- Properties of Vieth-Muller Circle
- Calculating Stereo Disparity
- Euclidean Transformation
- Affine Transformation

1 Stereo Disparity

1.1 Properties of Vieth-Muller Circle

We can define the following properties for given Vieth-Muller Circle

1. If the eye movement is a pure version eye movement, the fixation point stays on the same Vieth-Muller Circle. As in Figure. 1, the fixation point P moves to P'.
2. Now keeping the fixation point, we study disparity for various points. Disparity is defined as ϕ_R - ϕ_L. See Figure. 1.

Theorem 1 *If point Q is on Vieth-Muller Circle, its disparity is zero.*

As Q moves out (e.g. point P moves to point Q in Figure. 1), ϕ_L gets smaller and ϕ_R gets larger and vice versa if Q moves in.

Theorem 2 *Disparity is non-zero outside/inside Vieth-Muller Circle (with opposite sign for outside/inside).*

For human vision system, when the disparity is very large, you will see double objects (one from left eye and the other from right eye). It is called *Diplopia*. The maximum disparity without diplopia is termed as *Panum's Fusional Limit.*

1.2 Calculating Disparity

In Figure. 2, ϕ_L and ϕ_R are made by line of sight with the straight ahead direction.
We define *Gaze Angle* $\gamma = \frac{1}{2}(\phi_L + \phi_R)$,
define *Vergence Angle* $\mu = \frac{1}{2}(\phi_L - \phi_R)$. CE is the cyclopean eye.

Horizontal Disparity

$$h = \frac{Icos\gamma}{d}(\frac{\delta}{d+\delta} + \frac{dtan\gamma}{d+\delta}x + x^2)$$

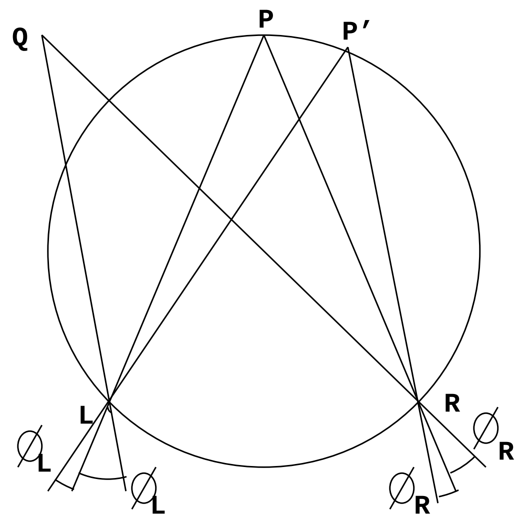

Figure 1: *Disparity Properties on Vieth-Muller Circle*

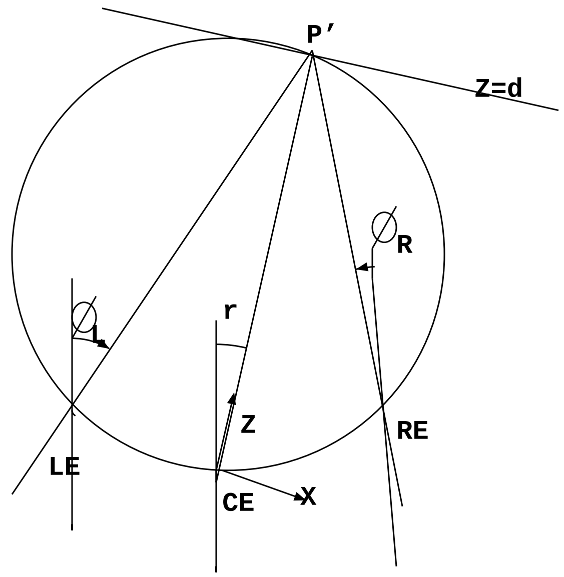

Figure 2: *Calculating the Disparity*

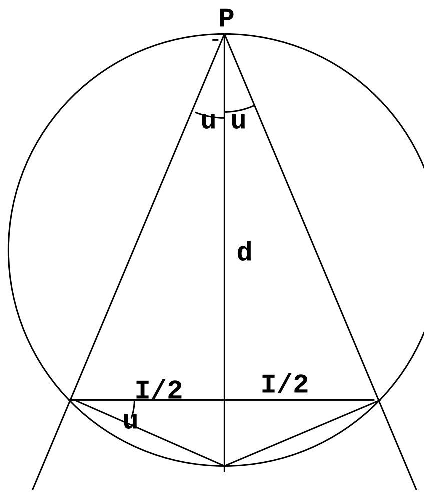

Figure 3: *Simple Justification*

Vertical Disparity

$$v = \frac{Icos\gamma}{d}(\frac{dtan\gamma}{d+\delta}y + xy)$$

where $(x,\ y)$ are cyclopean image coordinates, $x = X/Z,\ y = Y/Z,\ Z = d+\delta,\ I$ is the interocular distance. So the coordinate system changes with the cyclopean eye.

Theorem 3 $d = Icos\gamma/sin2\mu$

We have a simple justification for $\gamma = 0$, see Figure. 3.
$I/2 = d \times sin\mu \times cos\mu \Rightarrow d = Icos\gamma/sin2\mu$

The image transformed from the left eye to the right eye can be modeled as an euclidean transformation with translation vector t and rotation vector ω:

$$t = [\, t_x \quad 0 \quad t_z \,] = I[\, cos\gamma \quad 0 \quad sin\gamma \,]$$

$$\omega = [\, \omega_x \quad \omega_y \quad \omega_z \,] = [\, 0 \quad -2\mu \quad 0 \,] = [\, 0 \quad \tfrac{-Icos\gamma}{d} \quad 0 \,]$$

After plugging the above equation into the optical flow equation, we have

$$u(x,y) = [-\frac{t_x}{Z(x,y)} - \omega_y + \omega_z y] - x[-\frac{t_Z}{Z(x,y)} - \omega_x y + \omega_y x] = -\frac{I \times cos\gamma}{d+\delta} + \frac{I \times cos\gamma}{d} + \frac{x \times sin\gamma}{d+\delta} + \frac{I \times cos\gamma}{d} x^2 \quad (1)$$

So it is the same as horizontal disparity h. Similarly, we can verify for the vertical disparity. It is easy to see that when γ and x are zero and $\delta \ll d$, we have the special case as we discussed in the lecture before.

$$h = -\frac{I}{d+\delta} + \frac{I}{d} = \frac{I\delta}{d(d+\delta)} \doteq \frac{I\delta}{d^2}$$

2 Euclidean, Affine and Projective Transformations

Euclidean transformation, which is rotation reflection, followed by translation, can be modeled as $x' = Rx + d$, where R is an orthogonal matrix. Under Euclidean transformation, lengths, angels and areas are preserved. Also Euclidean transformation is closed under composition.

Affine transformation can be expressed by $x' = Ax + d$, where A is any non-singular matrix. So Euclidean transformation is a subset of affine transformation, similarly, affine transformation is a subset of projective transformation. We will come to that in the next lecture.

Let us count the Degree Of Freedom (DOF) for Euclidean transformation and affine transformation mapping between n dimension of space. From $R^1 \mapsto R^1$, Euclidean transformation has 1 DOF, affine transformation has 1+1=2 DOF. From $R^2 \mapsto R^2$, Euclidean transformation has 2+1=3 DOF, affine transformation has 4+2=6 DOF. From $R^1 \mapsto R^1$, Euclidean transformation has 3+3=6 DOF, affine transformation has 9+3=12 DOF.

We also can think affine transformation as a generalized linear transformation. So any linear composition of x, y will be mapped to linear composition of X, Y with the same coefficient as in the following formula:
If $x \mapsto X$ and $y \mapsto Y$ then $\lambda x + (1-\lambda)y \mapsto \lambda X + (1-\lambda)Y$.

Unlike Euclidean transformation, lengths, angels and areas are not preserved under affine transformation. Relationship of parallelism and relationship of middle-point, or more generally, ratio on the length are invariants under affine transformation. For example, if C is a point on line AB, then $|AC|/|CB| = |f(A)f(C)|/f(C)f(B)|$. Besides, incidence is also preserved under affine transformation, e.g., if a point A is on line l_1, then after affine transformation, it is still on line l_1.