacitance can be determined from the resonant frequency. The resistance is a characteristic of the load, but may be subject to an impedance transformation if it is in parallel with a capacitor, as is often the case. In practice, the system can be built without knowledge of the exact value of this resistance, and can then be adjusted to provide an appropriate output power.

The first objective of this project is to design a power supply capable of delivering hundreds of watts of power at approximately 100000V. The base-feed method is not practical for such a task. Although it has been demonstrated that the resonator is able to produce a step-up effect by itself, the exact voltage produced by such a circuit depends heavily on the characteristics of the load, is difficult to predict, and is limited to a factor of around 1:50. Much better results can be achieved by using the transformer action between a short primary coil and a long secondary coil. A step-up ratio of around 1:100 can be achieved using this method, and this will be multiplied by any resonant voltage rise that is also present.

III. THE AIR CORE TRANSFORMER

The transformer described in this section is designed to generate electric sparks, and can also be used to power a gas tube. It consists of the resonator described in the previous section, along with a primary coil. The primary coil is wound with 3 turns of 14AWG wire, and can be moved up and down to provide more or less mutual inductance. The top of the secondary coil is left disconnected, and is fitted with a small piece of wire that causes streamers to form from the sharp points.



One of most important parameters for an air core transformer is the coupling coefficient, which is closely related to the mutual inductance by the following equation [4]:

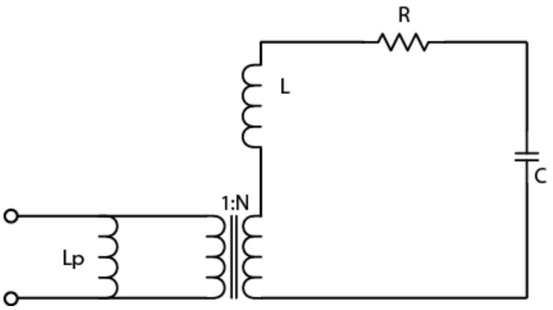
$$M = k\sqrt{L_{pri}L_{sec}}$$

Once the coupling coefficient is determined, the RLC resonator model explained in the previous section can be modified for use with the primary drive method. For modeling purposes, the inductance of the resonator is divided into two parts:

$$L_{s1} = k^2 L_s$$

$$L_{s2} = (1 - k^2) L_s$$

The first portion of the inductor forms the secondary coil of an ideal transformer that is fully coupled to the primary coil. The remaining fraction of the coil is the leakage inductance, which is named L in the new model:



As predicted by the MandK simulation program [2], the maximum coupling coefficient for this coil is around 0.35, and is achieved if the primary coil is a few inches above the base of the secondary. If the objective is to generate very high voltages, a high coupling coefficient is essential. The coupling coefficient represents the fraction of the secondary coil that is magnetically connected to the primary. Since we have 630 turns and a coupling coefficient of 0.35, we can model the air-core transformer as an ideal transformer with 3:221 turns in series with a leakage inductance L=10mH. This transformation is described in detail by Richard Burnett [5].

A short primary coil is used because this maximizes the turn ratio, resulting in high output voltages. Unfortunately, this advantage comes with a major tradeoff. The inductance of such a short primary coil is only 1.8uH. This number is known as the magnetizing inductance, and is much lower than that of a comparable ferrite transformer. Very large currents will flow through such a small inductor, which appears in parallel to the useful portion of the circuit model. This can cause real power to be dissipated in the driver circuit, especially during switching. If one end of the primary coil is driven with a 660kHz, 160Vp-p square wave, the magnetizing current is 16.8A, a much larger quantity than is commonly encountered with ferromagnetic cores. If the number of primary turns is decreased in an attempt to achieve a larger turn ratio, the magnetizing current for this drive waveform will rise to unacceptable levels.

$$I_{mag-peak} = \frac{V}{8fL}$$

Design of an air-core transformer is similar to that of a ferromagnetic transformer, but two additional tradeoffs must be considered.

As always, one objective is to minimize the AC resistance of the windings. This makes a large wire diameter desirable. However, if the wire diameter is too large, there will be fewer windings and the leakage inductance will be very low. This is actually undesirable because it will result in a high resonant frequency. This will increase the AC resistance because it will decrease the skin depth of the copper. The problem may be mitigated by increasing the capacitance of the load. For example, Tesla coils are often fitted with a toroidal output terminal because this structure has a very high capacitance to ground.

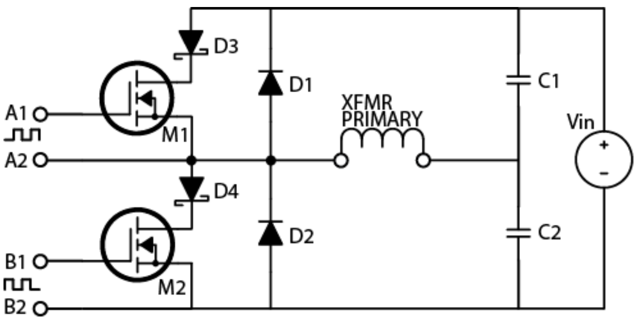
Special attention must also be paid to the design of the primary coil. First, it must be tightly coupled to the secondary coil. Second, the number of turns must be carefully chosen. A high inductance is desirable, but if too many turns are used in the primary coil, the device will no longer be able to produce high voltages.

These considerations apply mainly to single-layer solenoid transformers, which are simple and are able to withstand high output voltages. If a more compact structure is needed, it is also possible to wind an air-core coil using a multi-layer configuration. Such a transformer has actually been used commercially as a plasma arc initiator [6]. The same RLC model as that of the single-layer transformer can be used, but the resonant frequency and coupling coefficient will be more difficult to predict.

The air core transformer described in this section is able to produce high voltages, but must be driven at a high frequency. Therefore, the next step is to construct a circuit capable of generating the 660kHz, 160V drive waveform. SSTC enthusiasts have attempted to use several different circuit topologies for this purpose, and have found the H bridge to be the most robust option [7].

IV. THE HALF H BRIDGE

The system described in this section is based on a design by Richard Burnett [8]. It is similar to the resonant converter topology that is often used to drive induction heaters [9]. It is very simple to construct, and provides an excellent starting point for high power experiments. The maximum tested input voltage is 200VDC.

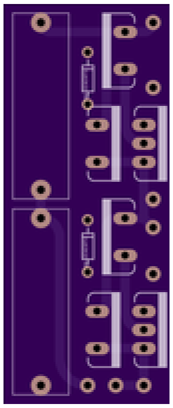


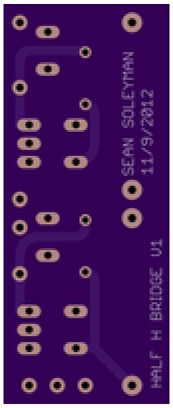
The two transistors are switched in opposition at just under 50% duty cycle each, driving one end of the primary coil with a square voltage waveform.

There is a short interval during which both transistors are turned off. This is known as the dead time. If the MOSFETs are both open-circuited while current is flowing through the primary coil, a large voltage spike will be created. This is why clamping diodes need to be placed in parallel with the transistors.

The MOSFET body diode is unacceptable for this purpose because it has a high reverse recovery time. If these diodes were to turn on, they would not be able to turn off in time for the next part of the cycle, and would short-circuit the two power supply rails. Instead, their operation is blocked by D3 and D4. Fast recovery rectifiers D1 and D2 can then be added in parallel.

All circuit boards used in this project were drawn in Eagle and fabricated by OSH Park. The layout was designed to minimize the leakage inductance of traces that carry large RF currents.





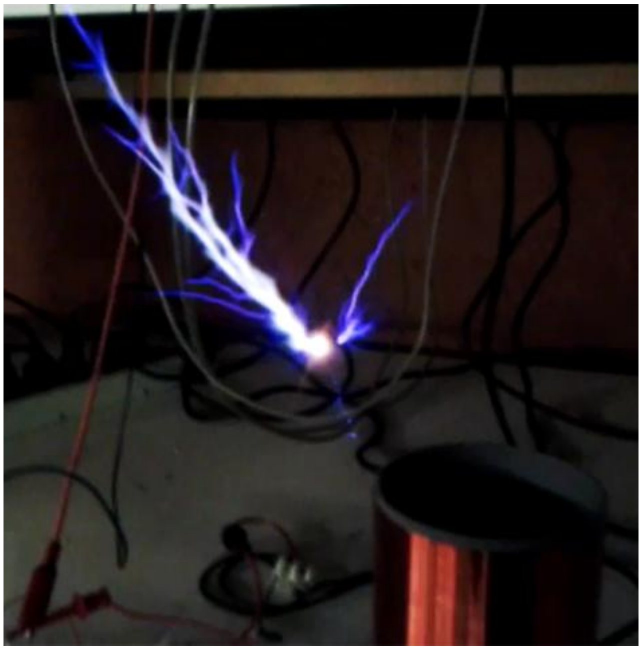
M1 and M2 are FDP33N25 MOSFET devices. They are rated at 250V, 33A. MOSFETs with lower current ratings were also tested, and worked very well. Switching devices with a large gate capacitance should be avoided because this will make fast switching times difficult to achieve.

The diodes D1 and D2 are BYC10DX-600 rectifiers with a reverse recovery time of 18ns. D3 and D4 are 15A Schottky diodes.

Film capacitors C1 and C2 are used to create a voltage divider for the non-driven end of the primary coil. They are 0.47uF, 275V.

The circuit can run directly from rectified 120VAC line current. If half wave rectification is used and no smoothing is applied, sword-like sparks are observed. This method produces a pulsed output of high peak power with reduced transistor heating. The system can run for several minutes with very small heat sinks. Most of the heating that does occur is due to the high magnetizing current of a 3-turn primary coil. Much greater efficiency could be achieved by using a larger number of turns, because the magnetizing current is actually the only drive current component that contributes significantly to switching losses. If the magnetizing current is eliminated and the circuit is perfectly tuned, the output current of the driver will be a sinusoid with zero crossings at the switching times.

The maximum spark length is estimated to be 15 cm. Since the discharge terminal is sharp, this corresponds to an output voltage of around 100kV [10]



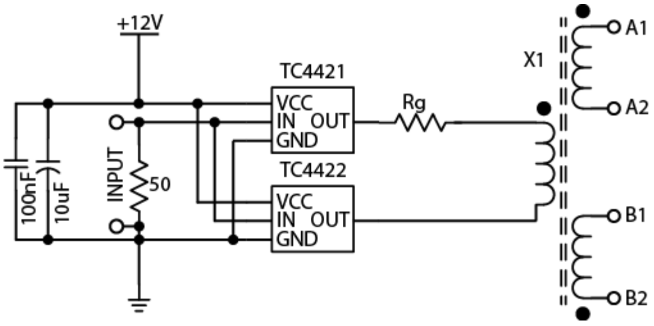
If a continuous output waveform is required, full wave rectification and smoothing can be used instead. Power draw is difficult to measure due to the distorted input current waveform of this experimental setup, but is estimated to be approximately 600W.

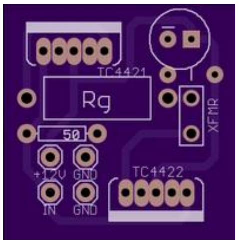
This circuit can be scaled up to a full H bridge, where both ends of the coil are driven in opposition. The effect is to double the voltage across the primary coil, thereby also doubling the output voltage without increasing the step up ratio of the air core transformer itself. Such a circuit can be constructed from two of the half bridges used here, although a longer coil must be used to prevent arcs from striking the primary coil and destroying the semiconductors. Even with a half bridge, arcs were observed between the final two windings of the secondary coil.

The only disadvantage of the H bridge is the need for a complex gate drive circuit. The source of the high side device is connected to the primary coil, and experiences voltage swings of 160VAC. This means that if a low voltage circuit is used to drive the transistor, it must be protected by an isolation transformer.

V. TRANSFORMER-COUPLED GATE DRIVE

The design of the gate drive system is actually more complicated than the design of the H-bridge itself. For optimal performance, the gate drive transformer needs to have a 1:1 turn ratio. However, the power MOSFET gate voltage needs to swing between +12V and -12V, and inexpensive gate drive integrated circuits are unable to achieve a 24V output swing. The solution is to use a double-ended gate drive circuit, which is itself a full H bridge [11].





The TC4421 (inverting) and TC4422 (non-inverting) drivers were chosen because of their low cost and high power handling capability. The gate drive transformer consists of 10 trifilar windings on an FT-50A-J core from Amidon Associates. This gate drive system was based on a design by Jan Wagner [12]. The most important design objective was to achieve low leakage inductances by using short traces, a small number of turns, and trifilar windings. If magnet wire is used, care must be taken to prevent damage to the insulation by the ceramic core. Wrapping wire is recommended as a more rugged alternative.

It is necessary to verify that the transformer does not exceed the saturation flux density of the chosen core [13]. The flux density can be calculated as follows. V is the voltage across the primary, t is the period times the duty cycle, N is the number of primary turns, and Ae is the core cross section.

$$\Delta B = \frac{V * t}{N * A_e}$$

The FT-50A-J core has a permeability of 5000, a saturation flux density of 0.43T, and a cross section of 0.152 cm^2. For V=12, t=0.5/660000, and N=10, we get ΔB = 0.06T, which is well below the acceptable limit.

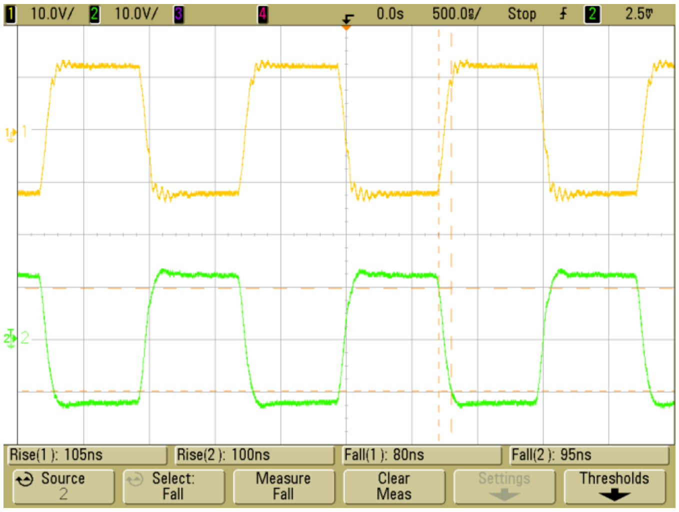
The magnetizing current should also be calculated.

$$L_{mag} = A_l * N^2$$
$$\Delta I_{mag} = \frac{V * t}{L_{mag}}$$

For this design, Lmag = 500uH and ΔImag = 18mA. These figures indicate that it would be possible to further decrease the size of the core and the number of turns, but that would preclude the operation of the circuit at lower frequencies.

The J type material works very well at 660kHz. Distortion due to harmonic attenuation is barely noticeable. For higher frequency operation, a material with higher volume resistivity would be preferred.

The gate resistor Rg creates a major tradeoff between switching time and overshoot. Since leakage inductance was successfully reduced in the previous design steps, a 5 Ohm Rg results in sufficiently low ringing. An even lower value could probably be used, but it is possible that increased ringing may occur when the 160VDC power supply is connected. This complication is caused by the Miller effect, which can effectively increase the gate capacitances [11]. With Rg=5, switching times are around 100ns, providing good efficiency at 660kHz.



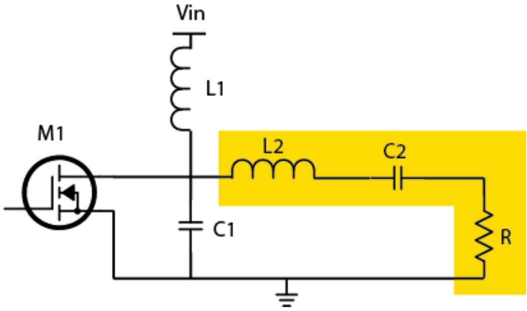
Rise(1): 105ns
Rise(2): 100ns
Fall(1): 80ns
Fall(2): 95ns

This concludes the description of the H bridge system. The next section describes a system that operates at much higher frequencies, and is therefore more compact.

VI. 4MHz Class E Amplifier HFSSTC

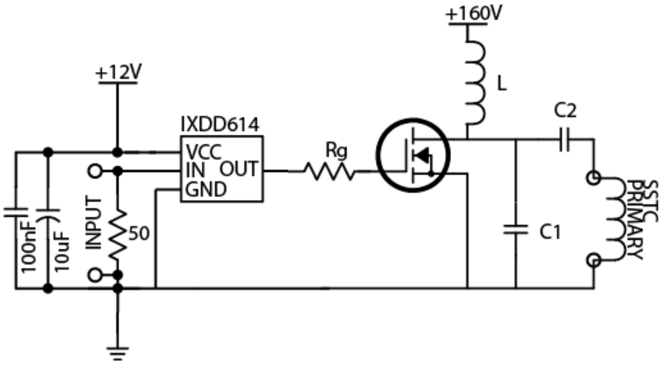
A class E Tesla coil was also constructed as part of this project, although it was not as successful as the H-bridge coil in the previous section.

The class E power amplifier was developed by Nathan and Alan Sokal in 1972 [14]. It is a single-transistor narrowband amplifier that allows for high efficiency operation even when the switching time is a significant portion of the cycle period. The following example circuit is one possible implementation of the class E amplifier.



It is possible to replace the highlighted part of the circuit with the primary coil of an air core transformer. One major design decision is the choice of gate drive method. In the traditional class E amplifier configuration, the transistor is driven with a sinusoidal waveform. Often, multiple class E stages are cascaded so that a very weak input signal can be amplified.

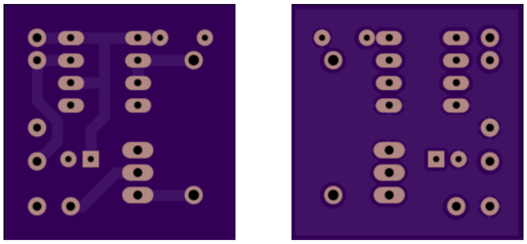
The other option is to use a gate drive integrated circuit. Most gate drive integrated circuits are optimized for the 100kHz to 1MHz operating range, but it is actually possible to use them at much higher frequencies. The IXDD614 IC has even been used in high power shortwave radios [15]. This drive method was chosen because it requires fewer components than a cascade of class E stages.



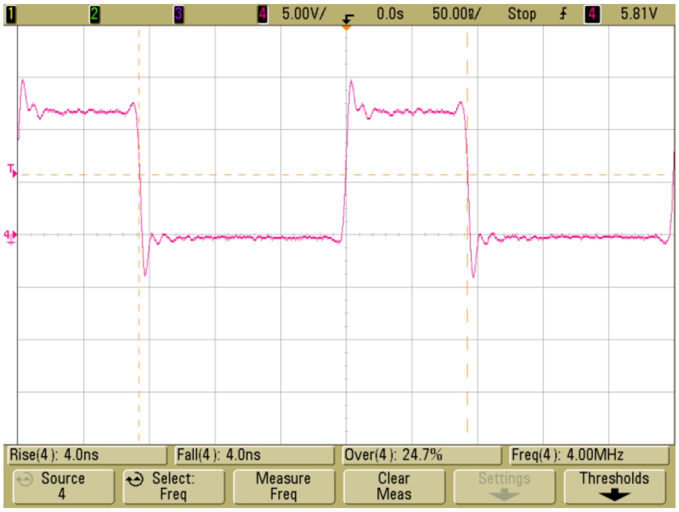
The transistor is an FDPF16N50 500V, 16A device. C1 is 560pF, and C2 is a DC blocking capacitor with a nominal value of 10nF. L is a DC choke with a nominal value of 100uH. The primary coil was 5 turns of 20AWG wire, and the secondary was single-layer of 22AWG magnet wire with a diameter of 6.1cm and a length of 9.5cm.

The system was designed for 160V operation, but the transistor failed when the circuit was supplied with more than 75VDC. Also, the circuit resonated at 3.8MHz instead of the predicted 4MHz. Sparks were produced, but were not of very high power. The low performance can most likely be attributed to the board design.

The gate drive board was designed using a ground plane and a very compact layout. It seems that there was insufficient isolation between the class E load network and the gate drive circuit. Capacitive coupling between the low voltage VCC supply for the IXDD614 and the drain of the MOSFET is suspected. Spurious oscillations were sometimes observed during switching.



When Vin is disconnected, acceptable gate drive waveforms are obtained for frequencies of up to 10MHz. These waveforms are degraded significantly once the power supply is connected to the DC choke. The non-corrupted waveform is shown below for 4MHz.



Rise(4): 4.0ns
Fall(4): 4.0ns
Over(4): 24.7%
Freq(4): 4.00MHz

Further experimentation with the class E Tesla coil was abandoned because it was determined that this system is not as practical as the much more simple H bridge coil. Although a small coil has some advantages, it is very easily detuned. Auto-tuning class E coils have been constructed by other experimenters, but they are not able to maintain class E efficiency if they drift far enough away from their designed operating frequency. Furthermore, high frequency radio emissions are more likely to cause FCC compliance problems than those in the 100kHz 400kHz range, and the skin effect also becomes a major problem for a 4MHz system. One major objective of this project is to evaluate the suitability of the SSTC for commercial applications, and the H bridge system is clearly more practical.

VII. COMMERCIAL APPLICATIONS

The concept of using a Tesla coil to power high voltage devices is not at all new. In the early 1900s, it was common for scientists to use Tesla coils to drive X-ray and gas filled tubes [16]. This setup fell into disfavor not because of limitations of the air core transformer, but because of the impracticality of the spark gap. Spark gaps produce broadband radio emissions that interfere with radios at a significant distance.

The SSTC partially solves this problem. The electromagnetic emission of the SSTC is of very narrow bandwidth, and does not seem to cause major interference with commercial radio equipment. In terms of potential electromagnetic interference, the SSTC is very similar to the RF induction furnace, which operates at up to 400KHz and is commonly found in industrial plants. Induction chargers are also able to drive air core coils without causing major radio interference. The electromagnetic emission of the SSTC must be considered, but is probably not prohibitive.

The traditional Tesla coil fell into commercial disfavor for other reasons as well. The spark gap is an inefficient switch, and the wasted energy is not dissipated in the form of benign heat. Spark gaps can produce loud noises, UV radiation, ozone, and other hazards. In contrast, the H bridge driver is very efficient. It is also much more reliable than a spark gap.

The most important benefit of the SSTC is its continuous wave output. Since the oscillator of a traditional Tesla coil needs to be recharged, it provides pulsed output power. The SSTC output is a very clean sine wave.

An additional problem with the traditional Tesla coil is the complex tuning procedure needed to achieve optimal performance. If two LRC resonant circuits are used, one of them must be tuned to match the other. This has to be done by tapping the primary coil at the correct point.

Since the SSTC described in this report is controlled by a function generator, adjusting its operating frequency is as simple as turning a knob. In fact, tuning the SSTC can be even easier than that.

VIII. DYNAMIC TUNING

Proper tuning is essential for achieving good power output from the SSTC. The resonator is a high-Q RLC circuit. If it is not driven at the correct frequency, the driver will see it as a high-impedance reactive load.

More importantly, proper tuning provides major efficiency benefits. The system described in this report is driven by an H bridge square wave generator. As described before, the resonator only responds to the fundamental frequency of the square wave. If the magnetizing current is ignored, this square wave gives rise to sinusoidal current flow at the fundamental frequency.

If the sinusoidal current flow is in phase with the voltage waveform, no current will be flowing through the H bridge during the switching intervals. However, even a minor tuning error will introduce a phase shift between the voltage and current waveforms. This eliminates the soft-switching effect.

It is very difficult to maintain the resonant frequency of the system to the required tolerance. If the circuit is being used to generate sparks, the sparks themselves can decrease the resonant frequency of the system by introducing additional capacitive coupling paths to ground. The situation is even worse if a high voltage load is connected to the coil. If the load has a very high capacitance, it may lower the resonant frequency of the coil by an order of magnitude.

The solution is to alter the drive frequency to maintain resonance. Several feedback-based methods for dynamic tuning have been developed.

The simplest solution is to measure the electric field near the resonator by using a small antenna [12]. This signal can be fed directly to the MOSFET drivers, turning the system into a simple oscillator.

Performance with this method is surprisingly very stable, but a starting pulse is needed. Alternatively, a feedback coil can be used to measure the magnetic field near the resonator. With the class E coil, it is even possible to drive the MOSFET directly using this method, eliminating the need for gate drive circuitry.

The disadvantage of such an approach is that delays will inevitably be introduced in the feedback path, and will cause the operating frequency to be slightly lower than the true resonant frequency. A more sophisticated solution uses a phase locked loop to adjust the operating frequency until the primary current is in phase with the primary voltage. Such systems are still being developed.

IX. CONCLUSION

Although the traditional Tesla coil has been removed from most commercial applications outside of the special effects industry, the SSTC may prove to be a much more practical device. The air core transformer described here uses only 300 feet of 30AWG magnet wire, and is much less costly than a comparable ferrite transformer. The H bridge driver is also fairly inexpensive to construct. Like the traditional Tesla coil, the SSTC can be used as a power supply for UV lamps, X-ray tubes, and other high voltage components. It can do this without introducing the disadvantages of a spark gap.

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