Lecture #21 (Correspondence) Guest speaker: Yair Weiss, DRAFT Notes by Ganping Sun

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Contents:

- Block matching by Least Mean Square Approach
- Lucas-Kanade approach
- Constant brightness assumption

The purpose of the correspondence is to find the corresponding points between two images and thus to define the optical flow. Here we introduce two approaches: Least Mean Square Approach and Lucas-Kanade approach. The difference between two approaches lies in their best matching functions.

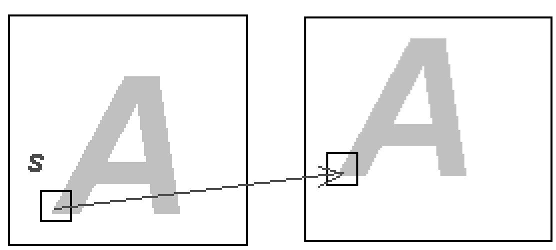
1.0 Block matching by Least Mean Square approach

The basic ideas is to

1) Define a window for each point in one image

2) Find a best match for that window in another image

1.1 Algorithm



$I1(x,y)$ $I2(x,y)$

1) Define a window

$$\forall (x, y), V$$

$$J(V, s) = \sum w(r)[I_2(r+v) - I_1(r)]^2$$

s: Center point of the window

w(r):

$$w(r) = e^{-\frac{\|r-s\|^2}{2\sigma^2}}$$

2) Choose

$$V(s) = \arg min J(V, s)$$

1.2 Advantages and disadvantages

Advantages:

- Intuitive
- Parallizable

Disadvantages:

- Slow scanning
- False matches
- Local approach
- Only applicable to translation and discrete rotation

Discrete rotation limitation can be improved by scaling the images. The slow scanning and false matches can improved by the Lucas-Kanade approach.

2.0 Lucas-Kanade approach

2.1 Mathematics

Talor series in R^1

$$f(x+d) \approx f(x) + f'(x)d$$

Similarly, Talor series in R^n

$$f(\bar{x}, \bar{d}) \approx f(\bar{x}) + \left(\frac{\partial f}{\partial \bar{x}}\right)\bar{d}$$

So:

$$I2(r+V) \approx I2(r) + \left[\frac{\partial}{\partial r} I2\right] V$$

$$J(V, s) = \sum_r w(r)[I2(r+v) - I1(r)]^2 = \sum_r w(r)\left[I2(r) - I1(r) + \left[\frac{\partial}{\partial r} I2\right]\Big|_r V\right]^2$$

w(r), I2(r)-I1(r) are independent of V. So only one minimum for this function. This minimum can be found following method.

$$J(V, s) = \sum_r w(r)[d(r)^t V - y(r)]^2$$

In which:

$$d = \frac{\partial I}{\partial r} = \begin{bmatrix} Ix \\ Iy \end{bmatrix}$$

$$y(r) = I1(r) - I2(r)$$

Then:

$$\frac{\partial J}{\partial V} = 2\sum_r w(r) d(r) d^t(r) V - 2\sum_r w(r) d(r) y(r) = 0$$

Obtain:

$$MV = b$$

In which:

$$M = \sum_r w(r) d(r) d^t(r) = \sum_{x, y} w(x, y) \begin{bmatrix} Ix^2 & IxIy \\ IxIy & Iy^2 \end{bmatrix}$$

$$b = \sum_r w(r) d(r) y(r) = \sum_{x, y} w(x, y) \begin{bmatrix} Ix(I1 - I2) \\ Iy(I1 - I2) \end{bmatrix}$$

M is a 2 by 2 matrix. b is a 2 by 1 matrix. V can be solved from this equation.

2.2 Algorithm

This algorithm is developed by Lucas and kanade(1981). in summery,

1) For each pixel r(x,y), obtain M(r) and b(r)

2)Solve MV=b to find V

2.3 Problems

The Lucas and kanade approach improve the match speed. But the discrete rotation limitation still exits.

In some cases, M is not invertible:

1) If rank(M)=0, this means Ix=Iy=0 in the window, a uniform block. Every where can be matched.

2)If rank(M)=1, this means Ix, Iy constant in the window. Every where in vertical can be matched.

3.0 Constant brightness assumption

Constant brightness assumption:

$$I(r+V, t+\delta t) = I(r, t)$$

$$I1 = I(r, t)$$

$$I2 = I(r+V, t+\delta t)$$

Then:

$$I(r, t+\delta t) + \frac{\partial I}{\partial r}V = I(r, t)$$

$$\frac{\partial I}{\partial r}V + I(r, t+\delta t) - I(r, t) = 0$$

Obtain:

$$\frac{\partial I}{\partial r}V + \frac{\partial I}{\partial t} = 0$$

This is the linearized constant brightness assumption.

(to be continued)