A New Derivation and Dataset for Fitts' Law of Human Motion

Ken Goldberg
Siamak Faridani
Ron Alterovitz

Electrical Engineering and Computer Sciences
University of California at Berkeley

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RESEARCH ARTICLE

A New Derivation and Dataset for Fitts' Law of Human Motion

Ken Goldberg[a*] , Siamak Faridani[b] and Ron Alterovitz[c]

[a]*Professor, UC Berkeley and UC San Francisco, CA, goldberg@berkeley.edu*;
[b]*Researcher, Microsoft, WA, faridani@live.com* [c]*Assistant Professor, University of North Carolina at Chapel Hill, NC, ron@cs.unc.edu*;

Human motion models for reaching tasks facilitate the design of many systems such as computer and cellphone interfaces, cockpits, and assembly lines. Fitts' Law specifies a logarithmic two-parameter relationship between motion duration and the ratio of target distance over target size, and more recent models consider square root and modified logarithmic relationships. This paper contributes new theory and new experiments. For the former, we provide a succinct derivation of the square-root model based on optimal control theory. Our derivation is intuitive, exact, makes fewer assumptions, and requires fewer steps than prior derivations. We present data from two experimental user studies, one a controlled (in-lab) study and the second an uncontrolled (online) study with a total of 94,580 timing measurements. We consider three two-parameter models that relate motion duration to the ratio of target distance over target size: LOG (the classic logarithmic function), SQR (square-root), and LOG' (logarithmic plus 1.0). We find that: (1) the data from the controlled and uncontrolled studies are remarkably consistent; (2) for homogeneous targets, the SQR model yields a significantly better fit than LOG or LOG', except with the most difficult targets (i.e., the ratio of target distance over target size is large) where the models are not significantly different; (3) for heterogenous targets, SQR yields a significantly better fit than LOG for easier targets and LOG yields a significantly better fit for more difficult targets, while the the LOG' model yields a significantly better fit than both LOG and SQR on more difficult targets.

Keywords: Fitts' law; human-computer interfaces; time and motion studies; human movement time

Statement of relevance:

1. Introduction

Many tasks from playing games to working on an assembly line to piloting a jet require timely and accurate human reaching movements between targets. In particular, many computer interfaces require users to move a mouse or related input device to manually direct a cursor to targeted areas (e.g., menu items, buttons) on a computer screen. Some motions can be performed more efficiently than others. To facilitate efficient human-computer interfaces, designers seek accurate predictive models of human reaching motion.

The inherent tradeoff between speed and accuracy of such movements was first quantified by Paul Fitts of Ohio State University in 1954 (Fitts 1954). Fitts studied reaching movements between "homogeneous targets" of fixed size and distance, which are common in industrial settings for tasks ranging from installing parts on an assembly line to stamping envelopes in an office. In a series of experiments, Fitts required human subjects to repetitively move a stylus between two fixed contact plates as quickly as possible for 15 seconds. Fitts set the width W of the plates and the amplitude (distance) A between the plates which were constant during each experiment and varied between experiments. Fitts measured the time T required to move back and forth between the two targets for 16 human volunteers.

Inspired by Shannon's Information Theory, Fitts empirically fitted a logarithmic model to the data yielding the now-classic "Fitts' Law". Since then, many researchers have repeated these experiments under varying conditions and proposed alternative models and derivations based on human perception and physiology.

This paper contributes new theory and new experiments. For the former, we reconsider the square-root model and provide a succinct derivation based on optimal control theory. Our derivation is intuitive, exact,

*Corresponding author.

Figure 1. Using an applet, sequences of rectangular and circular targets are presented to users, where target distance A and width W can remain constant (homogeneous) or vary (heterogeneous) after every click.

makes fewer assumptions, and requires fewer steps than the derivation presented in Meyer *et al.* (Meyer et al. 1988). We review related work in the next Section.

Since Meyer *et al.* used the original Fitts' data and performed new experiments with four human subjects performing wrist rotation movements to heterogenous targets, we undertook two comprehensive user studies to gather data on humans performing computer screen cursor motions. We designed and implemented a Java-based applet that can be easily downloaded from the web. It presents users with a sequence of visual targets for users to click on, records completion times, and sends the data back to our lab. Our first study is a standard controlled (in-lab) user study with volunteers using the applet with identical mouse and settings. Our second study is a uncontrolled (web-based, "in the wild") study based on an indeterminate number of volunteers who visited the website (many visit more than once) and used a variety of mouse types and settings.

Uncontrolled (also known as "in the wild") studies on the web do not provide the consistency of controlled in-lab studies but can collect data from large numbers of diverse human participants and are gaining acceptance, especially when confirmed by controlled experiments (Bigham et al. 2010, Bakshy et al. 2012). Uncontrolled studies gather data in a variety of settings with perhaps greater ecological validity than found in a laboratory.

As with any study design, there are disadvantages as well for a web-based uncontrolled study. One cannot obtain detailed data about the users, some users may perform the experiment multiple times, and one has no control over the user environment nor the input and display devices. In a survey, Andreasen et al. (Andreasen et al. 2007) systematically compare controlled and uncontrolled (web-based) usability studies and find that synchronous studies (with a live remote human monitor) are more reliable than asynchronous studies (akin to our uncontrolled experiments) but that both en- able collection of use data from a large number of participants. They note that "it would be interesting to perform comparative studies of remote usability testing methods" against controlled studies.

Uncontrolled experiments are gaining acceptance in the Computer Human Interaction community. Our uncontrolled study was motivated by our desire to learn how the results might vary between a controlled lab setting and online with many different experimental environments. We were surprised to find that data from the controlled and uncontrolled studies were remarkably consistent.

2. Related Work

There is a vast body of related work on this subject and we regret our inevitable errors of omission.

2.1 *Classic Fitts' Law*

In "choice reaction time tasks," a set of stimuli are assigned unique responses, and participants must give the correct response when receiving the stimulus (Jagacinski and Flach 2003). In 1885, J. Merkel designed an experiment in which the stimulus was a number selected from a set of size N with uniform probability and the participant was required to press a key corresponding to the number (Jagacinski and Flach 2003). As N increased, so did the reaction time T_R. Merkel found that the reaction time increased by a constant for every doubling of the set of possible numbers.

In 1948, Claude Shannon published the foundational paper on Information Theory, defining the information capacity for a communication channel, C, as:

$$C = B \log_2 \left(\frac{S+N}{N} \right), \tag{1}$$

where B is the channel bandwidth, S is signal strength, and N is noise power. Shannon also defined the information I of a symbol based on the probability of receiving the symbol, $I = \log_2 \frac{1}{p}$.

Adopting Shannon's model, Merkel's reaction time can be viewed as proportional to the amount of "information" received by the participant:

$$T_R = a + b \log_2 N$$

where a and b are experimentally determined constants.

In 1953, J. Hyman extended Merkel's work to the case where an element i in the set of possible numbers was selected with non-uniform probability p_i (Hyman 1953). Hyman found that the average reaction time was consistent with the Shannon's model (Jagacinski and Flach 2003).

In 1954, Fitts hypothesized that the information capacity of the human motor system is specified by its ability to produce consistently one class of movement from among several alternative classes of movements (Fitts 1954). Fitts then defined the difficulty of a task based on the minimum amount of "information" required to complete it on average. For the "tapping task", Fitts defined a tap between two targets of distance (amplitude) A with width of W as a movement class. Using Shannon's definition of information as a guideline, Fitts defined the *index of difficulty* (I) to be the "information" transmitted during the task:

$$I = \log_2 \left(\frac{2A}{W} \right).$$

Fitts noted that the choice of the numerator for this index "is arbitrary since the range of possible amplitudes must be inferred," so $2A$ was selected rather than A to ensure that the index is positive in "all practical situations."

Fitts then modeled movement time T as a linear function of the "information" transmitted, producing his classic two-parameter Logarithmic model:

$$T = a + b \log_2 \left(\frac{2A}{W} \right). \tag{2}$$

In this paper we refer to this as the LOG model.

2.2 *Alternative Information Theory Models of Human Motion*

In 1960, Welford proposed a revised model based on a Weber fraction where the user must select "a distance from a total distance extending from his starting point to the far edge of the target" (Welford 1960, 1968). For some constant b, Welford's formulation is given by:

$$T = b \log \left(\frac{A + \frac{1}{2}W}{W} \right) = b \log \left(\frac{A}{W} + 0.5 \right).$$

Researcher I. Scott MacKenzie developed a variation on Fitts' model that is more closely based on Shannon's Theorem (MacKenzie 1992). MacKenzie's communication channel model considers noise N to be the variation around a specific signal S, so the signal strength equals the movement amplitude ($S = A$) and the noise equals the width ($N = W$). By analogy to Shannon's model (equation 1), movement time is given by:

$$T = a + b \log_2 \left(\frac{A}{W} + 1 \right). \tag{3}$$

In this paper we refer to this as the LOG' model.

Other researchers such as Crossman and Welford (Welford 1968) explore alternative modifications to Fitts' model using communication theory. We welcome fellow researchers to apply such models to the dataset we provide.

2.3 *Applications of Fitts' Law*

Although Fitts' Logarithmic model was originally developed for industrial pick-and-place tasks (Fitts 1954), it has been applied to a variety of human reaching movements. The first application of Fitts' Logarithmic model to Human Computer Interaction dates from before the commercialization of modern personal computers. Card, English, and Burr at Xerox PARC studied the relative speed of four input devices: mouse, joystick, step keys, and text keys (Card et al. 1978). They found that Fitts' Logarithmic model accounts for the variation in movement time to select text on a CRT monitor using mice and joysticks. Subsequent studies applied Fitts' Logarithmic model to pen input devices (MacKenzie et al. 1991). Fitts' Logarithmic model has also been applied to robotics applications including telemanipulation tasks with remote video viewing (Drascic 1991) and pairs of participants performing tapping motions using a robot manipulator (Reed et al. 2004).

Friedlander et al. found that a linear model for movement time fits selection in a non-visual (tactile or auditory) bullseye menu more closely than Fitts' Logarithmic model (Friedlander et al. 1998). Also, Kristensson proposes using context information, such as pattern recognition of likely key presses on a stylus keyboard, to develop input devices that increase the speed of input beyond what would be predicted by Fitts' Logarithmic model (Kristensson 2005).

Plamondon and Alimi review a number of studies of speed/accuracy trade-off models and their applications (Plamondon and Alimi 1997). They categorize the experimental procedures used for the speed/accuracy trade-offs into two different categories: spatially constrained movements and temporally constrained movements. For the procedures in the first category, distance (A) and the width (W) are usually given and the time (T) is measured. In the temporal group, movement time is given and the accuracy of reaching the target is being measured. With this definition, Fitts' Law falls into the first category. They classify different studies on the Fitts' Logarithmic model based on different types of movements (tapping, pointing, dragging), limbs and muscles groups (foot, head, hand, etc), experimental conditions (underwater, in flight, etc), device (joystick, mouse, stylus, touchpad, etc), and participants (children, monkeys, adults of different ages, etc).

Hoffmann and Hui study reaching movements of fingers, wrist, forearm and shoulder. They show for the cases where an operator can choose which limb to reach a target, the limb with the smallest mass moment of inertia is often used to minimize energy needed to reach the target (Hoffmann and Hui 2010).

2.4 *Alternative Models of Reaching Movements*

Alternative models of human reaching movements consider properties of the neuromuscular system, such as minimizing jerk or a sequential impulse model. Crossman and Goodeve (Crossman and Goodeve 1983) proposed a movement time model based on a sequence of discrete positional corrective motion impulses, which resulted in a Logarithmic model like Fitts' law.

Flash and Hogan developed a mathematical model of voluntary reaching movements based on maximizing the smoothness of trajectories (Flash and Hogan 1985). They propose that the human motor system minimizes jerk, the derivative of acceleration. Using calculus of variations, they derive a polynomial formula for the time integral of the square of the magnitude of jerk is

$$C = \frac{1}{2}\int_0^{t_f} \sum_{i=1}^{n} \left(\left(\frac{d^3 x_i}{dt^3} \right)^2 \right) dt$$

where n is the dimension of the space, x is the vector coordinate of the pointer as a function of time, and t_f is the time to reach the end point. Minimizing this formula results in 5th order polynomials with 6 unknown parameters for each dimension. One can constrain the position of the start and end points and assume the velocity and acceleration are zero at the start and end of the movement, and then solve for the parameters. The resulting trajectories have smooth position and velocity curves qualitatively similar to experimentally measured data. However, this model differs from Fitts' Logarithmic variant because it has more than two parameters and does not explicitly consider the tradeoff resulting from varying the size of a target region.

In 1988, Hoffamann and Gan proposed a model for ballistic arm movements in which the movement time is only a function of the amplitude, $T = a + b\sqrt{A}$ (Hoffmann and Gan 1988). In 1992, Réjean Plamondon proposed an alternative to Fitts' Logarithmic model using a neuromuscular impulse response model (Plamondon 1992, 1995a,b). Plamondon's theory for rapid human movements is based on the synergy between the agonist and antagonist neuromuscular systems (Plamondon 1995a). In his model, the agonist and antagonist systems synchronously receive an impulse input $U_0(t - t_0)$ at time t_0 scaled by D_i, where $i = 1$ for the agonist system and $i = 2$ for the antagonist system. Each system independently responds in parallel to the input with impulse response functions $H_i(t)$ to generate output velocities $v_i(t)$ for $i = 1, 2$. Although the two systems may be coupled in reality, Plamondon assumed the output $v(t)$ of the synergy is obtained by subtracting the two parallel outputs. Plamondon proposed defining the impulse response using a log-normal function, a very general formulation based on 7 parameters that can qualitatively predict a variety of velocity profiles including single peaks, double peaks, triple peaks, asymmetric peaks, and multiple peaks with no zero crossing. Reaching movements from one point to another point terminate at a time T when the velocity of motion $v(T)$ equals zero. Solving the velocity equation for the zeros using constraints set by Fitts' experiment, Plamondon modeled movement time

$$T = K\left(\frac{2A}{W}\right)^{\alpha} \tag{4}$$

with parameters K and α.

Equation 4 defines a power model, an alternative two-parameter formulation based on a fitted log-normal approximation of the velocity profile. We welcome fellow researchers to apply such models to the dataset we provide.

2.5 *Extensions to Fitts' Law*

Fitts' Logarithmic model, which was originally developed for one-dimensional reaching movements, has been extended to the two-dimensional movements that are common in graphical user interfaces (MacKenzie and Buxton 1992, MacKenzie 1995). For general two-dimensional targets, both the shape of the target and angle of approach must be considered. For circular targets, the assumptions of the one-dimensional model remain largely intact with target width W being defined by the circle's diameter[1]. For rectangular targets, Card et al. propose the status quo model defines W by the width of the target while ignoring height. This model can result in a negative index of difficulty for near wide targets (MacKenzie 1995, Card et al. 1978). MacKenzie et al. proposed two models for rectangular targets (MacKenzie and Buxton 1992). The smaller-of model sets W to the smaller of the target width or height. The effective width model sets W by considering an additional parameter: the angle between the start point and the target center. MacKenzie tested the status quo, smaller-of, and effective width models and found that the linear correlation of movement time to Fitts' index of difficulty was significantly greater for both the smaller-of and effective width models compared to the status quo model (MacKenzie and Buxton 1992).

Gillan et al. examined how Fitts' Logarithmic model can be applied to point-drag movement sequences rather than simply point-click operations. They found that Fitts' Logarithmic model must first be applied as the user points to the left edge of the text object and then applied separately for the dragging distance (Gillan et al. 1990).

Accot et al. investigated extensions for Fitts' Logarithmic model for trajectory-based interactions, such as navigating through nested menus, drawing curves, or moving in 3D worlds (Accot and Zhai 1997). They developed a "steering law" similar to Fitts' Logarithmic model except the index of difficulty for steering a pointer through a tunnel is defined by the inverse of the width of a tunnel integrated over the length of the tunnel. They applied the steering law to participants using 5 input devices (tablet, mouse, trackpoint,

[1]In this paper we use the diameter of circular targets for W as suggested by MacKenzie (MacKenzie 1995).

touchpad, and trackball), and the linear correlation of movement time to the index of difficulty for steering exceeded 0.98 (Accot and Zhai 1999). Apitz et al. build a crossing-based interface, CrossY (Apitz et al. 2008). Unlike point-and-click interfaces, crossing-based interfaces allow participants to trigger an action by crossing a target on the screen instead of clicking on it. Apitz et al. show that a crossing task is as fast as, or faster than a point-and-click task on for the same index of difficulty (Apitz et al. 2008).

In a related line of research, Wobbrock et al. derive a predictive model for error rates instead of mean times (Wobbrock et al. 2008). Error rate models have practical applications in designing text entry devices and video games (Wobbrock et al. 2008).

2.6 *Input device settings and Fitts' Logarithmic Model*

The computer mouse and other pointing devices usually offer configurable parameters that adjust the mapping between movement of the device and movement of the cursor on the screen. The most common adjustment is mouse speed, a type of "control-display gain" (Moyle and Cockburn 1991). The control-display gain scales the distance d the mouse moves on the table to a distance p in pixels that the cursor moves on the screen. The setting of the gain can have a significant impact on movement time to a target. Thompson et al. experimentally verified that lower gains are better for low amplitude or small target movements while higher gains are better for large amplitude or large target movements (Thompson et al. 2004). This mixed result makes it difficult to select a single optimal gain for standard computer usage. Blanch et al. introduce semantic pointing, a technique that improves target acquisition by decoupling the visual size of a target from the motor size of the target by dynamically adjusting the control-display gain when the cursor moves over a target (Blanch et al. 2004). The technique is effective because user movement times are determined primarily by the motor rather than visual space.

Another input device configuration parameter that can be adjusted on many modern personal computers is mouse acceleration. In its most basic form, mouse acceleration includes two parameters, acceleration and threshold (Moyle and Cockburn 1991). When the mouse speed exceeds the threshold, the control-display gain is scaled by the acceleration parameter. Recent operating systems commonly use more complex mouse acceleration models, e.g. multiple thresholds (Microsoft Corporation 2003). Mouse speed is defined as the maximum of the x or y axis displacement of the mouse per unit time. When the mouse speed exceeds the first threshold, the operating system doubles the gain. When the speed exceeds the second threshold value, the system quadruples the gain.

Modern input devices like tablets have inspired new research. Hoffmann and Drury adjust the target width W by considering the width of the target, its proximity to another target and the width of the finger (Hoffmann and Drury 2011). They show that in the case that two keys are adjacent to each other and the width of the finger pad is larger than the clearance between the two key, W can be replaced by "Available W" whose value is $W_{avail} = 2S - W - F$, where W is the target size, F is the width of the finger pad on the device, and S is the target center spacing.

2.7 *The Square-Root Model*

Several researchers have considered a two-parameter square-root model:

$$T = a + b\sqrt{\frac{A}{W}}. \tag{5}$$

In this paper we refer to this as the SQR model. Kvalseth and Meyer *et al.* noted that the SQR model behaves similarly to the logarithmic model in the standard range of index of difficulty (Kvålseth 1980, Meyer et al. 1988).

Meyer *et al.* used the homogeneous target data from the original Fitts' paper (Fitts 1954), and showed that the SQR model fits the original data better than the LOG model (Meyer et al. 1988). Meyer *et al.* also performed experiments with 4 human subjects performing wrist rotation movements to heterogenous targets with similar results

Meyer *et al.* propose a complex derivation of the SQR model based on the assumption that reaching motion can be partitioned into two submovements, a primary ballistic submovement and a secondary corrective submovement, with near-zero velocity at the transition. The derivation is an approximation based on four strong assumptions: 1) two submovements with a stop between them, 2) submovement endpoints have Gaussian distributions around the center point of the target, and 3) the standard deviation of each Gaussian is linearly related to the average velocity during that submovement, and 4) there are strong numerical bounds on values of A and W for which the approximation holds.

Meyer *et al.* then derive the time T to reach the target as the sum of the average time for the primary submovement T_1 and for the corrective submovement T_2. They estimate T by minimizing its derivative with respect to the submovements and show that when $A/W > 4/z\sqrt{2\pi}$ the value of T can be approximated by the SQR function above where z is the z-score such that 95% of the area under a standard Gaussian distribution $N(0,1)$ falls inside $(-z,z)$.

In addition to its complexity vis a vis Occam's Razor, there are several other drawbacks to this derivation (Rioulo and Guiard 2012). As Meyer et al. note, if the participant reaches the target in a single movement, the derivation collapses to a linear model which fits the data very poorly. The approximation requires numerical bounds on values of A and W. Furthermore, Guiard et al. note that for a fixed positive value of A/W Meyer's model approaches 1 as the number of submovements n approaches infinity (Guiard et al. 2001, Rioulo and Guiard 2012). Meyer et al. evaluated their model with one-dimensional movements using wrist rotation of a dial that can be rotated to different angular targets. In their experiments, 4 participants are presented with 12 target conditions with A/W values ranging from 2.49 to 15.57. This range of A/W does not violate the assumption made for their derivation.

3. A Succinct Derivation of the Square-Root (SQR) Model

It is well known in control theory that the optimal time for a system to reach a target is obtained by "bang-bang" control, where maximal positive acceleration is maintained for the first half of the trajectory and then switched to maximal negative acceleration for the second half (Macki and Strauss 1982, Jagacinski and Flach 2003).

In this section we provide a new derivation for the SQR model that models acceleration as (1) piecewise constant as predicted by optimal control theory, and (2) proportional to target width: wider targets are perceived by humans as "easier" to reach and hence humans apply larger accelerations as they have a larger margin for error.

Given this model, we define the halfway point (the point reached at the switching time) for a human to reach a target at distance A as $x_{mid} = A/2$. Acceleration as a function of time for bang-bang control is shown in Figure 2(a), where the switching time between maximum acceleration and maximum deceleration is $s = T/2$.

As shown in Figure 2, Acceleration has only two values: full forward or full reverse, hence the term "bang-bang". Velocity is initially zero and then ramps up linearly during the first phase and ramps down during the second. Velocity is thus $\dot{x}(t) = \ddot{x}t$ during the acceleration phase $(t \le s)$ and $\dot{x}(t) = \ddot{x}s - \ddot{x}(t-s)$ during the deceleration phase $(t > s)$, where $\ddot{x}$ is the constant magnitude of acceleration.

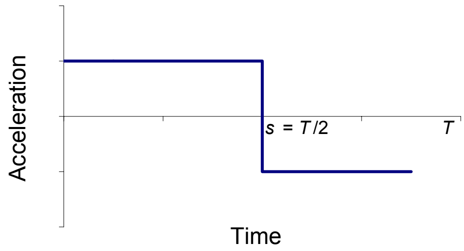
We can integrate this linear velocity with respect to time to get a quadratic function for position $x(t)$. At the switching time s, the position by integration will be $x(s) = \frac{1}{2}\ddot{x}s^2$. By symmetry, position after time $T = 2s$ will be $x(T) = \ddot{x}s^2 = \frac{1}{4}\ddot{x}T^2$. For cursor motion, we set the total distance traveled during movement time T as the amplitude $x(T) = A$. Hence, $A = \frac{1}{4}\ddot{x}T^2$ which implies

$$T = 2\sqrt{\frac{A}{\ddot{x}}}. \tag{6}$$

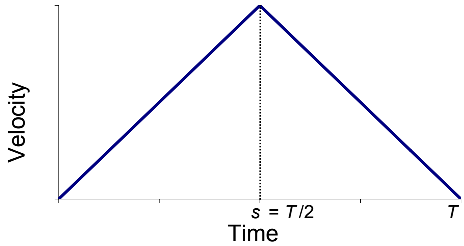
Now, from the second assumption, acceleration magnitude is proportional to the width of the target: $\ddot{x} = kW$ where k is a constant scalar and W is the target width. Substituting into equation 6, we get

$$T = 2\sqrt{\frac{A}{kW}}.$$

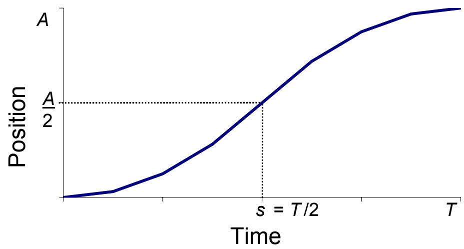
We now add an initial reaction time a and let $b = 2/\sqrt{k}$. The total movement time is then:



(a)



(b)



(c)

Figure 2. Acceleration vs. Time (a), Velocity vs. Time (b), and Position vs. Time (c) under symmetric optimal control. The "bang-bang" controller maintains the maximal positive acceleration in the first half of the motion and then switches to the maximal negative acceleration until the target is reached (a). The maximal velocity is reached in the middle of the path (b).

$$T = a + b\sqrt{\frac{A}{W}}. \tag{7}$$

This derivation is intuitive, exact, makes fewer assumptions, and requires fewer steps than the two-submovement derivation presented in Meyer *et al.* (Meyer et al. 1988).

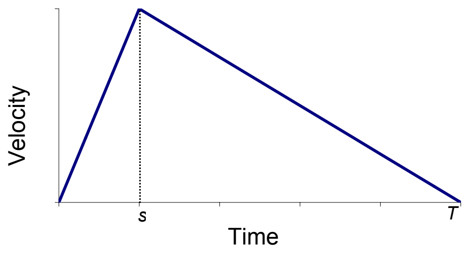
3.1 *The SQR Model with Asymmetric Acceleration*

In 1987, C. L. MacKenzie showed empirically that velocity profiles for reaching movements during Fitts' task are often asymmetric (MacKenzie et al. 1987). The derivation above does not require a symmetric motion profile.

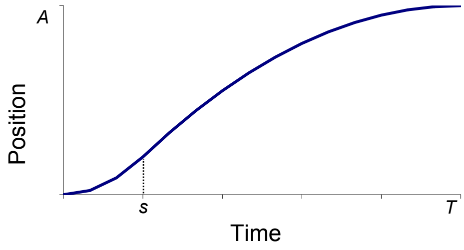
In this section we present a modified derivation based on an asymmetric velocity profile. Let s be the switching time between the acceleration phase and deceleration phase. The peak velocity will occur at the switching time. To complete the reaching movement of amplitude A with $\dot{x}(T) = 0$, the magnitude of constant acceleration $\ddot{x}_a$ before time s may be different from the constant deceleration $\ddot{x}_d$ after s.

MacKenzie showed that normalized time to peak velocity s/T increases roughly linearly as target width W increases and does not depend on amplitude A (MacKenzie et al. 1987). We approximate the normalized time to peak velocity as linearly proportional to W:

$$\frac{s}{T} = kW$$



(a)



(b)

Figure 3. Velocity vs. Time (a) and Position vs. Time (b) for the asymmetric acceleration model. Similar to MacKenzie we assume that the velocity profile is asymmetric and the peak velocity occurs at a switching time s that is not necessarily equal to $T/2$ (a) (MacKenzie et al. 1987).

where k is a scalar constant. We also assume that initial acceleration $\ddot{x}_a$ for an individual is a fixed maximum acceleration regardless of the task and the deceleration $\ddot{x}_d$ is set so velocity is 0 at time T. The maximum initial acceleration condition implies $|\ddot{x}_a| \geq |\ddot{x}_d|$, which is true according to empirical observations in MacKenzie's results (MacKenzie et al. 1987).

To obtain a relationship between T, A, and W, we first solve for the peak velocity $\dot{x}_{max} = \ddot{x}_a s$. The switching time constraint $s/T = kW$ implies $\dot{x}_{max} = \ddot{x}_a kWT$. Integrating the asymmetric velocity profile in Figure 3(a) with respect to time, we get position $x(t)$, shown in Figure 3(b).

At time T, position as a function of $\dot{x}_{max}$ is

$$x(T) = \frac{1}{2}\dot{x}_{max}s + \frac{1}{2}\dot{x}_{max}(T - s) = \frac{1}{2}\dot{x}_{max}T. \tag{8}$$

Setting $x(T) = A$ and substituting $\dot{x}_{max}$ into equation 8 yields:

$$A = \frac{1}{2}\ddot{x}_a kWT^2.$$

Hence,

$$T = \sqrt{\frac{2}{\ddot{x}_a k}\frac{A}{W}}.$$

Letting $b = \sqrt{\frac{2}{\ddot{x}_a k}}$ and adding a fixed initial reaction time a common to all trials for a given participant, we get

$$T = a + b\sqrt{\frac{A}{W}}. \tag{9}$$

Equation 9 is identical to equation 7 except for the definition of the constant term b. Both binary acceleration models were derived based on kinematic assumptions. The former model assumes switching time is fixed relative to T and acceleration is proportional to W while the latter model assumes switching time is proportional to W and initial acceleration is a fixed constant.

4. Experimental User Studies

We performed two experimental user studies, one a controlled (in-lab) study and the second an uncontrolled (web-based) study. Both studies include two conditions, a "homogeneous targets" condition where sequential targets are constant in distance and size, and a "heterogeneous targets" condition where sequential targets vary in distance and size. The experimental test and full dataset are available online at http://www.tele-actor.net/fitts/. The experiments consider targets of different difficulty, as defined by the ratio of target distance over target size.

4.1 *Experiment Conditions: The Java Applet*

For both the controlled and uncontrolled studies, we implemented a Java applet that asks each subject to complete two experiments by using his or her cursor to click on a sequence of rectangular or circular targets as they are presented on the screen. The Java applet is available online at http://www.tele-actor.net/fitts/.

The applet records the time in milliseconds between when the target appears until the subject clicks on the target. A subject may click when the cursor is outside the target, but the timer increments until the target is successfully clicked upon. To allow precise measurement of movement times without lag from Internet communications, movement times are measured locally by the applet and sent to our central server after completion of the trials. We did not attempt to capture the complete motion trajectory since the client computer may not have sufficient processing speed when running other processes to take reliable measurements.

4.1.1 Homogeneous Targets Experiment

This set of trials focuses on repetitive motions like the ones studied in the original Fitts papers. A sequence of 33 vertical rectangles are presented as illustrated in Figure 1(a). The first, second, and third set of the 11 rectangles have the same (homogenous) width and amplitude. They hence have the same difficulty, as defined by the ratio of target distance over target size. In other words after the 11th, 22nd, and 33rd repetition, the width and amplitude (and difficulty) of the rectangles are changed. To allow subjects to "warm-up" and become familiar with each set, the system discards timing data from the first 3 timing measurements out each set of 11, so data from the latter 8 rectangles for each difficulty is collected, producing 24 timing measurements.

4.1.2 Heterogeneous Targets Experiment

This set of trials focuses on changing targets as might be encountered in a game or computer human interface. A sequence of 25 circular targets are presented as illustrated in Figure 1(b). Each trial begins when the subject clicks inside a small "home" circle in the center of the window and ends when the user successfully clicks inside the target. Each of the circular targets varies in distance from the home circle and varies in diameter (and hence in difficulty).

The distance/amplitude and size/width of the targets (in pixels) are shown in Table 1. Note that the difficulty varies and is not strictly increasing or decreasing. Since the targets are measured in units of pixels, the distance and size of targets may appear different on computer systems with different display sizes and resolutions.

4.2 *Two User Studies*

User studies were conducted under UC Berkeley human subject certificate *IRB-2009-09-283*.

4.2.1 Controlled User Study

For the controlled user study, we posted ads on campus and Facebook offering an Amazon.com gift certificate for participation. Forty-six (46) people responded, including 17 female (37%) and 29 male (63%) participants. From a questionnaire, we learned that the distribution of their ages is as shown in Figure 4. The average age was 24.7 (variance = 23.8). We also learned that participants play video games for an average of 1.5 hours per week (the population has a high variance of 10.01 hours, suggesting that the majority do not play video games during the week). Out of the 46 subjects, 4 were left-handed, but opted to use their right hand to operate the pointing device. Although all of the left-handed participants

Trial	Homogeneous Targets		Heterogeneous Targets	
	A	W	A	W
1	370	50	67	20
2	370	50	184	38
3	370	50	280	14
4	370	50	230	29
5	370	50	144	55
6	370	50	249	29
7	370	50	255	14
8	370	50	96	50
9	240	10	225	19
10	240	10	263	12
11	240	10	259	25
12	240	10	229	20
13	240	10	215	31
14	240	10	198	83
15	240	10	301	16
16	240	10	194	66
17	180	70	260	12
18	180	70	296	14
19	180	70	180	44
20	180	70	278	11
21	180	70	283	37
22	180	70	40	32
23	180	70	233	10
24	180	70	191	50
25	-	-	179	18

Table 1. Target distance/amplitude (A) and size/width (W), in display pixels, for the 24 recorded Fixed Rectangles (Fixed Rectangles) trials and 25 Variable Circles trials.

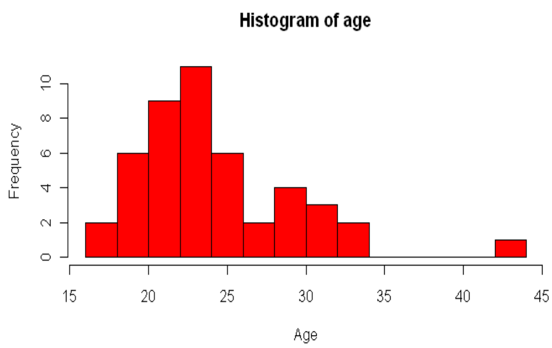


Figure 4. Age distribution for participants for the controlled study

were given the chance to customize their environment, none of them changed their mouse settings to left-handed; prior studies have shown that this does not disadvantage left-handed users (Hoffmann et al. 1997).

Each subject performed the homogenous target and the heterogeneous target experiments 10 times. Subjects were given breaks between experiments to reduce fatigue. The experiments were performed under supervision of lab assistants who encouraged subjects to repeat a trial if the subject became distracted.

For this controlled experiment, we collected a total of $22,540$ timing measurements ($11,040$ for homogenous targets and $11,500$ for heterogenous targets). We cleaned this raw dataset by keeping only timing measurements for cases where the subject successfully clicks on all presented targets within a "reasonable"

	LOG Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
2.57	224.16	147.90	120.22	64.04	7.68E-28	**SQR**
7.40	421.80	291.36	237.92	132.98	5.26E-23	**SQR**
24.00	704.86	489.78	553.09	329.48	3.74E-06	**SQR**

Table 2. Homogeneous Targets: Controlled Study: Prediction Error and Pairwise Fit between LOG and SQR models. SQR yields a significantly better fit than LOG.

time period (within 3 std dev of the global mean time). Our goal is to remove most cases where subjects were distracted or decided not to complete the experiment. After cleaning, the dataset contains 16,170 valid timing measurements (8,250 for homogenous targets and 7,920 for heterogenous targets).

4.2.2 Uncontrolled User Study

To conduct the uncontrolled study, we made the same applet available online and advertised by emails and postings on user groups. The experimental applet and datasets are available online at http://www.tele-actor.net/fitts/.

To comply with our Human Subjects approval, each online participant is asked to click an online consent box before starting the applet. An entry is created in the server database each time the consent box is clicked. We do not record IP addresses and cannot determine if the same person runs the experiment multiple times so we do not know the number of unique participants. We ask online visitors to indicate the type of mouse device they use (trackpad, mouse, trackball, etc), but cannot verify these responses.

The online applet presents visitors with 24 homogenous targets and 25 heterogenous targets and thus collects up to 49 timing measurements. Unlike the controlled experiment, online visitors were not asked to repeat each experiment 10 times. (The online applet includes a third experiment with variable-sized rectangular targets; we discovered a timing error in that experiment so we do not use data from that experiment.)

We collected timing data from 2,689 visits to the homogeneous target experiment and 2,811 visits to the heterogenous target experiment. As with the controlled study, our goal is to remove most cases where subjects were distracted or did not complete the experiment. We cleaned the raw dataset by keeping only timing measurements for cases where the subject successfully clicked on all presented targets within a "reasonable time" (i.e., within 3 standard deviations of the global mean time).

After cleaning, the online study dataset includes 78,410 valid timing measurements (39,360 for the homogeneous targets and 39,050 for the heterogenous targets).

4.3 *Experimental Results*

Using the data we collected, we compare three two-parameter models that relate motion duration to the difficulty: LOG (the classic logarithmic function), SQR (square-root), and LOG' (logarithmic plus 1.0 proposed by (MacKenzie and Buxton 1992)).

We use regression to fit the unknown a, b parameters for each subject and model and compute the resulting root-mean-squared (RMS) error and variance. We perform two-sided paired Student t-tests comparing the within-subject models using the $p = 0.05$ level of significance. As noted by R. A. Fisher in his classic text, Statistical Methods for Research Workers: "The value for which $p = 0.05$, or 1 in 20, is 1.96 or nearly 2; it is convenient to take this point as a limit in judging whether a deviation ought to be considered significant or not. Deviations exceeding twice the standard deviation are thus formally regarded as significant. Using this criterion we should be led to follow up a false indication only once in 22 trials, even if the statistics were the only guide available. Small effects will still escape notice if the data are insufficiently numerous to bring them out, but no lowering of the standard of significance would meet this difficulty."

4.3.1 Homogeneous Targets

Data from the controlled study are presented in Tables 2 and 3. Data from the uncontrolled study are presented in Tables 4 and 5. The last column indicates the model that fits better and is in bold face if the difference is statistically significant beyond the $p < .05$ level.

The results from the controlled and uncontrolled experiments are remarkably consistent. For homogeneous targets, the SQR model yields significantly better fit than LOG or LOG', except for the most difficult targets.

	LOG' Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
2.57	147.36	87.26	120.22	64.04	6.32E-06	**SQR**
7.40	299.46	191.16	237.92	132.98	2.00E-06	**SQR**
24.00	549.20	358.28	553.09	329.48	8.84E-01	LOG'

Table 3. Homogeneous Targets: Controlled Study: Prediction Error and Pairwise Fit between LOG' and SQR models. SQR yields a significantly better fit than LOG' except for the most difficult targets, where the two models are not significantly different.

	LOG Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
2.57	257.63	166.89	143.87	82.75	1.80E-120	**SQR**
7.40	484.39	322.55	296.63	177.29	8.84E-88	**SQR**
24.00	814.39	545.63	686.34	423.14	7.68E-14	**SQR**

Table 4. Homogeneous Targets: Uncontrolled Study: Prediction Error and Pairwise Fit between LOG and SQR models. As in the Controlled study, SQR yields a significantly better fit than LOG.

	LOG' Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
2.57	173.60	102.60	143.87	82.75	1.15E-19	**SQR**
7.40	351.91	218.18	296.63	177.29	2.33E-15	**SQR**
24.00	649.56	412.45	686.34	423.14	1.18E-02	**LOG'**

Table 5. Homogeneous Targets: Uncontrolled Study: Prediction Error and Pairwise Fit between LOG' and SQR models. As in the Controlled study, SQR yields a significantly better fit than LOG' except for the most difficult targets, where the two models are not significantly different.

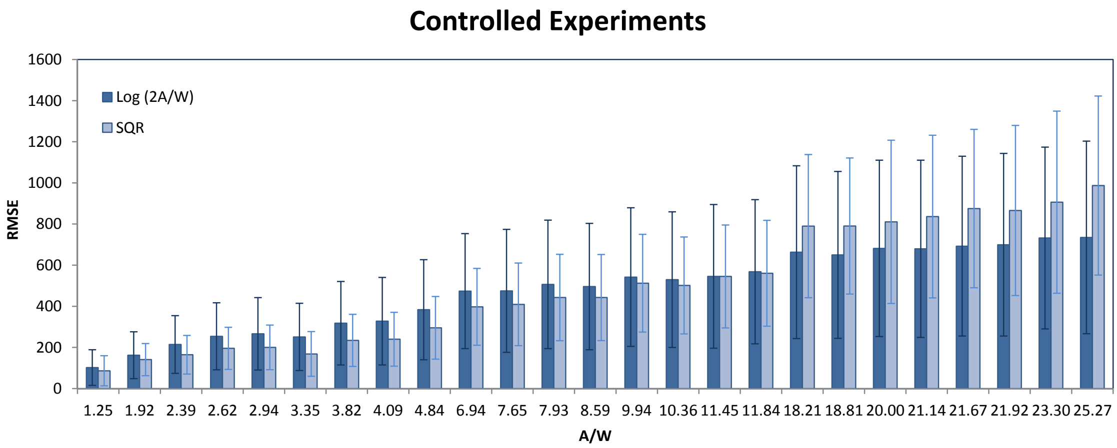


Figure 5. Heterogeneous Targets: Controlled user Study: LOG vs SQR models. See Tables VI through IX for numerical details.

4.3.2 Heterogeneous Targets Experiments

Data from the studies are presented first using four sets of plots and then in four numerical tables (Tables VI through IX). The plots show RMS Error and standard deviation for increasing values of difficulty for pairs (two models) at a time. The first two plots compare the LOG and SQR in the Controlled and Uncontrolled Experiments respectively. The third and fourth plots compare the LOG' and SQR in the Controlled and Uncontrolled Experiments respectively. In the tables, the last column indicates the model that fits better and is in bold face if the difference is statistically significant beyond the $p < .05$ level.

In both controlled and uncontrolled studies with heterogeneous targets, SQR yields a significantly better fit than LOG for easier targets and LOG yields a significantly better fit for harder targets. For heterogenous targets, the LOG' model yields significantly better fit than LOG or SQR, except for easier targets where the results are inconclusive.

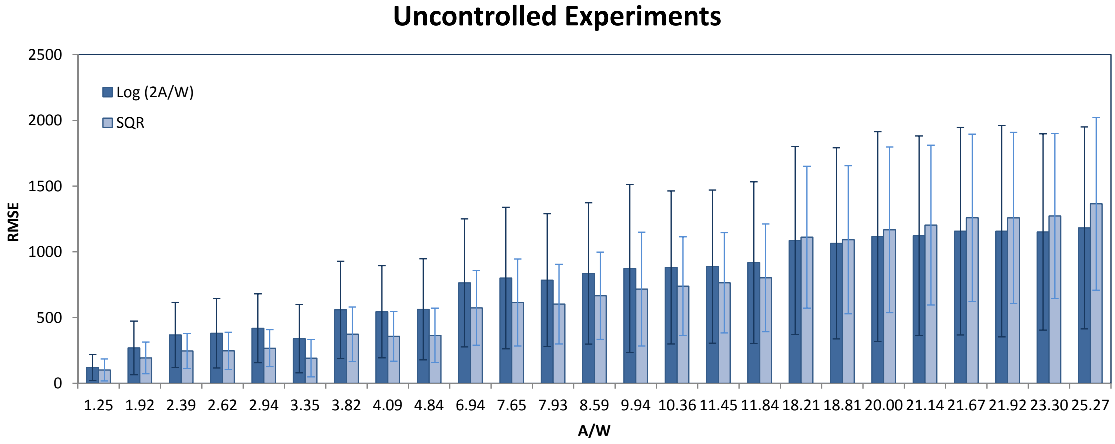


Figure 6. Heterogeneous Targets: Uncontrolled user Study: LOG vs SQR models.

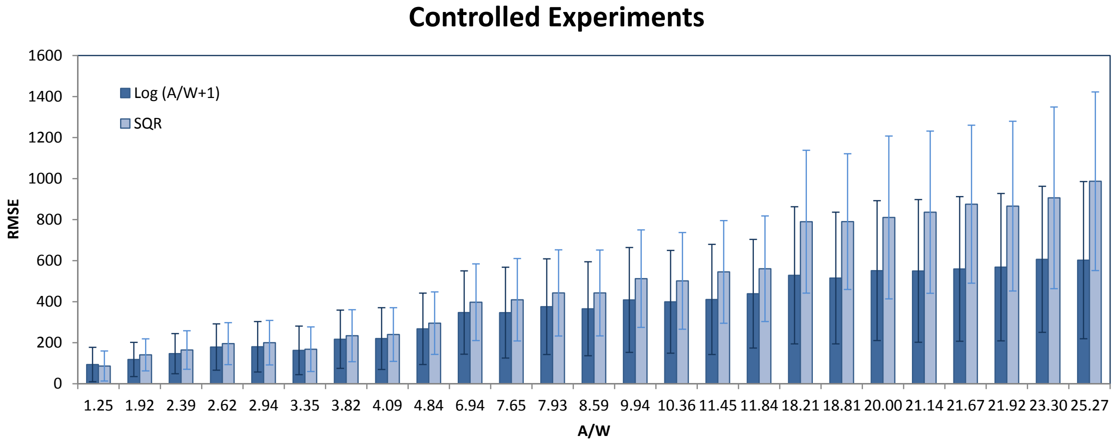


Figure 7. Heterogeneous Targets: Controlled user Study: LOG' vs SQR models.

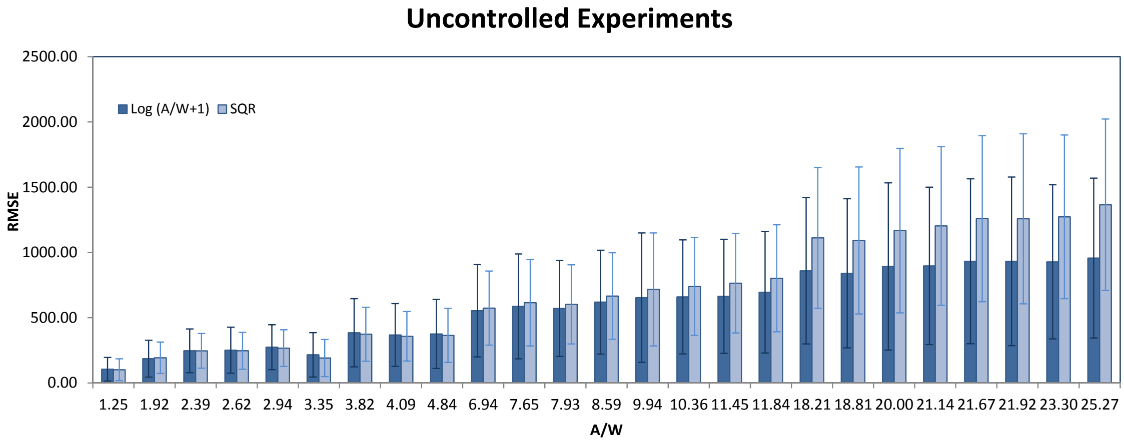


Figure 8. Heterogeneous Targets: Uncontrolled user Study: LOG' vs SQR models.

5. Discussion and Conclusion

We explore three two-parameter models that relate human motion duration to the difficulty (i.e., the ratio of target distance over target size) of the targets: LOG (the classic logarithmic function), SQR (square-root), and LOG' (logarithmic plus 1.0 proposed by (MacKenzie and Buxton 1992)). The latter two have been proposed as superior models.

We describe new theory and new experiments. For the former, we provide a succinct derivation of the SQR model based on optimal control theory. Our derivation is intuitive, exact, makes fewer assumptions,

	LOG Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
1.25	102.09	86.82	86.26	73.34	1.02E-02	**SQR**
1.92	162.18	113.97	140.68	78.22	4.42E-03	**SQR**
2.39	214.24	140.20	164.37	93.82	1.38E-07	**SQR**
2.62	253.96	162.95	195.47	102.42	6.79E-08	**SQR**
2.94	266.23	176.04	199.92	108.42	1.28E-08	**SQR**
3.35	250.99	163.39	168.21	108.85	1.00E-13	**SQR**
3.82	317.67	203.14	234.14	126.64	4.47E-10	**SQR**
4.09	327.71	212.77	239.75	130.97	4.62E-10	**SQR**
4.84	383.26	243.54	295.41	152.44	4.26E-08	**SQR**
6.94	473.81	279.80	397.26	186.73	4.90E-05	**SQR**
7.65	474.89	299.14	409.27	201.03	1.09E-03	**SQR**
7.93	506.52	312.39	442.76	209.92	2.41E-03	**SQR**
8.59	495.86	307.24	442.47	209.20	8.85E-03	**SQR**
9.94	541.84	337.11	512.23	237.91	2.04E-01	SQR
10.36	529.48	329.97	501.45	235.70	2.07E-01	SQR
11.45	545.50	349.61	545.01	250.04	9.92E-01	SQR
11.84	567.83	350.57	560.45	257.54	7.35E-01	SQR
18.21	663.13	419.94	789.96	348.02	2.55E-05	**LOG**
18.81	649.96	405.78	790.24	330.94	1.86E-06	**LOG**
20.00	681.42	428.81	810.65	397.08	6.07E-05	**LOG**
21.14	679.44	431.03	836.16	395.38	1.41E-06	**LOG**
21.67	692.45	437.13	875.12	385.34	1.74E-08	**LOG**
21.92	699.30	444.25	865.82	413.59	1.08E-06	**LOG**
23.30	732.01	442.11	906.26	442.91	7.10E-07	**LOG**
25.27	734.77	468.18	987.07	435.41	1.83E-12	**LOG**

Table 6. Heterogeneous Targets: Controlled user study: LOG vs SQR models. SQR yields a significantly better fit than LOG for easier targets and LOG yields a significantly better fit for harder targets.

and requires fewer steps than the derivation presented in Meyer *et al.* (Meyer et al. 1988).

We present data from two experimental user studies, one a controlled (in-lab) study and the second an uncontrolled (online) study. The controlled study collected $16,170$ valid timing measurements from 46 volunteers using the identical mouse and settings. The uncontrolled (online) study collected $78,410$ valid timing measurements from an indeterminate number of volunteers who visited the website with with a variety of mouse types and settings. Both studies include two conditions, a “homogeneous targets” condition where sequential targets are constant in distance and size, and a “heterogeneous targets” condition where sequential targets vary in distance and size.

We use regression to fit the unknown parameters for each model and compute the resulting root-mean-squared error and variance. We perform two-sided paired Student t-tests comparing the within-subject models using the $p = 0.05$ level of significance.

We find that (1) the data from the controlled and uncontrolled studies are remarkably consistent. Tables VIII and IX exhibit some inconsistency for easier targets. Although uncontrolled experiments do not provide the uniform setup of controlled in-lab studies, they are gaining popularity as they can collect data from large numbers of human participants. A few earlier studies have also shown consistent results from controlled and uncontrolled experiments (Bigham et al. 2010, Bakshy et al. 2012). Our study showed consistent results across the uncontrolled and controlled studies, suggesting that the relative performance of the evaluated models is robust to the variables introduced by actual computer users in the real world.

We find that (2) for homogeneous targets, the SQR model yields a significantly better fit than LOG or LOG’, except with the most difficult targets (with higher difficulty) where the models are not significantly different. That SQR is superior is surprising in these cases since Fitts’ original experiments were with homogenous targets but is consistent with more recent experiments.

We find that (3) for heterogenous targets, SQR yields a significantly better fit than LOG for easier targets and LOG yields a significantly better fit for more difficult targets. The results are inconclusive for

	LOG Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
1.25	119.57	98.92	100.97	83.80	1.56E-08	**SQR**
1.92	268.83	204.39	192.50	120.64	6.22E-36	**SQR**
2.39	367.02	248.20	245.90	133.16	3.02E-61	**SQR**
2.62	380.05	264.30	246.15	141.72	1.32E-65	**SQR**
2.94	417.87	261.97	266.70	140.79	4.45E-83	**SQR**
3.35	338.92	259.77	190.52	141.98	4.51E-81	**SQR**
3.82	558.07	369.86	373.06	206.70	4.52E-63	**SQR**
4.09	543.80	350.37	357.48	189.44	1.87E-71	**SQR**
4.84	562.20	384.60	364.24	207.13	2.01E-67	**SQR**
6.94	763.44	486.99	573.27	283.94	3.13E-39	**SQR**
7.65	800.26	538.79	614.45	330.99	1.97E-30	**SQR**
7.93	784.19	505.72	602.04	303.43	2.40E-33	**SQR**
8.59	835.37	537.14	665.74	331.57	8.00E-26	**SQR**
9.94	872.83	638.81	716.32	433.10	1.62E-15	**SQR**
10.36	881.19	582.18	738.77	374.93	6.56E-16	**SQR**
11.45	887.13	582.48	764.25	381.57	3.85E-12	**SQR**
11.84	917.90	614.22	801.91	409.63	6.14E-10	**SQR**
18.21	1085.70	715.08	1111.37	539.99	2.58E-01	LOG
18.81	1064.30	727.25	1091.28	563.31	2.46E-01	LOG
20.00	1116.00	797.88	1167.20	630.27	4.66E-02	**LOG**
21.14	1122.52	758.94	1203.31	607.69	1.03E-03	**LOG**
21.67	1157.07	790.10	1258.60	636.56	7.83E-05	**LOG**
21.92	1156.89	804.50	1258.05	651.53	1.15E-04	**LOG**
23.30	1151.02	746.97	1272.71	627.45	8.67E-07	**LOG**
25.27	1181.75	768.04	1364.91	657.32	1.00E-12	**LOG**

Table 7. Heterogeneous Targets: Uncontrolled user study: LOG vs SQR models. As in the Controlled study, SQR yields a significantly better fit than LOG for easier targets and LOG yields a significantly better fit for harder targets.

targets in the middle range of difficulty, while the the LOG' model yields a significantly better fit than both LOG and SQR on more difficult targets. This suggests that there might be an underlying difference in human motor processes for targets of different levels of difficulty and more work remains to be done.

Our applet records the time in milliseconds between when the target appears until the subject clicks on the target. We did not attempt to capture the complete motion trajectory as we were not confident that computer clients would have sufficient processing speed when running other processes to take reliable measurements, but this is an interesting avenue for future study.

To the best of our knowledge, the dataset of $94,580$ timing measurements is the largest dataset to date for human reaching motion. The experimental applet and dataset are openly available online at http://www.tele-actor.net/fitts/. The data may also contain patterns such as variations between subjects with overall faster response times and those that have slower response times. We encourage others to use this data with other metrics or to evaluate models beyond the three we study in this paper.

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	LOG' Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
1.25	93.39	83.74	86.26	73.34	2.33E-01	SQR
1.92	118.17	83.18	140.68	78.22	4.36E-04	**LOG'**
2.39	146.71	97.94	164.37	93.82	1.76E-02	**LOG'**
2.62	178.96	112.99	195.47	102.42	4.69E-02	**LOG'**
2.94	180.12	123.00	199.92	108.42	2.71E-02	**LOG'**
3.35	162.56	118.13	168.21	108.85	5.20E-01	LOG'
3.82	216.83	142.10	234.14	126.64	1.06E-01	LOG'
4.09	220.04	150.55	239.75	130.97	6.95E-02	LOG'
4.84	267.82	174.09	295.41	152.44	3.36E-02	**LOG'**
6.94	347.05	203.18	397.26	186.73	9.34E-04	**LOG'**
7.65	346.52	221.30	409.27	201.03	1.54E-04	**LOG'**
7.93	375.59	233.22	442.76	209.92	1.11E-04	**LOG'**
8.59	365.48	228.88	442.47	209.20	9.72E-06	**LOG'**
9.94	408.58	255.77	512.23	237.91	9.08E-08	**LOG'**
10.36	399.28	250.50	501.45	235.70	1.18E-07	**LOG'**
11.45	410.84	268.34	545.01	250.04	5.82E-11	**LOG'**
11.84	438.75	264.69	560.45	257.54	5.19E-09	**LOG'**
18.21	528.38	334.25	789.96	348.02	2.00E-21	**LOG'**
18.81	515.00	321.04	790.24	330.94	6.00E-25	**LOG'**
20.00	551.39	341.18	810.65	397.08	2.53E-18	**LOG'**
21.14	549.85	347.95	836.16	395.38	1.83E-21	**LOG'**
21.67	559.49	352.79	875.12	385.34	8.17E-26	**LOG'**
21.92	568.38	359.30	865.82	413.59	3.90E-21	**LOG'**
23.30	606.58	356.09	906.26	442.91	3.89E-20	**LOG'**
25.27	602.49	383.22	987.07	435.41	2.59E-30	**LOG'**

Table 8. Heterogeneous Targets: Controlled user study: LOG' vs SQR models. The LOG' model yields a significantly better fit than SQR on harder targets (with higher difficulty).

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	LOG' Model		SQR Model		Hypothesis Testing	
A/W	μ_{RMSE}	σ_{RMSE}	μ_{RMSE}	σ_{RMSE}	p-value	Best Fit
1.25	104.93	90.87	100.97	83.80	2.06E-01	SQR
1.92	185.80	141.69	192.50	120.64	1.55E-01	LOG'
2.39	245.99	167.69	245.90	133.16	9.87E-01	SQR
2.62	250.80	176.23	246.15	141.72	4.17E-01	SQR
2.94	273.89	172.17	266.70	140.79	2.02E-01	SQR
3.35	215.11	169.70	190.52	141.98	1.16E-05	**SQR**
3.82	384.23	260.92	373.06	206.70	1.85E-01	SQR
4.09	367.28	240.42	357.48	189.44	2.06E-01	SQR
4.84	375.31	264.38	364.24	207.13	1.93E-01	SQR
6.94	552.96	353.79	573.27	283.94	7.69E-02	LOG'
7.65	586.59	401.56	614.45	330.99	3.45E-02	**LOG'**
7.93	570.40	367.80	602.04	303.43	8.76E-03	**LOG'**
8.59	618.94	397.74	665.74	331.57	3.60E-04	**LOG'**
9.94	653.28	495.88	716.32	433.10	1.57E-04	**LOG'**
10.36	659.37	436.75	738.77	374.93	5.39E-08	**LOG'**
11.45	663.57	437.44	764.25	381.57	8.62E-12	**LOG'**
11.84	695.07	465.35	801.91	409.63	1.16E-11	**LOG'**
18.21	859.01	560.57	1111.37	539.99	1.13E-36	**LOG'**
18.81	839.83	571.25	1091.28	563.31	1.95E-34	**LOG'**
20.00	892.45	640.79	1167.20	630.27	7.12E-33	**LOG'**
21.14	896.95	603.33	1203.31	607.69	4.82E-44	**LOG'**
21.67	932.24	631.92	1258.60	636.56	1.89E-45	**LOG'**
21.92	931.79	646.30	1258.05	651.53	1.58E-43	**LOG'**
23.30	927.85	590.91	1272.71	627.45	3.09E-54	**LOG'**
25.27	956.81	612.56	1364.91	657.32	1.24E-68	**LOG'**

Table 9. Heterogeneous Targets: Uncontrolled user study: LOG' vs SQR models. As in the Controlled study, the LOG' model yields a significantly better fit than SQR on harder targets.

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