- Disparity measurement approaches
  - Discrete(block) Matching Approaches
  - Differential Approaches(not good)
  - Epipolar Constraint Approaches
  - Ordering Constraint Approaches
  - Piecewise Smoothing Approaches
  - Uniqueness Constraint Approaches
- Algorithms that have been used
  - Area Matching
  - Edge Matching

1 Disparity Measurement

1.1 Discrete(block) Matching Approaches

As shown in Fig.(1), this method computs matching error E between two images. It finds (u, v) which minimize the error, i.e.:

$$(u, v) = \min_{u,v}\{E\} \tag{1}$$

Where, the matching error E will be defined in section 1.3.

1.2 Differential Approaches (not good)

This method assume the brightness is conserved, and:

$$E_x u + E_y v + E_t = 0 \tag{2}$$

There is 1 equation and 2 unknowns—Aparture problem.
Lucas Kanade:

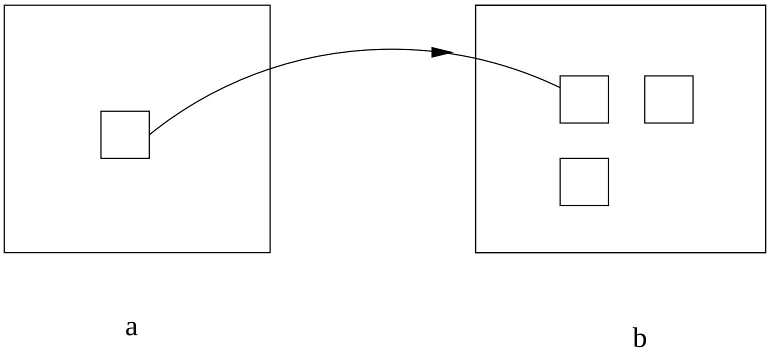


Figure 1: Block matching strategy

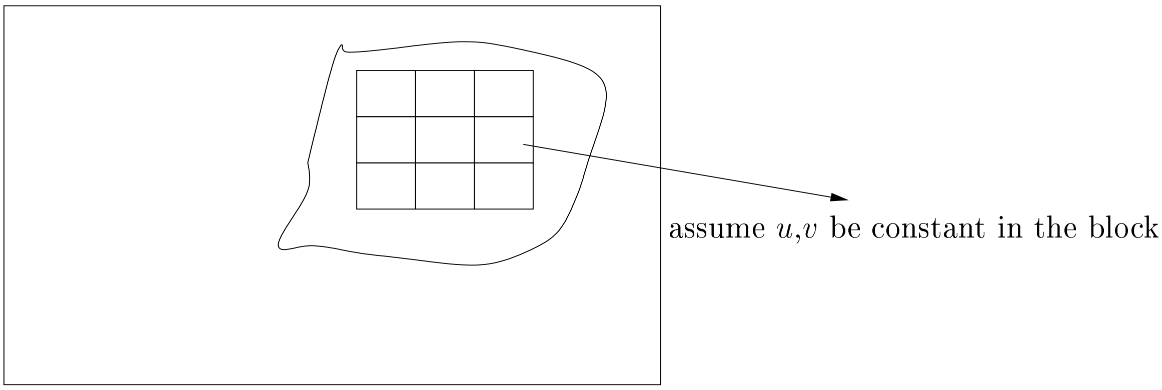
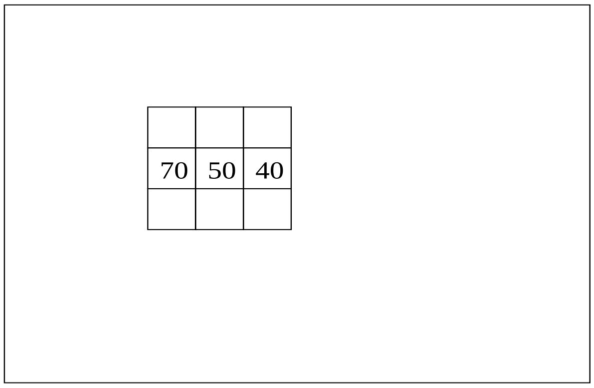


Figure 2: A typical block



$$E_x = \frac{40-70}{2} = -15$$

Figure 3: How to calculate E_x

As shown in Fig.(2), assume u, v be constant in the block. Thus there are totally 9 equations and two unknowns for the 9 pixels in the block, i.e.:

$$E_x^1 u + E_y^1 v + E_t^1 = 0$$
$$\vdots$$
$$E_x^9 u + E_y^9 v + E_t^9 = 0$$

How to get E_x, E_y and E_t? As shown in Fig.(3), $E_x = \frac{40-70}{2} = -15$.

Note:

- Digital way to get derivatives
- Smooth the image first in order to get bette results (i.e. small noise)

There are some points we must pay attention to:

- As shown in Fig.(4), consider only moving of cameras, and if it's Lambertian, then the brightness is conserved.
- Note for uniform white board, then E_x, E_y and E_t=0, there is no useful information.
- Also note that in stereo case, the differential method not good for the fact that the assumption that u, v be constant in a block doesn't hold because the motion is large.

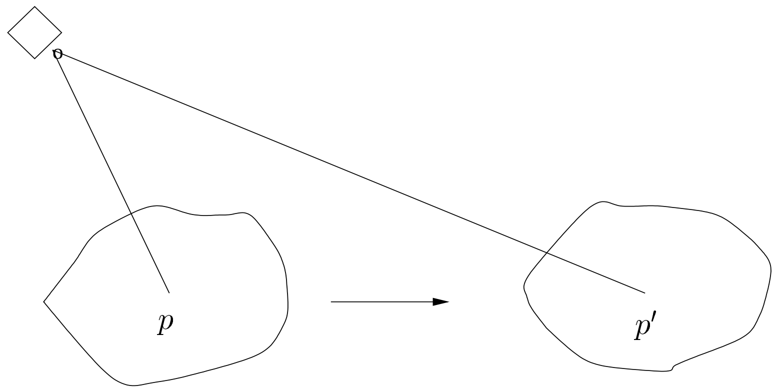


Figure 4: Lambertian condition

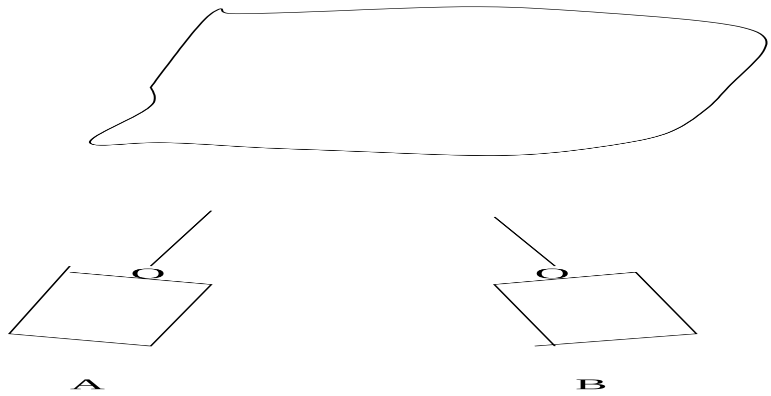


Figure 5: The relationship between two cameras

1.3 Epipolar Constraint Approaches

The idea here is using epipolar conatraint to reduce search to be along corresponding epipolar line.

As shown in Fig.(5), in stereosis, typically the geometry relationship between the two cameras are known. $\Rightarrow$ **Epipolar Constraint** can be used. Corresponding points in two planes lie on corresponding e[ipolar lines. And now we can search in 1-dimension instead of 2-dimension.

As shown in Fig.(6), from the fact that the point should lie in the Epipolar line we can get another equation.

As shown in Fig.(7), we can now search only on the line!

Blocking matching error criterion:

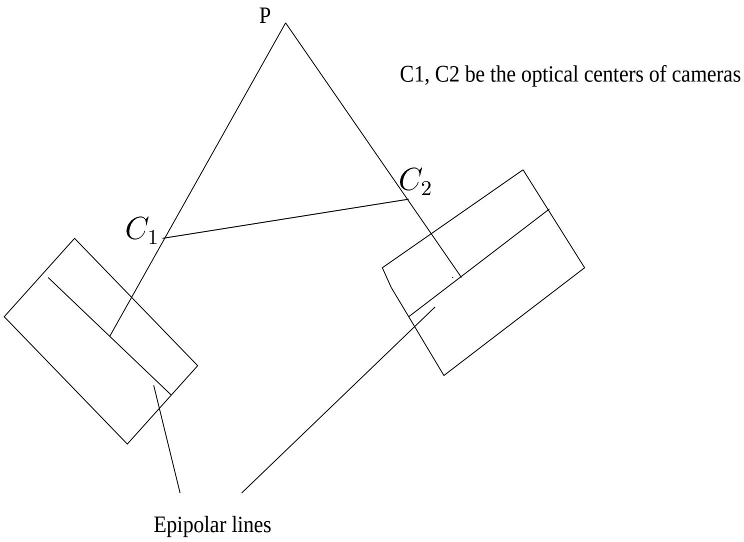


Figure 6: Illustration of epipolar constraint

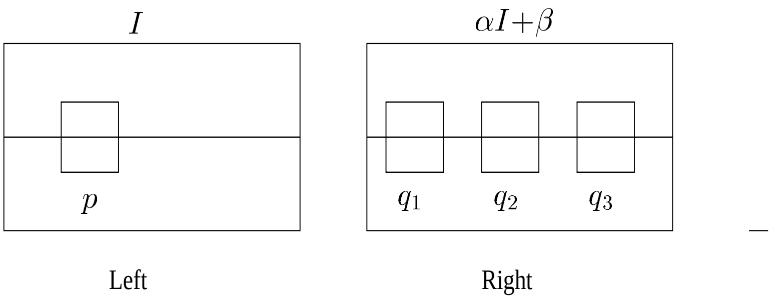


Figure 7: An example of epipolar constraint

v_1	v_2	v_3
v_4	v_5	v_6
v_7	v_8	v_9

p block

$\vec{v} \in \mathrm{R}^9$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

q block

$\vec{w} \in \mathrm{R}^9$

Figure 8: Two blocks and their elements

There are two kind of block matching error criterion:

- Sum of Squared Diference:

$$SSD \equiv \|\vec{v} - \vec{w}\|^2 \tag{3}$$

- Normalized Cross Correlation:

$$COR \equiv \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\|\|\vec{w}\|} \tag{4}$$

Note:

- The $SSD \neq 0$ if one bright and the other dark
- Correlation is ok because there is no effect of $(I,\, \alpha I)$, and in fact, this method takes into consideration of different gains of the two cameras.

1.4 Ordering Constraint Approaches

Ordering constraint says that if p is to the left of r, then parter(p) is to the left of parter(r).

As shown in Fig.(9), if p is to the left of r, then parter(p) is to the left of parter(r). Note that this is true unless points lie in the forbidden zone as described later.

As shown in Fig.(10), M is the fixation. The shadowed area is called forbidden zone in which the ordering constraint is wrong. And if M and N

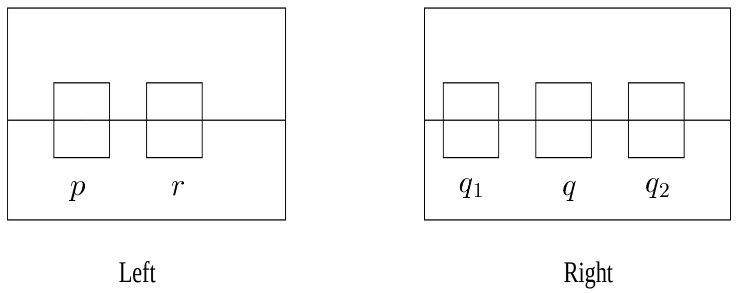


Figure 9: Illustration of ordering constraint

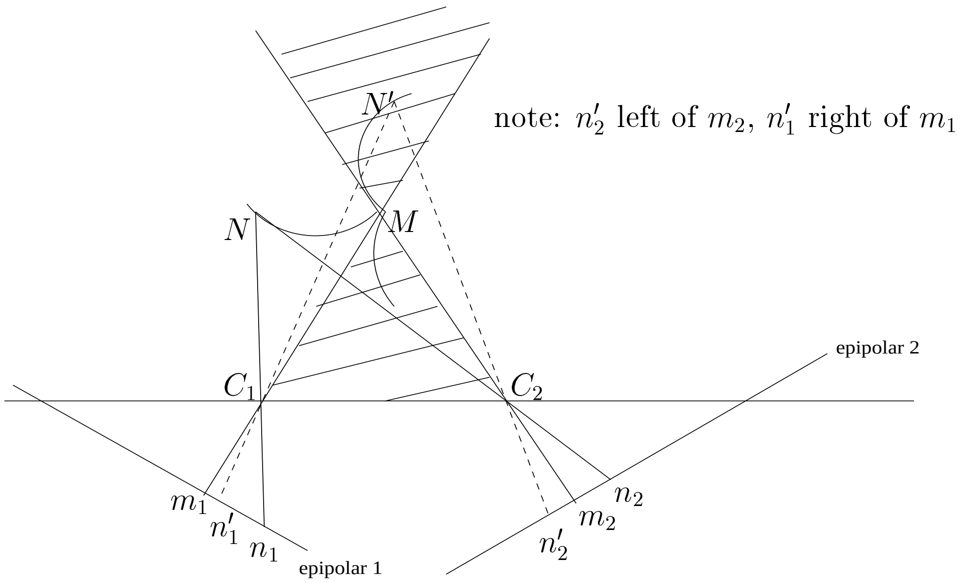


Figure 10: Area where ordering constraint works and area it doesn't work (i.e. forbidden zone)

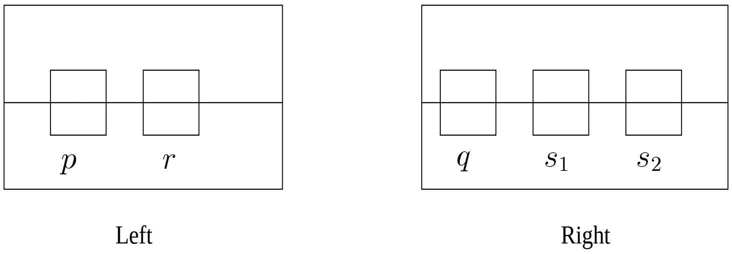


Figure 11: Illustration of piecewise constraint

are on a single smooth surface, points in the shaded region can only be seen by one eye.

Conclusions:

- The ordering constraint is wrong in forbidden zone
- The ordering constraint can be used in other zone

1.5 Piecewise Smoothing Approaches

Note the fact that the disparity values are piecewise smooth. If two points are on same smooth surface, then nearby points will have similar disparity.

As shown in Fig.(11), p and q mathch, r will match with s_1 or s_2.

1.6 Uniqueness Constraint Approaches

The idea is that a point in left image may match exactly one point in the right image (except for occlusion, thus ≤ 1).

As shown in Fig.(12), p and r in the left image cannot be matched to the same point in the right image.

2 Algorithms That Have Been Used

These two method are used frequently:

- Area matching
- Edge matching

For the problem shown in Fig.(13), you can use either area matching or edge matching to solve it.

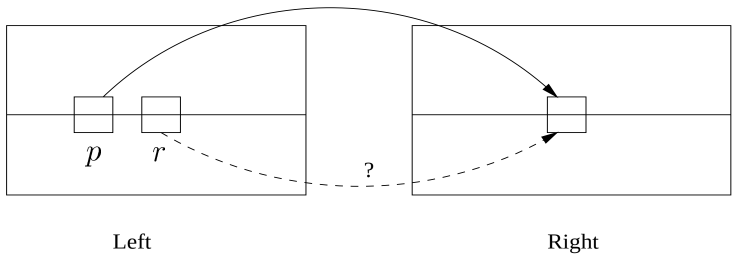


Figure 12: Illustration of uniqueness constraint

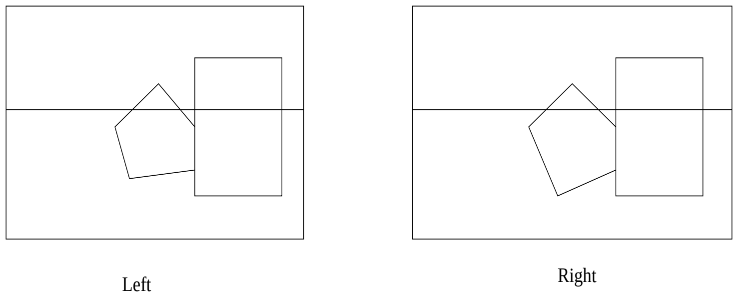


Figure 13: A problem that you can use either area or edge matching