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1 Reflection

1.1 BRDF

The reflectance property of a material can be characterized by the Bidirectional Reflectance Distribution Function (BRDF). BRDF is a function of both the incoming and outgoing directions of light [1] A formal defination of BRDF is:

$$BRDF = \frac{outgoing\ radiance\ in\ a\ given\ direction}{incoming\ irradiance\ in\ another\ given\ direction}$$

Notice that although the irradiance is a directionless quantity, the denominator in the above defination considers the irradiance contribution from a particular given direction.

Each of the incoming and outgoing direction is specified by two variables. Let θ_0, ϕ_0 be the outgoing direction, and θ, ϕ be the incoming direction. Then, the BRDF can be experessed as:

$$\rho_{bd}(\theta_0, \phi_0, \theta, \phi) = \frac{L(x, \theta_0, \phi_0)}{L_i(x, \theta, \phi) cos(\theta) d_\omega} \quad (1)$$

where the d_ω is the solid angle. For a perfect black material (no reflected light), the outgoing L is 0, and so is the ρ_{bd}. The measurements of L and L_i may be obtained by shining lights of known property on the material from many different angles.

There are two important special cases of BRDF:

- BRDF is a constant: a surface with this property is called a Lambertian Surface. In such case, the outgoing radiance is a constant, independent of direction.
- Perfect mirror: the BRDF for a perfect mirror is 0 in all directions except the direction of light reflection.

[1] In fact, the BRDF is also a function of λ, the wavelength, which we do not consider here for simplicity.

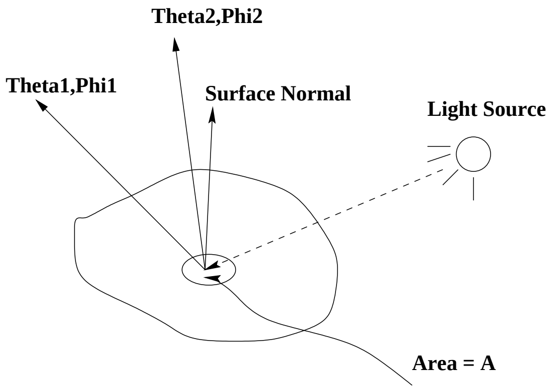

Figure 1: Reflection in different directions for a Lambertian surface.

1.2 Properties of Lambertian surface

Let's undertand more about Lambertian surface. Since BRDF is a constant, the surface looks equally bright from different directions (because our eyes measure radiance rather than power). However, the energy may differ. This can be seen from figure 1, where the power going out from direction 1 and 2 are:

$$L(x, \theta_1, \phi_1) A cos(\theta_1)(Solid\ angle)$$

and

$$L(x, \theta_2, \phi_2) A cos(\theta_2)(Solid\ angle)$$

, respectively, where A is the area of the local patch of interest. It is readily seen that the outgoing power peaks in the perpendicular direction, where $\theta = 0$.

Now, let's consider the effect of moving the light source for a Lambertian surface. As the light source moves, the angle θ between the surface normal and light source direction is changed, and so is the projected area of light. Although the BRDF is constant, the incoming irradiance changes as a function of $cos(\theta)$. So the brighness appears to be changed.

In conclusion, the brightness is related to $\hat{n} * \hat{s}$, where $\hat{n}$ is the surface normal, and $\hat{s}$ is the light source direction.

1.3 Computing BRDF

Idealy, if we know the detailed surface geometric structure and material's physical property, we can predict BRDF.

For a dielectric (non-conducting) material, one can use the Fresnel's Law to compute the amount of light that is reflected:

$$\rho(i) = \frac{1}{2}[[\frac{cos(i) - \sqrt{n^2 - sin^2(i)}}{cos(i) + \sqrt{n^2 - sin^2(i)}}]^2 + [\frac{n^2 cos(i) - \sqrt{n^2 - sin^2(i)}}{n^2 cos(i) + \sqrt{n^2 - sin^2(i)}}]^2] \tag{2}$$

where i is angular incidence, n is index of refraction, as in figure 2. Notice this does not apply to conductor.

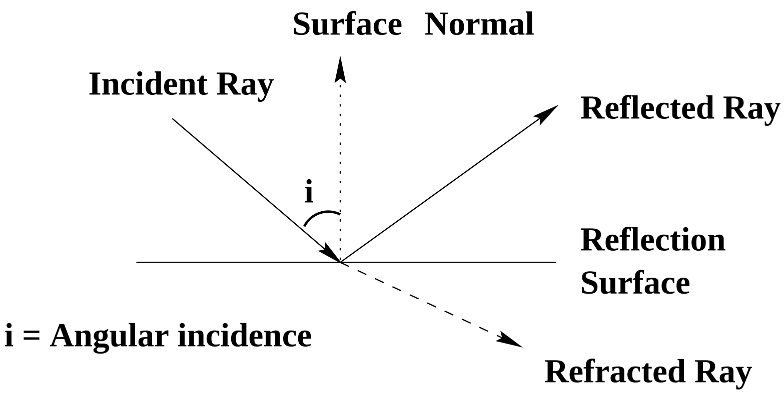

Figure 2: Reflection and refraction.

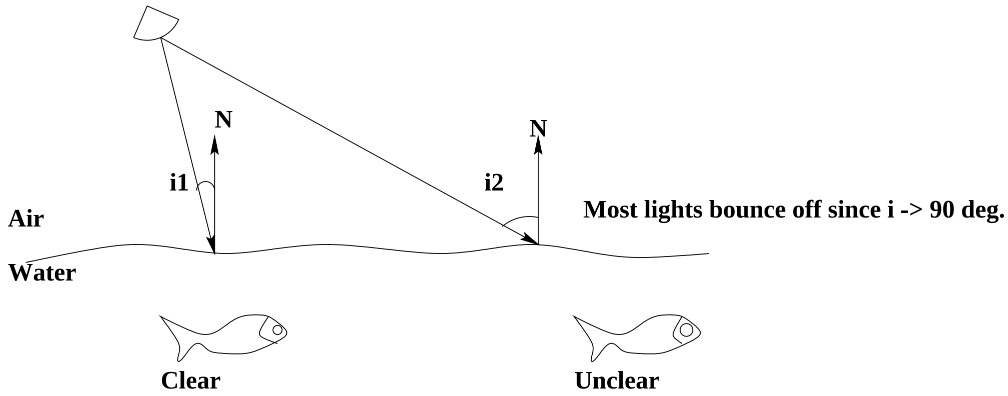

Figure 3: Illustration of Fresnel's Law.

When we have perpendicular incidence, i.e. $i = 0$:

$$\begin{aligned}\rho(i) &= \frac{1}{2}[[\frac{1-n}{1+n}]^2 + [\frac{n^2-n}{n^2+n}]^2] && (3)\\ &= [\frac{1-n}{1+n}]^2 && (4)\end{aligned}$$

For glass, $n = 1.5$, and $\rho(0) = 0.04$, i.e. only 4% of light reflected.

As i goes to $\pi/2$, $\rho(i)$ approaches 1,i.e. most of the lights are reflected. This can be used to explain the phenomenon shown in figure 3.

2 Scattering

Scattering of light occurs when we have very small particles (relative to the wavelengths of visible lights, which is from 400nm for blue light to 700nm for red light) in the medium of light.

Rayleigh's Law of scattering states that the scattering is proportional to $\frac{1}{\lambda^4}$, where λ is wavelength. It can be used to explain several natural phenomena associated with lights traveling in atomosphere.

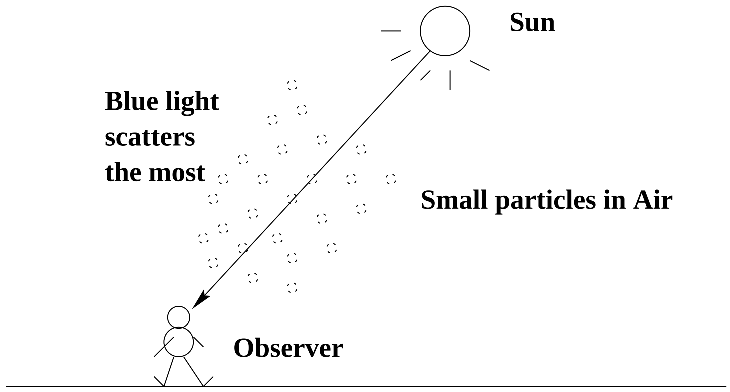

Figure 4: Sky appears blue.

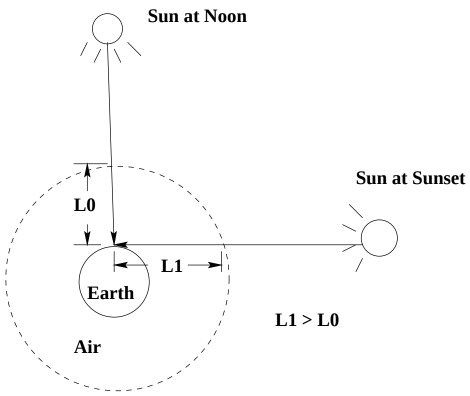

Figure 5: Sunlight is redder at sunset.

- Sky appears blue: See figure 4. Since the blue light has the shortest wavelength, it scatters the most as the sunlight travels through the earth atomospere,which contains many small molecules of air and dirt.

- Sunlight appears redder at sunset, and yellowish at noon: See figure 5. The path of sunlight in the atomosphere is longer at sunset, so more of the blue lights scatter off; at noon, the path is shorter and more blue lights remain to give the yellowish appearance of sunlight.

- Sky is brighter at horizon than at the zenith: The light from the horizon travels to the observer through a longer air column than the light from the zenith. The longer path creates more scattering and better brightness.

- Aerial Perspective: p_1 appears brighter than p_2 in figure 6, although p_1 is farther away from observer. Again, it's because the air column from p_1 is longer. This happens when the effect of air column dominates, and should be more obvious in foggy days.

Quantititavely, we can define the following:

$$Object\ light : L(r) = L_0 e^{-\sigma r} \tag{5}$$

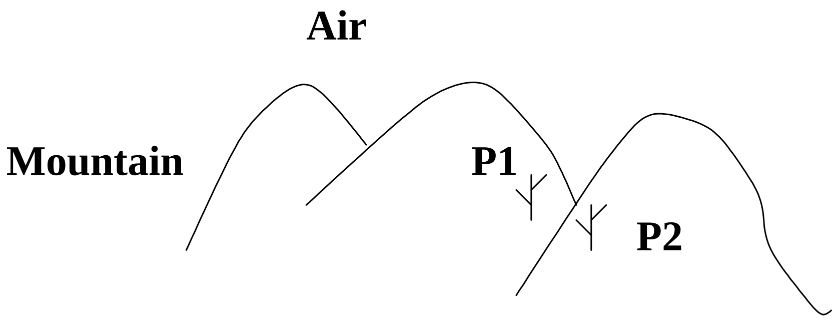

Figure 6: Aerial Perspective: object (P_1) farther away may appear brighter than object (P_2) closer to observer.

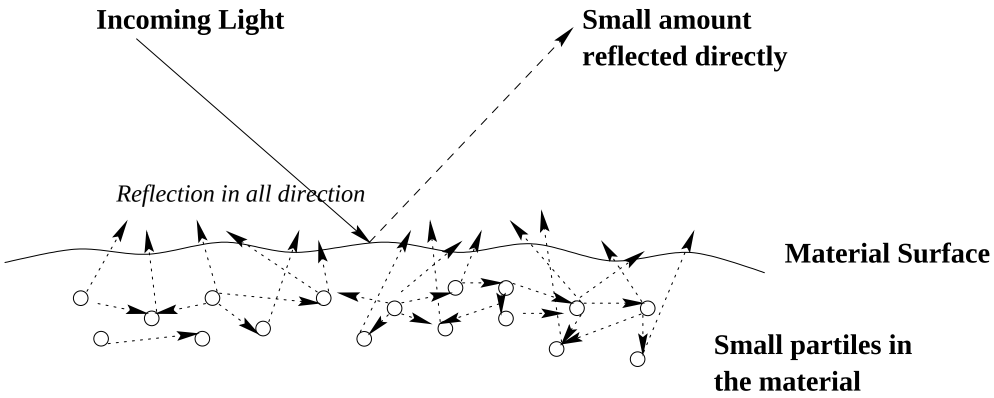

Figure 7: Lambertian surface.

where r is distance to the object, and L_0 is the light at the object.

$$Air\ light : L(r) = L_h(1 - e^{-\sigma r}) \tag{6}$$

where r is air column length, and L_h is the light at horizon.

$$Contrast : \frac{L_0 e^{-\sigma r} - L_h e^{-\sigma r}}{L_h} = \frac{L_0 - L_h}{L_h} e^{-\sigma r} \tag{7}$$

3 More on Lambertian Surface

Lambertian surface was first discovered by French physicist Lambert in 1740's. It's due to the large number of random reflection in all directions caused by small particles in the reflecting material, as illustrated in figure 7. Statistically, equal amount of light is reflected in each direction by the small particales.

Finally, the basic equation for today is:

$$L(x, \theta_0, \phi_0) = L_e(x, \theta_0, \phi_0) + \int_\Omega \rho_b d(x, \theta_0, \phi_0, \theta, \phi) L(x, \theta, \phi) cos(\theta) d_\omega \tag{8}$$

The first term L_e is the original emission from the patch, and the second term is integrated over all incoming directions.