Scribe Notes for Feb. 8

Topics:
1) Understanding the rotation matrix
2) Introducing optic flow

The rotation matrix can be represented as an exponential:

$$R=e^{\phi * \mathbf{s}_{\mathbf{hat}}}$$

$$\mathbf{S}_{\mathbf{hat}}=\begin{bmatrix} 0 & -s_3 & s_{2c} \\ -s_3 & 0 & s_1 \\ -s_2 & -s_1 & 0 \end{bmatrix}$$

$$\mathbf{s}=(s_1, s_2, s_3)$$

There are various ways that one can discover **s** and ϕ

1) Find the eigenvector of R corresponding to eigenvalue of 1 to uncover **s**

   so, find x such that $Rx=x$

   x then is your **s**, which corresponds to the axis of rotation, which is the one thing that doesn't change with rotation—hence the eigenvector with eigenvalue = 1

2) Use the trace of the rotation matrix to find ϕ

   For example, if you're dealing with rotation about the z-axis then

$$R=\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathrm{trace}R= 1+2\cos\phi$ (just add the diagonal elements)

then, you can easily solve for ϕ

the trace is invariant under similarity transformations:

$\mathrm{trace}(QRQ^{-1}) = \mathrm{trace}R$

Rodriquez' Formula

-- this is another way to come up with the rotation matrix given an axis of rotation ω (note the notation change, formerly known as s) and an angle of rotation θ (a.k.a. ϕ)

-- another notation note: ϖ will stand for the skew symmetric matrix corresponding to the rotation axis ω

$R(\omega,\theta) = I + \varpi\sin\theta + \varpi^2(1-\cos\theta)$
$R^T \quad = I - \varpi\sin\theta + \varpi^2(1-\cos\theta)$

A little help with the derivation of the Rodriguez formula:

-- think about the powers of $\mathbf{S_{hat}}$

$\mathbf{S_{hat}}^{\mathbf{2}} = \mathbf{s*s}^T - \|\mathbf{s^2}\| *I$

$\mathbf{S_{hat}}^{\mathbf{3}} = -\|\mathbf{s}\|^2 * \mathbf{S_{hat}}$

-- hint for proving formula: think about the Taylor expansion

$$e^{\phi*\mathbf{shat}} = I + \phi s + (\phi^2 * s^2)/2! + (\phi^3 * s^3)/3! + \ldots$$

Alternative representations of rotations

1) orthogonal matrices
2) Euler angles
3) $e^{\phi*\mathbf{shat}}$ **:** this is the most fundamental and the cleanest to use
4) complex numbers for rotation in 2D, quaterions in 3D

note: We are concerned with relative motion. The motion could be ego (camera) motion or it could be object motion.

Dynamic Perspective

- planar perspective
- spherical perspective

let's say the camera moves with some translational velocity **t** and some rotational velocity ω, then the relative motion between the camera and a world point **x** is the derivative of **x, x'**

$$\mathbf{x'} = -\mathbf{t} - \omega \otimes \mathbf{x}$$

Now, let's look at how this motion projects:

$\omega = (\omega_x, \omega_y, \omega_z)$
$\mathbf{t} = (t_x, t_y, t_z)$
$\mathbf{x} = (x, y, z)$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = - \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} - \begin{bmatrix} \omega_x z - \omega_z y \\ \omega_z x - \omega_x z \\ \omega_x y - \omega_y x \end{bmatrix}$$

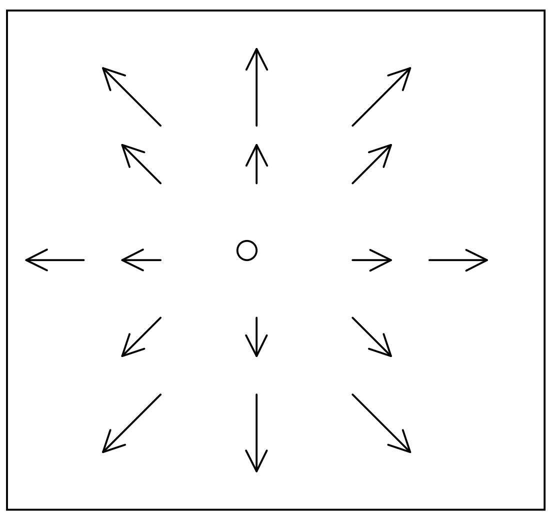
projections: $x = X/Z$ $\quad x' = (x'z - xz')/z^2$
$y = Y/Z$ $\quad y' = (y'z - yz')/z^2$

defn: $u = x'$
$v = y'$

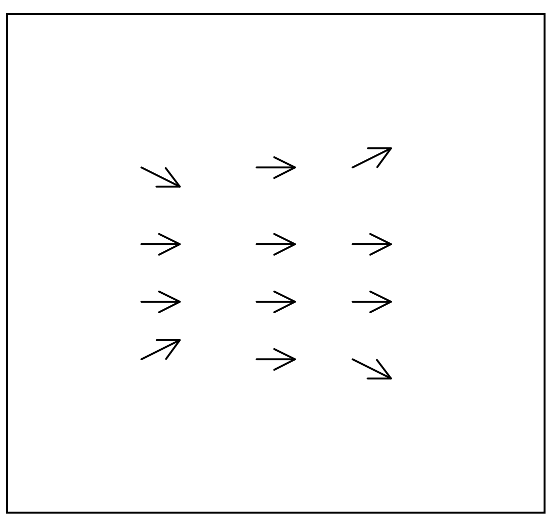
Optic flow is $\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}$

you can create optic flow diagrams by plotting the (u,v) vector at the appropriate (x,y) locations

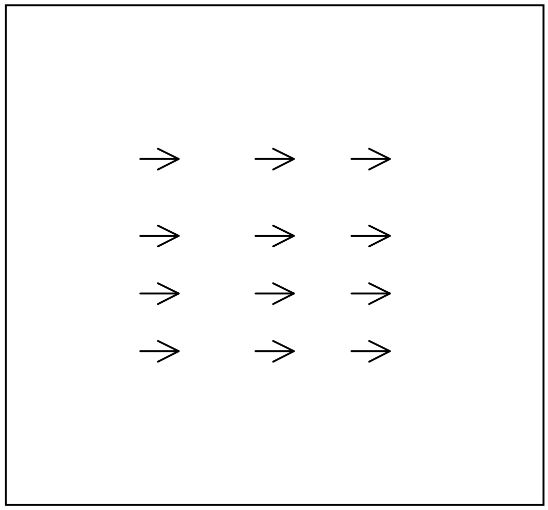
This is the optic flow field corresponding to pure translation along z



Rotation about y – axis



Translation along x-axis



Notice that translation along the x-axis and rotation about the y-axis look very similar

Focus of expansion (FOE): this is the point at which there is no optic flow (represented as the circle at the center of the "translation along z axis" diagram above). This value corresponds to the vanishing point associated with the direction of motion.

At FOE: $u_t = v_t = 0$

Scale factor ambiguity: it is impossible to distinguish between fast speeds at far distances from slow speeds at close distances

Spherical persective

In spherical perspective, all directions are treated equally.

1) Rotation about North-South axis : flow is along latitude lines
2) Rotation about radius through the equator : cyclorotation
3) Translation: flow fields for translation are great circles of constant longitude where the poles correspond to the FOE and the FOC

Deriving optical flow for spherical perspective:

This information is provided in the handout for this day. The general idea is to decompose x' into radial and transverse components and see how the transverse components project (the radial direction, along the line of sight, gives rise to no optic flow)

The conclusion:

$$\mathbf{q}' = \lambda \mathbf{q}_{\mathbf{hat}}^{\mathbf{2}} t + \mathbf{q}_{\mathbf{hat}}\, \omega$$

where λ is a term referred to as "reduced nearness"

The focus of expansion occurs where **q=t**
Again, we have the problem of scale factor ambiguity.

Solve the inverse problem

You have a bunch of points taken at two different times, you know what the correspondences are between the points, you would like to figure out how to find the reduced nearness, the translation and the rotation.

$\mathbf{q'} = \lambda \mathbf{q}_{\mathbf{hat}}^2 t + \mathbf{q}_{\mathbf{hat}}\, \omega$

So, λ, t, ω are what you're trying to solve for.

You know that you are going to need at least as many equations as unknowns.

When you have	you have	and
1 point	2 equations	6 unknowns
2 points	4 equations	7 unknowns
3 points	6 equations	8 unknowns
4 points	8 equations	9 unknowns
5 points	10 equations	10 unknowns

And, now you have hope of solving the system. Kruppa (1913) showed that this system can, indeed, be solved.