

CMOS Operational Amplifier Design

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EE240A Design Project

*Electrical Engineering and Computer Sciences Dept
UC Berkeley*

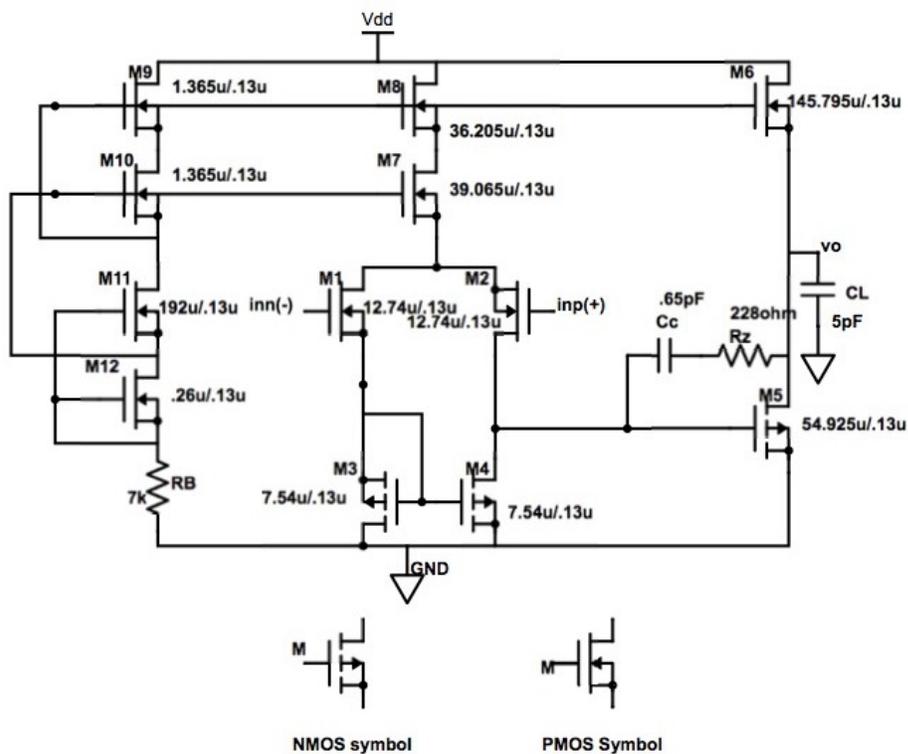
CMOS Operational Amplifier Design

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- Overview:**

A two stage compensated differential amplifier with self biased Cascode circuitry was designed in 130nm CMOS technology which achieved low power operation of 1.9mW with modern supply voltage of 1.2v, and fast 0.1% settling time of less than 4.9ns for load capacitance of 5pF, with output swing of .1v to 1.1v, and input Common Mode Range of 0.5v, with large CMRR and PSRR of more than 124dB and 74dB respectively due to cascode PMOS tail circuit. DC gain of over 2000v/v, with unity frequency of over 400MHz was designed. Only two small resistors of 7k and 228ohm was used. The schematic of the op-amp and bias circuitry is shown below with all transistor sizes next to them. Please note all NMOS bodies are connected to GND and PMOS bodies to VDD which are not shown here.



Please note all NMOS bodies are connected to GND and all PMOS bodies are connected to VDD

- **Design:**

As depicted in the circuit above, a two stage op-amp was designed with first stage as a differential single ended op-amp with current mirror loading, and second stage a common source stage. Tail of first stage was designed in PMOS to achieve high PSRR [1]. Cascode tail was designed for differential pair due CMRR requirements. As a result of tail cascode, Ssooch current mirror[2] was used to bias the cascode with low power consumption of only 11uW in bias circuit. To achieve fast slewing per 5ns settling time requirement, second stage was biased in large bias current. Discussion of the design will be provided in Discussion section of the report. To have fast settling time and stability in unity feedback configuration, phase margin of 75 degrees[3] was designed, using miller capacitance with nulling resistor technique[4]. Per compensation technique used, the zero generated was used to cancel the second pole, leaving first dominant pole and 3rd pole the only poles in the system. The details of design of each part will be given in Discussion section of the report.

- **Transistor and Bias Summary:**

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12
W(um)	12.74	12.74	7.54	7.54	54.925	145.795	39.065	36.205	1.365	1.365	0.192	0.26
L(um)	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Id(uA)	155.5	155.5	155.5	155.5	1280.1	1280.1	311.1	311.1	11.7	11.7	11.7	11.7
Vgs(mv)	480	480	401	401	401	407	414	407	407	418	710	589
gm(mS)	2.5	2.5	3.0	3.0	25.2	23.9	6.2	5.8	.2	.2	.03	.1
go(uS)	24.3	24.3	33.8	33.8	274	199	49	47	1.7	1.8	77	1.9

- **Performance:**

	Design Specification	Spice Results
DC Gain	≥ 1500 V/V	2228 v/v (67dB)
Common-Mode Input Range	0.5V (inside the output swing range)	1.1v (.1v to 1.2v)*
Output Swing	Within 0.15 V of each supply	Within 0.1 V of each supply
Power Dissipation (includes Biasing)	Minimize (less than 2 mW)	1.92mW
Unity Gain Frequency	≥ 100 MHz	439MHz
Settling Time for Unity Gain Buffer (± 0.4 V Input Step)	≤ 5 nsec to 0.1% for both rising and falling inputs	4.7ns rising, 3.8ns falling
CMRR at DC	≥ 75 dB	124.6 dB
PSRR	≥ 60 dB at DC ≥ 50 dB at 1 MHz	74 dB at DC 68 dB at 1 MHz
Load Capacitance	5pF	same
Supply Voltage	VDD = 1.2V, VSS = 0V	same
Lmin Wmin	130nm 195nm	same

* Common-Mode Input Range spice results are based on the range of input where unity feed back configuration keeps the gain of 1.

Total active area ⁺	249.39 μm^2
Total Resistance used	7.228 k
Total Capacitance used	.65 pF

+active area of each MOS is Width * (325nm (drain) + Length + 325nm (source)). Please note 325nm is for length of drain and source in .13 μm tech.

- **Discussion:**

In this section I will explain how I designed the op-amp to meet the spec listed in Performance section. I also provide hspice wave plots showing the results. This section is divided into subsections for each metric: 1. Open Loop DC gain, 2. pole-zero calculations (phase response) 3. Slewing and Settling time, 4. Bias circuit, 5. CMRR, 6. PSRR, 7. input CMR, and 8. output swing.

1. Open Loop Differential Mode DC Gain:

Open Loop Different Mode DC Gain, A_{dm} , is defined as:

$$A_{dm} = v_o / (i_{np} - i_{nn}) = v_o / v_{id} = A_{dm_first_stage} * A_{dm_second_stage} = g_{m1}(r_{o2} || r_{o4}) * g_{m5}(r_{o5} || r_{o6})$$

where,

$$g_{m1} = 2I_1 / v_{ov1} = I_7 / v_{ov1}$$

$$g_{m5} = 2I_6 / v_{ov5}$$

$$r_{o2} = 1 / \lambda_2 I_2 = 2 / \lambda_2 I_7$$

$$r_{o4} = 1 / \lambda_4 I_4 = 2 / \lambda_4 I_7$$

$$r_{o5} = 1 / \lambda_5 I_6$$

$$r_{o6} = 1 / \lambda_6 I_6$$

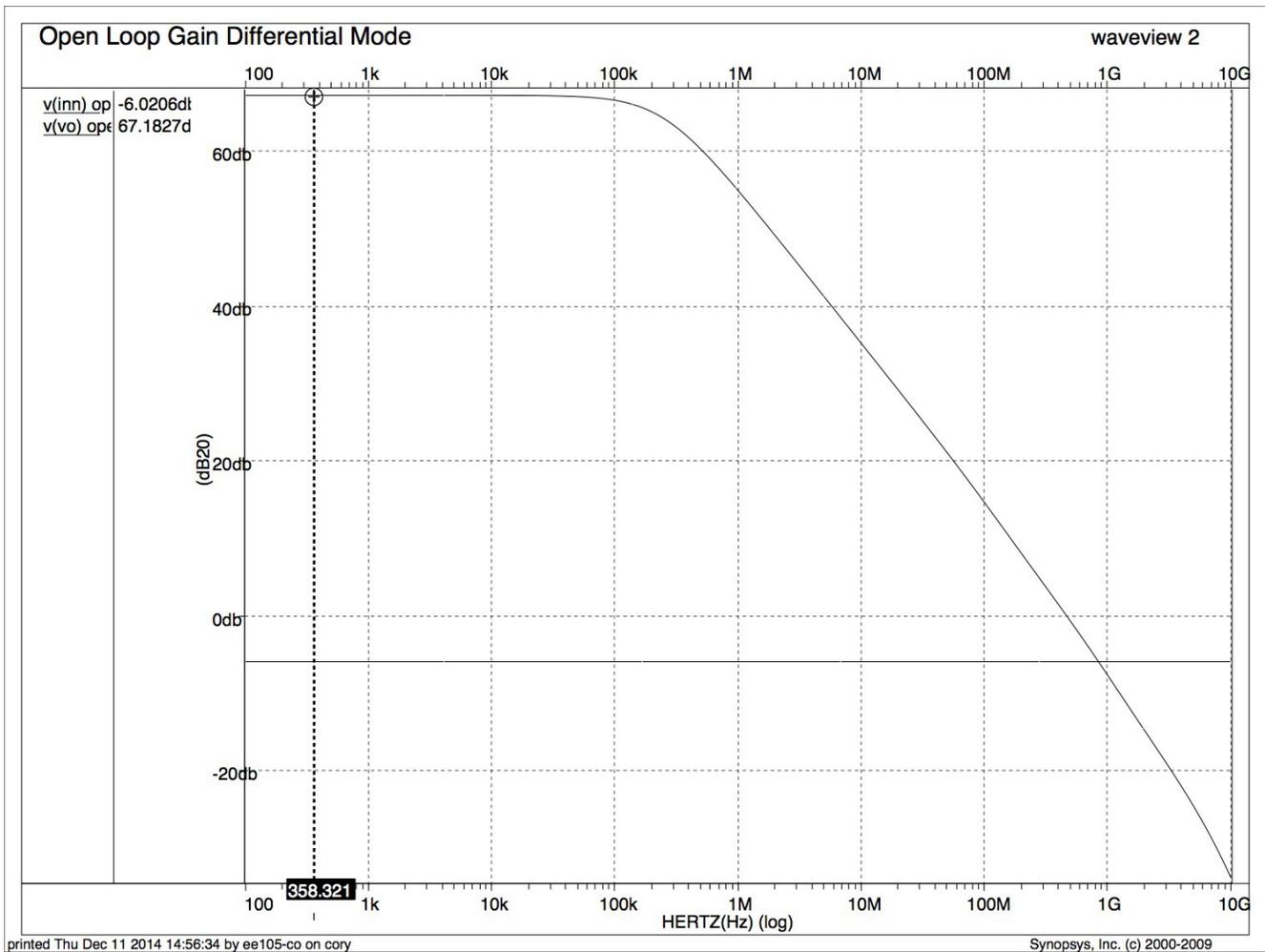
$$\Rightarrow A_{dm} = I_7 / v_{ov1} * (2 / I_7 * 1 / \lambda_2 || 1 / \lambda_4) * 2I_6 / v_{ov5} * (1 / I_6 * 1 / \lambda_6 || 1 / \lambda_5)$$

canceling I_6 and I_7 above:

$$A_{dm} = 4 / (v_{ov1} * v_{ov5}) * 1 / (\lambda_2 + \lambda_4) * 1 / (\lambda_6 + \lambda_5) \quad (1)$$

Due to design requirements as will be clear later in this section, v_{ov1} was chosen as .124v, v_{ov5} as .1v, L_2-6 as L_{min} ($\lambda_4 = \lambda_5 = .2v^{-1}$ and $\lambda_6 = \lambda_2 = .15v^{-1}$).

Plugging numbers in (1), will give gain of 2632 v/v. Hspice simulation has DC gain of 2228 v/v as depicted in picture-1 below.



Picture-1: Open Loop Differential Mode Gain

2. Pole and Zero Calculation:

There are three node that can have poles: output node, output of first stage, and gate of current mirror load in first stage, node 3. First let's see node 3 is a show stopper:

2.1 Pole at node 3:

I investigated possible pole frequency for the gate of current mirror load, and found out that the pole associated there are too fast to worry about. Here are the calculations:

$$C3 = \frac{2}{3}W3L3C_{ox} + \frac{2}{3}W4L4C_{ox}, \text{ where } c_{ox} \text{ is } e_{ox}/t_{ox} = 13.27 \text{ fF}/\mu\text{m}^2.$$

Before I came up with final W and L values which are listed in the table of Transistors Summary, I used smallest value for W and L, 190nm and 130nm respectively, and a typical value for $1/gm3$ which is the resistance seen by this cap, C3, .1k for 1mA current in M3 and 0.1v vov3. The value for pole frequency at this node came up to be 3.5THz. So, even making R and C associated for this pole 1000x larger which is not the case here, this pole is out of range of our operation, so it can be ignored. Simulation results later confirmed this assumption.

2.2 Poles and zeros after compensation:

As depicted in the circuit digram, compensation cap Cc is used in series with nulling resistor Rz, to move RHP zero created by Cc alone to LHP and cancel wp2[4]. Poles and zeros in this circuit topology are as follow:

To find ω_{p1} :

$$\omega_{p1} = 1/(R_I \cdot g_{mII} \cdot R_{II} \cdot C_c)$$

where, R_I is the output resistance of first stage: $r_{o2} || r_{o4}$,
 g_{mII} is the transconductance of second stage: g_{m5} ,
 R_{II} is the output resistance of second stage: $r_{o5} || r_{o6}$,
and, C_c is the compensation capacitance as depicted in the circuit.

ω_{p1} is also equal to $g_{m1}/(A_{dm} \cdot C_c)$ as $A_{dm} = g_{mI} \cdot R_I \cdot g_{mII} \cdot R_{II}$.

To find ω_{p2} :

$$\omega_{p2} = g_{mII} / C_{II}$$

where C_{II} is the output capacitance at output node, which is:

$$C_{II} = C_L + C_{db5} + C_{db6} + C_{gd5} + C_{gd6}$$

After calculating typical values for C_{db} and C_{gd} above I found that C_{II} is dominated by C_L . The following show my calculations:

$$C_{db0} = C_J \cdot A_S + C_{JSW} \cdot P_D + C_{JGATE} \cdot W$$

where, A_S is $2 \cdot H_{DIFF} \cdot W = 2 \cdot 130nm \cdot W$, and, P_D is $4 \cdot H_{DIFF} + 2W = 4 \cdot 130nm + 2W$.

Putting values in the C_{db0} above for W_{min} , we get:

$C_{db0}(W_{min}) = 0.067\text{fF}$, and this junction cap at 1v V_{db} , decreases to:

$$C_{db}(W_{min})|_{v_{db}=1v} = C_{db0}(W_{min}) / \sqrt{1 + 1v/2\phi_f} = 0.041\text{fF}$$

For case of C_{gd} , we have:

$$C_{gd} = LD.W.Cox, \text{ and for } W_{min} \text{ } C_{gd}(W_{min}) = 0.064\text{fF}$$

We can see from C_{db} and C_{gd} values above, even for large size transistors, they are much smaller than C_L of 5pF.

To find ω_{p3} :

$$\omega_{p3} = 1/R_z C_I,$$

where C_I is the output capacitance of first stage before putting C_c in the circuit.

$$C_I = C_{gd2} + C_{db2} + C_{db4} + C_{gd4} + g_{m5} R_{II} C_{gd5} + C_{gs5}$$

When I was designing the circuit, I didn't have values for W_s in the beginning, so I couldn't guess C_I , but it can be seen from this equation that C_I should be small value, and with small value R_z , ω_{p3} , which will become the second pole of the system after pole-zero cancellation of ω_{p2} and ω_z , is very fast. This assumption turned out to be true when I first simulated the circuit and found $PM=90$. However, due to slow settling which I will explain later, PM became 75 degrees. And it seems ω_{p3} came into frequency range of circuit.

By plugging final design values to ω_{p3} , I get:

$$\begin{aligned} R_z &= 228 \text{ ohm,} \\ g_{m5} &= 25.6\text{mS,} \\ R_{II} &= 1/474\mu\text{S} = 2.1\text{k,} \\ C_{gd5} &= 18.2\text{fF,} \end{aligned}$$

So, C_I is around 1pF, and $\omega_{p3} = 1/228 \times 1\text{pF} = 4.4\text{Grad/s}$, and $f_{p3} = 700\text{MHz}$. For $PM=75$, we get unity gain frequency of $f_u = \tan(15) \cdot f_{p3} = 200\text{MHz}$. f_u I measured in spice was around 400MHz. The followings are poles and zeros from hspice simulation:

$$\begin{aligned} f_{p1} &= -248.871\text{k} \\ f_{p2} &= -123.399\text{x} \\ f_{p3} &= -933.019\text{x} \\ f_z &= -146.884\text{x} \end{aligned}$$

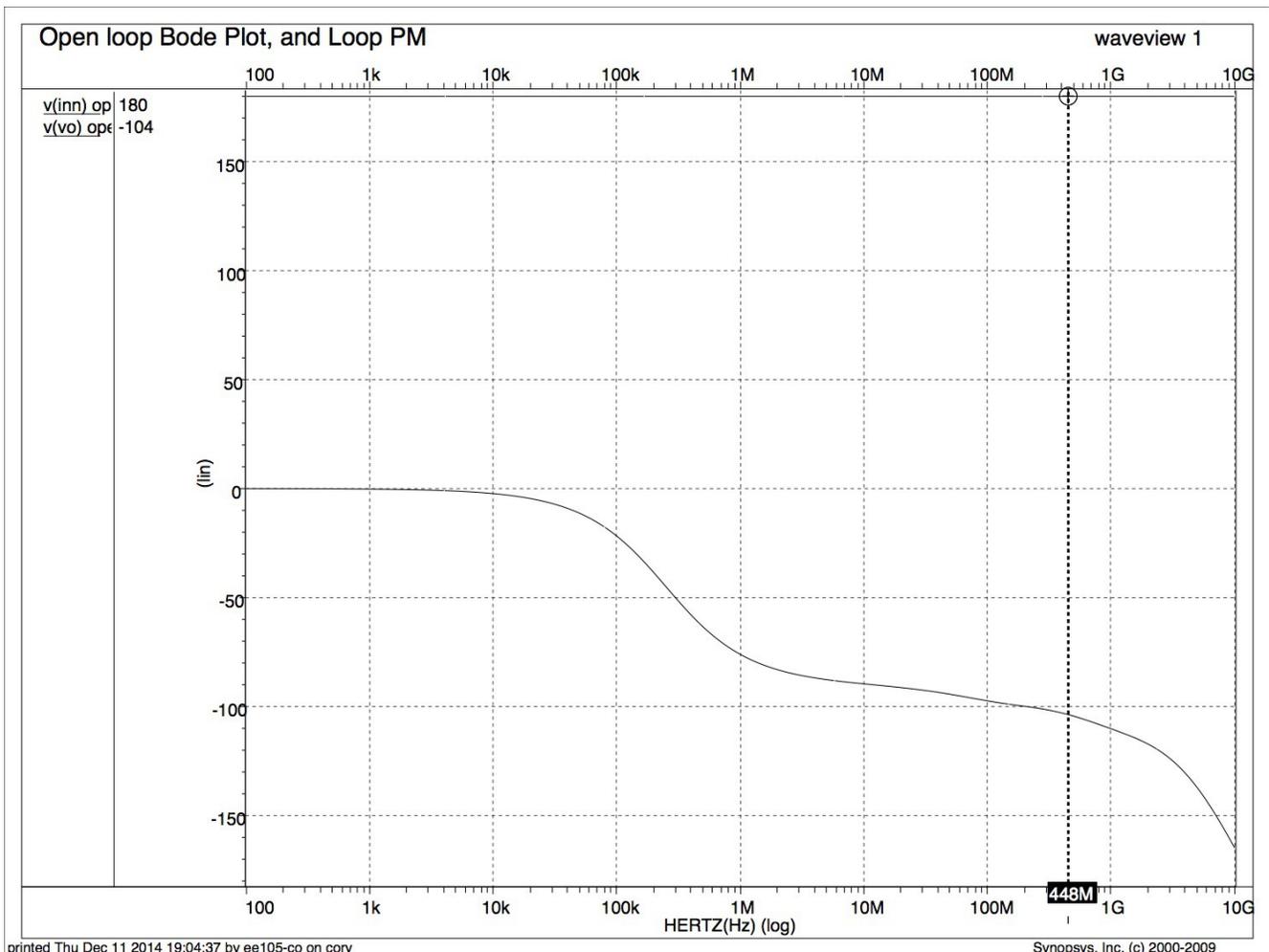
As we can see above, fz is almost canceling fp2, and fp3 is slightly higher than what I measured above, and spice and hand calculations are matching.

To find ω_z :

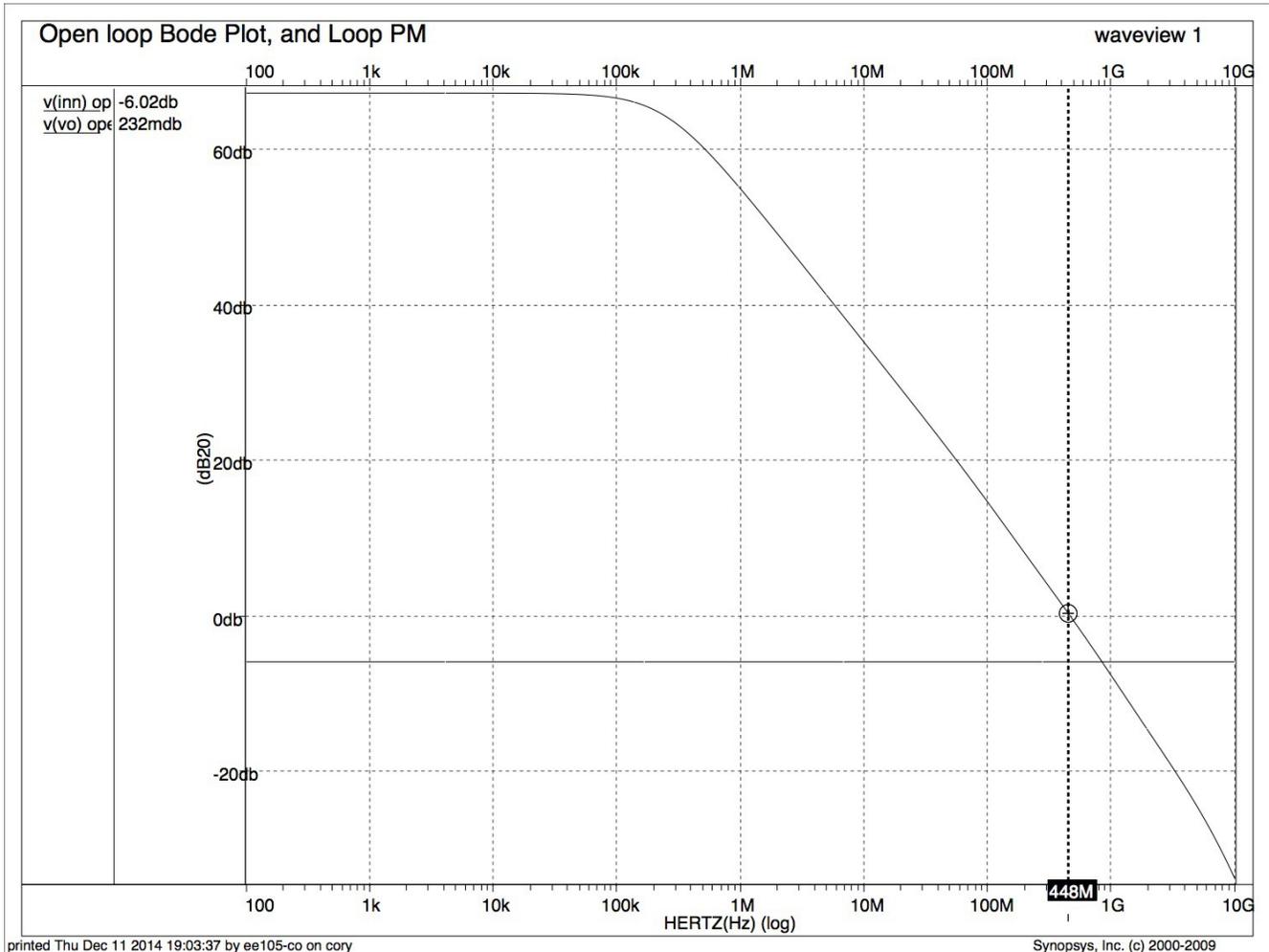
$\omega_z = 1/(R_z - 1/g_{mII})C_c$, and from here we have R_z as:

$$R_z = (C_L + C_c) / (C_c \cdot g_{m5}) \quad (3)$$

picture below shows Phase response of open loop amplifier, which is same as phase response of loop in feedback configuration, assuming frequency independent feedback factor (eg. Unity f/b). As we can see below phase margin is 76 degrees, measured at f_u (the next picture), which is 450MHz:



Picture below shows unity gain frequency of open loop amplifier which is 450MHz:



3. Settling time and Slewing:

Here I calculated rough estimate for slewing and estimate settling time from that. In unity feedback configuration, rising and falling due to a unit step input has two parts: first slewing and then exponential behavior in linear region of op-amps. Explanations below show why I chose values in my design, and why I changed slightly my design in a path to meet spec for t_{set} of 5ns.

if $v_{id} > 1.4v_{ov1}$ (step up input), M2 will go off, while M1, M3, and M4 are on and all tail current go through them, which in a rough estimate causes the output of the first stage to slew down. Here in most region I assumed M5 is off as output of first stage is slewing down. As the result, current going into CL (IL) will be I_6-I_7 , where I_6 is the current of second stage and I_7 is the tail current of the first stage.

So, I have, slew rate of CL as:

$$SR_2 = I_L / CL = I_6 - I_7 / CL$$

I ignore slew down of CL as it should be fast due to M5 current large while output of first stage is slewing up and causes v_{gs} of M5 to be large.

As $V_{id} > 1.4v_{ov1}$, we slew, I should choose values for v_{ov1} and v_{ov2} . The larger this value, the less slewing I will have as $v_{id} - 1.4v_{ov1}$ is the amount of dv for v_o where we slew in unity feedback (v_{id} step is .4v in this project). At the same time, Input Common Mode Range, CMR, put an upper limit for v_{ov1}/v_{ov2} . As I used cascode in my tail of diff pair and v_{od} requirement of 0.1v in this project, if we set v_{ov} of M3 (current mirror load) to 0.1v, which is a good choice in terms of CMR and as I explained before pole at gate of this current mirror load doesn't affect our performance (small v_{ov3} and larger W_3 is ok for this node), and we put v_{od} of 0.1v for cascode tail PMOS transistors, input CMR becomes $v_{dd} - v_{ov3} - v_{ov7} - v_{ov8} - v_{T1} - v_{ov1} - GND$. This CMR will be $1.2v - .1 - .1 - .1 - .3 - v_{ov1} > .5v$. So v_{ov1} shouldn't be more than 0.1v. Please note as input common mode comes lower toward gnd, V_T of M1-2 will be increased due to body effect (.05v increase in this technology), and this allow input common mode to come as low as 50mv instead of 100mv as explained above. However, spec says, CMR of .5v inside output swing range. From here v_{ov5} should be 0.1v. In this design final v_{ov1-2} I used was 124mv.

I want to have an idea how slewing will be:

dvo slews for amount of $v_{id} - 1.4v_{ov1}$ which from v_{ov1} of .15v (I originally used this v_{ov} in my first iteration) and v_{id} of .4v, $dvo = 188mv$ (v_{out} slews for 188mv in 400mv step input).

$SR_2 = 188mv / dt_{SR_2} = I_7 / CL$, where dt_{SR_2} is the amount time it take for output to slew into linear region (v_{id} becomes $< 1.4v_{ov1}$).

To find dt_{SR_2}

$$I_7 = CL \times 188mv / dt_{SR_2} = 940^{uA.ns} / dt_{SR_2}, \text{ where } 5pF \times 188mv = 940^{uA.ns}$$

Finally,

$$dt_{SR_2} = 940^{uA.ns} / I_7 \quad (2)$$

We have 5ns budget to go into .1% error bounds in .4v input step. After slewing it takes $6.9\tau[5]$ to reach .1% error bound in one pole

response, where $\tau=1/\omega_u$, (which is almost the case here), we have:

$$5ns - dt_{SR2} > 6.9\tau$$

from (2) above we have:

$$5ns - 940^{uA.ns}/I7 > 6.9/\omega_u$$

And we know from gain BW product that $\omega_u=gm1/Cc$, so:

$$5ns - 940^{uA.ns}/2I1 > 6.9Cc/gm1$$

And, $gm1=2I1/vov1$, so:

$$5ns - 940^{uA.ns}/2I1 > 6.9Cc.vov1/2I1$$

taking $vov1=.15v$, we have:

$$5ns > (940^{uA.ns} + 6.9 \times .15^v \times Cc)/2I1 \quad (4)$$

Here to find a value for $I1$, I calculated Cc based on $PM=60$, as follow:

$1.73 \times \omega_u = \omega_{p2}$, where ω_{p2} is $gmII/CL$. That is true that ω_{p2} will be cancelled by ω_z , but I chose as my starting point to find Cc . This approach can help if we get a doublet in case not exact canceling of pole and zero. So we have:

$$1.73\omega_u = gm5/CL$$

where ω_u is $gm1/Cc$, so:

$$1.73 \times (gm1/Cc) = gm5/CL \quad (a)$$

To have equal slew rate for outputs of first and second stage, we take current in second stage twice the current in first stage ($I6=2I7=4I1$) [6], if $CL=Cc$.

With $vov1=.15v$, $vov5=.1v$, and $I5=4I1$, we have:

$Cc=1.73 \times CL \times gm1/gm5 = 1.44pF$. (Please note my final Cc is $.65pF$. My final $I5=8.2 \times I1$, and $vov1=.12v$, and equation (a) above gives Cc of $.8pF$.)

With $Cc=1.4p$, from (4) above I get:

$I_1 > 245\mu\text{A}$, and $I_6 > 980\mu\text{A}$.

From equation (3):

$$R_z = I/gm_5 \times (CL+C_c)/C_c$$

with $v_{ov5} = .1\text{v}$ and $I_5 = I_6 = 980\mu\text{A}$, and $C_c = 1.44\text{pF}$, we get:

$$R_z = 228\text{ohm}.$$

By these values I sized transistors, using square law equation to find W/L ratio, and keeping L as L_{min} to save area and less parasitic caps (Please note per [7], L_{min} is not optimum value for analog design because of variation mismatches).

After simulating for first time, rising part of output for .4v step up input, was 30-40% slower than 5ns settling time budget, where fall down part was very fast. After discussion with GSI and Prof Allstot, I increased current of second stage by 40%, and to keep circuit below 2mW budget, I decreased first stage current by some amount. To keep ω_u intact ($\omega_u = gm_1/C_c$), I decreased C_c , which matches my C_c calculation above. Please note, these changes won't change gain due to equation (1) or other metrics such as CMRR or PSRR which I will explain later in this section. In my changing values, my PM was decreased to 75degrees which is actually the optimum PM for .1% error bound[3].

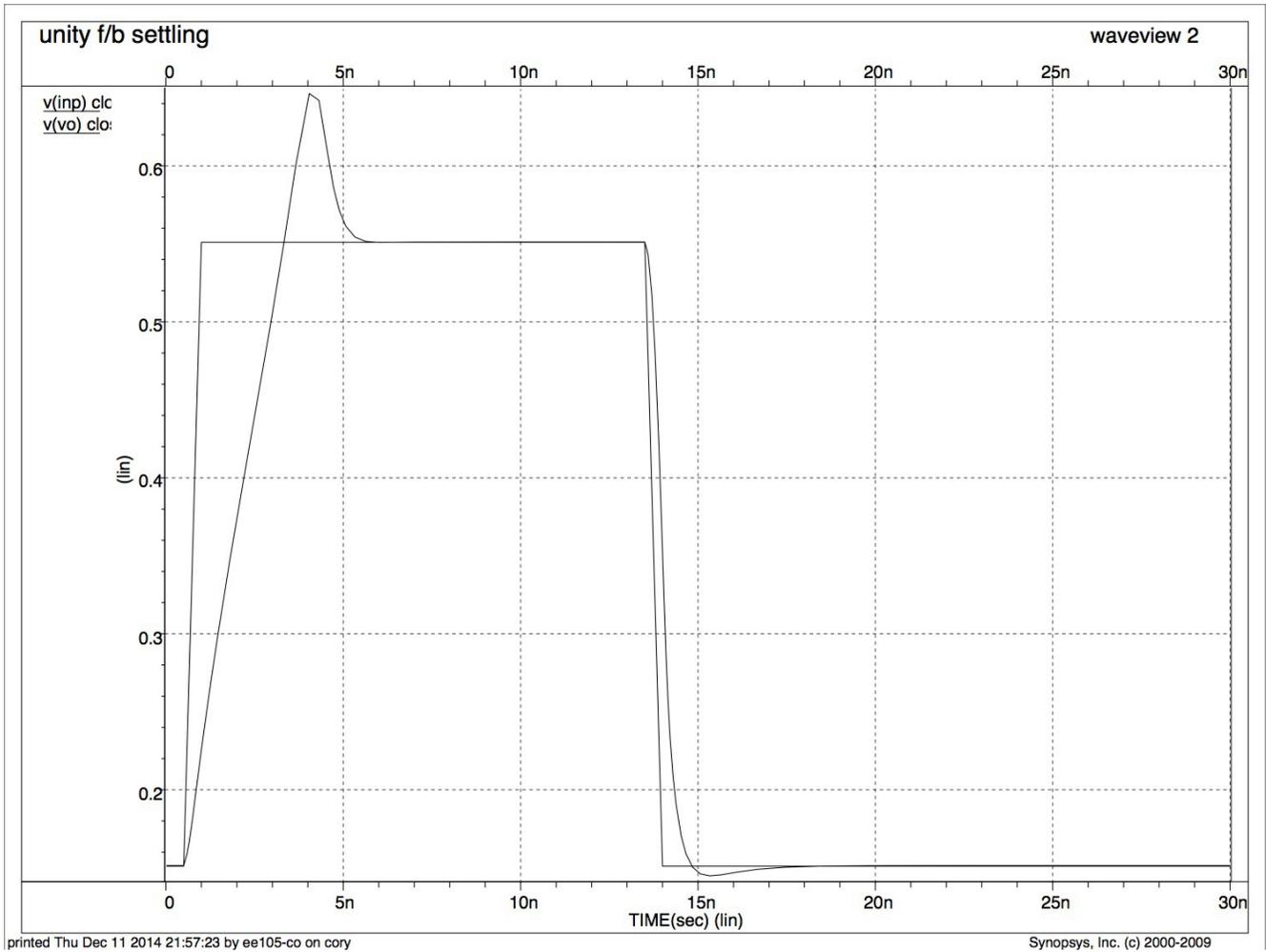
Please note, by paying attention to pole and zero locations explained above, R_z seems to stay unchanged as:

$$R_z = 1/gm_5 * (CL+C_c)/C_c$$

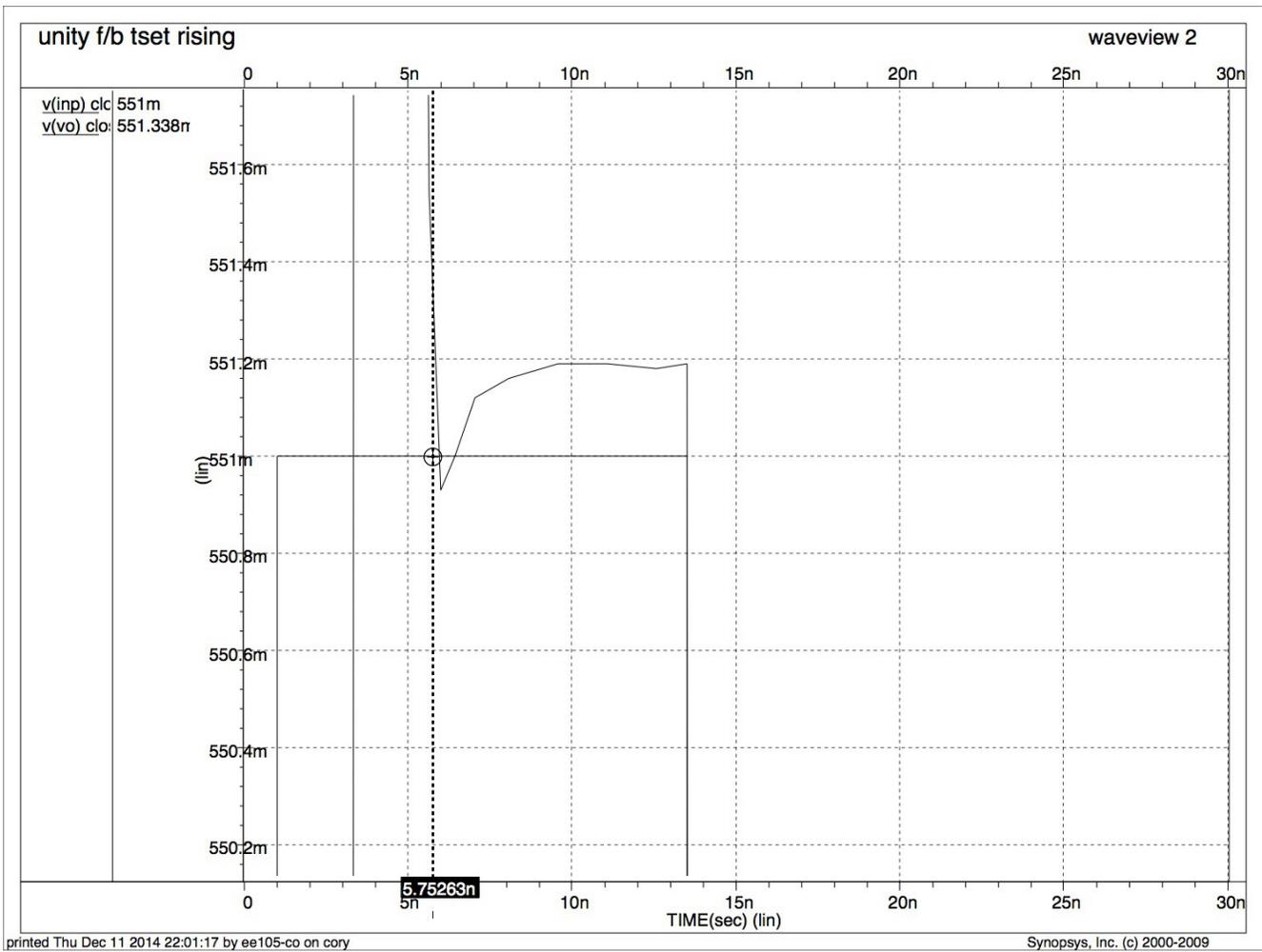
and, as gm_5 goes high due to I_5 increase, and C_c goes low, R_z shouldn't change that much which was the case for my design (note here $CL \gg C_c$).

Also, $\omega_{p2} = gm_5/CL$, and $\omega_z = 1/(R_z - 1/gm_5)C_c$. ω_{p2} increases as I_5 increases and ω_z also increases as C_c decreases (As $1/gm_5$ is a very small value even in compare to R_z) . So pole-zero cancellation is still happening. This was verified by pole zero values I showed in my hspice sim in section 2.2.

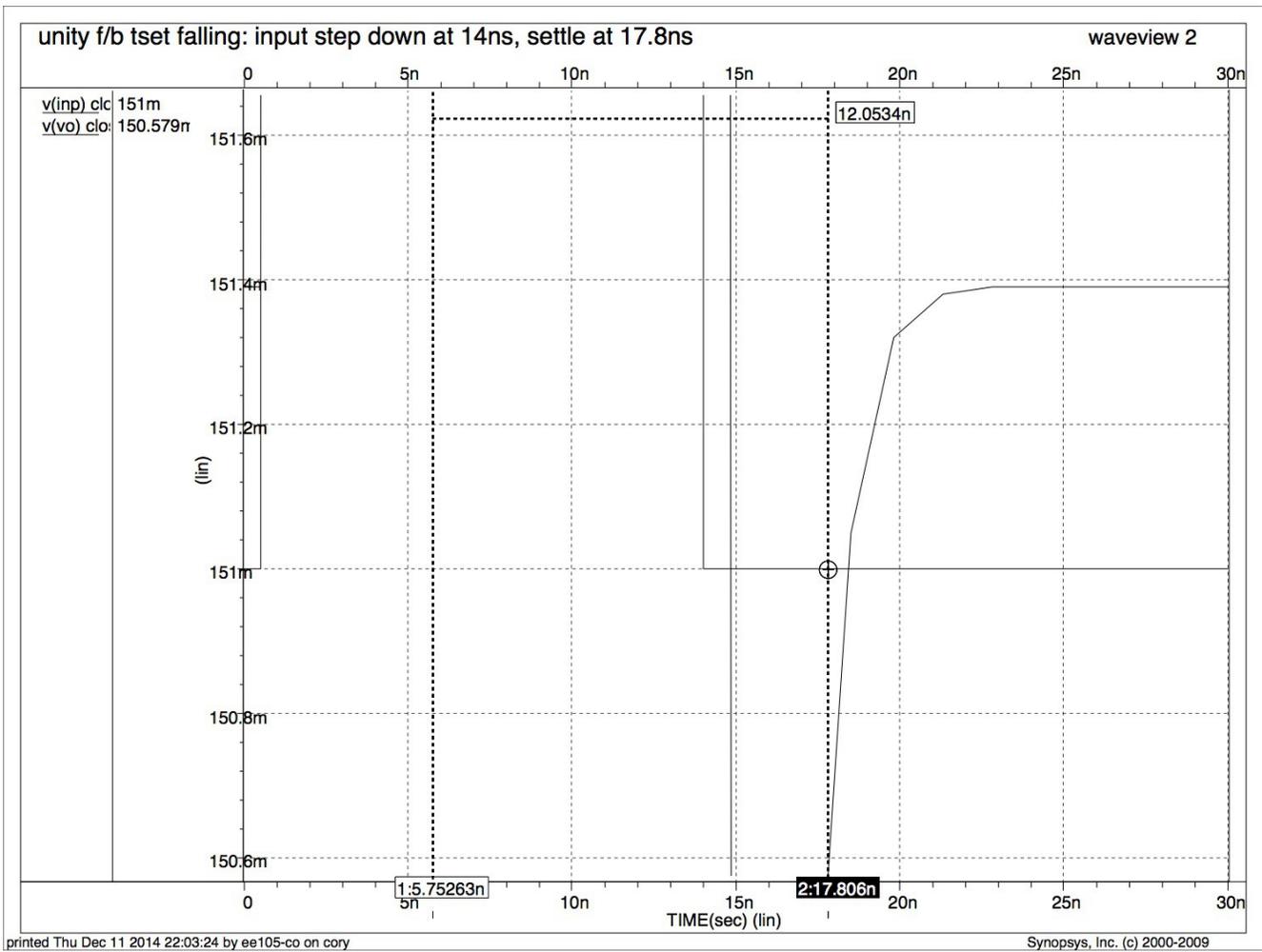
Below are waveview of settling time for vid=.4 step up at t=1ns, and step down at t=14ns:



The following picture is zoom in version for step-up input to find t_{set} .
 t_{set} is 4.75ns for rising:



The following picture is zoom in version for step-down input to find t_{set} . t_{set} is 3.8ns for falling. Please note .1% error bound for .4v input is equal to .4mv error bound:



4. Bias Circuit:

I used Sooch circuit current mirror[2] in PMOS fashion to bias the tail cascode of diff pair, with v_{ov} of 0.1v for each PMOS transistor in tail. Below is picture of bias circuit repeated:

acceptable, especially for area considerations.

$$A_{cm} = [g_{m1} \cdot r_{o4} / (1 + g_{m1} \cdot r_{o7} \times 2)] \cdot g_{m5} R_{II}$$

CMRR should be more than 75db, so (assuming $r_{o2} || r_{o4} \approx 1/2 r_{o4}$):

$$\begin{aligned} A_{dm}/A_{cm} &= [1/2 g_{m1} \cdot r_{o4}] / [(g_{m1} \cdot r_{o4}) / (1 + g_{m1} \cdot 2 \cdot r_{o7})] = g_{m1} r_{o7} \\ &= (2I_{I1} / v_{ov1}) \cdot 1 / \lambda_7 I_{I7} = 1 / (\lambda_7 v_{ov1}) > 75\text{dB} = 5623 \text{v/v} \end{aligned}$$

with $v_{ov1} = .15\text{v}$, $\lambda_7 < 1 / (5623 \times .15\text{v})$

so,

$$\lambda_7 < 0.0011 \text{v}^{-1}$$

this means that L_7 should be larger than $127 L_{min}$, which is a large number, especially once W_7 needs to be sized in a way that low v_{ov} and large current pass.

The following explains my cascode tail design to meet CMRR, and later once I explain PSRR, this topology helps too:

$$\begin{aligned} A_{cm} &= [g_{m1} \cdot r_{o4} / (1 + g_{m1} \cdot r_{o7} \cdot r_{o8} \times 2)] \cdot g_{m5} R_{II} \\ r_{o7} &= r_{o8} = 1 / I_7 \lambda_7 \end{aligned}$$

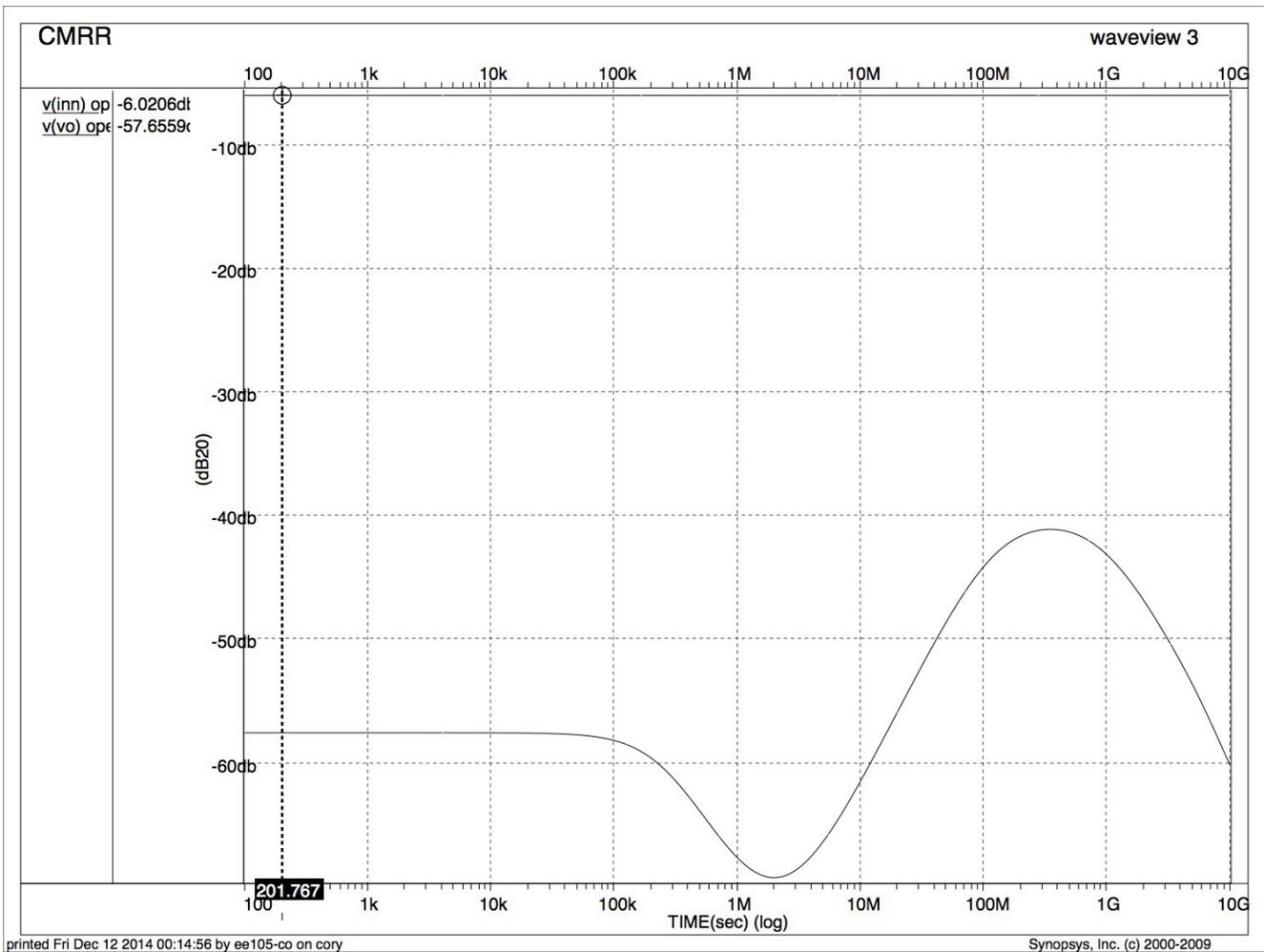
$$\begin{aligned} \text{CMRR} &= A_{dm}/A_{cm} = [g_{m1} (r_{o2} || r_{o4})] / [g_{m1} \cdot r_{o4} / (1 + g_{m1} \times 2 \times g_{m7} r_{o7}^2)] \\ &= \sim g_{m1} g_{m7} r_{o7}^2 \end{aligned}$$

Let's plug some values in CMRR equation to see if L_7 and L_8 needs to be larger than L_{min} or not:

$$\begin{aligned} g_{m7} &= 2I_7 / v_{ov7}, \text{ with our } I_7 \text{ calculated above as } .49\mu\text{A}, g_{m7} = 9.8\text{mS} \\ r_{o7} &= 1 / I_7 \lambda_7 = (L_7 / L_{min}) \times 1 / (I_7 \times .15\text{v}^{-1}) = (L_7 / L_{min}) 13.6\text{k} \\ g_{m1} &= 2I_{I1} / v_{ov1} = .49\text{mA} / .15\text{v} = 3.2\text{mS} \end{aligned}$$

So, $\text{CMRR} = 5921 \text{v/v}$ with $L_7 = L_8 = L_{min}$. This remained true even after I changed the current values as explained in settling time section.

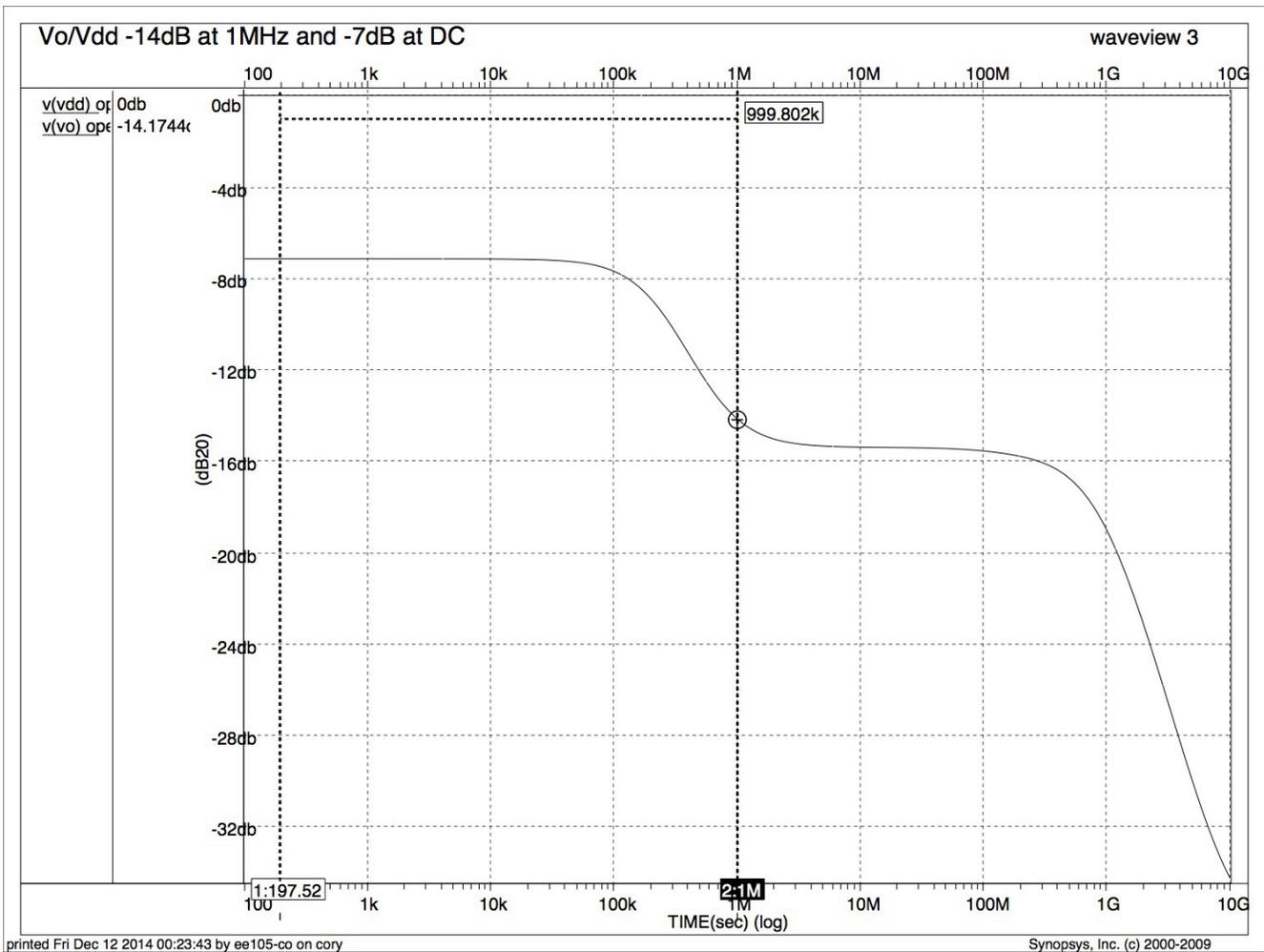
The following is the A_{cm} result from hspice:



6. PSRR:

To have high PSRR, PMOS/NMOS topology used in this design is desired[1]. In other words, if diff pair was used in NMOS tail fashion (complement design of current design), PSRR would be much worse[1].

The following is V_o/V_{dd} transfer function which shows gain of -7dB and -14dB at DC and 1MHz respectively.



7. Input CMR

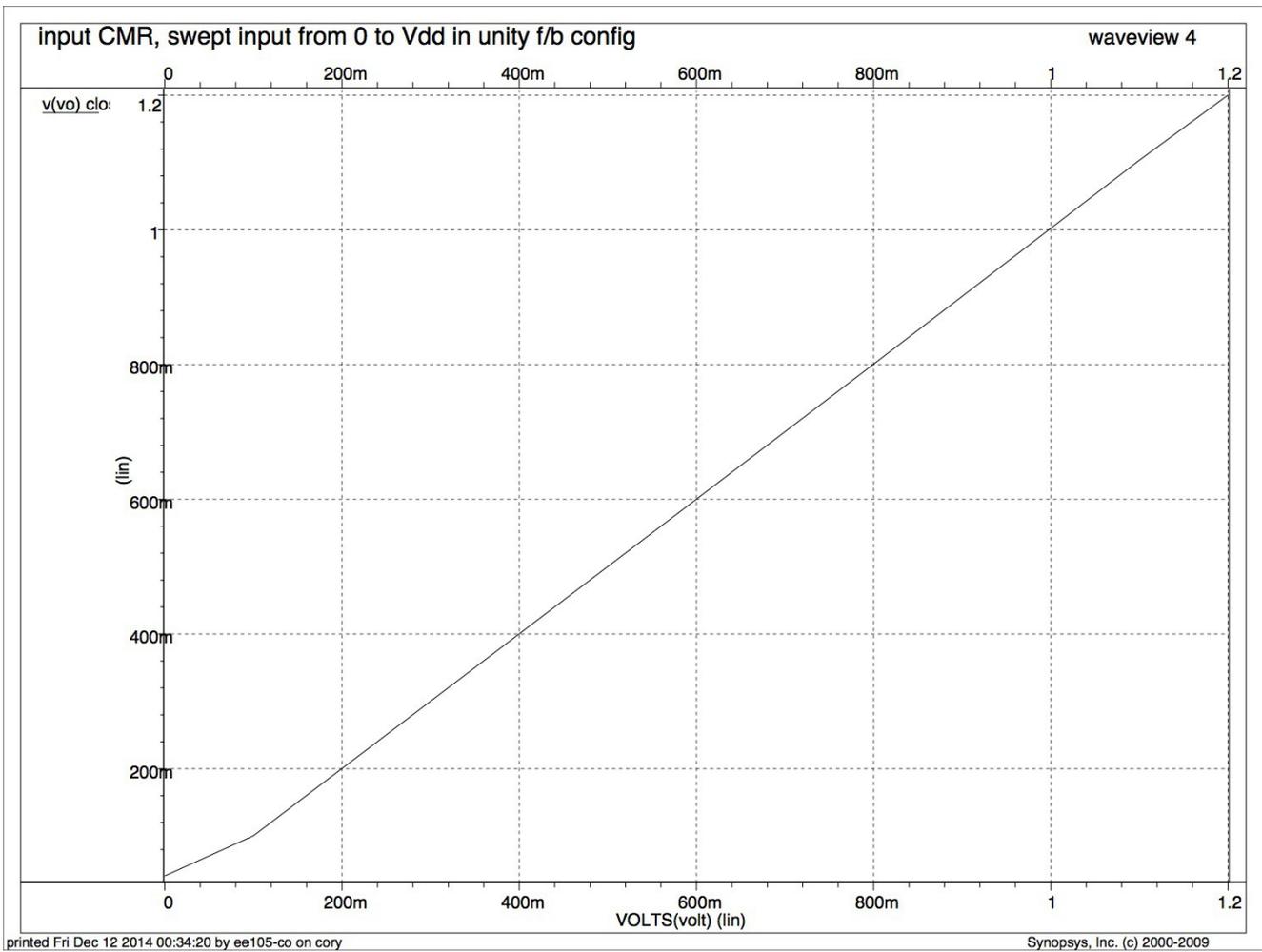
Lowest input common mode range is:

$$CMR^- = v_{ov3} = .1v$$

Highest input common mode range is:

$$CMR^+ = V_{dd} - v_{ov8} - v_{ov7} - v_{t7} - v_{ov1} - v_{t1} = 1.2 - .1 - .1 - .12 - .32 = .56v$$

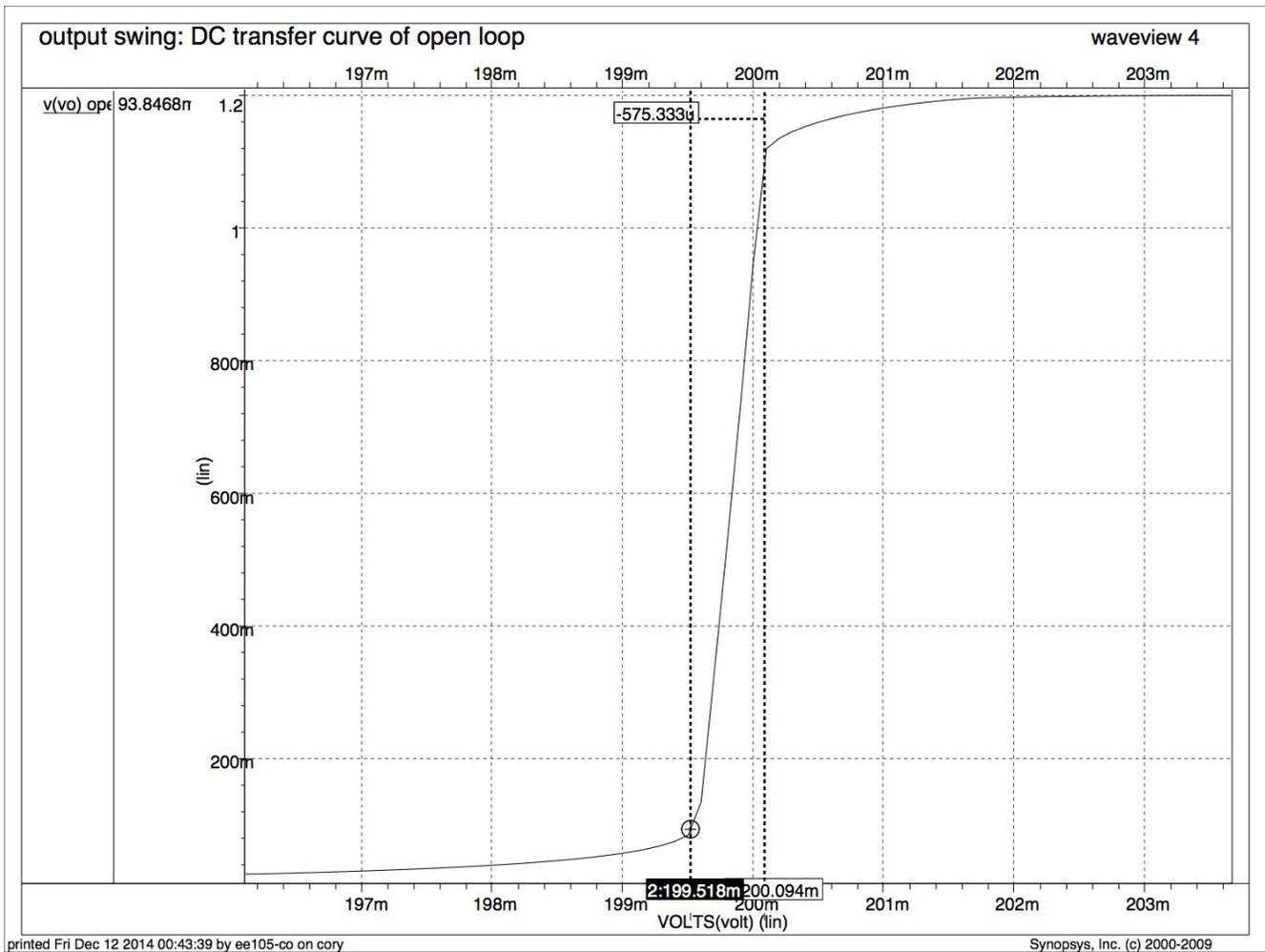
To test CMR in spice, I used unity feedback configuration, and swept input from 0 to 1.2v, to observe the range in which gain is 1. Obviously, in this test I get much larger CMR, as even open loop gain drops, we can still see gain of 1 in feedback.



8. Output swing:

$$V_o(\min) = v_{ov5} = .1\text{v}$$

$$v_o(\max) = 1.2 - v_{ov6} = 1.1\text{v}$$



- **Conclusion:**

What I learned from this design experience was hand calculations are starting point and they can guide us toward the goal, but at the same time we can use spice to reach the goal. In my slewing and settling time part, there was no closed form solution to slew rate and settling issue at the beginning, and I used spice to meet that part. Maybe more thorough methods, such as Convex Optimization[8], is helpful to actually find an optimum for power/area considerations.

- **References:**

[1] P. Gray and R. Meyer "Analysis and Design of Analog Integrated Circuits 5th ed." section 6.3.6

[2] P. Gray and R. Meyer "Analysis and Design of Analog Integrated Circuits 5th ed." page 269

[3] H. Yang and D. Allstot "Considerations for fast settling operational amplifiers", *IEEE Trans. Circuits Syst.*, vol. 37, no. 3, pp.326 -334 1990

[4] P. Gray and R. Meyer "Analysis and Design of Analog Integrated Circuits 5th ed." section 9.4.3

[5] DJ Allstot "EE240A lecture notes at UC Berkeley, Fall 2014" pg. 125

[6] DJ Allstot "EE240A lecture notes at UC Berkeley, Fall 2014" pg. 169

[7] DJ Allstot "EE240A lecture notes at UC Berkeley, Fall 2014" pg. 64

[8] S. S. Mohan, M. del Mar Hershenson, S. P. Boyd, and T. H. Lee, "Simple accurate expressions for planar spiral inductances", *IEEE J. Solid-State Circuits*, vol. 34, pp.1419 -1424 1999